Modal Predicates

Andrea Iacona
Università del Piemonte Orientale
andreai@lett.unipmn.it

Received by Greg Restall
Published August 2, 2004
http://www.philosophy.unimelb.edu.au/ajl/2004

Abstract: Despite the wide acceptance of standard modal logic, there has always been a temptation to think that ordinary modal discourse may be correctly analyzed and adequately represented in terms of predicates rather than in terms of operators. The aim of the formal model outlined in this paper is to capture what I take to be the only plausible sense in which 'possible' and 'necessary' can be treated as predicates. The model is built by enriching the language of standard modal logic with a quantificational apparatus that is "substitutional" rather than "objectual", and by obtaining from the language so enriched another language in which constants for such predicates apply to singular terms that stand for propositions.

I

The adjectives 'possible' and 'necessary' are used both in common parlance and in philosophical discourse. It seems essential to the way these adjectives are used that they can occur beside clauses of the form 'that so-and-so', that is, in constructions such as 'it is possible (necessary) that so-and-so' or 'that so-and-so is possible (necessary)'. The clauses of the form 'that so-and-so'—in short, *that-clauses*—are what grammarians call "nominalizations". Prefacing a declarative sentence with the word 'that', we create an expression that plays the syntactic role characteristic of nouns. Take the sentence 'that snow is white is true'. The expression 'that snow is white' occurs in it in the subject position, namely, in the same position occupied by 'snow' in the sentence 'snow is white'. Since that-clauses play the syntactic role characteristic of nouns, it is natural to treat them as semantically analogous to nouns. This means that it is natural to treat them as having a reference. It is a widely held hypothesis that propositions are the referents of that-clauses. According to this hypothesis, 'that snow is white' in the sentence above refers to the proposition that snow is white, and the adjective 'true' following it says something about that proposition, namely, that it is true. 'Possible' and 'necessary', just as 'true', take nouns and noun phrases as subjects. This grammatical fact encourages to think that when we assertively utter a sentence containing 'possible' or 'necessary', we refer to something and say of it that it is possible or necessary, that is, we predicate possibility or necessity of it. Therefore, it is quite natural to assume that expressions such as 'it is possible (necessary) that so-and-so' or 'that so-and-so is possible (necessary)' are used to describe certain things, propositions, as having a certain property, possibility (necessity). On the basis of this assumption one can easily account for inferences such as the following:

(1) whatever is necessary is possible
(2) it is necessary that every thing is identical to itself

---

(3) it is possible that every thing is identical to itself

It is quite natural to read (1) as having logical form $\forall x(Fx \rightarrow Gx)$. Accordingly, it seems that in order for (3) to follow from (1) and (2), (2) must have logical form Fa, that is, 'necessary' must be a predicate that applies to the referent of the expression 'that every thing is identical to itself'. More generally, it is quite natural to assume that the truth conditions of sentences in which 'possible' and 'necessary' occur beside that-clauses are analogous to those of sentences in which ordinary predicates occur beside ordinary singular terms. A sentence of logical form Fa is true just in case there is something to which a refers and this something belongs to the extension of F, or has the property of being F. Thus, 'snow is white' is true just in case there is something to which 'snow' refers and this something belongs to the extension of 'white', or has the property of being white. Similarly, one may be apt to think that 'it is possible that every thing is identical to itself' is true just in case there is something to which 'that every thing is identical to itself' refers and this something belongs to the extension of 'possible', or has the property of being possible.

Modal logic is intended to illuminate the notions of possibility and necessity by providing a systematization of the logical relations between sentences involving them. But if we open a book of modal logic we do not read that 'possible' and 'necessary' stand for properties of propositions. Rather, we find that the notions of possibility and necessity can be accounted for in terms of *operators* on formulas. The language of standard modal logic is obtained by adding the operators $\Diamond$ ('it is possible that' or 'possibly') and $\Box$ ('it is necessary that' or 'necessarily') to the language of classical logic, in such a way that if α is a formula of the language of classical logic, $\Box\alpha$ and $\Diamond\alpha$ are formulas of the language of standard modal logic. In other words, the syntactical treatment of modality is analogous to that of negation. Just as 'snow is not white' is treated as a sentence obtained by applying the operator $\neg$ ('it is not the case that') to a given sentence ('snow is white'), 'it is possible that snow is white' is treated as a sentence obtained by applying the operator $\Diamond$ to the same sentence. So if one thinks that modal logic provides a satisfactory and exhaustive clarification of the notions of possibility and necessity, one will be apt to talk of sentences and modal operators rather than of propositions and modal predicates. If we add to this that the notion of proposition is often regarded with distrust for the metaphysical commitments that seems to require, it is easy to see how one can come to the conclusion that the only acceptable modal talk is in terms of sentences and modal operators. The inclination towards this conclusion may be further reinforced by some limitative results that concern treatments of modality in terms of metalinguistic predicates attempted in the past, notably, the inconsistency results proved by Richard Montague[1]. On the basis of those results, one may be apt to think that modal talk in terms of predicates is not only unjustified but also inconsistent.

Thus, two divergent lines of thought seem practicable. On the one hand, one may follow the grammatical appearance of expressions such as 'that so-and-so is possible (necessary)', and take 'possible' and 'necessary' to be predicates that stand for properties of propositions. This amounts to saying that 'that so-and-so is possible (necessary)' is to be regarded as semantically analogous to 'snow is white'. Clear examples of this line of thought are provided by Gilbert Harman, George Bealer and Stephen Schiffer.[2] On the other hand, one may think that modal logic takes care of the notions of necessity and possibility, and nothing else need be assumed. Consequently, one may regard the grammar of expressions such as 'that so-and-so is possible (necessary)' as simply misleading: although it may *seem* that 'that so-and-so is possible (necessary)' breaks into 'that so-and-so' and 'is possible (necessary)', i. e., has logical form Fa, *in reality* it is to be treated as breaking into 'it is possible (necessary) that' and 'so-and-so', i. e. its logical form is analogous to that of a negated sentence.[3] I believe that neither of these lines of thought is fully satisfactory, although there is a grain of truth in both of them. The aim of what follows is to explain why. Section 2 outlines what I take to be the plausible sense in which 'possible' and 'necessary' can be said to apply to propositions. Sections 3 and 4 present a formal model that is intended to capture that sense. Section 5 shows how inferences such as that from (1) and (2) to (3) can be accounted for within the model. Lastly, section 6 deals with some philosophical implications of what is said in the previous sections.

II

There is no widespread agreement among philosophers about what exactly propositions are. But one thing that seems certain about them is that they are *truth bearers*, namely, entities to which truth can be ascribed. The condition at which a truth bearer is true is called its truth condition. For example, a

[1]See Montague 1963.

[2]See Harman 1972, p. 82, Bealer 1993, pp. 7–10, and Schiffer 2003, pp. 96–98.

[3]Here I don't have in mind someone in particular, but I'm pretty sure that many logicians would subscribe what has been just said.

truth bearer that is true just in case snow is white is a truth bearer that has snow being white as its truth condition. As snow is indeed white, such a truth bearer is true in the world as it is now: its truth condition is satisfied by the way things are. Truth bearers can also be evaluated with respect to states of affairs different from the actual and present one. Instead of wondering whether things as they are make a given truth bearer true, one may wonder whether the way things were in the past (will be in the future), or the way things could have been, would have been such as to make it true. That is, one may wonder whether those circumstances would have been such as to satisfy its truth condition. Just as for a truth bearer to be true in the present and actual state of affairs is for its truth condition to be satisfied in the present and actual state of affairs, for a truth bearer to be true in a state of affairs different from the present and actual one is for its truth condition to be satisfied in that state of affairs. This looks clear if we think of states of affairs as "possible worlds". For example, we can say that a truth bearer that has snow being white as its truth condition is true in a certain possible world just in case snow is white in that possible world.

Since propositions are truth bearers, the sense in which a proposition can be said to be possible or necessary must be that in which a truth bearer can be said to be possible or necessary. In accordance with the standard account of modality, it seems correct to assume that the conditions at which 'necessary' and 'possible' apply to truth bearers are to be given in terms of a quantification on possible worlds in which the truth bearers themselves are true. That is, a truth bearer is necessary just in case it is true in all possible worlds, possible just in case there is at least one possible world in which it is true. Therefore, the obvious way of making sense of sentences like '(the proposition) that so-and-so is necessary' or '(the proposition) that so-and-so is possible' is by taking them as meaning '(the proposition) that so-and-so is true in all possible worlds' or 'there is at least one possible world in which (the proposition) that so-and-so is true'. In substance, if 'possible' and 'necessary' are to be treated as predicates that apply to propositions, the plausible way of making sense of the sentences in which they occur beside that-clauses is in terms of a quantification on possible worlds in which the propositions to which those that-clauses refer are true, where the possible worlds in which they are true are the possible worlds in which their truth conditions are satisfied.[4]

This amounts to saying that the plausible way of making sense of sentences apparently involving ascription of possibility or necessity to propositions is one according to which they turn out to be equivalent to sentences containing modal operators attached to the sentences embedded in the that-clauses which

[4]To assume that 'necessary' is to be understood as 'true in all possible worlds' is not quite the same thing as to assume that it means 'logically true'. The issue of the connection between necessity and logical truth is deliberately left out of the picture. Accordingly, no attempt will be made to treat necessity in terms of provability in a system or of some other metalinguistic notion, as for example does Priest 1976 and 1977.

refer to those propositions. In terms of ordinary language, it amounts to saying that 'that p is possible' is equivalent to 'possibly, p', and 'that p is necessary' is equivalent to 'necessarily, p'. Here I assume that the truth condition of the proposition to which a that-clause refers is that of the sentence embedded in the that-clause itself. This seems uncontroversial. For example, the truth condition of the proposition to which the clause 'that snow is white' refers—the proposition that snow is white—is that of the sentence 'snow is white', i. e. both the proposition and the sentence are true just in case snow is white. I also assume, as usual, that the truth condition of a sentence of the form $\Diamond\alpha$ or $\Box\alpha$ is to be given in terms of a quantification on possible worlds in which α is true, where the possible worlds in which α is true are the possible worlds in which its truth condition is satisfied. As a sentence containing a sentence 'p' prefixed by the adverb 'possibly' or 'necessarily' has logical form $\Diamond\alpha$ or $\Box\alpha$, this means that its truth conditions depends on the satisfaction of the truth condition of 'p' in the relevant possible worlds just as the truth condition of a sentence in which 'that p' occurs beside 'possible' or 'necessary' depends on the satisfaction of the truth condition of the proposition that p in the relevant possible worlds.

III

Let L be a language of modal predicate logic. The vocabulary of L contains the standard first-order connectives, variables, constants, and the modal operators $\Diamond$ and $\Box$. The formation rules of L and the semantic interpretation of its formulas are given in the usual way. Now let L_M be a language obtained from L as follows. The vocabulary of L_M contains, in addition to the vocabulary of L, the universal substitutional quantifier Π and an infinite supply of variables $p_1, p_2, p_3 \ldots$ called ***substitutional variables***. The set of atomic formulas of L_M is specified as a set of expressions obtained by taking sentences of L and replacing zero or more sentences occurring in them with substitutional variables. Note that this way all the sentences of L turn out to be atomic formulas of L_M. Arbitrary formulas of L_M are defined inductively: an atomic formula is a formula; if α is a formula, so are $\neg\alpha$ and $\Pi p_n\alpha$; if α and β are formulas, so is $\alpha \to \beta$. Lastly, sentences of L_M are defined as formulas of L_M without free (substitutional) variables.

The substitutional quantifier Π is formally similar to the standard or "objectual" quantifier $\forall$. However, its semantics is quite different. Variables also play a different role. When we use the quantifier $\forall$ we presuppose a domain, that is, a class of objects that we take to be our "universe of discourse". $\forall$ is then said to range over the domain, and the variables it binds are taken to refer to the objects in it. On the contrary, Π does not range over a domain, and the variables it binds do not refer to objects. What we presuppose in using Π is a ***substitution class***, that is, a class of linguistic expressions that can be substituted for the variables it binds. A ***substitution instance*** of a sentence of the form $\Pi p_n\alpha$

is a sentence obtained by replacing the variable p_n occurring in α with an expression belonging to the substitution class. $\Pi p_n \alpha$ is said to be true just in case all its substitution instances are true. In our case the substitution class is that of the sentences of L. Accordingly, a sentence of the form $\Pi p_n \alpha$ turns out to be true just in case all the sentences obtained by replacing p_n with a sentence of L are true. A complete specification of the truth conditions of the sentences of L_M may be given as follows. Let α be a sentence of L_M. If α is an atomic formula of L_M, then α is true in L_M iff it is true in L; $\neg\alpha$ is true in L_M iff α is not true in L_M; $\alpha \to \beta$ is true in L_M iff either α is not true in L_M or β is true in L_M; $\Pi p_n \alpha$ is true in L_M iff all its substitution instances are true in L_M. In accordance with the assumption that the semantic interpretation of L is given in the usual way, the notion of truth is to be understood in terms of truth in a model. Note that formulas with free substitutional variables are assigned no semantic interpretation. This contrasts with the objectual case, where open formulas are normally interpreted in terms of satisfaction.[5]

Let L_{Mp} be a language obtained from L_M in the following way. The vocabulary of L_{Mp} contains, in addition to the vocabulary of L_M, the singular term forming operator [ ] (= 'that'), and the predicates P (= 'possible') and N (= 'necessary'). The set of atomic formulas of L_{Mp} is characterized as follows: every atomic formula of L_M is an atomic formula of L_{Mp}; if α is an atomic formula of L_{Mp}, so are $P[\alpha]$ and $N[\alpha]$; Px_n and Nx_n are also atomic formulas of L_M, where x_n is the nth objectual variable of L_{Mp}. Arbitrary formulas of L_{Mp} are defined inductively: an atomic formula is a formula; if α is a formula, so are $\neg\alpha$, $\forall x_n \alpha$ and $\Pi p_n \alpha$; if α and β are formulas, so is $\alpha \to \beta$. Sentences of L_{Mp} are defined as formulas of L_{Mp} without free variables (of any kind). It is easy to see that every formula of L_M turns out to be a formula of L_{Mp}. Conversely, there are formulas of L_{Mp} that are not formulas of L_M, namely, those containing symbols of the additional vocabulary of L_{Mp}. We call *rich* formulas of L_{Mp} the formulas of the latter kind. Accordingly, we call rich sentences of L_{Mp} the sentences of L_{Mp} that are rich formulas of L_{Mp}.

Let T be a relation on the set of formulas of L_{Mp}, that is, a set of ordered pairs $\langle \alpha, \alpha^* \rangle$ such that both α and α^* are formulas of L_{Mp}. For atomic formulas of L_{Mp}, T is defined as follows. If α a formula of L_M, then

(a) $\alpha^* = \alpha$

If α has the form $P[\beta]$ or $N[\beta]$, then

(b) $\alpha^* = \alpha(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$

where $\alpha(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$ is obtained from α by replacing $P[\beta]$ with $\Diamond\beta$ and $N[\beta]$ with $\Box\beta$. If α has the form Px_n or Nx_n, then

(c) $\alpha^* = (\alpha([p_n]/x_n))^*$

[5]Here I follow Kripke 1976, pp. 354–355.

where $\alpha([p_n]/x_n)$ is obtained from α by replacing the nth objectual variable with the nth substitutional variable. The following clauses complete the definition of T:

(d) $(\neg\alpha)^* = \neg(\alpha)^*$

(e) $(\alpha \to \beta)^* = (\alpha)^* \to (\beta)^*$

(f) $(\Pi p_n \alpha)^* = \Pi p_n (\alpha)^*$

(g) $(\forall x_n \alpha)^* = (\Pi p_n \alpha)^*$

The domain of T is the set of formulas of L_{Mp}, that is, every formula α of L_{Mp} is such that $\langle \alpha, \alpha^* \rangle \in T$ for some α^*. This simply follows from the definition of T in terms of (a)–(g). For given the formation rules of L_{Mp}, each formula of L_{Mp} falls under one of the cases specified by (a)–(g). The range of T is a subset of the set of formulas of L_M, that is, if α^* is a formula such that $\langle \alpha, \alpha^* \rangle \in T$ for some α, then α^* is a formula of L_M. This can be shown by induction on the complexity of the formulas of L_{Mp}, where the complexity of a formula of L_{Mp} is defined as the number of connectives of L_{Mp} it contains. Let ϕ be a formula of L_{Mp}. As a first step we assume that ϕ has complexity 0. In this case ϕ is atomic. If ϕ is a formula of L_M, then ϕ^* is a formula of L_M. For by (a) $\phi^* = \phi$. If ϕ is rich and has the form $P[\alpha]$ or $N[\alpha]$, then by (b) $\phi^* = \phi(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$. As $\phi(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$ is obtained from ϕ by replacing $P[\beta]$ with $\Diamond\beta$ and $N[\beta]$ with $\Box\beta$, it is a formula of L_M. If ϕ is rich and has the form Px_n or Nx_n, then by (c) $\phi^* = (\phi([p_n]/x_n))^*$. As $\phi([p_n]/x_n)$ has the form $P[\alpha]$ or $N[\alpha]$, ϕ^* turns out to be a formula of L_M for the reason seen above. We proved that if ϕ has complexity 0 then ϕ^* is a formula of L_M. Now we assume that the result holds for formulas of complexity $n-1$, and that ϕ has complexity n. Suppose that ϕ has the form $\neg\alpha$. Then, by (d) $\phi^* = \neg(\alpha)^*$. As the complexity of ϕ is n, the complexity of α is $n-1$. By hypothesis this entails that α^* is a formula of L_M. But then $\neg(\alpha)^*$ is a formula of L_M as well. Therefore, ϕ^* is a formula of L_M. The case in which ϕ has the form $\alpha \to \beta$ and that in which ϕ has the form $\Pi p_n \alpha$ are analogous. Lastly, the case in which ϕ has the form $\forall x_n \alpha$ reduces to the latter, as by (g) $(\forall x_n \alpha)^* = (\Pi p_n \alpha)^*$.

Fact 1 T is a *function*, in that for every formula α of L_{Mp} there is only one formula α^* of L_M such that $\langle \alpha, \alpha^* \rangle \in T$.

Proof We show that given any two formulas γ and δ of L_M, if $\langle \phi, \gamma \rangle \in T$ and $\langle \phi, \delta \rangle \in T$, then $\gamma = \delta$. As a first step we assume that ϕ has complexity 0. In this case ϕ is atomic. If ϕ is a formula of L_M, then by (a) $\gamma = \phi$ and $\delta = \phi$. Therefore, $\gamma = \delta$. If ϕ is rich and has the form $P[\alpha] or N[\alpha]$, then by (b) $\gamma = \phi(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$ and $\gamma = \phi(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$. Only one formula can be obtained by replacing in ϕ, $P[\beta]$ with $\Diamond\beta$ and $N[\beta]$ with $\Box\beta$. Therefore, $\gamma = \delta$. If ϕ is rich and has the form Px_n or Nx_n, then by (c) $\gamma = \phi([p_n]/x_n)$ and $\delta = \phi([p_n]/x_n)$. Only one formula can be obtained by replacing in ϕ the

nth objectual variable with the nth substitutional variable. Therefore, $\gamma = \delta$. We proved that if ϕ has complexity 0 then $\gamma = \delta$. Now we assume that the result holds for formulas of complexity $n - 1$, and that ϕ has complexity n. Suppose that ϕ has the form $\neg\alpha$. Then, by (d) $\phi^* = \neg(\alpha)^*$. This means that $\gamma = \neg(\alpha)^*$ and $\delta = \neg(\alpha)^*$. As the complexity of ϕ is n, the complexity of α is $n - 1$. By hypothesis this entails that there is a unique formula α^* such that $\langle \alpha, \alpha^* \rangle \in T$. Therefore, we get that $\gamma = \neg\alpha^*$ and $\delta = \neg\alpha^*$, and hence that $\gamma = \delta$. The case in which ϕ has the form $\alpha \to \beta$ and that in which ϕ has the form $\Pi p_n \alpha$ are analogous. Lastly, the case in which ϕ has the form $\forall x_n \alpha$ reduces to the latter, as by (g) $\gamma = (\Pi p_n \alpha)^*$ and $\delta = (\Pi p_n \alpha)^*$. We proved that for every formula ϕ of L_{Mp} there is a unique formula ϕ^* of L_M such that $\langle \phi, \phi^* \rangle \in T$. As usual, the unique ϕ^* is said to be the value $T(\phi)$ which T assumes at ϕ, and T is said to map the set of sentences of L_{Mp} into the set of sentences of L_M. T may be defined as a translation function from formulas of L_{Mp} to formulas of L_M. ♯

Fact 2 If ϕ is a sentence of L_{Mp} then $T(\phi)$ is a sentence of L_M.

Proof As a first step we assume that ϕ is a sentence of complexity 0. In this case ϕ is atomic. If ϕ is a formula of L_M, then by (a) $T(\phi) = \phi$. Therefore, $T(\phi)$ is a sentence of L_M. If ϕ is rich and has the form $P[\alpha]$ or $N[\alpha]$, then it contains a sentence γ of L prefixed by i occurrences of P and k occurrences of N. By (b) $T(\phi) = \phi(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$, where $\phi(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$ is a formula containing γ prefixed by i occurrences of $\Diamond$ and k occurrences of $\Box$. As γ is a sentence of L, $\phi(\Diamond\beta/P[\beta], \Box\beta/N[\beta])$ is a sentence of L_M. Therefore, $T(\phi)$ is a sentence of L_M. Now we assume that the result holds for sentences of complexity $n - 1$, and that ϕ is a sentence of complexity n. Suppose that ϕ has the form $\neg\alpha$. Then, by (d) $T(\phi) = \neg T(\alpha)$. As the complexity of ϕ is n, the complexity of α is $n - 1$. By hypothesis this entails that $T(\alpha)$ is a sentence of L_M. But if $T(\alpha)$ is a sentence of L_M then also $\neg T(\alpha)$ is a sentence of L_M. Therefore, $T(\phi)$ is a sentence of L_M. The case in which ϕ has the form $\alpha \to \beta$ is similar. Suppose now that ϕ has the form $\Pi p_n \alpha$. As the complexity of ϕ is n, the complexity of α is $n - 1$, where α is a formula in which p_n occurs free. Therefore, given any sentence s of L, $\alpha(s/p_n)$ turns out to be a sentence of complexity $n - 1$. By hypothesis this entails that $T(\alpha(s/p_n))$ is a sentence of L_M. It is easy to see that $T(\alpha)(s/p_n) = T(\alpha(s/p_n))$, and hence that $T(\alpha)(s/p_n)$ is a sentence of L_M. But then $T(\alpha)$ is a formula of L_M in which p_n occurs free. From this follows that $\Pi p_n T(\alpha)$ is a sentence of L_M. Since by (f) $T(\Pi p_n \alpha) = \Pi p_n T(\alpha)$, we get that $T(\phi)$ is a sentence of L_M. Lastly, suppose that ϕ has the form $\forall x_n \alpha$. Then, by (g) $T(\phi) = \Pi p_n \alpha$, which leads us back to the previous case. ♯

What has been said so far entails that there is a translation function from sentences of L_{Mp} to sentences of L_M. This can be seen as follows. We know that T is a translation function from formulas of L_{Mp} to formulas of L_M. T is defined as a set of ordered pairs, namely, the set of ordered pairs $\langle\alpha, \alpha^*\rangle$ such that α is a formula of L_{Mp} and α^* is the formula of L_M which is the value assumed by T at α. Let T' be the subset of T formed by all the ordered pairs $\langle\alpha, \alpha^*\rangle \in T$ such that α is a sentence of L_{Mp}. By the result just proved we get that if $\langle\alpha, \alpha^*\rangle \in T'$ then α^* is a sentence of L_M. Therefore, T' is a function that maps the set of sentences of L_{Mp} into the set of sentences of L_M. In other words, T' is a translation function from sentences of L_{Mp} to sentences of L_M.

Now we are in a position to fix the semantics of the sentences of L_{Mp}. As a first thing, we stipulate that every sentence of L_{Mp} that is a sentence of L_M has in L_{Mp} the same semantic interpretation that it has in L_M. Secondly, we impose a general constraint on the semantic interpretation of the sentences of L_{Mp}, namely, that the following condition is to be satisfied: for any sentence α of L_{Mp}, α is true in a given model if and only if $T'(\alpha)$ is true in that model. This way all the rich sentences of L_{Mp} turn out to be equivalent to sentences of L_M in accordance with the clauses (a)–(g). The clause (b) ensures that sentences containing P or N are equivalent to sentences containing $\Diamond$ or $\Box$ respectively. Informally speaking, this amounts to the view that a sentence containing 'possible' or 'necessary' beside a that-clause is equivalent to a sentence containing the corresponding modal adverb beside the sentence embedded in the that-clause. According to (c) and (g), objectually quantified sentences containing P or N besides "objectual" variables are equivalent to substitutionally quantified sentences containing P or N besides terms embedding substitutional variables. Informally speaking, the underlying assumption is that for every that-clause there is a proposition to which it refers, and for every proposition there is a that-clause that refers to it.[6]

IV

Let M be a set of sentences of L_M. We say that M is a *theory* whose language is L_M and that a sentence is a *theorem* of M just in case it belongs to M. For the purposes at hand we may assume that M is a theory that includes first-order predicate logic and modal propositional logic, say, a theory that includes a system of first-order logic extended with the axioms characteristic of KT. In this case L may be a language in which $\Diamond$ and $\Box$ do not occur in a formula unless they prefix a sentence, i. e., they do not occur within the scope of a quantifier. A set Γ of sentences of L_M is said to be *true* in a given model just in case every sentence in Γ is true in that model. Accordingly, M turns out to be true in a given model just in case every theorem of M is true in that model. A set Γ of sentences of L_M is said to *entail* a sentence ϕ of L_M just in case there is no model in which Γ is true but ϕ isn't true. We assume that M is closed under entailment. That is, if Γ is a subset of M and Γ entails ϕ, then ϕ belongs to M as well.

[6]Independent reasons for this assumption are provided in Iacona 2002.

Now let Λ be any set of rich sentences of L_{Mp} such that for every sentence $\alpha \in \Lambda$, $T'(\alpha) \in M$. We call Mp the set of sentences of L_{Mp} obtained by adding Λ to M, the result being closed under entailment. As in the case of M, we say that Mp is a theory whose language is L_{Mp}, and that a sentence of L_{Mp} is a theorem of Mp just in case it belongs to Mp. Accordingly, we say that Mp is true in a given model just in case all its theorems are true in that model. Closure under entailment is defined as in M. What has been said so far about M and Mp leaves indeterminate how they may be characterized in proof-theoretic terms. Here it suffices to say that they may be regarded as axiomatic systems whose theorems are the members of M and Mp respectively, and such that Mp includes some set of transformation rules which turn out to be valid on our assumptions about the semantic interpretation of the sentences of L_{Mp}. However a complete characterization of M and Mp may be given, it turns out that

Fact 3 Mp is equivalent to M

Proof To say that two theories are equivalent is to say that each of them is true in a given model just in case the other is true in that model. First we prove that if Mp is true in a given model then M is true in that model. Suppose that $\mathfrak{m}$ is a model in which Mp is true. As Mp is obtained by adding Λ to M, both Λ and M are true in $\mathfrak{m}$. Therefore, M is true in $\mathfrak{m}$. Second, we prove that if M is true in a given model then Mp is true in that model. Suppose that $\mathfrak{m}$ is a model in which M is true. Given our stipulations about the semantic interpretation of the sentences of L_{Mp}, every sentence $\alpha \in \Lambda$ is true in a model iff $T'(\alpha)$ is true in it. As we assumed that $T'(\alpha) \in M$ for every $\alpha \in \Lambda$, we get that Λ is true in $\mathfrak{m}$. Therefore, $M + \Lambda$ is true in $\mathfrak{m}$. But if $M + \Lambda$ is true in $\mathfrak{m}$ then every sentence entailed by $M + \Lambda$ is true in $\mathfrak{m}$. Therefore, Mp is true in $\mathfrak{m}$. ♯

From the equivalence result we get that

Fact 4 Mp is a consistent extension of M.

Proof It is easy to see that Mp is an extension of M. We say that a theory is an extension of another theory if every theorem of the second theory is a theorem of the first theory. As Mp is a set obtained by adding Λ to M, every sentence in M belongs to Mp. By definition this means that every theorem of M is a theorem of Mp. Now we prove that Mp is a consistent extension of M. We say that a consistent extension of a given theory is an extension of that theory such that the consistency of the latter entails the consistency of the former. Let us assume that M is consistent, i. e., that there is at least one model in which it is true. Call $\mathfrak{m}$ one such model. By the equivalence result every model in which M is true is a model in which Mp is true. Therefore, Mp is true in $\mathfrak{m}$. This means that there is at least one model in which Mp is true, and hence that Mp is consistent. ♯

V

The expressive power of L_{Mp} is rather limited. But it easy to see how the formal machinery presented might be enriched by adding to the vocabulary of L_M (hence of L_{Mp}) the connectives $\wedge$, $\vee$, $\exists$, and the existential substitutional quantifier Σ. There seems to be no claim in our ordinary modal discourse in terms of that-clauses that cannot appropriately be expressed within L_{Mp} or some unproblematic extension of it. This suggests that a theory such as Mp may be regarded as a basically adequate formal representation of that discourse. Accordingly, the additional linguistic resources that Mp has over M may be taken to formally represent the apparent "additional content" that our ordinary modal talk in terms of that-clauses has over the modal talk in terms of sentential operators involved in modal logic. The additional linguistic resources of Mp vindicate the intuitions on the basis of which one might be apt to regard the latter as departing from ordinary modal discourse. Take the inference considered at the beginning of the paper:

(1) whatever is necessary is possible
(2) it is necessary that every thing is identical to itself

---

(3) it is possible that every thing is identical to itself

We saw that it is natural to regard this inference as valid by assuming that (1) has logical form $\forall x(Fx \to Gx)$, (2) has logical form Fa, and (3) has logical form Ga. This is perfectly acceptable within a theory such as Mp. First of all, L_{Mp} enables us to represent (1), (2) and (3) as having the assumed logical form. The following are sentences of L_{Mp}:

(1a) $\forall x_n(Nx_n \to Px_n)$
(2a) $N[\forall x_i(x_i = x_i)]$
(3a) $P[\forall x_i(x_i = x_i)]$

The term $[\forall x_i(x_i = x_i)]$ occurring in (2a) and (3a) is the formal counterpart of the clause 'that every thing is identical to itself'. We saw that L_{Mp} allows objectual quantification over such terms. In the second place, the validity of the inference is preserved, in that (1a) and (2a) entail (3a). This can be shown as follows. Since every sentence α of L_{Mp} is true in a model iff $T'(\alpha)$ is true in that model, (1a), (2a) and (3a) are true in a model iff

(1b) $\Pi p_n(\Box p_n \to \Diamond p_n)$
(2b) $\Box\forall x_i(x_i = x_i)$
(3b) $\Diamond\forall x_i(x_i = x_i)$

are true in that model. That (1b) is the value that T' assumes at (1a) can be seen as follows. By clause (g) $T'(\forall x_n(Nx_n \to Px_n)) = T'(\Pi p_n(Nx_n \to Px_n))$. By clause (f) $T'(\Pi p_n(Nx_n \to Px_n)) = \Pi p_n T'(Nx_n \to Px_n)$. By clause (e) $\Pi p_n T'(Nx_n \to Px_n) = \Pi p_n T'(Nx_n) \to T'(Px_n)$. By clause (c)

$\Pi p_n T'(Nx_n) \rightarrow T'(Px_n) = \Pi p_n T'(N[p_n]) \rightarrow T'(P[p_n])$. By clause (b) $\Pi p_n T'(N[p_n]) \rightarrow T'(P[p_n]) = \Pi p_n(\Box p_n \rightarrow \Diamond p_n)$. It is easy to see that (2b) and (3b) are the values assumed by T' at (2a) and (3a) respectively. Now suppose that m is a model in which (1a) and (2a) are true. From the equivalence just considered we get that (1b) and (2b) are true in m. But every model in which (1b) and (2b) are true is a model in which (3b) is true. For (1b) entails $\Box\forall y(y = y) \rightarrow \Diamond\forall y(y = y)$ as one of its substitution instances, and from the latter together with (2b) we get (3b). Therefore, (3b) is true in m. From this plus the equivalence considered it follows that (3a) is true in m. We proved that (1a) and (2a) entail (3a). As Mp is closed under entailment, we get that if (1a) and (2a) are theorems of Mp then (3a) is a theorem of Mp. This amounts to saying that Mp enables us to formally represent the inference (1)–(3) as valid. At the same time, we saw that (1b) and (2b) entail (3b). As M is closed under entailment, we get that if (1b) and (2b) are theorems of M then (3b) is a theorem of M. This amounts to saying that the inference (1)–(3) may be appropriately "translated" into L_M and formally represented as valid within M.

VI

The foregoing considerations suggest we may plausibly steer a middle path between the two lines of thought outlined at the beginning of the paper. On the one hand, there is a sense in which it is right to say that 'possible' and 'necessary' are predicates that apply to propositions. It is the sense in which a language containing constants for such predicates and appropriate terms is a perfectly legitimate representation of our ordinary modal talk in terms of that-clauses. We saw that Mp turns out to be consistent on the assumption that M is consistent. Accordingly, it would be wrong to think that the only acceptable or legitimate modal language is that of modal operators. On the other hand, there is a sense in which it is right to say that nothing but modal logic is needed in order to account for our ordinary modal discourse. It is the sense in which there seems to be no interesting inference in our ordinary modal talk in terms of that-clauses that cannot be appropriately translated into the language of modal logic. Given the equivalence between rich sentences of L_{Mp} and sentences of L_M warranted by T', it seems right to say that Mp is a "purely linguistic" extension of M, or that Mp adds nothing "substantive" to M. It would be wrong to think that there is a substantive gap between our modal intuitions and modal logic, or that the language of modal logic significantly departs from our ordinary modal discourse.

What has been said so far leaves indeterminate whether or not it is right to say that propositions *exist*, and that 'possible' and 'necessary' stand for *properties* of propositions. There are different ways of accounting for the equivalence between sentences containing modal predicates and sentences containing modal operators. One is to say that propositions and their modal properties don't really exist, but there is nothing wrong in speaking "as if" they existed. This is essentially what Hartry Field takes to be fictionalism about intensional entities.[7] Another is to say that propositions and their modal properties are abstract entities that exist as a result of our way of speaking. This is a form of conceptualism that I find congenial.[8] The gap that separates the two views doesn't seem to be very big, and presumably no important philosophical issue turns on it. Rather, the significant divide is between the two views on the one hand, and the view that propositions are mind-independent and language-independent entities on the other, namely, Platonism. I take it that if one is a Platonist about propositions and their properties, one has to provide some non-trivial story according to which it cannot simply be assumed that for every that-clause there is a proposition to which it refers, and for every proposition there is a that-clause that refers to it. But this is just what happens with the formal model outlined.

A metaphor that has been invoked to suggest what things like properties or propositions turn out to be on a conceptualist construal is that of abstract entities as "shadows" of linguistic expressions: properties would be shadows of predicates, propositions would be shadows of sentences or that-clauses. The idea behind the metaphor is that properties or propositions are not objects belonging to an external reality, like lemon trees. Rather, they are the result of a projection, as it were, of our language and conceptual apparatus on the external reality. What there is to say about them is not to be found by investigating the external reality, as it happens with lemon trees, but can simply be extracted from an analysis of our linguistic and conceptual apparatus. The contrast is with Platonism, according to which properties or propositions *are* like lemon trees, and what there is to say about them is to be discovered by investigating the external reality. Obviously, metaphors are just metaphors. But there seems to be at least one respect in which propositions and their properties are like shadows. There can be an interesting question whether shadows exist or only the objects of which they are shadows exist. But independently of this question, one thing that is certain about shadows is that if they exist, their existence depends on that of the objects of which they are shadows. Similarly, there can be an interesting question whether propositions and their properties exist or only linguistic entities like that-clauses and predicates exist. But independently of this question, one thing that is certain about propositions and their properties as they have been characterized so far is that if they exist, their existence depends on that of our linguistic and conceptual apparatus.[9]

References

Bealer, G. 1993, 'Universals', *Journal of Philosophy*, 90, pp. 5–32.

[7] See Field 2001, p. 166.

[8] I advocate this view in Iacona 2002.

[9] Among those who have directly influenced this paper by discussing with me about its contents, I would like especially to thank Paolo Casalegno. Several of his comments on previous versions of the paper either exposed mistakes or suggested specific improvements.

Field, H. 2001, 'Attributions of meaning and content', in *Truth and the absence of fact*, Clarendon Press, Oxford, pp. 157–174.

Harman, G. 1972, 'Is modal logic logic?', *Philosophia*, 2, pp. 75-84.

Iacona, A. 2002, *Propositions*, Name, Genoa.

Kripke, S. 1976, 'Is there a problem about substitutional quantification?', in Evans, G. and McDowell, J. (eds.), *Truth and meaning: essays in semantics*, Clarendon Press, Oxford, pp. 325–419.

Montague, R. 1963, 'Syntactical treatments of modality', in *Acta Philosophica Fennica*, 16, Helsinki, reprinted in Thomason, R. E. (ed.), *Formal Philosophy: Selected Papers of Richard Montague*, New Haven, Yale, 1974, pp. 286–302.

Priest, G. 1976, 'Modality as a meta-concept', *Notre Dame Journal of Formal Logic*, 17, pp. 401–414.

Priest, G. 1977, 'A refoundation of modal logic', *Notre Dame Journal of Formal Logic*, 18, pp. 340–354.

Schiffer, S. 2003, *The things we mean*, Oxford University Press, Oxford and New York.

The *Australasian Journal of Logic* (ISSN 1448-5052) disseminates articles that significantly advance the study of logic, in its mathematical, philosophical or computational guises. The scope of the journal includes all areas of logic, both pure and applied to topics in philosophy, mathematics, computation, linguistics and the other sciences.

Articles appearing in the journal have been carefully and critically refereed under the responsibility of members of the Editorial Board. Only papers judged to be both significant and excellent are accepted for publication.

The journal is freely available at the journal website at

http://www.philosophy.unimelb.edu.au/ajl/.

All issues of the journal are archived electronically at the journal website.

Subscriptions Individuals may subscribe to the journal by sending an email, including a full name, an institutional affiliation and an email address to the managing editor at ajl-editors@unimelb.edu.au. Subscribers will receive email abstracts of accepted papers to an address of their choice. For institutional subscription, please email the managing editor at ajl-editors@unimelb.edu.au.

Complete published papers may be downloaded at the journal's website at http://www.philosophy.unimelb.edu.au/ajl/. The journal currently publishes in pdf format.

Submission The journal accepts submissions of papers electronically. To submit an article for publication, send the LaTeX source of a submission to a member of the editorial board. For a current list of the editorial board, consult the website.