Reply to Fritz

Timothy Williamson

In 'First-order modal logic in the necessary framework of objects', Peter Fritz starts from some remarks in *Modal Logic as Metaphysics* about first-order necessitist modal logic and develops a far more systematic theory. The paper is a technical *tour de force*. One of the main aims of my book was to open up new areas for investigation (not just debate). In that respect, Peter Fritz's work, along with that of Jeremy Goodman and others, is vindicating it more rapidly and extensively than I had dared hope.

As a simple and plausible working hypothesis, *Modal Logic as Metaphysics* assumes that the propositional modal logic of metaphysical modality is S5. In terms of Kripke frames for propositional modal logic, one can therefore drop the accessibility relation, in effect treating all worlds as mutually accessible. The logic of such a frame depends only on the number of worlds, irrespective of which is designated as the actual world. If the number is infinite, the logic of the frame is S5 itself. If the number is finite, the logic of the frame is a proper extension of S5; the extra theorems are implausibly restrictive on the metaphysical reading of the modal operators, so we can dismiss that case (Williamson 2013, p. 111).

When we move to first-order necessitist modal logic, we inhabit the frames simply by adding a single domain of individuals. The extra formulas validated if the domain is finite

are again implausibly restrictive on the metaphysically intended reading of the language, so we can dismiss that case too. Thus the structure of the frames of interest for first-order necessitist modal logic is fixed by two infinite cardinals, the number of individuals (Fritz's κ) and the number of worlds (Fritz's λ). However, by contrast with the propositional case, that they are infinite does not fix the logic of the frame. The logic also depends on their relative cardinality. I give an example of a first-order modal formula valid on such frames with more individuals than worlds but invalid on those with no more individuals than worlds (Williamson 2013, p. 145). Section 2 of Fritz's paper massively extends that observation, by reducing the issues to well-studied questions in non-modal model theory. Restricting attention to frames of the relevant sort and their logics, he shows that any frame with no more individuals than worlds determines the unique weakest logic, while any frame with enough more individuals than worlds determines the unique strongest logic. If there are only 'slightly' more individuals than worlds, the logic is very sensitive to how many more, and to delicate set-theoretic issues involving Cantor's Generalized Continuum Hypothesis; there are many intermediate logics. In general, increasing the number of worlds and decreasing the number of individuals never strengthen the logic; decreasing the number of worlds and increasing the number of individuals never weaken it. Fritz provides a helpful map of all the cases, and a wealth of information about the logics. He axiomatizes the weakest logic simply by adding as axioms 'There are at least n individuals' for all natural numbers n to a standard axiomatization of necessitist S5. He also shows that the strongest logic is recursively enumerable, so in principle recursively axiomatizable, and notes that it would be interesting to develop an axiomatization for it. Indeed: ideally, one wants a perspicuous schema for the additional axioms that somehow clarifies their metaphysical significance.

One upshot of Fritz's technical results is to show that, already in first-order modal logic, necessitism comes in many varieties, even when finite restrictions on the number of individuals or the number of worlds are excluded. Given the methodology of MLM, these logics are all genuine rivals of each other, for if one of them has all and only the metaphysically universal formulas of the language as theorems, then the others do not. The next step is to find robust considerations that favour one of them over the rest, or at least some of them over others. Of course, there is a tempting argument by the plural analogue of Cantor's theorem that there must be more individuals than worlds, because for any worlds there is an individual proposition true in all and only those worlds. There is also a tempting argument by the plural analogue of Cantor's theorem that there must be more worlds than individuals, because for any individuals there is a world in which all and only those individuals are thought about at the last moment of time. The challenge is to develop more stable and reliable ways of resolving the issues that overcome such conflicting temptations.

The frames in Fritz's section 2 are all set-sized, so none of them really provides the metaphysically intended interpretation of the first-order modal language, with absolutely unrestricted quantifiers. Thus it is by no means automatic that the metaphysically universal formulas are all and only those valid on a frame with κ individuals and λ worlds, for some fixed set-sized κ and λ. I sketch an argument to bridge the gap (Williamson 2013, pp. 117-18, 145). That is the topic of Section 3 of Fritz's paper. My argument relies on a sort of reflection principle suggested by Kreisel, which says that every theory in a standard non-modal second-order language satisfiable on the intended interpretation is satisfiable on a set (Premise 2, on Fritz's analysis). It also relies on an equivalence between the metaphysical universality of a formula of the first-order modal language, expressed by its universal

generalization in a corresponding second-order modal language, and its 'translation' into a non-modal second-order language with quantification over worlds, envisaged as a special case of propositions (Premise 1 on Fritz's analysis).

Concerning Premise 1, Fritz points out that it would be in the higher-order spirit of *Modal Logic as Metaphysics* to replace the first-order quantification over worlds (propositions, treated as individuals) by quantification into sentence position, along the lines of chapter 5. He shows how to carry out this modification, while neither making special assumptions about worlds nor losing philosophical plausibility. I welcome this refinement of the argument.

I supported Premise 2, Kreisel's Principle, by appeal to a consistency proof for it by Stewart Shapiro (for a universe of pure sets). As Fritz points out, such a result falls some way short of proving the Principle *true*. I did not suggest otherwise. In the book, I merely say that from a contemporary set-theoretic perspective, Kreisel's Principle 'looks quite plausible' (Williamson 2013, p. 118). At the very least, Shapiro's result shows that a refutation of Kreisel's Principle would in effect have to refute many other widely held assumptions about set theory too.

In the long run, we might hope to approach the problem from the other side, by arguing more directly for constraints on first-order necessitist modal logic, without the detour through set-sized frames, and then use Fritz's results to show that any logic meeting those constraints is the logic of some set-sized frame. It is not yet clear whether that hope is realistic.

References

Fritz, Peter 2016: 'First-order modal logic in the necessary framework of objects'. THIS VOLUME.

Williamson, Timothy 2013: *Modal Logic as Metaphysics*. Oxford: Oxford University Press.