

Is Evidence of Evidence Evidence? Screening-Off vs. No-Defeaters

William Roche

Department of Philosophy, Texas Christian University, Fort Worth, TX, USA, e-mail:
w.roche@tcu.edu

ABSTRACT: I argue elsewhere (Roche 2014) that evidence of evidence is evidence under screening-off. Tal and Comesaña (2017) argue that my appeal to screening-off is subject to two objections. They then propose an evidence of evidence thesis involving the notion of a defeater. There is much to learn from their very careful discussion. I argue, though, that their objections fail and that their evidence of evidence thesis is open to counterexample.

KEYWORDS: Comesaña, Evidence of Evidence, No-Defeaters Condition, Screening-Off Condition, Tal, Transitivity in Evidence

1 Introduction

I argue elsewhere (Roche 2014) that evidence of evidence is evidence under screening-off. I argue, in fact, for four distinct theses to that effect. The last of them can be put as follows:

- EEE_{SOC} If (i) $\Pr(H_1 | E) > \Pr(H_1)$ where H_1 is the proposition that H_2 's probability given S 's total evidence is high,
(ii) $\Pr(H_2 | H_1) > \Pr(H_2)$,
and
(iii) $\Pr(H_2 | H_1 \ \& \ E) \geq \Pr(H_2 | H_1)$
 $\Pr(H_2 | \neg H_1 \ \& \ E) \geq \Pr(H_2 | \neg H_1)$,
then (iv) $\Pr(H_2 | E) > \Pr(H_2)$.

(Here the tacit quantification is over any propositions E , H_1 , and H_2 and any subject S . Similar points are true of TIP_{SOC}, EEE_{NDC}, EEE*_{NDC}, TIP_{NDC}, EEE**_{NDC}, TIP*_{NDC}, and TIP**_{NDC} below.) Let “evidence-HP” be evidence in the sense of *high probability* and “evidence-IP” be evidence in the sense of *increase in probability*.¹ Then EEE_{SOC} says that if (i) E is evidence-IP for H_1 where H_1 is the proposition that S 's total evidence is evidence-HP

¹ These are but two senses of evidence in the literature. See Douven (2011), Roche (2012b, 2015), and Roche and Shogenji (2014b).

for H_2 , (ii) H_1 is evidence-IP for H_2 , and (iii) H_1 screens-off E from H_2 in that E has no negative impact on H_2 's probability given the truth or falsity of H_1 , then (iv) E is evidence-IP for H_2 .^{2,3}

The argument behind EEE_{SOC} is straightforward. EEE_{SOC} is a special case of the more general thesis that evidence-IP is transitive under screening-off:

TIP_{SOC} If (i) $\Pr(H_1 | E) > \Pr(H_1)$,
 (ii) $\Pr(H_2 | H_1) > \Pr(H_2)$,
 and
 (iii) $\Pr(H_2 | H_1 \ \& \ E) \geq \Pr(H_2 | H_1)$
 $\Pr(H_2 | \neg H_1 \ \& \ E) \geq \Pr(H_2 | \neg H_1)$,
 then (iv) $\Pr(H_2 | E) > \Pr(H_2)$.

The difference with TIP_{SOC} , as compared to EEE_{SOC} , is that the first condition in TIP_{SOC} 's antecedent can hold even if H_1 is not the proposition that H_2 's probability given S 's total evidence is high. TIP_{SOC} holds without exception (see Roche 2012a) and thus so too does EEE_{SOC} .

Imagine a case where initially you have a low credence in:

H_2 The departmental meeting is scheduled for 3:30 today.

You then learn the following about a trusted colleague:

E John has a high credence in H_2 .

Now consider the following:

H_1 John's total evidence is evidence-HP for H_2 .

You do not know all the details of John's total evidence. But, still, it seems clear that E is evidence-IP for H_1 and that H_1 , in turn, is evidence-IP for H_2 . It also seems clear that E is

² Moretti (2016b) argues for a related but distinct EEE thesis involving screening-off.

³ EEE_{SOC} 's screening-off condition is a "negative-impact" screening-off condition. There is also a "no-impact" screening-off condition to the effect that E has *no impact* on H_2 's probability given the truth or falsity of H_1 , and a "positive-impact" screening-off condition to the effect that E has *no positive impact* on H_2 's probability given the truth or falsity of H_1 . See Roche and Shogenji (2014a) for relevant discussion.

evidence-IP for H_2 . There are counterexamples, though, to the thesis that evidence-IP is transitive. Is the case at hand a case in point?

If what I argue elsewhere (Roche 2014) is right, then the answer is negative. This is because the case at hand is a case where H_1 screens-off E from H_2 (in that E has no negative impact on H_2 's probability given the truth or falsity of H_1) and thus the third condition in EEE_{SOC} 's antecedent holds.

Tal and Comesaña (2017), hereafter "T&C", argue that my appeal to screening-off is subject to two objections. They write:

First, it is not clear what the evidential relevance of the screening-off conditions is, and so even if they worked flawlessly we would not have a deep understanding of the resulting principle. Second, they do not work flawlessly, for the screening-off conditions are sufficient but not necessary for evidential transitivity. (T&C 2017, p. 107)

Here by "screening-off conditions" they have in mind, first, the condition that $\Pr(H_2 | H_1 \& E) \geq \Pr(H_2 | H_1)$ and, second, the condition that $\Pr(H_2 | \neg H_1 \& E) \geq \Pr(H_2 | \neg H_1)$. They call the former "the positive screening-off condition" and the latter "the negative screening-off condition".

T&C then propose an alternative EEE thesis. They continue the passage above as follows:

In the next section we argue that the right fix for the right version of the EEE principle appeals not to screening-off conditions, but to the notion of a defeater. Defeaters wear their epistemic relevance on their sleeves. Moreover, we will argue that a suitable addition of a no-defeaters condition provides a necessary as well as a sufficient condition for evidence of evidence to be evidence. (T&C 2017, p. 107)

Their proposed alternative EEE thesis can be put as follows:

EEE_{NDC} If (i) $\Pr(H_1 | E) > \Pr(H_1)$ to degree α where H_1 is the proposition that there is a proposition E^* such that E^* is true and is evidence-IP for H_2 to degree β ,
(ii) $\Pr(H_2 | H_1) > \Pr(H_2)$ to degree γ ,
and
(iii) $\Pr(H_2 | H_1 \& E) > \Pr(H_2)$ to degree δ ,
then (iv) there is a degree ϵ such that $\Pr(H_2 | E) > \Pr(H_2)$ to degree ϵ .

This thesis differs from EEE_{SOC} in at least four key respects.⁴ First, there is no mention in EEE_{NDC} of a subject. Second, the evidence at issue in H_1 is evidence-IP. Third, there is reference in EEE_{NDC} to degrees of evidence-IP.⁵ Fourth, the third condition in EEE_{NDC} 's antecedent is a “no-defeaters” condition.

There is much to learn from T&C's very careful discussion. I aim to show, though, that their objections to my appeal to screening-off fail and that EEE_{NDC} is open to counterexample.⁶

⁴ The second condition in EEE_{NDC} 's antecedent is suppressed in T&C's formulation of their EEE thesis (which they call “Existential $EEE1$ *de dicto* no defeat”). See T&C (2017, p. 110, en. 7).

⁵ See Moretti (2016a) for discussion of a thesis like EEE_{NDC} but with no reference to degrees of evidence-IP.

⁶ I take issue with certain other parts of T&C's discussion. First, consider:

What is this case [i.e., Fitelson's case] a counterexample to? It is uncontroversially a counterexample to a *de re* reading of $EEE1$ But Roche suggests that the case is not a counterexample to $EEE1$ *de dicto* According to Roche, Fitelson's case is not a counterexample to $EEE1$ *de dicto* because we already know the following existential proposition: *there is some proposition that John has which is evidence for H*'. (T&C 2017, p. 100, emphasis original)

T&C then argue that my argument fails. It is not the case, though, that I argue that Fitelson's case is not a counterexample to $EEE1$ *de dicto*. I have in mind a thesis in which there is no reference to degrees of evidence-IP. I call the thesis in question “ EEE_U ” (see Roche 2014, p. 120). Given this, and given that there is reference in $EEE1$ *de dicto* to degrees of evidence-IP, it follows that I have in mind a thesis distinct from $EEE1$ *de dicto*. Second, consider:

Roche seems to think that the point holds only under a Williamsonian conception of evidence possession, and thus hedges by saying that Fitelson's case only *may* be a counterexample to (in effect) $EEE1$ *de dicto*. But Roche's point holds (if at all) on any sane conception of evidence-possession, not only a Williamsonian one. (T&C 2017, p. 110, en. 8, emphasis original).

What I say (in effect) is that the point in question holds *if* a Williamsonian conception is assumed. This, of course, does not mean that the point holds *only if* a Williamsonian conception is assumed. See Roche (2014, p. 121, fn. 6). The reason why I hedge and use the expression “might” was because of a worry to the effect that if S believes E^* as a result of an

2 Transitivity in evidence-IP: SOC versus NDC

There are counterexamples to the thesis that evidence-IP is transitive. It will help in terms of understanding the rationale behind my appeal to screening-off to understand *why* there are counterexamples to the thesis that evidence-IP is transitive.

Consider the following theorem of the probability calculus:

$$\begin{aligned} \Pr(H_2 | E) - \Pr(H_2) &= [\Pr(H_1 | E) - \Pr(H_1)][\Pr(H_2 | H_1) - \Pr(H_2)] && + \\ &[\Pr(\neg H_1 | E) - \Pr(\neg H_1)][\Pr(H_2 | \neg H_1) - \Pr(H_2)] && + \\ &\Pr(H_1 | E)[\Pr(H_2 | H_1 \& E) - \Pr(H_2 | H_1)] && + \\ &\Pr(\neg H_1 | E)[\Pr(H_2 | \neg H_1 \& E) - \Pr(H_2 | \neg H_1)] \end{aligned}$$

This theorem is due to Shogenji (forthcoming) and thus shall hereafter be referred to as “Shogenji’s theorem”. Let the first addend on the right be “A1”, the second addend on the right be “A2”, the third addend on the right be “A3”, and the fourth addend on the right be “A4”. Suppose that E is evidence-IP for H_1 and that H_1 is evidence-IP for H_2 . Then each of A1 and A2 is positive and thus A1+A2 is positive. It does not follow, though, that A1+A2+A3+A4 is positive, because it could be that A3+A4 is negative and greater than or equal to in absolute value A1+A2. All counterexamples to the thesis that evidence-IP is transitive are cases where A1+A2 is positive but A3+A4 is negative and greater than or equal to in absolute value A1+A2.

Suppose, to illustrate, that a card is randomly drawn from a standard and well-shuffled deck of cards. Let E be the proposition that the card drawn is not a Heart, H_1 be the proposition that the card drawn is a Diamond, and H_2 be the proposition that the card drawn is a Red. Then, since E is evidence-IP for H_1 and H_1 is evidence-IP for H_2 , A1+A2 is positive:

$$\left[\frac{1}{3} - \frac{1}{4}\right] \left[1 - \frac{1}{2}\right] + \left[\frac{2}{3} - \frac{3}{4}\right] \left[\frac{1}{3} - \frac{1}{2}\right] = \frac{1}{18}$$

A3+A4, however, is negative and greater than or equal to in absolute value A1+A2:

$$\left[\frac{1}{3}\right] [1 - 1] + \left[\frac{2}{3}\right] \left[0 - \frac{1}{3}\right] = -\frac{2}{9}$$

inference, then E^* is not included in S 's total evidence even if there is a proposition E such that E is included in S 's total evidence and E^* is entailed by E .

It follows that $A1+A2+A3+A4$ is negative and that so too is $\Pr(H_2 | E) - \Pr(H_2)$. This case is thus a counterexample to the thesis that evidence-IP is transitive.⁷

It is clear from Shogenji's theorem than any condition that is sufficient for transitivity in evidence-IP is a condition that guarantees that, when E is evidence-IP for H_1 and H_1 is evidence-IP for H_2 , it is not the case that $A3+A4$ is negative and greater than or equal to in absolute value $A1+A2$. There is simply no way around it.

Now consider the condition:

$$\text{SOC} \quad \Pr(H_2 | H_1 \ \& \ E) \geq \Pr(H_2 | H_1) \text{ and } \Pr(H_2 | \neg H_1 \ \& \ E) \geq \Pr(H_2 | \neg H_1)$$

This is the screening-off condition in EEE_{SOC} and TIP_{SOC} . SOC guarantees that each of $A3$ and $A4$ is non-negative and thus guarantees that, when E is evidence-IP for H_1 and H_1 is evidence-IP for H_2 , it is not the case that $A3+A4$ is negative and greater than or equal to in absolute value $A1+A2$. It follows that TIP_{SOC} holds without exception and that so too does EEE_{SOC} .

It is important to note that neither conjunct of SOC is sufficient for transitivity in evidence-IP. Return to the card case above. E is evidence-IP for H_1 , H_1 is evidence-IP for H_2 , and the first conjunct of SOC holds (because each probability equals 1). But it is not the case that E is evidence-IP for H_2 . Hence the first conjunct of SOC is not sufficient for transitivity in evidence-IP. Now let E be the proposition that the card drawn is a Club, the Ace of Spades, or the Ace of Hearts, H_1 be the proposition that the card drawn is not a Spade, and H_2 be the proposition that the card drawn is a Red. Then E is evidence-IP for H_1 , H_1 is evidence-IP for H_2 , and the second conjunct of SOC holds (because each probability equals 0). But, as $\Pr(H_2 | E) = 1/15 < 1/2 = \Pr(H_2)$, it is not the case that E is evidence-IP for H_2 . Hence the second conjunct of SOC is not sufficient for transitivity in evidence-IP.

T&C suggest that the second conjunct of SOC be dropped and that the first conjunct be replaced by a no-defeaters condition:

$$\text{NDC} \quad \Pr(H_2 | H_1 \ \& \ E) > \Pr(H_2)$$

This is the no-defeaters condition in EEE_{NDC} . NDC is weaker than the first conjunct of SOC on the assumption that H_1 is evidence-IP for H_2 . Any case where H_1 is evidence-IP for H_2 and the first conjunct of SOC holds is a case where $\Pr(H_2 | H_1 \ \& \ E) \geq \Pr(H_2 | H_1) > \Pr(H_2)$ and thus is a case where NDC holds. But some cases where H_1 is evidence-IP for H_2 and

⁷ It is also a counterexample to Hempel's "Special Consequence Condition" (Hempel 1965) understood in terms of evidence-IP: If (i) E is evidence-IP for H_1 and (ii) H_1 entails H_2 , then E is evidence-IP for H_2 .

NDC holds are cases where $\Pr(H_2 | H_1) > \Pr(H_2 | H_1 \& E) > \Pr(H_2)$ and thus are cases where the first conjunct of SOC does not hold. Is NDC sufficient for transitivity in evidence-IP?

The answer, clearly, is no. NDC is weaker than the first conjunct of SOC on the assumption that H_1 is evidence-IP for H_2 and so holds in all cases where the first conjunct of SOC holds and H_1 is evidence-IP for H_2 . But, as noted above, there are cases where E is evidence-IP for H_1 , H_1 is evidence-IP for H_2 , the first conjunct of SOC holds, and yet it is not the case that E is evidence-IP for H_2 . All such cases are counterexamples to the thesis that NDC is sufficient for transitivity in evidence-IP.

T&C acknowledge that my appeal to SOC works. They claim, though, that it works by “brute force”. They want to know *why* it works. This in effect is their first objection to my appeal to SOC. It should be clear, at this point, why my appeal to SOC works:

SOC guarantees that, when E is evidence-IP for H_1 and H_1 is evidence-IP for H_2 , it is not the case that A_3+A_4 is negative and greater than or equal to in absolute value A_1+A_2 .

It should also be clear why an appeal to NDC would not work:

NDC fails to guarantee that, when E is evidence-IP for H_1 and H_1 is evidence-IP for H_2 , it is not the case that A_3+A_4 is negative and greater than or equal to in absolute value A_1+A_2 .

There is simply no getting around the point that any condition that is sufficient for transitivity in evidence-IP is a condition that guarantees that, when E is evidence-IP for H_1 and H_1 is evidence-IP for H_2 , it is not the case that A_3+A_4 is negative and greater than or equal to in absolute value A_1+A_2 .

There is more. Consider what can be called the “Trivial Condition”:

$$\text{TC} \quad \Pr(H_2 | E) > \Pr(H_2)$$

This condition, as with SOC, is sufficient for transitivity in evidence-IP. But TC is not at all interesting. TC is impotent to help explain in a given case why E confirms H_2 and it is impotent to help in verifying in a given case that E confirms H_2 . SOC is different. It can help explain in a given case why E confirms H_2 and it can help in verifying in a given case, for example, the case of John and the departmental meeting, that E confirms H_2 .^{8,9}

⁸ See Roche and Shogenji (2014b) for an argument to the effect that SOC holds in the context of Moore’s “proof” of the existence of a material world.

⁹ Consider the following conditions (the first of which is the so-called “Dragging Condition” from Kotzen 2012):

What about T&C's charge that SOC is not necessary for transitivity in evidence-IP? How exactly do T&C mean for this charge to be understood?

One answer is that they mean for it to be understood as the charge that SOC is not necessary for the relation of "being evidence-IP for" to be transitive, that is, for it to be such that any case where E is evidence-IP for H_1 and H_1 is evidence-IP for H_2 is a case where E is evidence-IP for H_2 . If this answer is right, then T&C's charge is clearly correct. This can be seen by considering what can be called the "Weaker Condition":

WC It is not the case that $A_3 + A_4$ is negative and greater than or equal to in absolute value $A_1 + A_2$.

This condition is weaker than SOC. Yet it is sufficient for transitivity in evidence-IP.

It turns out, though, that T&C mean for their charge to be understood differently. Their worry is that there are cases where E is evidence-IP for H_1 , H_1 is evidence-IP for H_2 , SOC does not hold, and yet E is evidence-IP for H_2 .¹⁰ They want a condition C such that (i) C is sufficient for transitivity in evidence-IP and (ii) if E is evidence-IP for H_1 , H_1 is evidence-IP for H_2 , and C does not hold, then E is not evidence-IP for H_2 .

It is true that SOC is not a condition of that sort. But note that the same is true of NDC. This can be seen by appeal to a variant of the card cases above. Let E be the proposition that the card drawn is a Two or a Three, H_1 be the proposition that the card drawn is a Three or a Four, and H_2 be the proposition that the card drawn is a Two or a Four. $\Pr(H_1 | E) = 1/2 > 2/13 = \Pr(H_1)$, so E is evidence-IP for H_1 . $\Pr(H_2 | H_1) = 1/2 > 2/13 = \Pr(H_2)$, so H_1 is

$$\Pr(H_2) < \Pr(H_1 | E)$$

$$\Pr(H_2 \ \& \ \neg H_1 | E) \geq \Pr(H_2 \ \& \ \neg H_1)$$

Each of these conditions is sufficient for transitivity in evidence-IP *in the special case* where H_1 entails H_2 . But, unlike SOC, neither of them is sufficient for transitivity in evidence-IP in the general case where it is not required that H_1 entails H_2 . They thus have no application in cases like the case of John and the departmental meeting. See Roche and Shogenji (2014b) for discussion.

¹⁰ T&C (2017, p. 111, en. 19) claim in effect that Roche and Shogenji (2014b) note that the second conjunct of SOC (what T&C call "the negative screening-off condition") is sufficient but not necessary for transitivity in evidence-IP. This is misleading. Roche and Shogenji (2014b, pp. 807-808) have in mind the special case where H_1 entails H_2 . It is not the case, and Roche and Shogenji (2014b) do not suggest otherwise, that the second conjunct of SOC is sufficient for transitivity in evidence-IP.

evidence-IP for H_2 . $\Pr(H_2 | H_1 \& E) = 0 < 2/13 = \Pr(H_2)$, so NDC does not hold. But $\Pr(H_2 | E) = 1/2 > 2/13 = \Pr(H_2)$, so E is evidence-IP for H_2 .

There are conditions of the sort that T&C want. One is TC. But this condition, as noted above, is not at all interesting. A better condition is WC. This condition, unlike TC, is distinct from the condition that E is evidence-IP for H_2 . There are cases where WC holds but, since in part E is not evidence-IP for H_1 or H_1 is not evidence-IP for H_2 , it is not the case that E is evidence-IP for H_2 .

I see no need to choose between SOC and WC or between theses involving SOC such as TIP_{SOC} and EEE_{SOC} and similar theses involving WC. They all have their place.

I turn now to EEE_{NDC} and some variants of it.

3 A counterexample to T&C's EEE thesis

Consider the following variant of EEE_{NDC} :

$\text{EEE}^*_{\text{NDC}}$ If (i) $\Pr(H_1 | E) > \Pr(H_1)$ where H_1 is the proposition that H_2 's probability given S 's total evidence is high,
(ii) $\Pr(H_2 | H_1) > \Pr(H_2)$,
and
(iii) $\Pr(H_2 | H_1 \& E) > \Pr(H_2)$,
then (iv) $\Pr(H_2 | E) > \Pr(H_2)$.

$\text{EEE}^*_{\text{NDC}}$ is a special case of:

TIP_{NDC} If (i) $\Pr(H_1 | E) > \Pr(H_1)$,
(ii) $\Pr(H_2 | H_1) > \Pr(H_2)$,
and
(iii) $\Pr(H_2 | H_1 \& E) > \Pr(H_2)$,
then (iv) $\Pr(H_2 | E) > \Pr(H_2)$.

TIP_{NDC} , though, is false, for it is not the case that NDC is sufficient for transitivity in evidence-IP. Does it follow that $\text{EEE}^*_{\text{NDC}}$ too is false?

The answer, it turns out, is negative. It could be that TIP_{NDC} is false but nonetheless holds without exception in special cases where H_1 is a proposition about evidence. Consider, for example, the thesis:

$\text{EEE}^{**}_{\text{NDC}}$ If (i) $\Pr(H_1 | E) > \Pr(H_1)$ where H_1 is the proposition that S has some evidence-IP for H_2 ,

- (ii) $\Pr(H_2 | H_1) > \Pr(H_2)$,
 and
 (iii) $\Pr(H_2 | H_1 \ \& \ E) > \Pr(H_2)$,
 then (iv) $\Pr(H_2 | E) > \Pr(H_2)$.

EEE**_{NDC} holds without exception if it is assumed both that all subjects have some propositions (with non-extreme unconditional probabilities) as evidence and that if a subject has a proposition as evidence, then she also has any implication (with a non-extreme unconditional probability) of that proposition as evidence. For, if those things are assumed, then there are no cases where the first condition in EEE**_{NDC}'s antecedent holds. This is easy to see. Suppose that *S* has *E** as evidence. Then, since *E** entails $E^* \vee H_2$, it follows that *S* has that disjunction as evidence. Given this, and given that $E^* \vee H_2$ is entailed by *H*₂, it follows that $E^* \vee H_2$ is evidence-IP for *H*₂ (assuming that *H*₂ has a non-extreme unconditional probability). Hence *S* has some evidence-IP for *H*₂. So, though EEE**_{NDC} is a special case of TIP_{NDC}, and though the latter is false, it is not the case that the former is false.^{11,12} Hence the mere fact that TIP_{NDC} is false leaves it open that it is not the case that EEE*_{NDC}, which is a special case of TIP_{NDC}, is false.

Consider now a different version of the case of John and the departmental meeting. Each of John's credences is either high or not high, either in a proposition for which his total evidence is evidence-HP or not in a proposition for which his total evidence is evidence-HP, and either in a true proposition or not in a true proposition. Suppose that you know that:

- (a) 3/5 of John's credences are high & in a proposition for which his total evidence is evidence-HP & in a true proposition
- (b) 1/15 of John's credences are high & in a proposition for which his total evidence is evidence-HP & *not* in a true proposition
- (c) 0 of John's credences are high & *not* in a proposition for which his total evidence is evidence-HP & in a true proposition
- (d) 2/27 of John's credences are high & *not* in a proposition for which his total evidence is evidence-HP & *not* in a true proposition

¹¹ These points about EEE**_{NDC} carry over to an EEE thesis put forward in Roche (2014). The thesis in question is called “(EEE' U)” and is like EEE**_{NDC} except that it has SOC where EEE**_{NDC} has NDC.

¹² See Comesaña and Tal (2015) for a similar point about an EEE thesis discussed in Fitelson (2012).

- (e) 0 of John's credences are *not* high & in a proposition for which his total evidence is evidence-HP & in a true proposition
- (f) 0 of John's credences are *not* high & in a proposition for which his total evidence is evidence-HP & *not* in a true proposition
- (g) 1/4 of John's credences are *not* high & *not* in a proposition for which his total evidence is evidence-HP & in a true proposition
- (h) 1/108 of John's credences are *not* high & *not* in a proposition for which his total evidence is evidence-HP & *not* in a true proposition

(Note that $3/5 + 1/15 + 0 + 2/27 + 0 + 0 + 1/4 + 1/108 = 1$.) Let E be the proposition that John has a high credence in H_2 , H_1 be the proposition that John's total evidence is evidence-HP for H_2 , and H_2 be the proposition that the departmental meeting is scheduled for 3:30 today. Suppose that H_2 is one of the propositions in which John has a credence, and that, from your perspective, John's credence in H_2 is a random member of the set of propositions in which he has a credence. Suppose, further, and consistent with all this, that your probability for H_2 is 0.85. Then you should have the following probability distribution:

E	H_1	H_2	Pr
T	T	T	$\frac{3}{5}$
T	T	F	$\frac{1}{15}$
T	F	T	0
T	F	F	$\frac{2}{27}$
F	T	T	0
F	T	F	0
F	F	T	$\frac{1}{4}$
F	F	F	$\frac{1}{108}$

It follows that:

- (i) $\Pr(H_1 | E) = 9/10 > 2/3 = \Pr(H_1)$
- (j) $\Pr(H_2 | H_1) = 9/10 > 85/100 = \Pr(H_2)$
- (k) $\Pr(H_2 | H_1 \ \& \ E) = 9/10 > 85/100 = \Pr(H_2)$

$$(1) \quad \Pr(H_2 | E) = 81/100 < 85/100 = \Pr(H_2)$$

So, E is evidence-IP for H_1 where H_1 is the proposition that John's total evidence is evidence-IP for H_2 , H_1 is evidence-IP for H_2 , and NDC holds, and yet it is not the case that E is evidence-IP for H_2 . Hence EEE^*_{NDC} , as with TIP_{NDC} , is false.

It is unclear at this point whether the same is true of EEE_{NDC} . This is because it is unclear at this point how exactly EEE_{NDC} is to be understood. T&C never specify how degree of evidence-IP is to be measured. They use expressions like “entailing evidence” and “evidence that raises the probability to at least 4/5”. But expressions like those leave it open how exactly evidence-IP is to be measured. Consider the following measures of evidence-IP:

$$e_d(H, E) = \Pr(H | E) - \Pr(H)$$

$$e_{KO}(H, E) = \frac{\Pr(E | H) - \Pr(E | \neg H)}{\Pr(E | H) + \Pr(E | \neg H)}$$

The first of these measures is the “difference” measure. The second is the “Kemeny-Oppenheim” measure.¹³ Suppose that E entails H and thus is evidence-IP for H (assuming that neither E nor H has an extreme unconditional probability). It follows by e_d that whether the degree of evidence-IP is high or low depends on whether $\Pr(H)$ is high or low. If $\Pr(H)$ is high, then the degree of evidence-IP is low. If $\Pr(H)$ is low, then the degree of evidence-IP is high. It follows by e_{KO} , in contrast, that the degree of evidence-IP is high, indeed, is maximal at 1, regardless of whether $\Pr(H)$ is high or low.

T&C presumably hold that EEE_{NDC} is true regardless of which of the leading measures of evidence-IP in the literature is assumed (otherwise, presumably, they would have specified how degree of evidence-IP is to be measured). I shall assume e_d and thus shall assume that in the case of John and the departmental meeting the degree to which H_1 is evidence-IP for H_2 is 0.05. But nothing of importance for my purposes hinges on this, since all the main points below would hold *mutatis mutandis* if e_{KO} or any alternative measure of evidence-IP (in the literature) were assumed instead.

Now, continuing with the second version of the case of John and the departmental meeting, let H^* be the proposition that there is a proposition E^* such that E^* is true and is evidence-IP for H_2 to degree 0.05. There is no reference to H^* in the probability distribution above. It turns out, though, that H^* is logically equivalent (in the context) to H_1 . (See Appendix for details.) It follows immediately that:

¹³ See Roche and Shogenji (2014a) for a list of the main measures of evidence-IP (or “confirmation”) in the literature.

- (m) $\Pr(H^* | E) = 9/10 > 2/3 = \Pr(H^*)$
- (n) $\Pr(H_2 | H^*) = 9/10 > 85/100 = \Pr(H_2)$
- (o) $\Pr(H_2 | H^* \& E) = 9/10 > 85/100 = \Pr(H_2)$
- (p) $\Pr(H_2 | E) = 81/100 < 85/100 = \Pr(H_2)$

By (m) it follows that E is evidence-IP for H^* to degree $7/30$ where H^* is the proposition that there is a proposition E^* such that E^* is true and is evidence-IP for H_2 to degree 0.05. By (n) it follows that H^* is evidence-IP for H_2 to degree 0.05. By (o) it follows that $H^* \& E$ is evidence-IP for H_2 to degree 0.05. By (p) it follows that E is not evidence-IP for H_2 . Hence EEE_{NDC} , as with EEE^*_{NDC} , is open to counterexample.

T&C close their discussion by giving a gloss of EEE_{NDC} and writing (where notation has been slightly modified):

Evidence that there is ... evidence for H_2 is itself evidence for H_2 when it is not at the same time a defeater for the support that the proposition that there is evidence for H_2 provides to H_2 . Doesn't quite roll off the tongue, but it has not yet been shown false. (T&C 2017, p. 110)

They can no longer say that.

Now return to the point from Section 2 that there are counterexamples to the thesis that NDC is sufficient for transitivity in evidence-IP. It turns out that things are different in the special case where E entails H_1 . The following holds without exception:

- TIP^*_{NDC} If (i) $\Pr(H_1 | E) > \Pr(H_1)$,
(ii) $\Pr(H_2 | H_1) > \Pr(H_2)$,
(iii) $\Pr(H_2 | H_1 \& E) > \Pr(H_2)$,
and
(iv) E entails H_1 ,
then (v) $\Pr(H_2 | E) > \Pr(H_2)$.

Suppose that (i)-(iv) in TIP^*_{NDC} all hold. It follows from (iv) that E is logically equivalent to $H_1 \& E$. Hence, given (iii), it follows that $\Pr(H_2 | E) > \Pr(H_2)$. Should this be welcome news to T&C?

No. Any case where (iv) in TIP^*_{NDC} holds is a case where (v) holds if and only if (iii) holds. It does not matter whether (i) and (ii) hold. This can be seen by noting that the following variant of TIP^*_{NDC} also holds without exception:

TIP**_{NDC} If (i*) $\Pr(H_1 | E) = \Pr(H_1)$,
(ii*) $\Pr(H_2 | H_1) = \Pr(H_2)$,
(iii) $\Pr(H_2 | H_1 \ \& \ E) > \Pr(H_2)$,
and
(iv) E entails H_1 ,
then (v) $\Pr(H_2 | E) > \Pr(H_2)$.

TIP*_{NDC}, as with TIP**_{NDC}, is a TIP thesis in name only.¹⁴

T&C could modify EEE_{NDC} along the lines of TIP*_{NDC}. Then it would not be open to counterexample. But, at the same time, it would be an EEE thesis in name only.

A better tack would be to abandon the no-defeaters approach in favor of the screening-off approach. EEE_{SOC} is not open to counterexample and is not an EEE thesis in name only.

4 Conclusion

SOC is sufficient for transitivity in evidence-IP. Further, unlike TC (the condition that E is evidence-IP for H_2), SOC can help explain in a given case why E confirms H_2 , and can help in verifying in a given case that E confirms H_2 . Thus SOC is fit for use in EEE theses. NDC, by contrast, is not sufficient for transitivity in evidence-IP and thus is not fit for use in EEE theses. These points are borne out by the fact that EEE_{SOC} holds without exception whereas EEE_{NDC} does not.

Acknowledgments

I wish to thank an anonymous reviewer, an audience at the 12th Annual Formal Epistemology Workshop, Juan Comesaña, Tomoji Shogenji, and Eyal Tal for helpful very feedback.

¹⁴ There are cases where TIP**_{NDC}'s antecedent holds. Take a card case where E is the proposition that the card drawn is a Red, H_1 is the proposition that the card drawn is a Jack or not a Jack, and H_2 is the proposition that the card drawn is a Diamond. Here $\Pr(H_1 | E) = 1 = \Pr(H_1)$, $\Pr(H_2 | H_1) = 1/4 = \Pr(H_2)$, $\Pr(H_2 | H_1 \ \& \ E) = 1/2 > 1/4 = \Pr(H_2)$, and E entails H_1 .

Appendix

Take the probability distribution given above in Section 3. First, it follows that H^* is logically equivalent (in the context) to a disjunction with the following disjuncts:

- (q) H_1
- (r) $(E \ \& \ H_2) \vee H_1$
- (s) $(E \ \& \ H_1) \vee (E \ \& \ H_2) \vee (H_1 \ \& \ H_2)$
- (t) $(E \ \& \ H_1) \vee (E \ \& \ H_2) \vee (H_1 \ \& \ \neg H_2)$
- (u) $(E \ \& \ H_1) \vee (E \ \& \ H_2)$
- (v) $(E \ \& \ H_1) \vee (H_1 \ \& \ H_2)$
- (w) $(E \ \& \ H_1) \vee (H_1 \ \& \ \neg H_2)$
- (x) $(E \ \& \ H_1)$

This is because there are exactly 256 distinct Boolean propositions constructible from one or more of E , H_1 , and H_2 , each of (q)-(x) is evidence-IP for H_2 to degree 0.05, and none of the remaining 248 distinct Boolean propositions constructible from one or more of E , H_1 , and H_2 is evidence-IP for H_2 to degree 0.05.¹⁵ Second, each of (q)-(x) entails and is entailed by H_1 . The key here is that H_1 entails (in the context) E and is entailed by (in the context) $E \ \& \ H_2$. QED

References

- Comesaña, J., and Tal, E. (2015). Evidence of evidence is evidence (trivially). *Analysis*, 75, 557-559.
- Douven, I. (2011). Further results on the intransitivity of evidential support. *Review of Symbolic Logic*, 4, 487-497.

¹⁵ This is all straightforward (though tedious) to verify on *Mathematica* using “BooleanFunction”, “BooleanConvert”, and Fitelson’s “PrSAT”. See Fitelson (2008) for discussion of PrSAT.

- Fitelson, B. (2008). A decision procedure for probability calculus with applications. *Review of Symbolic Logic*, 1, 111-125.
- Fitelson, B. (2012). Evidence of evidence is not (necessarily) evidence. *Analysis*, 72, 85-88.
- Hempel, C. (1965). Studies in the logic of confirmation. In C. Hempel, *Aspects of scientific explanation and other essays in the philosophy of science* (pp. 3-46). New York: Free Press.
- Kotzen, M. (2012). Dragging and confirming. *Philosophical Review*, 121, 55-93.
- Moretti, L. (2016a). Tal and Comesaña on evidence of evidence. *The Reasoner*, 10, 38-39.
- Moretti, L. (2016b). Evidence of expert's evidence is evidence. *Episteme*, 13, 209-218.
- Roche, W. (2012a). A weaker condition for transitivity in probabilistic support. *European Journal for Philosophy of Science*, 2, 111-118.
- Roche, W. (2012b). Transitivity and intransitivity in evidential support: Some further results. *Review of Symbolic Logic*, 5, 259-268.
- Roche, W. (2014). Evidence of evidence is evidence under screening-off. *Episteme*, 11, 119-124.
- Roche, W. (2015). Evidential support, transitivity, and screening-off. *Review of Symbolic Logic*, 8, 785-806.
- Roche, W., and Shogenji, T. (2014a). Dwindling confirmation. *Philosophy of Science*, 81, 114-137.
- Roche, W., and Shogenji, T. (2014b). Confirmation, transitivity, and Moore: The Screening-Off Approach. *Philosophical Studies*, 168, 797-817.
- Shogenji, T. (forthcoming). Mediated confirmation. *British Journal for the Philosophy of Science*.
- Tal, E., and Comesaña, J. (2017). Is evidence of evidence evidence? *Nous*, 51, 95-112.