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TRANSPARENT QUANTIFICATION INTO HYPERPROPOSITIONAL CONTEXTS *DE RE*

MARIE DUŽÍ AND BJØRN JESPERSEN

Abstract

This paper is the twin of (Duží and Jespersen, in submission), which provides a logical rule for transparent quantification into hyperpropositional contexts *de dicto*, as in: Mary believes that the Evening Star is a planet; therefore, there is a concept c such that Mary believes that what c conceptualizes is a planet. Here we provide two logical rules for transparent quantification into hyperpropositional contexts *de re*. (As a by-product, we also offer rules for possible-world propositional contexts.) One rule validates this inference: Mary believes of the Evening Star that it is a planet; therefore, there is an x such that Mary believes of x that it is a planet. The other rule validates this inference: the Evening Star is such that it is believed by Mary to be a planet; therefore, there is an x such that x is believed by Mary to be a planet. Issues unique to the *de re* variant include partiality and existential presupposition, substitutivity of co-referential (as opposed to co-denoting or synonymous) terms, anaphora, and active vs. passive voice. The validity of quantifying-in presupposes an extensional logic of hyperintensions preserving transparency and compositionality in hyperintensional contexts. This requires raising the bar for what qualifies as co-denotation or equivalence in extensional contexts. Our logic is Tichý's Transparent Intensional Logic. The syntax of TIL is the typed lambda calculus; its highly expressive semantics is based on a procedural redefinition of, *inter alia*, functional abstraction and application. The two non-standard features we need are a hyperintension (called *Trivialization*) that presents other hyperintensions and a four-place substitution function (called *Sub*) defined over hyperintensions.

0. *Introduction*

Quine (1956) and Kaplan (1968), (1986) introduced the topic of quantifying-in — existential quantification into modal and attitudinal contexts — into the repertoire of philosophical logic and continue to shape the discussion of it today. Quine and Kaplan, as well as their commentators, such as Forbes (1996), (2000) and Crawford (2008), remain baffled by quantifying-in for both logical, ontological and hermeneutic reasons.[i] This paper demonstrates, in full logical detail, how to quantify into attitude contexts of the toughest kind, namely hyperintensional ones. Following Cresswell (1975), hyperintensional attitude contexts are those in which the complements of an agent's attitude are more finely individuated than up to necessary equivalence. Quantifying into intensional attitude contexts — in which the complements are individuated up to necessary equivalence, as in possible-world semantics — falls out as a by-product of our solution to the hyperintensional variant.

The cornerstone of our approach is that we avail ourselves of an extensional logic of hyperintensions. Only an extensional logic will validate the rule of existential generalization, hence only an extensional logic of hyperintensions will stand a chance of validating quantifying-in. We assign to terms and expressions occurring in hyperintensional contexts the very same semantics that we assign to those very same terms and expressions when occurring in intensional and extensional contexts. As a result of this top-down approach, the logical rule of existential generalization applies indiscriminately to all contexts. The upside of our top-down approach is that referential transparency and compositionality of meaning are preserved throughout, together with semantic innocence, since we have no recourse to reference shift.[ii] At no point do we invoke contextualist epicycles to somehow create

[i] Quine's stance towards quantifying into modal and attitudinal contexts is a convoluted one, though. Both Kaplan (1986), Crawford (2008), and Forbes (1996), (2000) point out that whereas Quine is dismissive of quantifying into modal, especially modal *de re*, contexts (which would yield 'Aristotelian essentialism' sentences of the form, "There is an x such that x is necessarily an F"), he strives to make good sense of what he calls *relational* (roughly, *de re*) attitude ascriptions like, "There is a sloop that I want", which presuppose the validity of quantifying into attitude contexts. That is, whereas Quine *rejects* the argument $\Box Fa \therefore \exists x \Box Fx$, he *embraces* the argument $\delta Gb \therefore \exists y \delta Gy$. See Crawford (2008) and Forbes (1996), (2000) for detailed discussion of Quine's excursions into dyadic versus triadic belief relations, the latter being an attempt of his to recuperate transparency for relational beliefs. For streamlined accounts of Quine's take on quantifying-in, see Hookway (1988, Chs. 5–7) and Kemp (2006, Ch. 6).

[ii] See Duží et al. (2010, p. 12) where we contrast Davidson's 'paratactic' conception of semantic innocence with a 'hypotactic' one. See also Tichý (1975).

a secondary semantics for ‘non-extensional’ contexts. The perceived downside would be that we revise the prevalent extensionalist semantic theory of terms and expressions, in that we universalize Frege’s semantics earmarked for *Sinn*-sensitive contexts to all contexts, including those that are merely *Bedeutung*-sensitive. Be that as it may, it is a strength of our solution that it is emphatically not tailor-made specifically for validating quantifying-in. Instead it is just yet another application of a large-scale background theory. So our solution to quantifying-in is principled and not *ad hoc*.

Laying out the required semantics requires a fair amount of footwork. Once this is in place, however, all that remains is filling in the nitty-gritty details of quantifying-in. The devil is in the detail, as ever, and quantifying into hyperintensional contexts is far from being technically trivial. But it is feasible. Showing one way of how to exactly go about this is the task of this paper. Our solution marks an advance for philosophical logic in general and hyperintensional logic in particular. Kaplan (1990, p. 14) lists quantifying-in as one of the challenges facing the program of pure semantics. So does Bealer (1982, p. 13), who lists quantifying-in as one of ten ‘classical puzzles’ any (hyper-) intensional logic worth its name must be capable of addressing adequately. And the community-wide project of establishing a general hyperintensional logic gains in credibility from cracking a hard nut.[iii]

The rest of the paper is organized as follows. Section 1 details the three aspects of quantifying-in: the logical, the ontological, and the hermeneutic one. This paper is devoted to the first aspect. Section 2 presents the relevant theoretical foundations of our extensional logic of hyperintensions. Section 3 sets out our theory of hyperpropositional and intensional attitudes *de dicto* and *de re*. Section 4 provides the respective rules of quantifying into hyperpropositional and possible-world propositional attitude contexts *de re*.

1. *The Three Aspects of Quantifying-in*

Quantifying-in spans three issues. The first is the logical one whether it is formally possible to quantify *into* so-called non-extensional contexts. This is, narrowly speaking, a question of the technical resources of a logical symbolism. The second is the philosophical one whether it is ontologically acceptable to quantify *over* non-extensional entities. This is, in essence, a question of the ontological commitments of a given logical theory. The third is the likewise philosophical, or hermeneutic, question of to what extent it

[iii] See Bealer (1982, §7 and §11).

makes sense to say, for instance, that there is somebody that somebody believes to be happy; that there is somebody such that they are believed by somebody to be happy; or that there is somebody of whom somebody believes that they are happy. This question concerns the link between philosophy and logic: if some logical symbolism enables quantifying-in then what philosophical notion has been symbolized? Or conversely, exactly what philosophical notion are we supposed to adequately interpret logically?

Let δ be a generic attitude operator.[iv] Then the first question broached above translates into whether these two argument schemata (in the notation of predicate logic) are valid:

$$\frac{\delta Fa}{\exists x \delta Fx}$$

where the individual a is being quantified away, and

$$\frac{\delta Fa}{\exists f \delta fa}$$

where the property F is being quantified away.

Both schemata are valid without qualification — *provided* the principle of existential generalization is untrammelled by considerations of which sort of position in which sort of context one is attempting to quantify into. But riding roughshod over contextual embedding is no option, of course. The logical problem is that δ generates a context that seals f and x off from $\exists$, which needs to reach across δ to catch f and x. (Definition 4 in Section 2 defines *hyperintensional, intensional* and *extensional context* by defining what it means for a hyperintension to occur either *mentioned* or *used*.) This is one reason why it would be naïve to assume that if, e.g., b believes that a is an F then it logically follows that there is somebody that b believes to be an F, or that there is a property that b believes a to have. Another reason is that the truth of an ascription of a non-factive attitude such as *believing* need not discharge existential presuppositions, as does *knowing*.[v]

[iv] For ease of exposition, we shall contrast existential *quantifiers* with attitude *operators* in our informal discussion. However, formally speaking, TIL does not have attitude operators. Instead TIL has binary *relations*-in-intension between agents and either propositions or hyperpropositions (propositional constructions). These relations are partial functions, such that it is either true or false or neither, at some world/time pair of evaluation, that the relevant agent entertains the relevant attitude to the relevant (hyper-)proposition.

[v] To know that Pegasus is a horse is not to know something factual. It is to know something *conceptual*. To understand the concept of Pegasus is to know, *inter alia*, that Pegasus is a horse. Such knowledge does not presuppose the existence of an individual that is the horse Pegasus. It does presuppose the existence of a set of properties jointly defining the individual

It may be true that the attributee b believes that a is an F while it is at the same time false that a exists.[vi] But, if b knows that a is an F then it needs to be true already that a exists. The set of worlds at which Fa is known is a (proper) subset of the set of worlds at which Fa is true. The set of worlds at which Fa is believed is the union of the set of worlds at which Fa is true and the set of worlds at which Fa is not true (because either false or without truth-value). This is how room is created for believing falsehoods, whether contingent or necessary (as with inconsistent beliefs), and believing propositions without truth-value.[vii]

concept of Pegasus. The property of being a horse will be in that set, so will the property of being winged. See Duží et al. (2010, pp. 286–88) on 'Sherlock Holmes' and (ibid., §4.1 'Requisites defined') on necessary relations-in-extension between intensions.

[vi] TIL pursues a Fregean tack in questions of attribution of existence: saying of an individual that it exists is trivially true. See Duží et al. (2010, §§2.3.1–2.3.2 'Existence and extensions', 'Existence and intensions'). In the main text we are pretending, for ease of exposition, that it be conceivable for a not to exist.

[vii] There is an interesting direct parallelism between factive attitude operators and subsective modifiers and between non-factive attitude operators and modal modifiers. Both the rule of factivity and the rule of subsection are, syntactically, elimination rules:

$$\textit{Known } A \therefore A$$

and

$$[\textit{Modifier Property}](x) \therefore \textit{Property } (x)$$

For instance, since the modifier *Skillful* is subsective, it follows that a skillful musician is a musician. Set-theoretically, any set of skillful logicians must be a subset of a set of logicians. Modal modifiers are somewhat elusive, oscillating as they do between being subsective and being privative. (See Jespersen and Primiero (2012) on the logic of modal modification.) The disjunction of A and $\neg A$ is a classical tautology and so is too weak to capture what is characteristic of non-factive operators and modal modifiers (that they are hit-or-miss). But the conjunction of two mutually exclusive possibilities (matching the limiting case of union where the intersection is empty) sums up the little that can be inferred (other than various instances of existential generalization):

$$\textit{Believed } A \therefore \textit{Possibly}, A \wedge \textit{possibly}, \neg A$$

and

$$[\textit{Modifier}' \textit{ Property}](x) \therefore \textit{Possibly}, \textit{Property } (x) \wedge \textit{possibly}, \neg\textit{Property}(x)$$

For instance, if x is an alleged terrorist then x is a terrorist or x is not a terrorist: there is a world/time pair at which x is a terrorist and there is another world/time pair at which x is not a terrorist. Both non-factive attitude operators and modal modifiers leave it open which side truth comes down on.

The second, ontological, question is exemplified by the second schema. Whereas it is uncontroversial to quantify over individuals, it is controversial, in some quarters, to quantify over so-called intensional entities such as properties, propositions, relations-in-intension, and individual concepts. Besides, if we do not assume that a is an individual but an individual concept or role or office, such that b believes that the incumbent of that office is an F, then the first schema involves quantification over individual concepts. We are not going to argue independently for the acceptability or indispensability of non-extensional entities here. Instead we simply assume them, introducing two kinds of non-extensional entities: possible-world intensions and hyperintensions.[viii] Our notion of hyperintension will be defined in Definition 2 in Section 2. For now, our hyperintensions serve in the capacities as modes of presentation of other entities, linguistic senses, and the complements of hyperintensional attitudes. In particular, hyperpropositions are sentential senses and are modes of presentation of truth-conditions, or possible-world propositions, or empirical states-of-affairs. Our logic contains the resources to quantify over hyperintensions. This is thanks to our ramified type hierarchy, in which our hyperintensions are organized, since there is always going to be a hyperintension presenting another hyperintension located one step down. (Definition 3 in Section 2 defines *ramified type hierarchy*.)

The basic idea informing the rule of existential generalization is simple enough. The conclusion of the rule of existential generalization makes explicit an existential commitment incurred by the premise. If the individual Mary is happy then there is an element in the domain of individuals that is happy:

$$\frac{Fa}{\exists x Fx}$$

This much is uncontroversial. Now what if the pope — the incumbent of the papal office — is happy? Then we would not hesitate to infer that there is an

[viii] A valid objection, however, is that possible-world intensions are intensions on the cheap, because they are extensionally individuated:

$$\forall fg(\forall w(f(w) = g(w)) \rightarrow f = g)$$

Co-intensionality amounts to nothing other than necessary co-extensionality, because f, g are mappings. This explains why TIL ranks possible-world intensions as *first*-order objects and types them in the *simple* type theory (provided they do not have domain or range in hyperintensions, i.e. TIL constructions, in which case they are *higher*-order objects and must be typed in the *ramified* type theory). Outside the idiom of possible-world semantics, 'intensional' tends to equate 'hyperintensional' (unless used in the condescending sense of 'non-extensional'). Just one example: Hindley and Seldin (1986, p. 72).

individual office whose occupant is happy. What if "as ølglas er halvfuldt" means that a's glass of beer is half-full, but not that a's glass is half-empty? Then we would not hesitate to infer that there is a hyperproposition presenting an empirical state-of-affairs, such that that hyperproposition is the meaning of that Danish sentence.

Quantifying over hyperintensions yields weaker propositions than does quantifying over intensional or extensional entities, because if we infer that there is a hyperintension we still have not inferred, and so do not yet know, whether this hyperintension presents anything, be it an intension or an extension. Let f be an individual office.[ix] Then if a believes that the occupant of f is an F, it follows that there is a *hyperintension* presenting an intensional entity belonging to the type of individual offices, such that a believes that the occupant of some office is an F. This inference would be a trivial one to draw. A more interesting inference to draw is that there is an *individual office*, such that a believes that its occupant is an F. A still more interesting inference to draw is that there is an *individual*, such that a believes that that individual is an F. Yet this conclusion is not always forthcoming.[x] If Mary believes that the King of Canada is happy, does it follow that there is an x such that

[ix] There is a substantial difference between proper names and definite descriptions. This distinction is of crucial importance due to their vastly different logical behaviour. Independently of any particular theory of proper names, it should be granted that a *proper* proper name (as opposed to a definite description grammatically masquerading as a proper name) is a rigid designator of a numerically particular individual. On the other hand, a definite *description* like, for instance, 'the Mayor of Dunedin', 'the King of France', 'the pope', 'the first man to run 100 m in less than 9 seconds', 'the Evening Star', etc., offers an *empirical criterion* that enables us, in principle, to establish which individual, if any, satisfies the criterion in a particular state of affairs. We model such criteria as possible-world intensions, which are functions that for each possible world and each time return at most one individual. Proper names do not come with an empirical criterion for fixing their bearers: it is purely a matter of linguistic *fiat* which name has which bearer (so proper names are no empirical terms for us).

[x] The set of inferences we just considered amounts to what Klement (2002, p. 157) calls 'at least a *minimally* adequate treatment' of quantifying-in. Klement's discussion of quantifying-in is faithful to the historical Frege, such that existential quantification will be over either a 'saturated' *Gegenstand* or an 'unsaturated' *Funktion*. Our *neo*-Fregean setting allows for quantification over extensional or intensional or hyperintensional objects. Frege did not have the intermediate level (on the almost uncontroversial assumption that *Sinn*, which Frege frequently, though not always, argued to be individuated in terms of cognitive significance, is hyperintensional). We agree with Klement that Frege's analysis of quantifying into "Gottlob believes that the Morning Star/Vulcan is a planet" amounts to, "There is some *Sinn* that, when saturated with the incomplete *Sinn* of "ξ is a planet", yields a *Gedanke* believed by Gottlob", with no *Bedeutung* being invoked (i.e. without executing something akin to Church's Δ mapping from a *Sinn* to its *Bedeutung*), so the fact that 'The Morning Star' picks out a *Bedeutung*, and 'Vulcan' does not, is of no logical import. What matters is that the *Eigennamen* (in Frege's inclusive notion of proper name) 'The Morning Star' and 'Vulcan' both have a *Sinn*. (*Ibid*., pp. 156–57.)

Mary believes that x is happy? Absent the additional premise that the King of Canada exists, the suggested inference will be a fallacy in case x ranges over individuals. However, if *believing* is replaced by a factive attitude like *knowing* then Mary's knowledge that the King is happy presupposes that it be true that Canada has exactly one king. With Mary's factive attitude as a premise, existential generalization over individuals in the conclusion is straightforward — from an ontological, though not logical, point of view; for there remains the problem of how the quantifier is to bind an x inside the scope of an attitude operator.

The third question concerns the hermeneutic comprehensibility of predicating of a random individual the property of being believed (in a hyperintensional manner) by some attributee to be such-and-such. For instance, Kaplan (1969, p. 221) pauses to reflect on the meaningfulness of the quantified *de re* ascription, "Someone is such that Ralph believes that he is a spy" (cf. entry (10), *ibid.*, p. 210). The notion of having an attitude *of* X lends itself to basically two different construals. On one construal, an attitude *de re* requires an exceptionally intimate epistemic relation between agent and X, perhaps in a manner that circumvents, or is prior to, most or all propositional knowledge about X, and may not be easy to come by. On the other construal, an attitude *de re* is parasitic on other attitudes, ultimately with an attitude *de dicto* at the origin, and much easier to come by.[xi] Ours is the second approach, which is devoid of the enigma integral with the first one.

This paper is the twin of another paper setting out the logic of transparent quantification into hyperpropositional contexts *de dicto*. These are the last two so far in a string of papers beginning with Tichý (1986), which quantifies into intensional contexts and over intensions; Materna (1997), which quantifies into hyperintensional contexts and over hyperintensions; Duží (2000), which corrects and simplifies Materna (1997), but has a flaw of its own;[xii] Duží et al. (2010, §5.3 'Quantifying in'), which puts forward a technically correct but philosophically strained solution that is replaced by a technically more elegant and philosophically appealing solution in Duží and Jespersen (submitted). In the twin paper we analyze, in particular, the following argument, for which we consider four analyses, two of which are valid, and one of which is the final analysis:

$$\frac{a \text{ believes that the Evening Star is a planet}}{\begin{array}{c}\text{There is a hyperintension presenting an individual office}\\ \text{such that } a \text{ believes that the occupant of that office is a planet}\end{array}}$$

[xi] See Duží et al. (2010, p. 435) for discussion.
[xii] See Duží et al. (2010, p. 500, n. 106).

We also show how to validate the following inference:

$$\frac{a \text{ believes that the Evening Star is a planet}}{\begin{array}{c}\text{There is an individual office such that } a \text{ believes that}\\ \text{the occupant of that office is a planet}\end{array}}$$

We leave out of consideration non-empirical 'that'-clause attitudes, such as "a knows that π is a transcendental number", and notional attitudes, such as "a seeks the fountain of youth" and "a calculates the first one million decimals of π". The logic of quantifying into possible-world propositional attitudes falls directly out of our logic of hyperpropositional attitudes, since the former attitudes are technically less demanding and are obtained by lifting various restrictions. The current paper follows a similar pattern. We also restrict ourselves to hyperpropositional empirical attitudes *de re*, but since intensional attitudes *de re* are a by-product, as it were, thanks to the trickle-down effect of how our theory is set up, we display, for illustration, how to quantify into intensional attitudes *de re*.[xiii]

We will concentrate on quantifying into the singular-reference position of δFa. Our main focus is on singular terms having the semantics of definite descriptions, because they are logically more intricate, and so more interesting, than singular terms with the semantics of proper names. We assume, in keeping with prevalent theories, that the meaning of a proper name (whatever the details of one's favourite theory of the meaning of proper names) is such that a proper name rigidly refers to one and the same individual whatever the contextual embedding. Hence, the transfer from the attributee's perspective to the attributer's, and vice versa, goes smoothly. On the other hand, the meaning of a definite description may occur either *de dicto* or *de re*, and a definite description does not refer rigidly to a particular individual. Rather it denotes an empirical *condition* that may be satisfied by individuals.

Here we are assuming, rather than arguing, that 'The Evening Star' and 'The Morning Star' are not two different names for the same individual. Instead 'The Evening Star' names one individual office and 'The Morning Star' names another individual office. When we say that the Evening Star is the Morning Star, we mean to say that these two offices are contingently co-occupied by the same individual. That is, "The Evening Star is the Morning Star" does not express the self-identity of an individual bearing two names,

[xiii] Also hyperintensional notional ('hypernotional') attitudes are amenable to being quantified into. See Duží et al. (2010, §§5.2–5.3 'Notional attitudes', 'Quantifying in').

but the contingent convergence of two named offices in one anonymous individual.[xiv] We say of 'The Evening Star' and 'The Morning Star' that they are not synonymous and do not co-denote, or are not equivalent (because they do not denote the same office), but co-refer (because their respective denotations happen to be co-extensional). For another standard example, although Quine's 'the man in the brown hat' and 'the man on the beach' happen to co-refer to Bernard J Ortcutt, because their offices happen to co-describe him, they do not co-denote him. Rather they denote, in every context, two distinct individual offices. There are worlds and times at which these two offices are co-occupied, e.g. by Ortcutt, but this empirical fact has no bearing on the semantic properties of these two definite descriptions.[xv] Their semantic properties concern instead whether they are synonymous (hence co-denoting) or merely co-denoting. Our strategy is to raise the bar somewhat for what qualifies as identity and equivalence of senses and synonymy vs. co-denotation of words and apply substitution of identicals and equivalents to those (fewer) pairs of words that do pass muster. The opposite, and common, strategy is to maintain a lower bar, which, however, generates referential opacity and inapplicability of substitution and quantification in various modal and attitudinal contexts.[xvi] Definition 7 in Section 2 defines *synonymy*, *equivalence* (co-denotation) and *co-reference*.

We shall analyze the sentence

"Mary believes of the Evening Star that it is a planet"

and its 'exported' variant, which introduces anaphoric reference:

"The Evening Star is such that Mary believes of *it* that *it* is a planet"

The exported variant can also be transformed from active to passive voice, from

"The Evening Star is believed by Mary to be a planet"

to

[xiv] For justification, see Duží et al. (2010, §3.3.1 'Hesperus is Phosphorus: co-occupation of individual offices').

[xv] For further details, see Duží et al. (2010, pp. 301–11).

[xvi] Duží et al. (2010, Ch. 1, esp. §1.2, §1.4.2.3, §1.5.2 'The top-down vs. bottom-up approach to logical semantics', 'The top-down approach to semantics revisited', 'Supposition *de dicto* and *de re* vs. reference shift') provides details on this project of universal transparency.

"The Evening Star is such that *it* is believed by Mary to be a planet"

The strongest conclusion is that there is an *individual* such that Mary believes that that individual is a planet. This conclusion is forthcoming in two cases. The first case is factive attitudes: you cannot know that the pope is a German unless there is exactly one pope of whom it is true that he is a German. The second case is attitudes *de re*, including *belief*: you cannot believe of the pope that he is a Protestant unless there is exactly one pope of whom to believe that he is a Protestant.

Two other conclusions, one weaker than the other, can be inferred as well. The weaker of the two is that there is a hyperintension presenting an office such that the attributee believes that its occupant is a planet. The stronger of the two is that there is an office such that the attributee believes that its occupant is a planet. While hyperintensional attitudes *de dicto* and *de re* both validate these last two conclusions, the important difference is that attitudes *de re* validate, furthermore, the strongest conclusion that there is an individual of whom an attitude is being entertained. In our logic, if there is no individual of whom or which to have the relevant attitude *de re*, the proposition that the attributee believes, or knows, of the F that he/she/it is an G will be without truth-value (rather than false). Contrast this with the attribution of a belief *de dicto* that the F is a G, which will have a truth-value. When a *partial* function such as a possible-world proposition trades a world for a gap, we say that the hyperintension being used to present the proposition is *improper*. Definition 2 in Section 2 defines *proper* and *improper* hyperintensions.

2. *Theoretical Foundations*

The background theory we have implicitly presupposed so far is Tichý's Transparent Intensional Logic. What makes TIL suitable for the job of quantifying into hyperintensional contexts is that the theory construes the semantic properties of the sense and denotation relations of terms and expressions as remaining invariant across linguistic contexts and that its ramified type theory enables quantification up to any order.[xvii]

[xvii] *Indexicals* being the only exception: while the sense of an indexical remains constant, its denotation trivially varies in keeping with its contextual embedding. See Duží et al. (2010, §3.4 'Pragmatically incomplete meanings').

Formally, TIL is an extensional logic of hyperintensions based on the partial, typed λ-calculus enriched with a ramified type structure to accommodate hyperintensions. The syntax of TIL is the familiar one of the λ-calculus, with the addition of a hyperintension called Trivialization (symbolized by a superscripted nought). The semantics is a *procedural* (as opposed to denotational) one. Thus, functional application, in TIL, is not the result of applying a function to an argument, but instead the very *procedure* of applying function to argument; and functional abstraction, in TIL, is not the result of forming a function, but instead the very *procedure* of sorting two selections of entities into functional arguments and values, respectively. Furthermore, variables are not terms, but hyperintensions: 'x' denotes the atomic hyperproposition x that presents the value that an assignment function has accorded to x (relative to a type assignment and a sequence of variable/value pairs). The TIL concept of procedurally construed hyperintensions is *construction*. Thus, hyperintensional attitudes translate into *constructional attitudes* and hyperintensional contexts into *constructional contexts*. Of the six different kinds of constructions that their inductive definition enumerates, we shall need four altogether in order to quantify into hyperpropositional contexts *de re* and a fifth for the intensional ones.

The first three definitions below constitute the logical heart of TIL. The subsequent five definitions build upon those. Taken together, they make up the logical tools needed to pull off quantifying into hyperpropositional and possible-world propositional contexts *de dicto* and *de re*. These two variants of quantifying-in are importantly different, so we exhibit the former for illustration (the fourth and final analysis alluded to in Section 1). The definitions are as follows.

Definition 1: (types of order 1) *Let B be a* base, *where a base is a collection of pair-wise disjoint, non-empty sets. Then:*

(i) *Every member of B is an elementary* type of order 1 over B.
(ii) *Let α, $\beta_1, \ldots, \beta_m (m > 0)$ be types of order 1 over B. Then the collection $(\alpha\ \beta_1 \ldots \beta_m)$ of all m-ary partial mappings from $\beta_1 \times \ldots \times \beta_m$ into α is a functional* type of order 1 over B.
(iii) *Nothing is a* type of order 1 over B *unless it so follows from (i) and (ii).* $\square$

Remark. For the purposes of natural-language analysis, we are currently assuming the following base of ground types, each of which is part of the ontological commitments of TIL:

o: the set of truth-values $\{\mathsf{T}, \mathsf{F}\}$;
ι: the set of individuals (a constant universe of discourse);
τ: the set of real numbers (doubling as temporal continuum);
ω: the set of logically possible worlds (the logical space).

Definition 2: (construction)

(i) *The* variable x *is a* construction *that constructs an object* O *of the respective type dependently on a valuation* v*:* x v*-constructs* O.

(ii) Trivialization*: Where* X *is an object whatsoever (an extension, an intension or a* construction*),* ${}^0X$ *is the* construction Trivialization*. It constructs* X *without any change in* X.

(iii) *The* Composition $[X\ Y_1 \ldots Y_m]$ *is the following* construction*. If* X v-constructs *a function* f *of type* $(\alpha\beta_1 \ldots \beta_m)$*, and* $Y_1, \ldots, Y_m$ v-construct *entities* $B_1, \ldots, B_m$ *of types* $\beta_1, \ldots, \beta_m$*, respectively, then the* Composition $[X\ Y_1 \ldots Y_m]$ v-constructs *the value (an entity, if any, of type* α*) of* f *on the tuple argument* $\langle B_1, \ldots, B_m \rangle$*. Otherwise the* Composition $[X\ Y_1 \ldots Y_m]$ *does not* v-construct *anything and so is* v-improper.

(iv) *The* Closure $[\lambda x_1 \ldots x_m\ Y]$ *is the following* construction*. Let* $x_1, x_2, \ldots, x_m$ *be pair-wise distinct variables* v*-constructing entities of types* $\beta_1, \ldots, \beta_m$ *and* Y *a construction* v*-constructing an* α*-entity. Then* $[\lambda x_1 \ldots x_m\ Y]$ *is the* construction λ-Closure *(or* Closure*). It* v-constructs *the following function* f *of the type* $(\alpha\beta_1 \ldots \beta_m)$*. Let* $v(B_1/x_1, \ldots, B_m/x_m)$ *be a valuation identical with* v *at least up to assigning objects* $B_1/\beta_1, \ldots, B_m/\beta_m$ *to variables* $x_1, \ldots, x_m$*. If* Y *is* $v(B_1/x_1, \ldots, B_m/x_m)$*-improper (see iii), then* f *is undefined on* $\langle B_1, \ldots, B_m \rangle$*. Otherwise the value of* f *on* $\langle B_1, \ldots, B_m \rangle$ *is the* α*-entity* $v(B_1/x_1, \ldots, B_m/x_m)$*-constructed by* Y.

(v) *The* Single Execution ${}^1X$ *is the* construction *that either* v*-constructs the entity* v*-constructed by* X *or, if* X v*-constructs nothing, is* v*-improper.*

(vi) *The* Double Execution ${}^2X$ *is the following* construction*. Where* X *is any entity, the* Double Execution ${}^2X$ *is* v-improper *(yielding nothing relative to* v*) if* X *is not itself a construction, or if* X *does not* v*-construct a construction, or if* X v*-constructs a* v*-improper construction. Otherwise, let* X v*-construct a construction* Y *and* Y v*-construct an entity* Z*: then* ${}^2X$ v*-constructs* Z.

(vii) *Nothing is a* construction*, unless it so follows from (i) through (vi).* $\square$

The definition of the ramified hierarchy of types decomposes into three parts. Firstly, simple types of order 1, which were already defined by Definition 1. Secondly, constructions of order n, and thirdly, types of order $n + 1$.

Definition 3: (ramified hierarchy of types)
T_1 (types of order 1). *See Definition 1.*
C_n (constructions of order n)

(i) *Let x be a variable ranging over a type of order n. Then x is* a construction of order n over B.
(ii) *Let X be a member of a type of order n. Then ${}^0X$, ${}^1X$, ${}^2X$ are* constructions of order n over B.
(iii) *Let $X, X_1, \ldots, X_m (m > 0)$ be constructions of order n over B. Then $[X\ X_1 \ldots X_m]$ is a* construction of order n over B.
(iv) *Let $x_1, \ldots x_m, X (m > 0)$ be constructions of order n over B. Then $[\lambda x_1 \ldots x_m\ X]$ is a* construction of order n over B.
(v) *Nothing is a* construction of order n over B *unless it so follows from* C_n *(i)–(iv).*

T_{n+1} (types of order $n + 1$). *Let $*_n$ be the collection of all constructions of order n over B. Then*

(i) *$*_n$ and every type of order n are* types of order $n + 1$.
(ii) *If $0 < m$ and $\alpha, \beta_1, \ldots, b_m$ are types of order $n + 1$ over B, then $(\alpha\ \beta_1 \ldots \beta_m)$ (see T_1 ii)) is a* type of order $n + 1$ over B.
(iii) *Nothing is a* type of order $n + 1$ over B *unless it so follows from (i) and (ii).* $\square$

Empirical languages incorporate an element of *contingency* that non-empirical ones lack. Empirical expressions denote *empirical conditions* that may or may not be satisfied at some empirical index of evaluation. Non-empirical languages have no need for an additional category of expressions for empirical conditions. We model these empirical conditions as *possible-world intensions*. Intensions are entities of type $(\beta\omega)$: mappings from possible worlds to an arbitrary type β. The type β is frequently the type of the *chronology* of α-objects, i.e. a mapping of type $(\alpha\tau)$. Thus α-intensions are frequently functions of type $((\alpha\tau)\omega)$, abbreviated as '$\alpha_{\tau\omega}$'. We shall typically say that an index of evaluation is a world/time pair $\langle w, t\rangle$. *Extensional entities* are entities of some type α where $\alpha \neq (\beta\omega)$ for any type β.

Examples of frequently used intensions are: *propositions* of type $\mathrm{o}_{\tau\omega}$, *properties of individuals* of type $(\mathrm{o}\iota)_{\tau\omega}$, binary *relations-in-intension between individuals* of type $(\mathrm{o}\iota\iota)_{\tau\omega}$, *individual offices* of type $\iota_{\tau\omega}$. As for individual offices, they are simply partial functions which, relative to a world/time pair $\langle w, t\rangle$, return at most one individual as value. The notion of office is broader than, say, social role, though social roles like *the pope* and *the King of France* were the original sources of inspiration. Given a $\langle w, t\rangle$, there is a function from *individual offices* to *individuals*. This function is neither a surjection, nor an injection, but a properly partial function, for some offices will go vacant. Given a $\langle w, t\rangle$, there is no function from *individuals* to *individual offices*, for some individual may occupy more than one office. Conversely, it is not given that each individual occupies at least one office. Importantly, the logical traffic does not flow from attributees (individuals) to individual offices, for not every attributee is guaranteed to occupy an office presenting a particular attributee.

Individual offices may be denoted by definite descriptions, but may just as well be denoted by proper names not reducible to definite descriptions. It depends on the linguistic quirks of a particular natural language in which manner a particular office is denoted. For instance, on our analysis of 'Hesperus', 'Phosphorus' these Latin/English names are better off naming two distinct offices (rather than the same individual), whereas 'Prague' is arguably a proper name for Prague, containing no descriptive material. (See Duží et al. (2010, §3.2 'Proper names'.))

Our *explicit intensionalization and temporalization* enables us to encode constructions of possible-world intensions, by means of terms for possible-world variables and times, directly in the logical syntax.[xviii] Where w ranges over ω and t over τ, the following logical form (to be explained below) essentially characterizes the logical syntax of any empirical language:

$$\lambda w \lambda t\, [\ldots w \ldots . t \ldots]$$

Logical objects like *truth-functions* and *quantifiers* are extensional: $\wedge$ (conjunction), $\vee$ (disjunction) and $\supset$ (implication) are of type (ooo), and $\neg$ (negation) of type (oo). *Quantifiers* $\forall^\alpha$, $\exists^\alpha$ are type-theoretically polymorphous, total functions of type $(\text{o}(\text{o}\alpha))$, for an arbitrary type α, defined as follows. The *universal quantifier* $\forall^\alpha$ is a function that associates a class A of α-elements with T if A contains all elements of the type α, otherwise with F. The *existential quantifier* $\exists^\alpha$ is a function that associates a class A of α-elements with T if A is a non-empty class, otherwise with F. Below all type indications will be provided outside the formulae in order not to clutter the notation. Furthermore, 'X/α' means that an object X is (a member) of type α. '$X \rightarrow_v \alpha$' means that the type of the object *valuation*-constructed by X is α. We write '$X \rightarrow \alpha$' if what is v-constructed does not depend on a

[xviii] See Duží et al. (2010, §2.4 'Explicit intensionalization and temporalization') or Jespersen (2005).

valuation v. Throughout, it holds that the variables $w \rightarrow_v \omega$ and $t \rightarrow_v \tau$. If $C \rightarrow_v \alpha_{\tau\omega}$ then the frequently used Composition $[[C\ w]t]$, which is the intensional descent (a.k.a. extensionalization) of the α-intension v-constructed by C, will be encoded as 'C_{wt}'.

When assigning constructions to expressions as their context-invariant meanings, we use a particular *method of semantic analysis*. The method consists in three steps, which are (a) type-theoretical analysis, (b) synthesis, and (c) type-theoretical checking. For illustration, here is the analysis of the sentence

"Mary believes that the Evening Star is a planet"

(a) The types of the objects that receive mention in the sentence are: *Mary*$/\iota$; *Believe**$/(\mathrm{o}\iota{*}_n)_{\tau\omega}$: a relation-in-intension of an individual (a doxastic agent) to a *propositional construction*; *Evening_Star*$/\iota_{\tau\omega}$: an individual office; *Planet*$/(\mathrm{o}\iota)_{\tau\omega}$: a property of individuals. *Hyperpropositional* belief, *Believe** (with asterisk) of type $(\mathrm{o}\iota{*}_n)_{\tau\omega}$, contrasts with *propositional* belief, *Believe* (without asterisk) of type $(\mathrm{o}\iota\mathrm{o}_{\tau\omega})_{\tau\omega}$, which is how possible-world semantics (perhaps skipping the temporal parameter) types propositional attitudes.[xix]

(b) We combine constructions of the objects *ad* (a) in order to construct the proposition of type $\mathrm{o}_{\tau\omega}$ denoted by the sentence. Here is how. First, since a property of individuals is not a type-theoretically proper object to predicate of an individual office, we must first extensionalize both intensions: $[[{}^0\mathit{Planet}\ w]\ t] \rightarrow_v (\mathrm{o}\iota)$, $[[{}^0\mathit{Evening\_Star}\ w]\ t] \rightarrow_v \iota$, or '${}^0\mathit{Planet}_{wt}$' and '${}^0\mathit{Evening\_Star}_{wt}$' for short. Now the Composition

[xix] So the type-theoretic difference between *propositional* and *hyperpropositional* attitudes is the difference between $(o\iota o_{\tau\omega})_{\tau\omega}$ and $(o\iota{*}_n)_{\tau\omega}$. One major philosophical difference is that the former are used to model *implicit* attitudes and the latter to model *explicit* attitudes, which translates into the logical difference between those attitudes that are *deductively closed* and those that are not. If a knows implicitly/propositionally that the Morning Star is a planet then a knows implicitly/explicitly every proposition entailed by the proposition that the Morning Star is a planet. Implicit knowledge notoriously leads to one form or other of logical omniscience (arguably *the* problem plaguing epistemic logic). If a knows*, explicitly/hyperpropositionally, that the Morning Star is a planet then much less is entailed, depending on what sort of logical intelligence in the shape of command of rules of inference has been assigned to a. (See Duží et al. (2010, §5.1.5 'Epistemic closure and inferable knowledge') for the notion of *inferable* knowledge, which charts the amount of explicit knowledge a would be able to harvest if a were to apply his entire logical intelligence maximally to his existing stock of explicit knowledge.) In this paper our concern is not *why* hyperpropositional attitudes *de re* would or could or should be attributed to an agent, but rather how to obtain a particular conclusion from them.

$[{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}]$ v-constructs T or F according as the individual that occupies the office of Evening Star at a given $\langle w, t\rangle$-pair of evaluation belongs to the class of planets at the same $\langle w, t\rangle$-pair.[xx] To obtain the proposition that the Evening Star is a planet we must abstract over the values of the variables w, t:

$$\lambda w \lambda t\ [{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}]$$

This is the predication of the property of being a planet of the occupant of the office of Evening Star.

Since an empirical hyperpropositional attitude is always a relation to a construction constructing a possible-world proposition, we model Mary's act of believing as the Composition

$$[{}^0\mathit{Believe*}_{wt}\ {}^0\mathit{Mary}\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}]]]$$

In other words, the nested Closure constructing the proposition that the Evening Star is a planet must be Trivialized, as *per* ${}^0[\lambda w \lambda t[{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}]]$. This Composition constructs T or F, according as Mary believes* at $\langle w, t\rangle$ that the Evening Star is a planet. Yet what is denoted by the sentence does not depend on contingent facts like Mary's being related or not to a particular hyperproposition. Thus we must again abstract over the values of w, t in order to construct the proposition denoted by the sentence (cf. explicit intensionalization and temporalization):

$$\lambda w \lambda t\ [{}^0\mathit{Believe*}_{wt}\ {}^0\mathit{Mary}\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}]]]$$

(c) We check whether the particular constituents of the above Closure are combined in compliance with the type-theoretical rules. To this end we draw a type-theoretical tree. In the interest of economy, we omit the steps of checking extensionalizations like $[{}^0\mathit{Planet}\ w] \rightarrow_v ((\mathrm{o}\iota)\tau)$ and $[[{}^0\mathit{Planet}\ w]t] \rightarrow_v (\mathrm{o}\iota)$. Thus we directly draw ${}^0\mathit{Planet}_{wt} \rightarrow_v (\mathrm{o}\iota)$. Moreover, instead of two steps of intensionalization, e.g. $\lambda t\ [{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}] \rightarrow_v (\mathrm{o}\tau)$ and $\lambda w \lambda t\ [{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}] \rightarrow ((\mathrm{o}\tau)\omega)$, we will directly draw $\lambda w \lambda t\ [{}^0\mathit{Planet}_{wt}\ {}^0\mathit{Evening\_Star}_{wt}] \rightarrow \mathrm{o}_{\tau\omega}$. The type-theoretically annotated tree is depicted by Figure 1.

[xx] See Duží et al. (2010, §2.4.2 'Predication as functional application') for details.

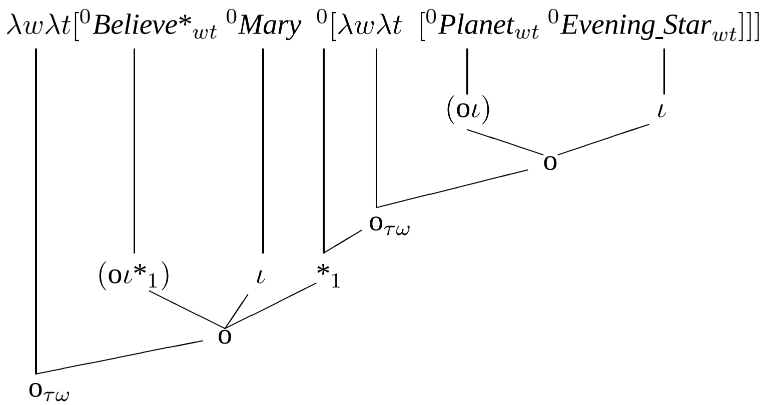

Figure 1. Type-theoretical tree

The above example illustrates also the difference between a *sub-construction simpliciter* and a *constituent sub-construction*, i.e. between a sub-construction which is *mentioned* as a functional argument, thereby displaying itself, and a sub-construction which is *used* to construct an object different from the sub-construction. The former case constitutes a hyperintensional context, the latter an extensional/intensional context. In our example the sub-construction that is mentioned within the whole analysis is the Closure $[\lambda w \lambda t\ [{}^0\textit{Planet}_{wt}\ {}^0\textit{Evening\_Star}_{wt}]]$. This is so because this hyperproposition is the second argument of the function $\textit{Believe*}_{wt}$, the first argument being *Mary*. The hyperproposition is mentioned by the constituent ${}^0[\lambda w \lambda t\ [{}^0\textit{Planet}_{wt}\ {}^0\textit{Evening\_Star}_{wt}]]$, and thus the mentioned Closure and all its sub-constructions occur hyperintensionally.

Figure 2 illustrates using/mentioning entities at the three different levels.

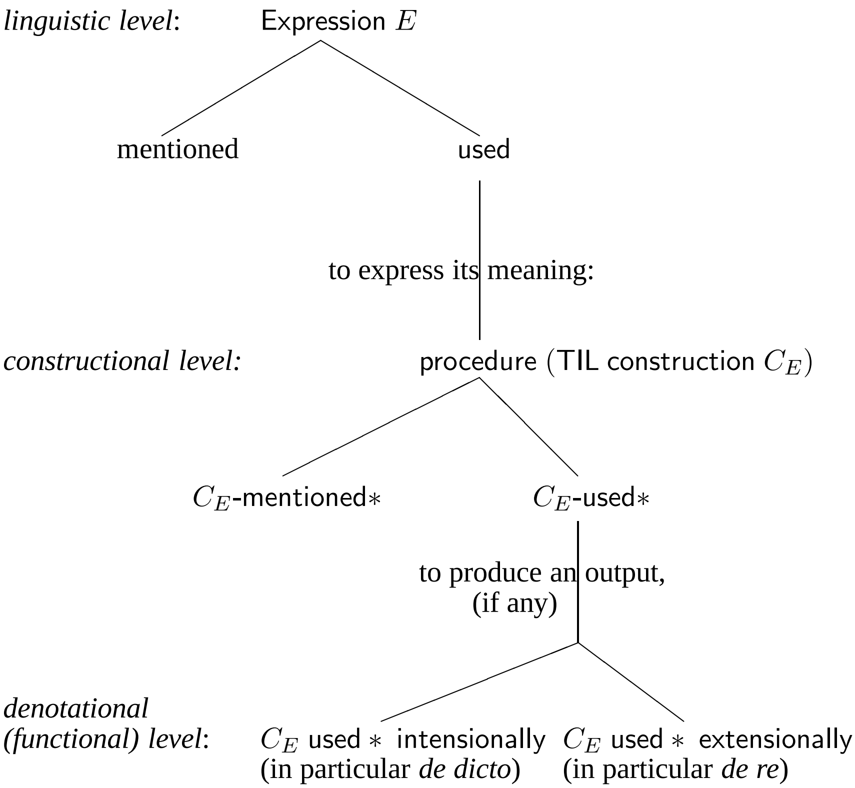

Figure 2. Using/mentioning entities

The three kinds of context are defined in Duží et al. (2010, §2.6). Here we need only the definition of the distinction between using and mentioning constructions:

Definition 4: (construction mentioned vs. used as a constituent) *Let C be a construction and D a sub-construction of C.*

(i) *If D is identical to C (i.e., ${}^0C = {}^0D$) then the occurrence of D is* used as a constituent *of C.*

(ii) *If C is identical to $[X_1\ X_2 \ldots X_m]$ and D is identical to one of the constructions $X_1, X_2, \ldots, X_m$, then the occurrence of D is* used as a constituent *of C.*

(iii) *If C is identical to $[\lambda x_1 \ldots x_m\ X]$ and D is identical to X, then the occurrence of D is* used as a constituent *of C.*

(iv) *If C is identical to ^1X and D is identical to X, then the occurrence of D is* used as a constituent *of C.*
(v) *If C is identical to ^2X and D is identical to X, or ^0D occurs as a constituent of X and this occurrence of D occurs as a constituent of Y v-constructed by X, then the occurrence of D is* used as a constituent *of C.*
(vi) *If an occurrence of D is* used as a constituent *of an occurrence of C' and this occurrence of C' is* used as a constituent *of C, then the occurrence of D is* used as a constituent *of C.*
(vii) *If an occurrence of a sub-construction D of C is not* used as a constituent *of C then the occurrence of D is* mentioned *in C.*
(viii) *No occurrence of a sub-construction D of C is* used/mentioned *in C unless it so follows from (i)–(vii).* □

Remark. In theory, a construction may be mentioned by another kind of construction than Trivialization; but in this paper we limit ourselves to Trivialization. Thus the Trivialization $^0[\lambda w \lambda t\ [^0Planet_{wt}\ ^0Evening\_Star_{wt}]]$ *mentions* the Closure $[\lambda w \lambda t\ [^0Planet_{wt}\ ^0Evening\_Star_{wt}]]$ and all the constituents of this Closure.[xxi]

Traditionally, the validity of quantifying-in has been fielded as a logical criterion for distinguishing (i) extensional/ transparent/'relational' (Quine) contexts from (ii) non-extensional/opaque/'notional' (Quine) contexts. The idea is that extensional (etc.) contexts are those that validate quantifying-in. And conversely, if a context resists quantifying-in, it is deemed to be in violation of one or more of the laws of extensional logic and as eluding full logical analysis. What we are saying is that also intensional and hyperintensional contexts may be quantified into, but that the feasibility of doing so presupposes that it be done within an extensional logic of hyperintensional contexts. Deploying a non-extensional logic of hyperintensions in order to quantify into hyperintensional contexts would, indeed, be a non-starter, generating opacity and thereby making hyperintensional attitude contexts logically intractable: it would be left logically lawless which terms (or meanings, as we would have it) could be substituted for which ones inside an attitude report. However, whether one accepts quantifying into (hyper-) intensional contexts or wants to restrict quantification to extensional contexts, like "Mary is happy", the logical question still remains which sort of context validates which sort of quantifying-in. Tichý issues in (1986, p. 256; 2004, p. 654) a warning against inter-defining the notion of extensional context and the validity of the rules of substitution of co-referring terms and existential

[xxi] The use/mention distinction normally applies only to *words*; in TIL it applies also to the *meanings* of words (i.e., constructions). See Duží et al. (2010, §2.6 'Three kinds of context').

generalization on pain of circularity (where TIL and Quine agree on the use of ‘co-referential’):

Q: When is a context extensional?
A: A context is extensional if it validates (i) the rule of substitution of co-referential terms and (ii) the rule of existential generalization.
Q: And when are (i), (ii) valid?
A: Those two rules are valid when applied to extensional contexts.

We steer clear of the circle by defining extensionality for (i) *hyperintensions* presenting functions, for (ii) *functions* (including possible-world intensions), and for (iii) *functional values*. These three levels are squared off with three kinds of context:

(i′) *hyperintensional contexts*, in which a hyperintension is not *used* to present an object, but is itself *mentioned* as functional argument (though a hyperintension of one order higher needs to be used to mention this lower-order construction);
(ii′) *intensional contexts*, in which a hyperintension is *used* to present a function without presenting a particular value of the function (moreover, the hyperintension does not occur within another hyperintensional context);
(iii′) *extensional contexts*, in which a hyperintension is *used* to produce a particular value of the function at a given argument (moreover, the hyperintension does not occur within another intensional or hyperintensional context).

The leading idea is that transparency in hyperintensional contexts requires identity of senses (hence pairs of synonymous words), while transparency in intensional contexts requires only equivalence of senses (hence pairs of co-denoting words).

If a construction C occurs mentioned in D, then all its sub-constructions (including C) occur *hyperintensionally* in D. Moreover, all the variables occurring within such a *hyperintensional context* are ^{0}bound. Thus in TIL we have two ways of binding variables: λ-binding (as in any λ-calculus) and ^{0}binding (which is unique to TIL). The latter is dominant over the former, because a higher-order context is dominant over a lower-order one. Thus we define:

Definition 5: (free and bound variables) *Let C be a construction with at least one occurrence of a variable ζ.*

(i) *Let C be ζ. Then the* occurrence of ζ *in C is* free.

(ii) *Let* C *be* ^0X. *Then every* occurrence of ζ *in* C *is* ^{0}bound *('Trivialization-bound').*

(iii) *Let* C *be* $[\lambda x_1 \dots x_n\ Y]$. *Any* occurrence of ζ *in* Y *that is one of* $x_i, 1 \leq i \leq n$, *is* λ-bound *in* C *unless it is* ^{0}bound in Y. *Any* occurrence of ζ in Y *that is neither* ^{0}bound *nor* λ-bound *in* Y *is* free *in* C.

(iv) *Let* C *be* $[X\ X_1 \dots X_n]$. *Any* occurrence of ζ *that is* free, ^{0}bound, λ-bound *in one of* $X, X_1, \dots, X_n$ *is, respectively,* free, ^{0}bound, λ-bound in C.

(v) *Let* C *be* ^1X. *Then any* occurrence of ζ *that is* free, ^{0}bound, λ-bound *in* X *is, respectively,* free, ^{0}bound, λ-bound in C.

(vi) *Let* C *be* ^2X. *Then any* occurrence of ζ *that is* free, λ-bound *in a constituent of* C *is, respectively,* free, λ-bound in C. *If an occurrence of* ζ *is* ^{0}bound *in a constituent* ^0D *of* C *and this occurrence of* D *is a constituent of* X' v*-constructed by* X, *then if the occurrence of* ζ *is free,* λ*-bound in* D *it is* free, λ-bound in C. *Otherwise, any other occurrence of* ζ *in* C *is* ^{0}bound *in* C.

(vii) *An* occurrence *of* ζ *is* free, λ-bound, ^{0}bound *in* C *only due to (i)–(vi).*

A construction with at least one occurrence of a free variable is an open construction. A construction without any free variables is a closed construction. □

The next notion we need to define is that of *synonymy*. Our notion of synonymy is defined in virtue of *procedural isomorphism*. Only procedurally isomorphic constructions, to the exclusion of merely equivalent ones, are substitutable *salva veritate* in hyperintensional contexts. The term 'procedural isomorphism' is a nod to Carnap's *intensional isomorphism* and Church's *synonymous isomorphism*. Church's Alternatives (0) and (1) leave room for additional Alternatives in between. One would be Alternative (1/2), another Alternative (3/4). The former includes α- and η-conversion while the latter adds a restricted form of β-conversion.[xxii] If we must choose, we would prefer Alternative (3/4) to soak up those differences between β-transformations that lack natural-language counterparts. The exact calibration of procedural isomorphism is less pressing here. What is important is that we should have a formal theory of synonymy and that, since we decide to include some form of β-conversion in the mix, we should have a means to block instances of invalid β-conversion.

One reason for excluding unrestricted $beta$-conversion is the well-known fact that β-conversion is not an equivalent transformation in logics boasting

[xxii] For Alternative (1/2), see Jespersen (2010).

partial functions, such as TIL. Another reason is that occasionally even β-equivalent constructions have different natural-language counterparts; witness the difference between attitude reports *de dicto* vs. *de re*. Thus, the difference between "a believes that b is happy" and "b is believed by a to be happy" is just the difference between β-equivalent meanings. The former (*de dicto*) receives the possible-world analysis

$$\lambda w \lambda t\ [{}^0\mathit{Believe}_{wt}\ a\ \lambda w \lambda t\ [{}^0\mathit{Happy}_{wt}\ b]]$$

while the latter (*de re*) receives the possible-world analysis

$$\lambda w \lambda t\ [\lambda x [{}^0\mathit{Believe}_{wt}\ a\ \lambda w \lambda t\ [{}^0\mathit{Happy}_{wt}\ x]] b]$$

Types: *Happy*/$(o\iota)_{\tau\omega}$; $x \to_v \iota$; $a, b \to \iota$.

Note that attitudes *de dicto* and *de re* are in general *not* equivalent. The following two sentences denote different propositions:

"a believes that the pope is happy";
"The pope is believed by a to be happy"

Their propositional analyses are (*Pope*/$\iota_{\tau\omega}$):

$$\lambda w \lambda t\ [{}^0\mathit{Believe}_{wt}\ a\ \lambda w \lambda t\ [{}^0\mathit{Happy}_{wt}\ {}^0\mathit{Pope}_{wt}]];$$

$$\lambda w \lambda t\ [\lambda x [{}^0\mathit{Believe}_{wt}\ a\ \lambda w \lambda t\ [{}^0\mathit{Happy}_{wt}\ x]]{}^0\mathit{Pope}_{wt}]$$

While the former Closure constructs a proposition that may well be *true* even when there is no pope (the papal office going vacant), the proposition constructed by the latter Closure will have a truth-value *gap* at such a world/time pair. This is because at such a world/time pair at which the office is vacant the Composition ${}^0\mathit{Pope}_{wt}$ is v-improper. Due to compositionality, the whole Composition $[\lambda x [{}^0\mathit{Believe}_{wt}\ a\ \lambda w \lambda t\ [{}^0\mathit{Happy}_{wt}\ x]]{}^0\mathit{Pope}_{wt}]$ comes out v-improper and so does not v-construct what it is typed to construct, namely a truth-value.

The *restricted* version of *equivalent* β-conversion we have in mind consists in substituting free variables for λ-bound variables of the same type, and will be called β_r*-conversion*. For instance, we see little reason to differentiate semantically or logically between "b is believed by a to be happy" and "b

has the property of being believed by a to be happy".[xxiii] The latter sentence expresses

$$\lambda w\lambda t\ [\lambda w'\lambda t'\lambda x[{}^0\mathit{Believe}_{w't'}\ a\ \lambda w\lambda t\ [{}^0\mathit{Happy}_{wt}\ x]]_{wt}\ b]$$

This is merely a β_r-expanded form of

$$\lambda w\lambda t\ [\lambda x[{}^0\mathit{Believe}_{wt}\ a\ \lambda w\lambda t\ [{}^0\mathit{Happy}_{wt}\ x]]\ b]$$

Thus we define:

Definition 6: (procedurally isomorphic constructions: (A3/4)) *Let C, D be constructions. Then C, D are* α-equivalent *iff they differ at most by deploying different λ-bound variables. C, D are* η-equivalent *iff one arises from the other by η-reduction or η-expansion. C, D are* β_r-equivalent *iff one arises from the other by β_r-reduction or β_r-expansion. C, D are* procedurally isomorphic, *denoted '${}^0C \approx {}^0D'$, $\approx/(o{*_n}{*_n})$, iff there are closed constructions $C_1, \ldots, C_m, m \geq 1$, such that ${}^0C = {}^0C_1$, ${}^0D = {}^0C_m$, and all $C_i, C_{i+1} (1 \leq i < m)$ are either α-, η- or β_r-equivalent.* $\Box$

Remark. The four constructions ^{0}*Prime*, $\lambda x[{}^0\mathit{Prime}\ x]$, $\lambda y[{}^0\mathit{Prime}\ y]$, $\lambda z[\lambda x[{}^0\mathit{Prime}\ x]z]$ are procedurally isomorphic, while $\lambda x[[{}^0\mathit{Card}\ \lambda y[{}^0\mathit{Divide}\ y\ x]] = {}^0 2]$ is only equivalent to them; it constructs the set of primes, to be sure, but does so in a non-isomorphic manner. (*Types*: x, y, $z \rightarrow \nu$, the type of natural numbers; *Card*/$(\nu(o\nu))$: the number of elements of a final set of natural numbers; *Divide*/$(o\nu\nu)$: the relation of x being divisible by y.)

Remark. Given a set of procedurally isomorphic constructions, we privilege the element that is in *normal form* to serve as a representative of the set. That element is the simplest one in the set and is defined as the alphabetically first, non-η-reducible construction. Thus, of the four constructions mentioned in

[xxiii] This is not to say we see no reason at all not to differentiate. For instance, if the believer is a self-assured nominalist then he may protest that while he does believe that a is happy he does not believe that a has any properties. Or it could be argued that one thing is to believe that a is happy and another is to believe that a has the property of being happy, because the latter at least appears to presuppose that the believer have the additional conceptual resources to master the notion of *property*. Furthermore, Soames (2010, Ch. 2) takes issue with Frege's claim, in 'Über Begriff und Gegenstand', that "das Prädikat 'fallend unter den Begriff *Mensch*' [dasselbe bedeutet wie] 'ein Mensch"'. This would namely mean that "Jesus ist ein Mensch" and "Jesus fällt unter den Begriff *Mensch*" would express the same Thought, i.e. be synonymous, i.e. share the same logical structure, which is hardly true of these two sentences. Further research is required. See Duží et al. (2010) and also Duží and Jespersen (2012) for discussion.

the previous Remark, ^{0}*Prime* is the one in normal form. See Duží et al. (2010, §2.2.1 'Concepts and synonymy', esp. p. 155). When in this paper we speak of *the* construction of something we intend a construction in normal form.

Since merely co-referential expressions can be substituted *salva veritate* only in extensional contexts, merely co-denotational or equivalent expressions in intensional and extensional contexts, and synonymous expressions in all contexts, we define:

Definition 7: (synonymous, equivalent and co-referential expressions) *Expressions E_1 and E_2 are* synonymous *if their meanings are procedurally isomorphic. Expressions E_1 and E_2 are* equivalent *(or* co-denoting*) if their meanings v-construct one and the same object for every valuation v. Finally, empirical expressions E_1 and E_2 are* co-referential *if their meanings construct intensions whose values are the same at the $\langle w, t\rangle$ of evaluation.* □

To summarize, the relevant tenets of TIL are these five:

1. (*hyperintensional syntax*) its syntax explicitly mentions hyperintensions;
2. (*anti-contextualism*) a non-indexical term's sense and denotation remain constant for *all* contexts;
3. (*mention versus use*) what is context-sensitive is whether a hyperintension occurs *mentioned* or *used* in a given context: if mentioned, it itself is operated on; if used, what it yields is operated on;
4. (*ramified type hierarchy*) the ramified type hierarchy enables higher-order hyperintensions to present lower-order hyperintensions (which in turn enables quantifying into hyperintensional contexts of any order);
5. (*mode of presentation*) it displays in logical terms how n-order constructions construct n-1-order constructions and first-order objects, by defining the various ways in which various constructions construct objects of various types.

Our neo-Fregean semantic schema, which applies to all contexts, is this:

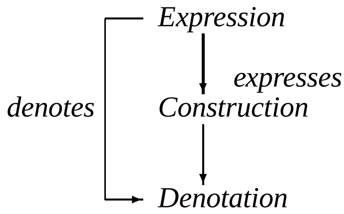

Figure 3. Semantic schema

The most important relation in this schema is between an expression and its meaning (a construction). We can investigate *a priori* what (if anything) a construction constructs and what is entailed by it. Once a construction is explicitly given as a result of logical analysis, the entity (if any) it constructs is already implicitly given, whereas it requires inquiry *a posteriori* to establish the reference of an empirical term at a given world/time pair. As a limiting case, the logical analysis may reveal that the construction fails to construct anything because it is improper. And if the construction is not improper, the denotation can be either a first-order object (i.e. a non-construction) or a lower-order construction. Intensional constructions (constructions of objects of type $(\beta\omega)$) are always proper, since they always construct an intension (including degenerate ones, which return no value at all or always the same value). In linguistic terms, every word whose sense is an intensional construction has a denotation, but will lack a reference at some or all $\langle w, t\rangle$ pairs, in case its denotation (a partial function) fails to return a value. This applies to, *inter alia*, ‘The pope’, ‘The Morning Star’ and ‘The Evening Star’.

3. *Hyperintensional and Intensional Attitudes* de dicto *and* de re

We begin by explaining how we understand the distinction between attitudes *de dicto* and *de re*. The philosophical difference between attitudes *de dicto* and *de re* is pivoted on an *inversion of perspective*: an attribution *de dicto* reproduces the attributee’s perspective; an attribution *de re*, the attributer’s. Let $F/\iota_{\tau\omega}$ be an office and $G/(o\iota)_{\tau\omega}$ a property. Consider then the sentence

“a believes that the F is a G”

Let a’s belief be an intensional attitude; *Believe*/$(o\iota o_{\tau\omega})_{\tau\omega}$. The meaning of the definite description ‘the F’ occurs with supposition *de dicto*. The reason is this: a is related to the *whole* proposition that the F is a G. Hence

a's attitude depends not only on the actual and current truth-value of this proposition but on all its values. Consequently, a's attitude concerns also the entire office F and not only its actual and current value. Even if there is no such value, a may well believe that the F is a G. The analysis of the sentence is this Closure:

$$\lambda w \lambda t\ [{}^0\mathit{Believe}_{wt}\ a\ [\lambda w^* \lambda t^* [{}^0G_{w^*t^*}\ {}^0F_{w^*t^*}]]]$$

Remark. *-superscripted letters for w, t variables represent the attributee's perspective, while those without superscript represent the attributer's. In the interest of full generality, throughout the rest of this paper we will use $a \rightarrow \iota$ as an arbitrary construction of an individual agent. Derivatively, 'a' will be an arbitrary name for an arbitrary individual.)

In case a's belief is a hyperintensional attitude, *Believe**/$(\mathrm{o}\iota{*}_n)_{\tau\omega}$, then a is related to a hyperproposition, i.e. a propositional construction, and the sentence encodes this construction:

$$\lambda w \lambda t\ [{}^0\mathit{Believe*}_{wt}\ a\ {}^0[\lambda w^* \lambda t^* [{}^0G_{w^*t^*}\ {}^0F_{w^*t^*}]]]$$

As before, a's attitude does not concern only the value of 'the F' in the actual world at the present moment. Now the hyperproposition $[\lambda w^* \lambda t^* [{}^0G_{w^*t^*}\ {}^0F_{w^*t^*}]]$ is itself the object of a's belief. Regardless of whether there is a value of the office F, the whole office has a role to play, as it is embedded within the attributee's perspective $\langle w^*, t^* \rangle$.

One worry, though, that one may have concerning attitudes *de dicto*, both intensional and hyperintensional, is the following. Constructions (usually Trivializations) of individual offices occur as part of the hyperpropositions that either construct a possible-world proposition or are themselves constructed to figure as complements of attitudes. Does this demand of the attributees that they possess a notion of individual office? The worry is that this may be asking too much, as it would require attributees to have conceptual resources they may actually, and reasonably, lack. Our position is this. If an attributee lacks any concept of intensions in their personal conceptual repertoire then indeed no attribution of an intension-involving attitude is an option. Such an attribution would fail to respect the attributee's perspective, whatever alternative perspective that might happen to be, as the attribution would 'hyper-intellectualize' the attributee's actual attitude.[xxiv] To have intensions in one's personal conceptual repertoire means comprehending the intensional character of some entity, e.g. the Evening Star. Its intensional character amounts in essence to the fact that it is not the numerically same individual that is the Evening Star in all possible empirical circumstances (all pairs of worlds and times), whenever a celestial body happens to be the Evening Star. But suppose the designated attributee uses 'The Evening Star' as a name for the *individual* Venus, because Venus is the actual Evening Star, while the designated attributer uses 'The Evening Star' as a name for the *individual office* of Evening Star. Then the consequence, relative to our framework, is that attributer and attributee speak at cross purposes, 'The Evening Star' being an instance of homonymy. The designated attributer is, therefore, not in a position to make a faithful attribution of an attitude *de dicto* to the designated attributee. Our framework observes the constraint that attributer and attributee must find themselves within the same *conceptual system* (see Duží et al. (2010, §2.2.3 'Conceptual system'). This involves according the same logical character to the Evening Star, be it as an individual or as an individual office, hence according the same semantic character to 'The Evening Star', as a name for an individual and a name for an individual office, respectively.

The situation is completely different with attitudes *de re*. To explain the nature of the attitude clearly, we use the passive form:

"The F is believed by a to be a G"

Now the attributer uses the office F as a pointer to a particular individual. The attributer might just as well have used any other office also occupied by that individual to single out the individual. For the proposition denoted by the sentence to be true, there must be a specific individual to whom a is related by believing that this individual has the property of being a G. This does not, however, mean that a is aware of the fact that this individual occupies the office F. There are namely two *independent* questions: "*Who* holds the office F" and "*What* does a think of that individual?" The attributer, not the attributee, needs to make the connection between office and occupant.

Now for the analysis of hyperpropositional attitudes *de re*. As above, let *Believe**/$(\text{o}\iota{*}_n)_{\tau\omega}$ be a relation-in-intension of an individual agent a to a

[xxiv] We are indebted to a referee for the apt phrase 'hyper-intellectualize' and for urging us to clarify our stance on the topic under what circumstances attributees must possess a notion of individual office.

hyperproposition. Then the active variant of a hyperpropositional attitude *de re* is this:

"a believes* of the F that *it* is a G"

The embedded clause "*it* is a G" contains the anaphoric reference 'it' to its antecedent 'the F'.[xxv] The meaning of this clause is an *open* construction with a free variable *it* $\rightarrow_v$ ι: $\lambda w \lambda t$ $[{}^0G_{wt}$ *it*]. Recall that our assignment of constructions to expressions as their meaning is context-*in*variant. Hence the meaning of this clause is the same in all contexts, whether extensional, intensional or hyperintensional.

Its meaning is any construction procedurally isomorphic with $\lambda w \lambda t$ $[{}^0G_{wt}$ *it*], e.g. $\lambda w^* \lambda t^*$ $[{}^0G_{w^*t^*}$ *it*]. The latter is an α-equivalent variant of the former with superscripted variables w^*, t^*, in order to represent the attributee's perspective. The attributer wants to express the fact that the individual who is the value of *it* is the holder of F. If F goes vacant, then there is no such holder and the proposition denoted by the sentence has a truth-value gap. On the other hand, if there is a holder of F, we must *pre-process* the Closure $\lambda w \lambda t$ $[{}^0G_{wt}$ *it*] in such a way that we substitute a construction of the holder of F (if any) for the variable *it* in the Closure. To this end we apply a *substitution technique* using the function Sub. Sub is of the polymorphous type $(*_n *_n *_n *_n)$ and operates on constructions in the following way. Let X, Y, Z be constructions of order n. Then Sub is a mapping which, when applied to $\langle X, Y, Z \rangle$, returns the construction that is the result of correctly substituting X for Y in Z. A correct substitution is one that does not make any variable occurring free in X bound in the resulting construction (no 'collision'). For illustration, the Composition $[{}^0Sub\ {}^{00}1\ {}^0x\ {}^0[{}^0{>}\ x\ {}^00]]$ constructs the result of substituting ^{01} for x into $[{}^0{>}\ x\ {}^00]$, which is the Composition $[{}^0{>}\ {}^01\ {}^00]$. Therefore, the Composition $[{}^0Sub\ {}^{00}1\ {}^0x\ {}^0[{}^0{>}\ x\ {}^00]]$ is equivalent to ${}^0[{}^0{>}\ {}^01\ {}^00]$, both constructing the Composition $[{}^0{>}\ {}^01\ {}^00]$ that constructs T.

Another polymorphous function we need when applying this substitution method is $Tr/(*_n\ \alpha)$ defined as follows.[xxvi] Let α be a type of order n, o an object of type α. Then Tr is a function which, when applied to o, returns the Trivialization of o. There is an essential difference between the *construction* Trivialization and the *function* Tr. For instance, whereas the Trivialization

[xxv] For our method of the analysis of sentences with anaphoric references, see Duží et al. (2010, §5.3 'Quantifying in').

[xxvi] Tichý introduces Sub in (1988, p. 75) and Tr in (*ibid.*, p. 68), where Tr is typed to take natural numbers to their respective Trivialization. Tr is easily generalized as a polymorphous function, however.

${}^0 3$ constructs the number 3, the Composition $[{}^0 Tr\ {}^0 3]$ constructs the Trivialization ${}^0 3$. Whereas the Trivialization ${}^0 x$ ^{0}binds the variable x and constructs just x, the variable x is free in the Composition $[{}^0 Tr\ x]$, which v-constructs the Trivialization of the number that v assigns to x. Thus, $[{}^0 Tr\ x]$ $v(2/x)$-constructs the construction ${}^0 2$. To illustrate the application of Sub and Tr, consider the schematic Composition $[{}^0 Sub\ [{}^0 Tr\ {}^0 A_{wt}]\ {}^0 y\ {}^0 [\ldots y \ldots]]$, where $A/\iota_{\tau\omega}$; $y \rightarrow_v \iota$; b/ι. This Composition either v-constructs the Composition $[\ldots {}^0 b \ldots]$, in case ${}^0 A_{wt}$ v-constructs b, or is v-improper, in case ${}^0 A_{wt}$ is v-improper, the individual role A going vacant at $\langle w, t\rangle$. In order to obtain the product of the substitution result, if any, we must execute the resulting Composition. To this end we apply *Double Execution*: ${}^2[{}^0 Sub\ [{}^0 Tr\ {}^0 A_{wt}]\ {}^0 y\ {}^0 [\ldots y \ldots]]$.

In our case we need to substitute the Trivialization of the individual v-constructed by ${}^0 F_{wt}$ for the variable *it* into the Closure $\lambda w^* \lambda t^*\ [{}^0 G_{w^*t^*}\ \textit{it}]$. Thus we get

$$[{}^0 Sub\ [{}^0 Tr\ {}^0 F_{wt}]\ {}^0 \textit{it}\ {}^0[\lambda w^* \lambda t^*\ [{}^0 G_{w^*t^*}\ \textit{it}]]]$$

If ${}^0 F_{wt}$ is v-improper (i.e., F is vacant at $\langle w, t\rangle$), then due to compositionality (cf. Definition 2. iii), the whole Composition is v-improper. Let ${}^0 F_{wt}$ v-construct the individual b. Then the result of the substitution is the *Closure* $[\lambda w^* \lambda t^*\ [{}^0 G_{w^*t^*}\ {}^0 b]]$. Since we analyze *hyper*propositional attitude, the agent a is related to this very Closure. Hence the analysis of "a believes* of the F that *it* is a G" is this Closure:

$$\lambda w \lambda t\ [{}^0 \textit{Believe*}_{wt}\ a\ [{}^0 Sub\ [{}^0 Tr\ {}^0 F_{wt}]\ {}^0 \textit{it}\ {}^0[\lambda w^* \lambda t^*\ [{}^0 G_{w^*t^*}\ \textit{it}]]]]$$

The truth-value of this proposition at a given $\langle w, t\rangle$-pair of evaluation depends on the holder of the office at $\langle w, t\rangle$; it is irrelevant who occupies it at worlds/times other than $\langle w, t\rangle$. We say that the meaning of 'the F', i.e. ${}^0 F$, occurs with *de re* supposition here. In this case *the two de re principles* are valid. They are (i) the principle of *existential presupposition* and (ii) the principle of *substitution of co-referential* expressions. It is not only entailed but also *presupposed* that the F *exists*, which means that the office F is occupied at a given $\langle w, t\rangle$. For, if the office goes vacant then for the same reason also the negated sentence "a does *not* believe* of the F that *it* is a G" denotes a proposition with a truth-value gap. Hence, in order that the denoted proposition have *any* truth-value, the office F must be occupied.

As for the substitution principle, if the office succeeds in picking out an individual b, then we can validly infer that b is believed* by a to be a G.

And if this individual happens to occupy another office $H/\iota_{\tau\omega}$, then we can validly infer that the H is believed* to be a G. We have got the following *valid* argument, based on the principle of predication *de re*, which is derived from the principle of substitution of co-referential terms:

$$\frac{\lambda w \lambda t\ [{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ {}^0F_{wt}]\ {}^0\textit{it}\ {}^0[\lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ \textit{it}]]]] \qquad \lambda w \lambda t\ [{}^0F_{wt} = {}^0H_{wt}]}{\lambda w \lambda t\ [{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ {}^0H_{wt}]\ {}^0\textit{it}\ {}^0[\lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ \textit{it}]]]]}$$

The rationale informing the principle of predication *de re* is that if two offices are co-occupied at $\langle w, t\rangle$ then what, at $\langle w, t\rangle$, is predicated of the occupant of one office is *eo ipso* what is predicated, at $\langle w, t\rangle$, of the occupant of the other office.[xxvii]

The *passive* variant of the hyperpropositional attitude *de re* is phrased in this manner:

"The F is believed* by a to be a G"

The property of being believed* by a to be a G is ascribed to the holder of F (if any). In order to construct that property, we must again apply the substitution technique. The reason is this. The seemingly straightforward construction of the property — the Closure $\lambda w \lambda t\ \lambda x[{}^0\textit{Believe*}_{wt}\ a\ {}^0[\lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ x]]]$ — is *not* correct. The variable x occurs within the scope of a Trivialization in the Closure $[\lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ x]]$, because a is related to the *construction* of a proposition rather than the proposition so constructed. Hence x occurs ^{0}bound, which amounts to being hyperintensionally mentioned by the Trivialization, and so x is not amenable to a direct logical operation like λ-binding. As already mentioned, ^{0}binding, which raises a context to the hyperintensional level, is dominant over λ-binding, which creates a lower-level intensional context. The substitution technique yields the correct construction of the property:

$$\lambda w \lambda t\ [\lambda x[{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ x]\ {}^0x\ {}^0[\lambda w^* \lambda t^*\ [{}^0G_{w^*t^*}\ x]]]]]$$

We now need to apply this property to the holder of F. Hence, we extensionalize the property (by β_r-reduction of the above Closure) and Compose the

[xxvii] See Duží et al. (2010, p. 123), the *rule of substitution of v-congruent constructions* (*ibid.*, p. 124) and the *extensional rule of substitution* (*ibid.*, p. 274).

result with ${}^0F_{wt}$. The resulting analysis of the passive *de re* variant becomes

$$\lambda w\lambda t\ [\lambda x[{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ x]\ {}^0x\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ x]]]]\ {}^0F_{wt}]$$

Next we generalize the results to intensional attitudes *de re*. Let *Believe*/ $(o\iota o_{\tau\omega})_{\tau\omega}$ be a propositional attitude, as above. The critical difference between intensional and hyperintensional attitudes *de re* consists in the presence or absence, respectively, of Double Execution. The analysis of the active form "a believes of the F that *it* is a G" is as above, with one difference. Let ${}^0F_{wt}$ *v-construct* an individual b. Then the result of the Composition $[{}^0Sub\ [{}^0Tr\ {}^0F_{wt}]\ {}^0\textit{it}\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ \textit{it}]]]$ is the *Closure* $\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ {}^0b]$. But this time a is related to the *proposition* constructed by this Closure. Thus we have to execute the Closure to obtain what it constructs. Enter Double Execution (boldfaced):

$$\lambda w\lambda t\ [{}^0\textit{Believe}_{wt}\ a\ {}^{\mathbf{2}}[{}^0Sub\ [{}^0Tr\ {}^0F_{wt}]\ {}^0\textit{it}\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ \textit{it}]]]]$$

The passive variant could be analysed in the same way. Yet there is an easier solution, which does not apply the substitution method. In order to construct the property of being believed by a to be a G, we use this Closure:

$$\lambda w\lambda t\lambda x\ [{}^0\textit{Believe}_{wt}\ a\ [\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ x]]]$$

We Compose this property construction (extensionalized with respect to the attributer's perspective) with ${}^0F_{wt}$ and abstract again over the values of w, t in order to obtain a proposition. The resulting analysis is this:

$$\lambda w\lambda t\ [\lambda x\ [{}^0\textit{Believe}_{wt}\ a\ [\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ x]]]\ {}^0F_{wt}]$$

When we compile them all, we have a hyperintensional and an intensional variant of attitudes *de dicto*, and a hyperintensional and an intensional variant of attitudes *de re* in their respective active and passive forms:

(*de dicto*)

"a believes/believes* that the F is a G"
$\lambda w\lambda t\ [{}^0\textit{Believe}_{wt}\ a\ [\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ {}^0F_{w^*t^*}]]]$
$\lambda w\lambda t\ [{}^0\textit{Believe*}_{wt}\ a\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ {}^0F_{w^*t^*}]]]$

(*de re active*)

"a believes/believes* of the F that it is a G"

$\lambda w \lambda t\ [^0\textit{Believe}_{wt}\ a\ ^2[^0Sub\ [^0Tr\ ^0F_{wt}]\ ^0\textit{it}\ ^0[\lambda w^* \lambda t^*\ [^0G_{w^*t^*}\ \textit{it}]]]]$

$\lambda w \lambda t\ [^0\textit{Believe*}_{wt}\ a\ [^0Sub\ [^0Tr\ ^0F_{wt}]\ ^0\textit{it}\ ^0[\lambda w^* \lambda t^*\ [^0G_{w^*t^*}\ \textit{it}]]]]$

(*de re passive*)

"The F is believed/believed* by a to be an F"

$\lambda w \lambda t\ [\lambda x\ [^0\textit{Believe}_{wt}\ a\ [\lambda w^* \lambda t^*\ [^0G_{w^*t^*}\ x]]]\ ^0F_{wt}]$

$\lambda w \lambda t\ [\lambda x\ [^0\textit{Believe*}_{wt}\ a\ [^0Sub\ [^0Tr\ x]\ ^0x\ ^0[\lambda w^* \lambda t^*\ [^0G_{w^*t^*}\ x]]]]\ ^0F_{wt}]$

Let us take stock. Hyperpropositional attitudes *de re*, as we just described them, have a string of characteristic features that set them apart from their *de dicto* counterparts. First, they are attitudes *of* somebody or something. Second, they are parasitic on preceding attitudes, either another attitude *de re* or ultimately an attitude *de dicto*. Third, they occasion an *inversion of perspective*, from attributee's to attributer's. Fourth, they have an integral tension built into them: *qua* hyperintensional they reproduce the attributee's perspective; qua *de re* the individual objects about whom or which the attitudes are entertained are picked out from the attributer's perspective. Says Quine,

> Spelling dissolves the syntax and lexicon of the content clause and blends it with that of the ascriber's language. So long as we rest with the unanalyzed quotational form, on the other hand, the inverted commas mark an opaque interface between two ontologies, two worlds: that of the man in the attitude, however benighted, and that of our responsible ascriber of the attitude. (1992, pp. 69–70.)

Transpose Quine's language-centric approach into our construction-centric one. Then explicit intensionalization and temporalization is what enables us to keep separate the two 'worlds' or 'ontologies' that Quine alludes to; namely, the perspective of the attributee and that of the attributer. One set of λ-bound w, t variables represents the attributer's perspective and another set the attributee's. The inversion of perspective explains why the following argument, sporting an attitude *de re* in the first premise as well as in the conclusion, is *valid* and its *de dicto* counterpart is invalid:

(1) The Evening Star is believed by a to be a planet

(2) The Evening Star $=$ the Morning Star

---

(3) The Morning Star is believed by a to be a planet

The rationale is that the respective meanings of 'The Evening Star' and 'The Morning Star' occur *de re*: the respective occupants of the offices, which are individuals, are picked out rather than the respective offices, which are mappings. The first premise means that, at the index of evaluation, the occupant of the office of Evening Star has the property of being believed by a to be a planet. The second premise means that, at the same index, the two offices of Evening Star and Morning Star are co-occupied. The conclusion means that, at the same index, the occupant of the office of Morning Star has the property of being believed by a to be a planet. (1) and (3) are two different truth-*conditions* (possible-world propositions), for sure, but for any dual index at which (2) is true, (1) and (3) will share the same truth-*value*. It is immaterial to the validity of the argument whether a's belief be intensional or hyperintensional.

By contrast, this argument is *invalid*:

(1′) a believes that the Evening Star is a planet
(2′) The Evening Star $=$ the Morning Star
(3′) a believes that the Morning Star is a planet

The premises are too weak to sustain the conclusion, whether a's belief be intensional or hyperintensional. In (1′) the sense of 'The Morning Star' occurs *de dicto*, in (2) *de re*, and in (3′) *de dicto* again.[xxviii] It is irrelevant that the two offices happen to be co-extensional when what is wanted is that they should be co-intensional. But, on the other hand, if there were but one office and not two then the argument would come out trivially valid, because the conclusion would simply be a rephrasing of the first premise. The invalidity of the existing argument feeds on the classical confusion of function and functional value (or concept and instance, to use another vernacular).

Hence as soon as (1) is true, the two principles unique to contexts *de re* kick in. The validity of substituting terms that co-refer has been illustrated by the first valid argument. The principle of *existential presupposition*, that the

[xxviii] There is a structural analogy with *Partee's* puzzle of 90°F rising: the temperature is 90°F; the temperature is rising $\therefore$90°F is rising. What just went wrong? A temperature is a magnitude (a number-in-intension, like *the number of the planets*), hence a mapping, while 90°F is one of its values (a number). In the first premise 'the temperature' occurs *de re* because a particular value of the function is identified. In the second premise 'the temperature' occurs *de dicto* because the mapping is the subject of predication. So where the conclusion, with its predicate 'is rising', demands a reference to a mapping, Partee's puzzle has a reference to a functional value. Hence the fallacy. See Duží et al. (2010, pp. 124–125).

relevant office(s) must be occupied, yields this *valid* argument when applied to (1):

(1) The Evening Star is believed by a to be a planet

(4) The Evening Star exists

On the other hand, the following argument is *invalid*:

(1′) a believes that the Evening Star is a planet

(4′) The Evening Star exists

In the *de dicto* case a may well believe that the Evening Star is a planet even if there is no Evening Star, the office going vacant. This is so, because this office now receives mention within a's perspective, in the intensional/hyperintensional context of what is believed by a.

4. *Rules for Quantifying into Hyperpropositional and Propositional Contexts* de re

The technical challenge of untying an x occurring mentioned in the context $\delta[\ldots x \ldots]$ and $\exists$-binding it requires three non-standard devices. The first is *Trivialization*. The second is Sub. The third is Tr. We say that Trivialization is *used* to *mention* other constructions (cf. Definition 4 and the subsequent Remark). The point of mentioning a construction is to make it, rather than what it presents, a functional argument. TIL interprets δ as a binary relation-in-intension between an agent a entertaining an attitude and a construction in its capacity as attitude relatum. Relations, in turn, are construed as functions, such that δ is typed as a function from worlds to a function from times to a function from agents and hyperpropositions to truth-values: given a world/time pair, it is either true or false that a particular agent has a particular attitude to a particular hyperproposition. In order for the relevant construction to figure as the second argument of the relevant attitude relation, it itself needs to be mentioned. If a given (first-order) hyperproposition is not mentioned but used, the resulting relatum is a possible-world proposition, thus reinstalling the logic of attitudes known from modal logic. A hyperpropositional attitude context is one in which the second argument of the attitude relation is a propositional *construction*.

In Duží and Jespersen (submitted) *the rule for quantifying into hyperpropositional attitudes de dicto* is the following. Let the types be: $Att^{*} \rightarrow$

$(o\iota *_n)_{\tau\omega}$: an arbitrary construction of a hyperpropositional attitude relation; $a \to \iota$; $C(X)/*_n \to_v o_{\tau\omega}$: a propositional construction with a constituent $X/*_n \to_v \alpha$; $c/*_{(n+1)} \to_v *_n$; ${}^2c/*_{(n+2)} \to_v \alpha$; $\exists */(o(o*_n))$. Then the rule is this:

$$\frac{[Att*_{wt}\ a\ {}^0C(X)]}{[{}^0\exists * \lambda c\ [Att*_{wt}\ a\ [{}^0Sub\ c\ {}^0X\ {}^0C(X)]]]}$$

Proof. The Composition $[{}^0Sub\ c\ {}^0X\ {}^0C(X)]$ $v(X/c)$-constructs the construction $C(X)$. Hence at any $\langle w, t\rangle$ at which $[Att*_{wt}\ a\ {}^0C(X)]$ v-constructs T the class v-constructed by $\lambda c\ [Att*_{wt}\ a\ [{}^0Sub\ c\ {}^0X\ {}^0C(X)]]$ is non-empty and the conclusion $[{}^0\exists * \lambda c\ [Att*_{wt}\ a\ [{}^0Sub\ c\ {}^0X\ {}^0C(X)]]]$ v-constructs T as well. □

Remark. Contrast $[Att*'_{wt}\ a\ {}^0[{}^0Sub\ c\ {}^0X\ {}^0C(X)]]$ with $[Att*_{wt}\ a\ [{}^0Sub\ c\ {}^0X\ {}^0C(X)]]$. The latter constructs a truth-value that depends on whether the agent has the attitude $Att*$ to the *result* of executing a substitution. The former also constructs a truth-value, but this time it depends on whether the agent is related by $Att*'$ to the very *procedure* of executing that substitution, i.e. the agent needs to become a practicing logician. $Att*'$ would have to change its type to $(o\iota *_{n+1})_{\tau\omega}$, because the result of the Trivialization ${}^0[{}^0Sub\ c\ {}^0X\ {}^0C(X)]$ is the Composition $[{}^0Sub\ c\ {}^0X\ {}^0C(X)]$ that belongs at least to $*_{n+1}$, i.e. a construction of order $n+1$ that v-constructs a construction of order n.

Now the final analysis of the valid argument

$$\frac{a \text{ believes that the Evening Star is a planet}}{\begin{array}{c}\text{There is a construction of an individual office such that}\\ a \text{ believes that the occupant of that office is a planet}\end{array}}$$

is as follows. Let $ES \to \iota_{\tau\omega}$ be a construction of the office of Evening Star. Then the resulting analysis is this:

$$\frac{\lambda w \lambda t\ [{}^0\mathit{Believe}*_{wt}\ a\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Planet}_{wt}\ ES_{wt}]]]}{\lambda w \lambda t\ [{}^0\exists \lambda c\ [{}^0\mathit{Believe}*_{wt}\ a\ [{}^0Sub\ c\ {}^0ES\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Planet}_{wt}\ ES_{wt}]]]]]}$$

Quantifying into hyperpropositional attitudes *de dicto* requires but one rule. Not so with quantifying into hyperpropositional attitudes *de re*, which all come in an active and a passive variant. The twin arguments and their twin analyses are as follows.

(*active*)

$$\frac{a \text{ believes* of the } F \text{ that } \textit{it} \text{ is a } G}{\text{There is an individual of which } a \text{ believes* that it is a } G}$$

$$\frac{\lambda w\lambda t\ [{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ {}^0F_{wt}]\ {}^0\textbf{\textit{it}}\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ \textbf{\textit{it}}]]]]}{\lambda w\lambda t\ [{}^0\exists\lambda y\ [{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ y]\ {}^0\textbf{\textit{it}}\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ \textbf{\textit{it}}]]]]]}$$

(*passive*)

$$\frac{\text{The } F \text{ is believed* by } a \text{ to be an } F}{\text{There is an individual that is believed* by } a \text{ to be a } G}$$

$$\frac{\lambda w\lambda t\ [\lambda x\ [{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ x]\ {}^0x\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ x]]]]\ {}^0F_{wt}]}{\lambda w\lambda t\ [{}^0\exists\lambda y\ [{}^0\textit{Believe*}_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ y]\ {}^0x\ {}^0[\lambda w^*\lambda t^*\ [{}^0G_{w^*t^*}\ x]]]]]}$$

We are now in a position to formulate the corresponding *rules for quantifying into hyperpropositional attitudes de re*. Let the types be: $Att^* \rightarrow (\text{o}\iota^*_n)_{\tau\omega}$; $a \rightarrow \iota$; $x, y, \textit{it}/{*_n} \rightarrow_v \iota$; $\exists/(\text{o}(\text{o}\iota))$; $X/\iota_{\tau\omega}$ (an individual office); $C(x), C(\textit{it}) \rightarrow \text{o}_{\tau\omega}$ propositional constructions with at least one free occurrence of variables x, *it*, respectively. Then the rules are:

(*active hyperintensional variant*)

$$\frac{[Att^*_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ {}^0X_{wt}]\ {}^0\textbf{\textit{it}}\ {}^0C(\textit{it})]]}{[{}^0\exists\lambda y\ [Att^*_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ y]\ {}^0\textbf{\textit{it}}\ {}^0C(\textit{it})]]]}$$

(*passive hyperintensional variant*)

$$\frac{[\lambda x\ [Att^*_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ x]\ {}^0x\ {}^0C(x)]]\ {}^0X_{wt}]}{[{}^0\exists\lambda y\ [Att^*_{wt}\ a\ [{}^0Sub\ [{}^0Tr\ y]\ {}^0x\ {}^0C(x)]]]}$$

Proof. By the definition of Composition, if the antecedent v-constructs T, then it is v-proper, and so is ${}^0X_{wt} \rightarrow_v \iota$. Hence there is an individual occupying X at $\langle w, t\rangle$. Let this individual be b/ι. Then:

(*active variant*)

1. $[Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ ^0X_{wt}]\ ^0\textit{it}\ ^0C(\textit{it})]]$ $\varnothing$
2. $[Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ y]\ ^0\textit{it}\ ^0C(\textit{it})]]$ 1, $v(b/y)$-constructs $\top$
3. $\lambda y\ [Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ y]\ ^0\textit{it}\ ^0C(\textit{it})]]$ v-constructs a non-empty class
4. $[^0\exists\lambda y\ [Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ y]\ ^0\textit{it}\ ^0C(\textit{it})]]$ 3, EG

(*passive variant*)

1. $[\lambda x[Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ x]\ ^0x\ ^0C(x)]]\ ^0X_{wt}]$ $\varnothing$
2. $[Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ ^0X_{wt}]\ ^0x\ ^0C(x)]]$ 1, β-reduction
3. $[Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ y]\ ^0x\ ^0C(x)]]$ 2, $v(b/y)$-constructs $\top$
4. $\lambda y\ [Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ y]\ ^0x\ ^0C(x)]]$ v-constructs a non-empty class
5. $[^0\exists\lambda y\ [Att{*}_{wt}\ a\ [^0Sub\ [^0Tr\ y]\ ^0\textit{it}\ ^0C(\textit{it})]]$ 4, EG

Those are the twin rules for quantifying into *hyperpropositional* contexts *de re*. Notice that whereas in the *de dicto* case quantifying-in is technically complicated and, without auxiliary assumptions, we are guaranteed to infer only that there is a *construction* such that ..., in the *de re* case quantifying-in is straightforward; moreover, we can infer that there is an *individual* such that ...

The corresponding pair of rules for quantifying into *propositional* contexts *de re* can now be easily stated. Let $Att \to (o\iota o_{\tau\omega})_{\tau\omega}$ be a construction of a propositional attitudes *de re*; the other types as above. Then:

(*active intensional variant*)

$$\frac{[Att_{wt}\ a\ ^2[^0Sub\ [^0Tr\ ^0X_{wt}]\ ^0\textit{it}\ ^0C(\textit{it})]]}{[^0\exists\lambda y\ [Att_{wt}\ a\ ^2[^0Sub\ [^0Tr\ y]\ ^0\textit{it}\ ^0C(\textit{it})]]]}$$

(*passive intensional variant*)

$$\frac{[\lambda x\ [Att_{wt}\ a\ C(x)]\ ^0X_{wt}]}{[^0\exists\lambda y\ [Att_{wt}\ a\ C(y)]]}$$

Proof. By the definitions of Composition and Double Execution, if the antecedent v-constructs $\top$, then it is v-proper, and so is $^0X_{wt} \to_v \iota$. Hence there is an individual occupying X at $\langle w, t\rangle$. Let this individual be b/ι. Then:

(*active variant*)

1. $[Att_{wt}\ a\ {}^2[{}^0Sub\ [{}^0Tr\ {}^0X_{wt}]\ {}^0\textit{it}\ {}^0C(\textit{it})]]$	$\varnothing$
2. $[Att_{wt}\ a\ {}^2[{}^0Sub\ [{}^0Tr\ y]\ {}^0\textit{it}\ {}^0C(\textit{it})]]$	1, $v(b/y)$-constructs $\top$
3. $\lambda y\ [Att_{wt}\ a\ {}^2[{}^0Sub\ [{}^0Tr\ y]\ {}^0\textit{it}\ {}^0C(\textit{it})]]$	v-constructs a non-empty class
4. $[{}^0\exists\lambda y\ [Att_{wt}\ a\ {}^2[{}^0Sub\ [{}^0Tr\ y]\ {}^0\textit{it}\ {}^0C(\textit{it})]]$	3, EG

(*passive variant*)

1. $[\lambda x\ [Att_{wt}\ a\ C(x)]\ {}^0X_{wt}]$	$\varnothing$
2. $[Att_{wt}\ a\ C({}^0X_{wt})]$	1, β-reduction
3. $[Att_{wt}\ a\ C(y)]$	2, $v(b/y)$-constructs $\top$
4. $\lambda y\ [Att_{wt}\ a\ C(y)]$	v-constructs a non-empty class
5. $[{}^0\exists\lambda y\ [Att_{wt}\ a\ C(y)]]$	4, EG

(*example of active variant*)

$$\frac{\text{Mary believes of the Evening Star that it is a planet}}{\text{There is an individual such that Mary believes that it is a planet}}$$

$$\frac{\lambda w\lambda t\ [{}^0B_{wt}\ {}^0\textit{Mary}\ {}^2[{}^0Sub\ [{}^0Tr\ {}^0\textit{Evening\_Star}_{wt}]\ {}^0\textit{it}\ {}^0[\lambda w\lambda t\ [{}^0\textit{Planet}_{wt}\ \textit{it}]]]]}{\lambda w\lambda t\ [{}^0\exists\lambda x\ [[{}^0B_{wt}\ {}^0\textit{Mary}\ {}^2[{}^0Sub\ [{}^0Tr\ x]\ {}^0\textit{it}\ {}^0[\lambda w\lambda t\ [{}^0\textit{Planet}_{wt}\ \textit{it}]]]]}$$

(*example of passive variant*)

$$\frac{\text{The Evening Star is believed by Mary to be a planet}}{\text{There is an individual such that it is believed by Mary to be a planet}}$$

$$\frac{\lambda w\lambda t\ [\lambda y\ [{}^0B_{wt}\ {}^0\textit{Mary}\ \lambda w\lambda t\ [{}^0\textit{Planet}_{wt}\ y]]\ {}^0\textit{Evening\_Star}_{wt}]}{\lambda w\lambda t\ [{}^0\exists\lambda z\ [[{}^0B_{wt}\ {}^0\textit{Mary}\ \lambda w\lambda t\ [{}^0\textit{Planet}_{wt}\ z]]]}$$

Types: $B/(o\iota o_{\tau\omega})_{\tau\omega}$; Mary/$\iota$; *Planet*/$(o\iota)_{\tau\omega}$; *Evening_Star*/$\iota_{\tau\omega}$.

This completes our exposition of quantifying into hyperpropositional and possible-world propositional contexts *de re*.

Conclusion

Above we presented and proved two rules for quantifying into hyperpropositional attitude contexts *de re* and two rules for quantifying into propositional attitude contexts *de re*. We also presented the rule for quantifying into hyperpropositional contexts *de dicto*, for comparison. Quantifying into hyperintensional contexts requires an extensional logic of hyperintensions in order to be executed in a principled manner. Quantifying into intensional contexts requires an extensional logic of intensions in order to be executed in a principled manner. Transparent Intensional Logic is one such theory. Much nontrivial footwork is required to lay out such a large-scale logical semantics. Once this is done, though, quantifying into hyperintensional and intensional contexts turns out to be as trivially valid as quantifying into non-attitudinal (and non-modal), or extensional, contexts. The difference with extensional contexts is only a matter of logical complexity. In order to relate an agent to a hyperproposition, the hyperproposition needs to occur mentioned in order to present itself rather than what it constructs, namely a proposition. The complication is that, since every constituent of a mentioned construction itself occurs mentioned, the quantifier cannot bind any variable occurring inside the mentioned context. The solution consists in pre-processing the mentioned construction by means of a substitution technique that makes variables amenable to binding. In this manner quantifying into hyperintensional contexts is rendered valid while at the same time observing compositionality and transparency. This marks an advance for philosophical logic in general and for hyperintensional logic in particular.

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Marie Duží: Department of Computer Science FEI
VSB-Technical University Ostrava
Czech Republic
E-mail: `marie.duzi@vsb.cz`

Bjørn Jespersen: Department of Philosophy
Institute of Logic
Czech Academy of Sciences
Prague
Department of Computer Science FEI
VSB-Technical University Ostrava
Czech Republic
E-mail: bjornjespersen@gmail.com

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