

Deontic logic as a study of conditions of rationality in norm-related activities

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Abstract

The program put forward in von Wright's last works defines deontic logic as "a study of conditions which must be satisfied in rational norm-giving activity" and thus introduces the perspective of logical pragmatics. In this paper a formal explication for von Wright's program is proposed within the framework of set-theoretic approach and extended to a two-sets model which allows for the separate treatment of obligation-norms and permission norms. The three translation functions connecting the language of deontic logic with the language of the extended set-theoretical approach are introduced, and used in proving the correspondence between the deontic theorems, on one side, and the perfection properties of the norm-set and the "counter-set", on the other side. In this way the possibility of reinterpretation of standard deontic logic as the theory of perfection properties that ought to be achieved in norm-giving activity has been formally proved. The extended set-theoretic approach is applied to the problem of rationality of principles of completion of normative systems. The paper concludes with a plaidoyer for logical pragmatics turn envisaged in the late phase of Von Wright's work in deontic logic.

Keywords: Deontic logic, logical pragmatics, reinterpretation of standard deontic logic, G.H. von Wright.

1 Von Wright's reinterpretation of deontic logic

The foundational role and crucial influence of Georg Henrik von Wright (1916–2003) in the development of deontic logic is beyond dispute.² Recently, Bulgin [3] has divided von Wright's work in deontic logic in four phases: 1) dogmatic phase of 1950s marked by ignoring the fact that norms do not have truth-value; 2) eclectic phase of "Norm and Action" introducing the distinction between logic of norms and logic of norm propositions; 3) sceptic phase marked

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² Although usually called "the founding father of deontic logic", Von Wright preferred and used the term 'midwife' to denote his role in the development of the discipline.

by the thesis that logic of norms is impossible; 4) logic without truth-phase with the reinterpretation of deontic logic as the study of rationality conditions of the norm-giving activity. Von Wright's reinterpretation of deontic logic given in his later works (from 1980s onwards) has remained a non-formalized manifesto which so far has not received a fuller elaboration. In this paper the reinterpretation will be understood as the turn towards logical pragmatics. An exemplar programmatic statement is given in the following quote.

Deontic logic, one could also say, is neither a logic of norms nor a logic of norm-propositions but a study of conditions which must be satisfied in rational norm-giving activity. [13, p.111]

Von Wright's reinterpretation of deontic logic developed gradually and has introduced important conceptual distinctions and theses, among which the following stand out: the distinction between prescriptive and descriptive use of deontic sentences [10]; the thesis that relation between permission and absence of prohibition is not conceptual but normative in character [12]; this relation is one among "perfection properties" of normative system that the norm-giver ought to achieve in norm-giving activity [14,15]. These theses are mutually supporting. A normative systems can come into existence thanks to the prescriptive use of language . The logical properties of real normative systems can be described using the language of the "logic of norm-propositions". Some logical properties are "perfection-properties" of a normative-system. The absence of a certain perfection-property does not deprive a normative system of its normative force. In the prescriptive use of language the norm-giver ought to achieve some perfection properties of the normative system. Deontic logic is a study of logical perfection properties; properties which act as the normative source of requirements to which the norm-giver is subordinated.

Von Wright's reinterpretation of deontic logic can be formally explicated within the set-theoretic approach. The set-theoretic approach has been introduced into the logic of normative systems by Alchourrón and Bulygin [1]. Within this framework deontic sentences are treated as claims on membership in the set of consequences $Cn(\mathcal{N})$ of "explicitly commanded propositions" \mathcal{N} . Thus, $O\varphi$ in their approach means $\varphi \in Cn(\mathcal{N})$, while $P\varphi$ is explicated as $\neg\varphi \notin Cn(\mathcal{N})$. More recently a refinement and generalization of the set-theoretic approach has been developed by Broome [2] where the set of requirements is equated with the value of a code function, which takes as its arguments a normative source, an actor and a situation. The major point of divergence within the set-theoretic approaches lies in the properties that are assigned to sets of norms or requirements [17]. It is in accord with the approach proposed by Von Wright to treat real norm-sets, the one corresponding to obligation-norms and the other to permission-norms, as simple sets consisting just of affirmed and negated propositional contents of explicitly promulgated norms, not presupposing any a priori given properties. Rather, it is the question of compliance with second-order normativity whether a real normative system posses desirable logical properties and approximates an ideal system. In the

approach of this paper it is neither assumed that a norm-set is deductively closed nor that it is closed under equivalence.

If permission and obligation are not interdefinable, then two types of consistency should be distinguished. External consistency deals with the relation between obligation-norms and permission norms: $\neg(O\varphi \wedge P\neg\varphi)$. Internal consistency deals with obligation-norms alone: $\neg(O\varphi \wedge O\neg\varphi)$. According to Von Wright, the set of obligation-norms ought to have perfection properties.

... classic deontic logic, on the descriptive interpretation of its formulas, pictures a gapless and contradiction-free system of norms. A factual normative order *may* have these properties, and it may be thought desirable that it *should* have them. But can it be a *truth of logic* that a normative order has (“must have”) these “perfection”-properties? [14, p.20]

Perfection properties produce “normative demands on normative systems” and define “rationality conditions of norm-giving activity”. If the relation between permission and absence of prohibition is not a conceptual relation, then an addition to Von Wright’s outline is required. It is not sufficient to determine perfection properties of the set of obligation-norms. Perfection properties of the set corresponding to permission-norms must be taken into account, too, as well as perfection relations between obligation-norms and permission-norms, like external consistency. The needed extension of the set-theoretic approach can be obtained by the addition of the set related to permission norms. The formal explication of the relation between standard deontic logic and the theory of normative system perfection properties requires a provision of the translation function from the language of standard deontic logic without iterated operators to the language of extended set-theoretic approach. The translation function should reveal the fact that axioms of standard deontic logic are descriptions of an ideal normative system, a system endowed with “perfection properties”. These are the properties that a normative system ought have, as von Wright noted, and, as will be argued here, these are the properties to which the norm-giver and the norm-recipient relate in their corrective activities when faced with an imperfect normative system.

2 Perfection properties of a normative system

According to the extended Von Wright’s reinterpretation of deontic logic the norm-giver and the norm-recipient relate to the ideal concepts of obligation and permission.

Definition 2.1 Let \mathcal{L}_{pl} be the language of propositional logic. A set $\mathcal{N} \subseteq \mathcal{L}_{pl}$ is called norm-set and contains contents of obligation-norms. A set $\bar{\mathcal{N}} \subseteq \mathcal{L}_{pl}$ is called counter-set and contains negated contents of permission-norms. A normative system is the pair $\langle \mathcal{N}, \bar{\mathcal{N}} \rangle$.

The ideal concepts of obligation and permission can be explicated by pointing out the “perfection properties” of their corresponding sets, namely, of the norm-set and the counter-set. Since the filter structure and the weak-ideal structure of the respective sets will be later recognized as responsible for their

perfection properties these terms must be introduced. The first one is a well-known concept while the second will be introduced here.

2.1 Filter and weak ideal

It is well-known fact that the set of truth-sets of sentences belonging to a consistent and deductively closed set of sentences exemplifies a “filter” structure [4]. A filter F is a set of subsets of a given set W satisfying the following conditions [6, p.73]: (i) $\emptyset \notin F$, (ii) $W \in F$, (iii) if $X \in F$ and $Y \in F$, then $X \cap Y \in F$, (iv) for all $X, Y \subseteq W$, if $X \in F$ and $X \subseteq Y$, then $Y \in F$. In classical propositional logic the set of sets of valuations $\{\llbracket \varphi \rrbracket \mid \varphi \in Cn(T)\}$ is a filter if $Cn(T)$ is consistent, where $T \subseteq \mathcal{L}_{pl}$ is a theory, $Cn(T) = \{\varphi \mid T \vdash_{pl} \varphi\}$ is its deductive closure, and $\llbracket \varphi \rrbracket = \{v \mid v(\varphi) = t\}$ is the truth-set of φ . The properties of a filter can be reformulated in terms of logical syntax. In particular, reformulated condition (iii) expresses the closure under conjunction; reformulated condition (iv) expresses closure under entailment, i.e., if $\varphi \in Cn(T)$ and φ entails ψ , then $\psi \in Cn(T)$.

On the other hand, the set-theoretic structure corresponding to the “counter-theory”, $\mathcal{L} - Cn(T)$ is closed in the opposite direction: if $\llbracket \varphi \rrbracket$ corresponds to some $\varphi \in (\mathcal{L} - Cn(T))$, then so does any subset of it. In the syntactic reformulation: if $\varphi \in (\mathcal{L} - Cn(T))$ and ψ entails φ , then $\psi \in (\mathcal{L} - Cn(T))$. The structure of an ‘ideal’ is a particular kind of structure, which can be found in some but not all sets of truth-sets of sentences in counter-theories. An ideal I is defined as a set of subsets of a given set W satisfying the following conditions [6, p.73]: (i) $\emptyset \in I$, (ii) $W \notin I$, (iii) if $X \in I$ and $Y \in I$, then $X \cup Y \in I$, (iv) for all $X, Y \subseteq W$, if $X \in I$ and $Y \subseteq X$, then $Y \in I$. Conditions can be reformulated in syntactic terms. In particular, condition (iv) can be reformulated as the ‘closure under implicants’.

The complement of a filter needs not be an ideal, but if a theory is complete and consistent, its corresponding filter will be maximal, and its complement will be an ideal. The complement of any filter shares an essential property of the structure of an ideal, namely the property of “closure under implicants” as the first item in Proposition 2.2 shows.

Proposition 2.2 *If $S = W - F$ and F is a filter, then*

- (i) *if $\llbracket \varphi \rrbracket \subseteq \llbracket \psi \rrbracket$ and $\llbracket \psi \rrbracket \in S$, then $\llbracket \varphi \rrbracket \in S$,*
- (ii) *if $\llbracket \varphi \rrbracket \cap \llbracket \psi \rrbracket \in S$, then $\llbracket \varphi \rrbracket \in S$ or $\llbracket \psi \rrbracket \in S$,*

In this paper the question of logical structure of “counter-theory” will play an important role in the determination of perfection properties of the counter-set. Therefore, the new notion of weak ideal will be introduced.

Definition 2.3 A structure S is a weak ideal iff (i) $\llbracket \varphi \rrbracket \subseteq \llbracket \psi \rrbracket$ and $\llbracket \psi \rrbracket \in S$, then $\llbracket \varphi \rrbracket \in S$, and (ii) if $\llbracket \varphi \rrbracket \cap \llbracket \psi \rrbracket \in S$, then $\llbracket \varphi \rrbracket \in S$ or $\llbracket \psi \rrbracket \in S$.

Syntactic conditions corresponding to a weak ideal structure are: (i) inclusion of at least one conjunct for each conjunction contained, and (ii) closure under implicants, respectively.

Proposition 2.4 *Let $T \subseteq \mathcal{L}_{pl}$ and $Cn(T) \neq \mathcal{L}_{pl}$. The set $\{\llbracket \varphi \rrbracket \mid \varphi \in Cn(T)\}$ is a filter. The set $\{\llbracket \varphi \rrbracket \mid \varphi \in \mathcal{L}_{pl} - Cn(T)\}$ is a weak ideal.*

3 Translations, theorems of standard deontic logic and ideal normative systems

A formal explication of Von Wright’s reinterpretation of deontic logic asks for the establishment of a connection between the theorems of standard deontic logic and properties of ideal normative systems. For this purpose the translation function has been introduced in [17] connecting theorems of standard deontic logic with the properties of the norm-set. Now an additional translation function will be introduced, connecting deontic theorems also with the perfection properties of the counter-set and perfection relations between the norm-set and the counter-set.

Definition 3.1 Language \mathcal{L}_{sdl} is a deontic language without iterated modalities: $\varphi ::= p \mid O\varphi \mid P\varphi \mid \neg\varphi \mid (\varphi \wedge \psi) \mid$, where p is a sentence of language \mathcal{L}_{pl} of propositional logic. The definitions of deontic modality F and of truth-functional connectives are standard.

Definition 3.2 Language \mathcal{L}_{ns} is the language of the norm-set and counter-set membership within the extended set-theoretic approach: $\varphi ::= p \mid \ulcorner p \urcorner \in \mathcal{N} \mid \ulcorner p \urcorner \in \overline{\mathcal{N}} \mid \neg\varphi \mid (\varphi \wedge \psi) \mid$ where $p \in \mathcal{L}_{pl}$.³

Definition 3.3 Functions $\tau^+, \tau^-, \tau^* : \mathcal{L}_{sdl} \mapsto \mathcal{L}_{ns}$ translate formulas of the deontic language \mathcal{L}_{sdl} without iterated modalities to the language \mathcal{L}_{ns} of the extended set-theoretic approach.

$$\begin{aligned} \tau^+(O\varphi) &= \ulcorner \varphi \urcorner \in \mathcal{N} \\ \tau^+(P\varphi) &= \ulcorner \neg\varphi \urcorner \notin \mathcal{N} \\ \tau^+(P\neg\varphi) &= \ulcorner \varphi \urcorner \notin \mathcal{N} \\ \tau^-(O\varphi) &= \ulcorner \varphi \urcorner \notin \overline{\mathcal{N}} \\ \tau^-(P\varphi) &= \ulcorner \neg\varphi \urcorner \in \overline{\mathcal{N}} \\ \tau^-(P\neg\varphi) &= \ulcorner \varphi \urcorner \in \overline{\mathcal{N}} \\ \tau^*(O\varphi) &= \tau^+(O\varphi) \\ \tau^*(P\varphi) &= \tau^-(P\varphi) \end{aligned}$$

For $\star = +, -, *$

$$\begin{aligned} \tau^*(\varphi) &= \varphi \text{ if } \varphi \in \mathcal{L}_{pl} \\ \tau^*(\neg\varphi) &= \neg\tau^*(\varphi) \\ \tau^*(\varphi \wedge \psi) &= \tau^*(\varphi) \wedge \tau^*(\psi) \end{aligned}$$

³ “Quine quotes”, $\ulcorner \dots \urcorner$, will be omitted at most places in the subsequent text for the ease of reading and writing.

POSTULATES OF STANDARD DEONTIC LOGIC	NORM-SET PROPERTIES	COUNTER-SET PROPERTIES
(D) $O\varphi \rightarrow P\varphi$	consistency $\tau^+(D) = \varphi \in \mathcal{N} \rightarrow \neg\varphi \notin \mathcal{N}$	completeness $\tau^-(D) = \varphi \notin \overline{\mathcal{N}} \rightarrow \neg\varphi \in \overline{\mathcal{N}}$
(2*) $(O\varphi \wedge O\psi) \rightarrow O(\varphi \wedge \psi)$	closure under conjunction $\tau^+(2^*) = (\varphi \in \mathcal{N} \wedge \psi \in \mathcal{N}) \rightarrow (\varphi \wedge \psi) \in \mathcal{N}$	having at least one conjunct for each conjunction contained $\tau^-(2^*) = (\varphi \wedge \psi) \in \overline{\mathcal{N}} \rightarrow (\varphi \in \overline{\mathcal{N}} \vee \psi \in \overline{\mathcal{N}})$
(Rc) $\frac{\vdash_{pl} \varphi \rightarrow \psi}{O\varphi \rightarrow O\psi}$	deductive closure $\tau^+(O\varphi \rightarrow O\psi) = \varphi \in \mathcal{N} \rightarrow \psi \in \mathcal{N}$ if $\vdash_{pl} \varphi \rightarrow \psi$	“closure under implicants” $\tau^-(O\varphi \rightarrow O\psi) = \psi \in \overline{\mathcal{N}} \rightarrow \varphi \in \overline{\mathcal{N}}$ if $\vdash_{pl} \varphi \rightarrow \psi$
(D*) $O\varphi \rightarrow \neg P\neg\varphi$	RELATIONAL PROPERTIES external consistency $\tau^*(D^*) = \varphi \in \mathcal{N} \rightarrow \varphi \notin \overline{\mathcal{N}}$	
(Com) $O\varphi \vee P\neg\varphi$	“gaplessness” $\tau^*(Com) = \varphi \in \mathcal{N} \vee \varphi \in \overline{\mathcal{N}}$	

Table 1

Perfection properties come in non-equal pairs where each member is characterized by the same axiom or rule.

On counter-sets Although counter-intuitive at the first glance, the adequate metaphor for permitting is that of putting the negation of the content into the counter-set. This corresponds to the standard definition “ φ is permitted iff it is not obligatory that $\neg\varphi$ ” in the following way: since $\neg\varphi$ cannot go into the norm-set it must be placed into the counter-set. The perfection properties are different for different sets since “ideal concepts” of obligation and permission have different logical structure. For example, having a contradictory pair is an imperfection property of the norm-set, but for the counter-set this is neither a perfection nor an imperfection property. Similarly, completeness is a perfection property for permissions but not for obligations: it is indifferent whether $\varphi \in \mathcal{N} \vee \neg\varphi \in \mathcal{N}$ holds, whereas $\varphi \in \overline{\mathcal{N}} \vee \neg\varphi \in \overline{\mathcal{N}}$ ought to hold. This model, as will be shown, can account for the fact that perfection properties come in pairs, one for obligations, another for permissions, both of which are characterized by the same theorems of standard deontic logic, as shown in Table 1. The difference in logical structure of the two sets is also visible from the following facts: A perfect counter-set can have a contradictory pair of (negations) of permission-norm contents, which means that a certain state of affairs is optional. This fact does not cause an “explosion” since the principle *ex contradictione quodlibet* does not hold for the ideal counter-set.

The proposed two-sets model bears resemblance to the relation between a theory T and its counter-part $\mathcal{L} - Cn(T)$. The counter-theory has logical properties such as “closure under the implicant” (if $\psi \in \mathcal{L} - Cn(T)$ and φ entails

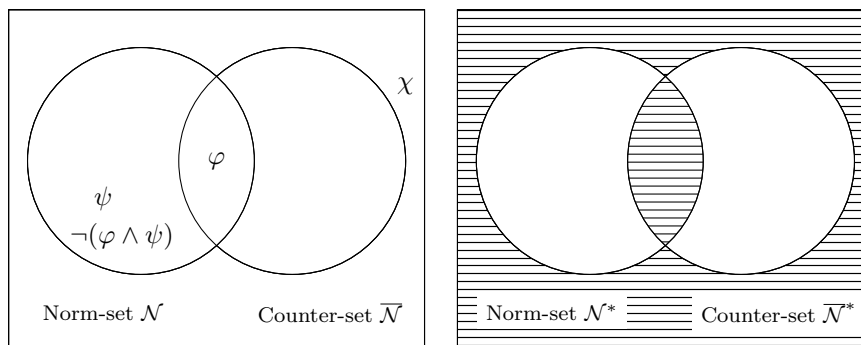


Fig. 1. A comparison between imperfect system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ and system $\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle$, which is endowed with some perfection properties. System $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is internally inconsistent under deductive closure; it is externally inconsistent as the presence of φ at the intersection shows and makes both $O\varphi$ and $P\neg\varphi$ true; it is also incomplete since $\chi \notin \mathcal{N} \cup \overline{\mathcal{N}}$. System $\langle \mathcal{N}^*, \overline{\mathcal{N}}^* \rangle$ is externally consistent and complete.

ψ , then $\varphi \in \mathcal{L} - Cn(T)$). The perfection properties of the descriptive theory have been well investigated within the logic of natural sciences. For example, the completeness of a theory, if attainable, counts as a perfection property, but the completeness of the (obligation) norm-set is not its perfection property. The mismatch holds also on the side of “counter-sets”: the completeness of the descriptive counter-part $\mathcal{L} - Cn(T)$ is an indifferent property, while in the realm of normativity it is a perfection property of the “counter-set” representing permission-norms. The construction is different, too: there is no “exclusion” part in building a theory since rejecting a sentence equals accepting its negation. This need not be the case with normative systems, whose obligation and permission parts are separately built. These facts shows that deontic logic as the study of “rationality conditions of norm-giving activity” or “perfection properties of normative systems” is a sui generis logic. If one accepts, together with von Wright, the central position of the phenomenon of normativity in humanities and social sciences, then deontic logic plays the prominent role in the philosophy of the science of man by revealing the logical basis of its methodological autonomy.

4 Deontic logic as the theory of ideal normative systems

As Aristotle famously wrote in *Nicomachean Ethics*, “it is possible to fail in many ways ... while to succeed is possible only in one way”. The same goes for constructing a normative system by prescriptive use of language: there are many imperfect normative systems in reality, but only one ideal system, the one, as will be proved here, described by standard deontic logic; compare Table 1.

An ideal normative system (*ins*) is internally (IntC) and externally consistent (ExtC), its obligation norm-set is closed under conjunction (2^*) and entailment (Re), and it is complete (Comp). An ideal normative system is

characterized by the following axioms and rules:

- (1) $\vdash_{ins} \top \in \mathcal{N}$
(2*) $\vdash_{ins} (\varphi \in \mathcal{N} \wedge \psi \in \mathcal{N}) \rightarrow \varphi \wedge \psi \in \mathcal{N}$
(IntC) $\vdash_{ins} \varphi \in \mathcal{N} \rightarrow \neg\varphi \notin \mathcal{N}$
(ExtC) $\vdash_{ins} \varphi \in \mathcal{N} \rightarrow \varphi \notin \overline{\mathcal{N}}$
(Comp) $\vdash_{ins} \varphi \in \mathcal{N} \vee \varphi \in \overline{\mathcal{N}}$
(RcO) $\frac{\vdash_{pl} \varphi \rightarrow \psi}{\vdash_{ins} \varphi \in \mathcal{N} \rightarrow \psi \in \mathcal{N}}$
(RcP) $\frac{\vdash_{pl} \varphi \rightarrow \psi}{\vdash_{ins} \psi \in \overline{\mathcal{N}} \rightarrow \varphi \in \overline{\mathcal{N}}}$

Other properties of an ideal normative system are consequences of these axioms and rules. In particular, perfection properties of the counter-set, closure under implicant and inclusion of at least one conjunct for each conjunction contained, can be derived in \vdash_{ins} . The three translation functions when applied to theorems of standard deontic logic (*sdl*) yield the following descriptions:

- if $\vdash_{sdl} \varphi$, then $\tau^+(\varphi)$ describes a perfection property of the (obligation) norm-set;
- if $\vdash_{sdl} \varphi$, then $\tau^-(\varphi)$ describes a perfection property of the (permission) counter-set;
- if $\vdash_{sdl} \varphi$ and both O and P occur in φ , then $\tau^*(\varphi)$ describes a perfection relation between the norm-set and the counter-set.

Since translations of axioms and rules of standard deontic logic yield truths about an ideal normative system, they can be understood as the theory of ideal normative system thus confirming Von Wright's conjecture.

Theorem 4.1 *If $\vdash_{sdl} \varphi$, then $\vdash_{ins} \tau^+(\varphi)$, $\vdash_{ins} \tau^-(\varphi)$, and $\vdash_{ins} \tau^*(\varphi)$.*

Proof. All axioms and rules of standard deontic logic can be derived in \vdash_{ins} . Therefore, any step of a proof in \vdash_{sdl} can be reproduced within \vdash_{ins} . \square

For the purpose of illustration the proofs for $\vdash_{ins} \tau^+(6.11)$, $\vdash_{ins} \tau^-(KD)$, and $\vdash_{ins} \tau^*(DD')$ are given in the Appendix.⁴ It should be noted that having a norm-set with contingent content, i.e., $\mathcal{N} \cap \{\varphi \mid \not\vdash_{pl} \varphi \text{ or } \not\vdash_{pl} \neg\varphi\} \neq \emptyset$ does not count as a perfection property of a normative system. Therefore, a nihilistic normative system in which any contingent state of affairs is permitted and none prohibited counts as an instance of an ideal normative system.

Von Wright's "pilgrim's progress" [13] from standard deontic logic to the position he held in his later works in 1990s may look as a circle, but the ending point is not the same. The theorems from 1950s still remain as theorems in 1990s deontic logic, but their position and character has been changed. They cease to be theorems of the "logical syntax" of deontic language, and become

⁴ The notations for theorems (6.11), (KD), and (DD') are taken over from [5].

the theorems of the “logical pragmatics” of deontic language use. What had been previously understood as a conceptual relation, later becomes a normative relation; a norm for the norm-giving activity, and not the logic of the norms being given.

4.1 Rationality of sealing principles

Completeness (“gaplessness”) of normative system is a perfection property that is hardly achievable for any non-nihilistic system. Von Wright gives a vivid definition of the problem and possible ways of solving it.

What is the difference “in practice” between a state of affairs not being prohibited and its being permitted? Suppose there is a code of norms in which there is no norm Pp . Now someone makes it so that p . What should be the law-giver’s reaction to this, if any? Could he say: “You were not permitted to do this and you must not do that which you are not permitted to do”? He could say this, making it a meta-norm that everything not-permitted is thereby forbidden. “Logically” this would be just as possible, even though perhaps less reasonable, as to have a meta-norm permitting everything which is not forbidden. But one can also think of some “middle way” between these two principles, a meta-norm to the effect that if something is not permitted by the existing norms of a code one must, as we say, ‘ask permission’ of the law-giver to do it. [12, p.280]

According to von Wright there are three principles by use of which a normative system can be completed: 1. $(\neg F \triangleright P)$ everything not forbidden is permitted, 2. $(\neg P \triangleright F)$ everything not permitted is forbidden, 3. normative gaps are filled in communication between the norm-recipient and the norm-giver. The third principle will be left aside because of its complexity. Using the two-sets model of normative system it can be shown why the first principle is to be preferred over the second one, i.e., why the first principle is “more reasonable”. In addition to this it can be proved that the mere “logical possibility” of the second mode $(\neg P \triangleright F)$ of filling normative gaps is not a sufficient condition of its rationality, according to von Wright’s own criterion of rationality of norm-giving activity.

Definition 4.2 A norm-system $\langle \mathcal{N}, \overline{\mathcal{N}} \rangle$ is gapless iff $\{\varphi, \neg\varphi\} \subseteq \mathcal{N} \cup \overline{\mathcal{N}}$ for all doable states of affairs φ and $\neg\varphi$, i.e., $\mathcal{L}_{\text{doable}} = \mathcal{N} \cup \overline{\mathcal{N}}$.

The notion of “doable state of affairs” is taken over from von Wright’s works. The notion of doability introduces complex problems of logic of action. Here the set of sentences describing doable states of affairs will be simplified and identified with the set of contingent sentences, $\mathcal{L}_{\text{doable}} = \mathcal{L}_{pl} - \{\varphi \mid \vdash_{pl} \varphi \text{ or } \vdash_{pl} \neg\varphi\}$.

The easy way of making a normative system complete is by applying the principle *everything which is not forbidden is permitted*. The way of filling in the gaps is straightforward, consisting in adding the missing sentences to the counter set and thus obtaining its extension $\overline{\mathcal{N}}^*$, as formula (1) shows.

$$\overline{\mathcal{N}}^* = \overline{\mathcal{N}} \cup \{\varphi \mid \varphi \notin Cn(\mathcal{N})\} \quad (1)$$

A completion of the normative system under the principle *everything which is not permitted is forbidden* is not a functional relation. In this mode the process is under-determined and so does not result in a unique system. The completion proceeds in two steps, each of which includes a choice.

The first step In the first step the counter-set must be completed in the view of perfection relations and properties. Also, the perfection-relation between the obligation norm set and its counter-set ought to be preserved if present and so their intersection must remain empty. This means that it will be expanded to achieve perfection-properties of being closed under implicants and under the rule of having at least one conjunct for each member conjunction. Since the last condition has the disjunctive consequent there may be different ways of performing the closure. Therefore, the weak-ideal expansion of a counter-set results in a set of sets.

Definition 4.3 The minimal weak-ideal closure $\text{WI}(\overline{\mathcal{N}})$ of a counter-set is the set of the smallest sets a satisfying the following conditions:

- (i) a includes $\overline{\mathcal{N}}$: $\overline{\mathcal{N}} \subseteq a$,
- (ii) if $\psi \in \overline{\mathcal{N}}$ and φ entails ψ and $\varphi \in \mathcal{L}_{\text{doable}}$, then $\varphi \in a$,
- (iii) a satisfies one of the following conditions:
 - (a) if $\varphi \wedge \psi \in \overline{\mathcal{N}}$, $\psi \notin \overline{\mathcal{N}}$, $\varphi \notin \text{Cn}(\mathcal{N})$ and $\varphi \in \mathcal{L}_{\text{doable}}$, then $\varphi \in a$,
 - (b) if $\varphi \wedge \psi \in \overline{\mathcal{N}}$, $\varphi \notin \overline{\mathcal{N}}$, $\psi \notin \text{Cn}(\mathcal{N})$ and $\psi \in \mathcal{L}_{\text{doable}}$, then $\psi \in a$.

Definition 4.4 Function γ picks an arbitrary member of the set $\text{WI}(\overline{\mathcal{N}})$ of weak-ideal sets: $\gamma(\overline{\mathcal{N}}) \in \text{WI}(\overline{\mathcal{N}})$.

Example 4.5 Let $\mathcal{L}_{\text{doable}} = \{p, q\}$. Let $\mathcal{N} = \emptyset$. Let the only norm be the norm-permission $\text{P}(\neg p \vee \neg q)$. It follows that: $p \wedge q \in \overline{\mathcal{N}}$; $\text{WI}(\overline{\mathcal{N}}) = \{\{p \wedge q, p, p \wedge \neg q\}, \{p \wedge q, q, \neg p \wedge q\}\}$

The second step The second step in a completion of normative system is also under-determined and complex in itself. It consists of two phases. In each of the two phases lists of sentence are being used in the construction. Lists will be understood as lists of equivalence classes $[\varphi] = \{\psi \mid \vdash_{pl} \psi \leftrightarrow \varphi\}$ $[\varphi_1], \dots, [\varphi_n], \dots$

- (i) In the first phase the obligation norm-set and its counter-set are closed under appropriate relations by taking into account “partially placed” sentences, i.e., those where only one sentence from a pair of contradictory sentences belongs to the closure of the system.

$$\begin{aligned} \overline{\mathcal{N}}_0 &\in \text{WI}(\gamma(\overline{\mathcal{N}})) \cup \{\varphi \mid \vdash_{pl} \varphi \leftrightarrow \neg\psi \text{ and } \psi \in \text{Cn}(\mathcal{N})\} \\ \mathcal{N}_0 &= \text{Cn}(\mathcal{N} \cup \{\varphi \mid \vdash_{pl} \varphi \leftrightarrow \neg\psi \text{ and } \psi \in \overline{\mathcal{N}}_0\}) \end{aligned}$$

- (ii) In the second phase “unplaced sentences” are being added in an iterative manner to the system. “Unplaced sentences” are those where no sentence

from a pair of contradictory sentences belongs to the system.

$$\begin{aligned} \langle \mathcal{N}_{n+1}, \overline{\mathcal{N}_{n+1}} \rangle &= \\ &= \begin{cases} \langle \text{Cn}(\mathcal{N}_n \cup \{\varphi_n\}), \overline{\mathcal{N}_n \cup \{\neg\varphi_n\}} \rangle, & \text{if } \mathcal{N}_n \cup \{\varphi\} \text{ is consistent,} \\ \langle \text{Cn}(\mathcal{N}_n \cup \{\neg\varphi_n\}), \overline{\mathcal{N}_n \cup \{\varphi_n\}} \rangle, & \text{otherwise.} \end{cases} \\ \langle \mathcal{N}^*, \overline{\mathcal{N}^*} \rangle &= \langle \bigcup_{0 \geq i} \mathcal{N}_i, \bigcup_{0 \geq i} \overline{\mathcal{N}_i} \rangle \end{aligned}$$

There is no preferred ordering of unplaced sentences. The outcome of the iterative process depends on the chosen ordering. In most cases the resulting systems are radically different.

The systems completed by the application of the principle *everything not permitted is forbidden* do not necessarily end in one and the same “ideal state of things”.

Example 4.6 Let $\mathcal{N} = \{p \vee q\}$ and $\overline{\mathcal{N}} = \emptyset$. Expansion of the counter-set by partially placed sentences and weak-ideal closure of the counter-set together yield the following set of sets: $\{\{\neg p \wedge \neg q, \neg p, \neg p \wedge q\}, \{\neg p \wedge \neg q, \neg q, p \wedge \neg q\}\}$. So, a choice must be made. Consequently expansion of the obligation-set with respect to the counter set depends on the set being chosen, and thus it yields either $\mathcal{N}'_0 = \{p \vee q, p, p \vee \neg q\}$ or $\mathcal{N}''_0 = \{p \vee q, q, \neg p \vee q\}$. Finally, the expansion by unplaced sentences depends on the list used in the construction. Suppose that List 1 is given by: $[q], \dots$, and List 2 by: $[\neg p], \dots$. Then $\mathcal{N}'_1 = \text{Cn}(\mathcal{N}'_0 \cup \{q\})$, while $\mathcal{N}''_1 = \text{Cn}(\mathcal{N}''_0 \cup \{\neg p\})$. Therefore, $(p \wedge q) \in \mathcal{N}'_1$, and $\neg(p \wedge q) \in \mathcal{N}''_1$. The completion results in incompatible ideal states as translations show: $O(p \wedge q)$ w.r.t. $\mathcal{N}^{*'}$, while $F(p \wedge q)$ w.r.t. $\mathcal{N}^{*''}$.

A critique of Von Wright: how many ideal states? Von Wright did not consider completion under the principle *everything not permitted is forbidden* as not rational, but only as less reasonable than the completion under the principle *everything not forbidden is permitted*.

Generally speaking: a legal order and, similarly, any coherent code or system of norms may be said to envisage what I propose to call an ideal state of things when no obligation is ever neglected and everything permitted is sometimes the case. If this ideal state is not logically possible, i.e., could not be factual, the totality of norms and the legislating activity which has generated it do not conform to the standards of rational willing. Deviations from these standards sometimes occur — and when they are discovered steps are usually taken to eliminate them by ‘improved’ legislation. [11, p.39]

If a normative system is completed under the principle *everything not permitted is forbidden*, then, if consistent, it can “envisage more than one ideal state”, each equally acceptable as the other. Thus, there will be no unique ideal state with respect to obligation-norms. If intending a unique ideal state is essential to rational willing on the side of the norm-giver, then the the principle *everything not permitted is forbidden* is not only “less reasonable”, as von Wright claimed, but also not (instrumentally) rational.

5 Concluding remarks and further research

The term ‘pragmatics’ indicates the study of language-use: the norm-giver is engaged in the prescriptive *use of language* while constructing a normative system; the norm-recipient *uses a system constructed by language use* as the basis of her/his normative reasoning. The term ‘social’ indicates that more than one language-user (or social role) should be taken into account: the (role of) norm-giver, the (role of) norm-recipient, the (role of) norm-evaluator. Social pragmatics of deontic logic studies the norms that apply to norm-related activities of social actor roles. These norms can be properly called ‘second-order norms’ since they cover the activities that are related to a normative-system. There is a difference between second-order norms which require construction (envisaging) a logical possible description of an ideal state (e.g., consistency norms) and second-order norms which are related to the will of the norm-giver. In the latter case if the aim is to construct a description of exactly one ideal state, then the second-order norm *everything not permitted is forbidden* is not acceptable since it might end in a multitude of equally valid ideal states. The language of modal deontic logic can be (and perhaps should be) understood as the language in which perfection properties of a normative system are being described. The norm-giver and the norm-recipient are related both to the actual normative-system, which may be imperfect, and to its, possibly missing, perfection properties (from which second-order norms spring). Logic has sometimes been understood as the *ethics of thinking*. Von Wright’s reinterpretation of deontic logic prompts us to understand logic also as the *ethics of language use*. In understanding deontic logic the perspectives of different social roles of should be taken into account as well as the purpose of norm giving activity. In this way deontic logic ceases to be a “zero-actor logic” and becomes the logic of language use which requires the presence of “users”. This fact redefines deontic logic as a research which necessarily includes the stance of *logical pragmatics*.

This paper is a continuation of the previous research [18] in which the extension of the pragmatic reading of deontic axioms has been introduced with respect to the difference of roles of the norm-subject and the norm-applier, but without separate treatment of permission-norms. Further research should extend the logical pragmatics approach and address the interrelated topics of normative reasoning based on an inconsistent normative system, the problem of conditional norms, and, at the most general level, the determination of the source of the second-order norms of norm-giving activity and provision of an adequate logical framework for their formalization. A sketch of possible directions of further research follows.

A normative vacuum does not appear if the norm-recipient is subordinated to an inconsistent normative system, in which there is no way out of the normative conflict on the basis of the metanormative principles on the priority order over norms. On the other hand, the norm-recipient cannot reason using classical logic since it would lead to the logical “explosion” (on the side of the norm-set). The only remaining option is logic revision. In the view of perfection properties, some postulates of logic revision can be outlined. The first

condition that a logic change ought to satisfy is to restore coherence (=non-explosiveness) of the set whose logic is being changed. Secondly, the change of logic ought to preserve desirable logical properties. The two conditions of the logic revision, restoration condition and preservation condition, resemble the content contraction, but the difference lies in the fact that instead of consistency it is the coherence that is being restored, and, instead of maximal preservation of the content, it is the desirable logical properties that are being saved. So, the norm-recipient faced with an inconsistent normative system ought to adopt an inconsistency-tolerant logic under which the normative properties will be preserved, namely, closure under entailment and adjunction of the norm-set together with correlated properties of the counter-set (closure under implicants and closure under having at least one conjunct for each conjunction). Is there such a logic? The deontic dialethic logic of G. Priest [8] seems to be adequate for the purpose.

The set-theoretic approach must be refined in order to capture the problem of conditional norms, which requires a more refined treatment of interaction between “is” and “ought”. The application of the generalized treatment of a code of requirements as a three-place functions introduced by Broome [2]. It has been proved in [17] that “for each world-relative code there is a realization equivalent world-absolute code”, or, in other words, that the narrow-scope and wide-scope reading have the same effects. The approach should be extended so to include also “necessary condition conditionals” having the form $O\varphi \rightarrow \psi$ and investigate perfection properties in this respect.

The third topic for the further research has a philosophical character because of its high level of generality. The relevant theoretical basis for this line of research can be found in dynamic logic as a logic of effects of language use, developed by J. van Benthem [9] and the vast group of related researchers. The essential formula of the theory has the form $[C]E$ and it describes communicative act C by its effect E . The inclusion of the actor’s identity in the C -part has been introduced by Ju and Liu [7], while Yamada [16] has added deontic effects to the E -part of the formula. Within this framework obligations of the norm-giver in the prescriptive use of deontic language can be captured by the formula $[g : \underline{!}\Delta_r\varphi]\mathbf{D}_g g : \underline{!}\Delta'_g\psi$, where g and r are the norm-giver and the norm-recipient, respectively, $!$ indicates the prescriptive use of deontic sentence, Δ and Δ' stand for deontic operators of the first-order, while \mathbf{D} stands for a deontic-operator of the second order. For example, in this perspective to the perfection property of external consistency there corresponds a second-order norm type forbidding creation of an externally inconsistent system, which can be formalized by the formula $[g : \underline{!}O_r\varphi]\mathbf{F}_g g : \underline{!}F_r\varphi$. A hypothesis worth considering is the one stating that the use of language in creation of a normative system is subordinated to the requirements of the second-order normativity, the normativity of language use, which ought to be studied within logical pragmatics of deontic logic.⁵

⁵ The author wishes to thank anonymous reviewers for helpful criticism and suggestions.

Appendix

Proposition .1 $\vdash_{ins} \tau^+(O\varphi \wedge P\psi) \rightarrow P(\varphi \wedge \psi)$

Proof. $\tau^+((O\varphi \wedge P\psi) \rightarrow P(\varphi \wedge \psi)) = (\varphi \in \mathcal{N} \wedge \neg\psi \notin \mathcal{N}) \rightarrow \neg(\varphi \wedge \psi) \notin \mathcal{N}$

1	$\varphi \in \mathcal{N} \wedge \neg\psi \notin \mathcal{N}$	
2	$\neg(\varphi \wedge \psi) \in \mathcal{N}$	
3	$\varphi \in \mathcal{N}$	1/ Elim \wedge
4	$(\varphi \wedge \neg(\varphi \wedge \psi)) \in \mathcal{N}$	2, 3/ 2*
5	$(\varphi \wedge \neg(\varphi \wedge \psi)) \rightarrow \neg\psi$	\vdash_{pl}
6	$\neg\psi \in \mathcal{N}$	4, 5/ RcO
7	$\neg\psi \notin \mathcal{N}$	1/ Elim \wedge
8	$\neg(\varphi \wedge \psi) \notin \mathcal{N}$	2-7/ Intro \neg
9	$(\varphi \in \mathcal{N} \wedge \neg\psi \notin \mathcal{N}) \rightarrow \neg(\varphi \wedge \psi) \notin \mathcal{N}$	1-8/ Intro \rightarrow

□

Proposition .2 $\vdash_{ins} \tau^-(O(\varphi \rightarrow \psi) \rightarrow (O\varphi \rightarrow O\psi))$

Proof. $\tau^-(O(\varphi \rightarrow \psi) \rightarrow (O\varphi \rightarrow O\psi)) = (\varphi \rightarrow \psi) \notin \overline{\mathcal{N}} \rightarrow (\varphi \notin \overline{\mathcal{N}} \rightarrow \psi \notin \overline{\mathcal{N}})$

1	$(\varphi \rightarrow \psi) \notin \overline{\mathcal{N}}$	
2	$\varphi \notin \overline{\mathcal{N}}$	
3	$\varphi \in \mathcal{N}$	2/ Comp
4	$(\varphi \rightarrow \psi) \in \mathcal{N}$	1/ Comp
5	$(\varphi \wedge (\varphi \rightarrow \psi)) \in \mathcal{N}$	3, 4/ 2*
6	$(\varphi \wedge (\varphi \rightarrow \psi)) \rightarrow \psi$	\vdash_{pl}
7	$\psi \in \mathcal{N}$	5, 6/ RcO
8	$\psi \notin \overline{\mathcal{N}}$	7/ ExtC
9	$\varphi \notin \overline{\mathcal{N}} \rightarrow \psi \notin \overline{\mathcal{N}}$	2-8/ Intro \rightarrow
10	$(\varphi \rightarrow \psi) \notin \overline{\mathcal{N}} \rightarrow (\varphi \notin \overline{\mathcal{N}} \rightarrow \psi \notin \overline{\mathcal{N}})$	1-9/ Intro \rightarrow

□

Proposition .3 $\vdash_{ins} \tau^*(O\varphi \rightarrow P\varphi)$

Proof. $\tau^*(O\varphi \rightarrow P\varphi) = \varphi \in \mathcal{N} \rightarrow \neg\varphi \in \overline{\mathcal{N}}$

1			$\varphi \in \mathcal{N}$	
2				
			$\neg\varphi \notin \overline{\mathcal{N}}$	
3				
			$\neg\varphi \in \mathcal{N}$	2/ Comp
4				
			$\neg\varphi \notin \mathcal{N}$	1/ IntC
5			$\neg\varphi \in \overline{\mathcal{N}}$	2-4/ Elim \neg
6		$\varphi \in \mathcal{N} \rightarrow \neg\varphi \in \overline{\mathcal{N}}$ 1-5/ Intro \rightarrow		

□

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