

## Aristotle on Mathematical and Eidetic Number\*

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Since, therefore, while there are numbers and a one both in affections and in qualities and in quantities and in movement, in all cases the number is a number of some things and the one is one something (ἐν ἅπασιν ὁ τε ἀριθμὸς τινῶν καὶ τὸ ἐν τὶ ἓν), but its substance is not this itself (οὐχὶ τοῦτο αὐτὸ ἢ οὐσία), the same must be true of substances; for it is true of all cases alike. That the one, then, in every genus is a certain nature, and in no case is its nature this itself—the one—, is evident. (*Meta.* X.2.1054a5–11)<sup>1</sup>

With these words Aristotle formulates part of his resolution of the eleventh aporia from Book III, which raised the problem of whether one and being are the substance of things or one and being belong somehow to some other nature (III.4.1001a4–b25). In the course of addressing this aporia, Aristotle identifies many senses of one, and he is led also to this statement relating one and number. We see here that numbers are pluralities of ones, and there are different ones for different numbers. The one is always some definite or determinate being, to be which includes being one, and in consequence of which each number is *a number of some beings* (cf. XIII.6.1080a15–16). Aristotle contrasts the unity of each one with the plurality of each number; in a word, each number is not τὶ ἓν (“one something” or “some one”).

These relations of unity and multiplicity obtain also in mathematical numbers. Each monad is indivisibly one, and each number is irreducibly plural. As I argue below, for Aristotle the indivisibility of the monad is ultimately traced to the sort of unity that is found in a dog or

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<sup>1</sup> Translation by W. D. Ross (with modifications), in Jonathan Barnes, ed., *The Complete Works of Aristotle*, 2 vols. (Princeton, N.J.: Princeton University Press, 1984). Translations of Aristotle are taken from this edition, with modifications as noted, except for Books XIII and XIV of *Metaphysics*, where, in most cases and with some modifications, I have used Julia Annas, *Aristotle's Metaphysics: Books M and N* (Oxford: Clarendon Press, 1976). For the Greek text, see W. D. Ross, ed., *Aristotle: Metaphysics*, 2 vols., (Oxford: Clarendon Press, 1924). Otherwise unidentified citations are to Aristotle's *Metaphysics*.

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horse, i.e., the unity that follows each determinate kind of being. Thus, the plurality of any number (mathematical or otherwise) is the necessary consequence of the unity of each being. That there is a natural or ontological source of the indivisibility of the monad lies at the heart of the irreducible difference between discrete and continuous quantity. The significance of preserving this difference in the manner Aristotle does becomes clear in view of efforts to deny or diminish this difference, as occurs, for example, with Descartes. Support for these claims requires (A) a summary exposition of Aristotle's account of mathematical number, followed by (B) a consideration of the contrast between Aristotle and Descartes. I conclude with a consideration of (C) eidetic number and (D) Aristotle's many senses of being in opposition to Parmenidean monism.

#### Preliminary Consideration

Aristotle's account of the mode of being of mathematical entities occupies a certain place in the inquiry for a sense of substance separate from the sensible substances (see, e.g., VII.2.1028b13–32 and VII.11.1037a10–20), namely, a place after the inquiry into sensible substance and before the inquiry into separate divine substance. Aristotle does not undertake a comprehensive reflection on the nature of mathematics but a relatively narrow inquiry into the mode of being of mathematical objects.<sup>2</sup> Given the nature of Aristotle's inquiry, calling attention to one aspect of his prior treatment of sensible substance helps to focus our remarks. In the first chapter of Book X, Aristotle distinguishes several senses in which something may be one in itself ( $\kappa\alpha\theta' \alpha\upsilon\tau\acute{o}$ ), i.e., indivisible per se. The primary sense of being one as indivisible belongs

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<sup>2</sup> See XIII.1.1076a8–37. John Cleary takes account of this by trying to recover “the cosmological context for problems about the foundations of mathematics” (*Aristotle and Mathematics: Aporetic Method in Cosmology and Metaphysics*, vol. LXVII of *Philosophia Antiqua* (Leiden: E. J. Brill, 1995), xxvii).

to the cause of substance's being one (X.1.1052a28–29). The central books make it clear that this is formal cause (εἶδος). Form is the actuality (ἐνέργεια) causing the substantial composite to be a unified whole. The alternative to being one in this sense is being a heap (σωρός), a disordered mass. In Book X, Aristotle turns from the senses of one in itself to another sense of one: one as a measure of a multitude.<sup>3</sup> The measure is *taken as* indivisible in relation to the multitude, irrespective of whether it is one in itself. The alternative to being one in this sense is not precisely being a heap but being a multitude of ones, i.e., *a number* in the sense of a counted or countable plurality. Careful attention to this distinction between being one per se and being one relative to a multitude reveals the inseparability of mathematics and metaphysics in Aristotle.<sup>4</sup>

#### A. Aristotle on Mathematical Number

**1. A mathematical number is a multitude of monads, i.e., a measured multitude of undifferentiated, indivisible, and interchangeable monads** (see XIII.8.1083b16–17, XIII.9.1085b22, XIII.9.1085b33–34, and XIV.1.1088a4–8). In mathematical number, no monad differs in any way from any other (XIII.6.1088a22–23, XIII.7.1081a4–7, 19–21, 1082b1–19, XIII.8.1083a1–17), and mathematical number is counted, after the one *two*, adding to the former one another one, and then *three*, adding to these two another one, and so on in the same fashion,

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<sup>3</sup> Technically, one as a measure has two senses: (a) first in a class of beings or (b) the measure of an amount. The exact sense of the former is disputed. Edward Halper has given this sense an extended and unusual treatment in his essay, "Metaphysics I and the Difference it Makes," in *Proceedings of the Boston Area Colloquium in Ancient Philosophy XXII, 2006*, edited by John J. Cleary and Gary M. Gurtler, S.J., (Leiden: Brill, 2007), 69–103.

<sup>4</sup> See Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*, trans. Eva Brann (New York: Dover Publications, 1992), 61. For another appreciation of the centrality of this theme, see Edward C. Halper, *One and Many in Aristotle's Metaphysics: Alpha–Delta* (Las Vegas: Parmenides Publishing, 2009) and *One and Many in Aristotle's Metaphysics: The Central Books* (Las Vegas: Parmenides Publishing, 2005).

each number incorporating the monad in the numbers preceding it (XIII.6.1080a30–33; cf. XIII.7.1081b12–20).<sup>5</sup> Mathematical numbers are not discovered but generated (γίγνεται) or counted by adding monads. Any two monads constitute the mathematical number two in the same sense, and any mathematical two consists of precisely two monads (XIII.6.1080b28–30)<sup>6</sup> because the monads are present actually and not merely potentially.<sup>7</sup> Monads are not in a number in the way that points and the half-line are in a whole line; monads are not in a number like Hermes in the unsculpted stone.<sup>8</sup> The monads are not connected (XIII.9.1085a3–4) but discrete. Whereas continuous quantities can be divided to yield any number whatever, a discrete multiplicity is fixed by the strict indivisibility of the monad. The monad is not a multitude (XIII.9.1085b15–17), which is to say that one is not a number, but two is a multitude and thus essentially not one.

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<sup>5</sup> Julia Annas regards this as an error (*M and N*, 168), and she sees Frege as having exceeded Aristotle (*ibid.*, 2 and 77). However, as she recognizes (*ibid.*, 1) and as Frege announces explicitly, Frege's account of number is ordered to serve modern science (Gottlob Frege, *The Foundations of Arithmetic*, trans. J. L. Austin, 2nd rev. ed. (Evanston, Ill.: Northwestern University Press, 1980), 69). On the new understanding of number ingredient to modern science, see Klein's *Greek Mathematical Thought*. Klein derived the basis for his analysis from Husserl; see "Phenomenology and the History of Science," in *Jacob Klein: Lectures and Essays*, ed. Robert B. Williamson and Elliott Zuckerman (Annapolis, Md.: St. John's College, 1985), 65–84. For a penetrating examination of Husserl and Frege on number, see Burt C. Hopkins, "Authentic and Symbolic Numbers in Husserl's *Philosophy of Arithmetic*," *New Yearbook for Phenomenology and Phenomenological Philosophy II* (2002): 39–71. By clarifying the inherently symbolic character of number in modern science, Klein and Husserl help to show that Aristotle must be approached independently of modern mathematics.

<sup>6</sup> Contrast Annas: "Number is a 'collection' only of potential units, not of actually existing ones" (*M and N*, 36, in note 40).

<sup>7</sup> "For either the dyad is not one or there is not a monad in it actually (ἐντελεχεία)" (VII.13.1039a13–14). If the dyad is not one but two, this will be because two monads are present actually; "for actuality divides" (VII.13.1039a7).

<sup>8</sup> See III.5.1002a20–25: "Besides these, any figure whatever or none at all is in the solid similarly; so that if Hermes is not in the stone, neither is the half of the cube in the cube thus as determinate; neither then is a surface (for if there is any surface whatever, also the surface itself marking off the half would be), and the same argument also in the case of a line and of a point and of a monad." This belongs to the statement of an aporia in Book III, which means that we are invited to question its adequacy.

2. **Numbers have no internal structure.** A number is not “something one” (ἓν τι), but more like a heap of ones (ὡς σωρός) (see XIII.8.1084b21–22). The accumulation of monads in a number (XIII.6.1080a30–33, XIII.7.1081b12–20) does take the structure of a Russian doll or a Chinese box.<sup>9</sup> These images suggest more unity than a number has (cf. XIII.7.1082b19–37). The three—to which one is added to produce four—are just three ones that have been counted together; they do not form one being. No new form obtains when one monad is added to three or when three are added to four. In the case of a sensible substance with several parts, it is possible to inquire into its unity by asking, ‘Why are these material parts this one being?’ (see VII.17.1041a21–b9). This question cannot be asked about number (XIII.2.1077a20–24 and XIII.7.1082a15–17). Whereas an organic substance has a cause unifying its material parts, no intrinsic cause (analogous to form) unites monads into one number. A number is not a *this* but, rather, *these*—an amount, not a substance (XIV.2.1089b32–1090a2)—, and the monads, if they must be considered parts, are not like parts in an organic whole.

3. **A mathematical number has determinate properties.** A mathematical number is not one being, and yet it has properties we can study arithmetically (even, odd, prime, etc.). These properties are qualities that belong to numbers *after* quantity (XIII.8.1083a10–11). The stability of these properties depends upon the number’s being just so many ones. The specific multiplicity of seven makes it both odd and prime, and that of sixteen makes it both even and square. Still, a number is not only the many ones but ones that have been counted up: number

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<sup>9</sup> Conceiving numbers as generated in counting does not require that numbers arise only in this way. Numbers might also be the result of calculations. Ten is also two fives or five twos (cf. XIII.7.1082a1–11 and b9–11). Klein (*Greek Mathematical Thought*, 17–25) discusses the distinction between counting and calculating, the difficulties with this distinction, and its relation to the Platonic distinction between arithmetic and logistic. Calculation is also central to Husserl’s analysis. See Hopkins, “Authentic and Symbolic Numbers,” esp. 45–49 and 53–63.

is a measured (μεμετρημένον) multitude (XIV.1.1088a5); its amount has been determined by its relation to the measure. As mentioned in the previous paragraph, we cannot quite ask why these monads are this one number, but we can ask why they are these many. The cause of the seven monads' being determinately seven is the unit measure, the one.<sup>10</sup> One is the principle of number (XIV.1.1088a6–8), which is always two or more. Nevertheless, insofar as the monads are collected together, the number may be seen to have some peculiar wholeness. When Aristotle alludes to this question of the unity of a number, he implies that there is at most an *extrinsic* cause, namely, soul (a theme to which we return below).<sup>11</sup> Even if we accept the essential role of the soul in constituting number, it must be emphasized that the soul *collects* the monads; it does not produce their multiplicity. The unit or monad is the cause of the number's being just so many.

**4. Number depends on the indivisibly one.** The fact that a number is not a being but beings establishes the priority of one to number.<sup>12</sup> One and number are relatives that are not by nature relative to one another; i.e., many is necessarily relative to one, but one is not necessarily relative to many (X.6.1056b32–34; cf. XIV.1.1088a15–b13). A number is always divisible into its

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<sup>10</sup> See Klein, *Greek Mathematical Thought*, 108–9.

<sup>11</sup> See, e.g., *Physics* IV.14.223a21–29, *Meta.* VIII.3.1044a2–9, XII.10.1075b34–37, XIII.2.1077a20–24, and XIII.7.1082a20–24. See Hopkins (“Authentic and Symbolic Numbers” and additional references therein) for an explanation of Husserl’s complicated analysis of the role of psychic acts in constituting number of different types. It seems that Husserl, in some sense, accepted Frege’s critique that this involved “the influx of psychology into logic” (quoted in *ibid.*, 40), and he came to regard his own initial approach as inadequate for *modern* mathematics. Its adequacy for Aristotle’s analysis is another question.

<sup>12</sup> The mathematical one is prior to mathematical number in being. It is a more difficult question whether it is also prior for us. On the one hand, Aristotle describes a kind of generation of mathematical number in the activity of counting (XIII.6.1080a30–33, XIII.7.1081a4–7, XIII.7.1081b12–20, and XIII.7.1082b11–19). This could support (even if it does not prove) the priority of the monad for us. On the other hand, Aristotle identifies senses of one by reference to the multiplicity or division each denies, as if that multiplicity were better known (X.1–2).

discrete ones, whereas the monad cannot be divided (XIII.8.1084b34, XIII.9.1085b15–17, 22, 33–34). The number’s divisibility is a function of the indivisibility of its monads, which ultimately depends upon the indivisibility of natural beings.

A man is one and indivisible as a man, and the arithmetician posits one indivisible, then studies what is incidental to man as indivisible; the geometer, on the other hand, studies him neither as man nor as indivisible, but as a solid object. For clearly properties he would have had even if he had not been indivisible can belong to him without them. (XIII.3.1078a23–28; trans. Annas, with modifications)<sup>13</sup>

In the ‘Preliminary Consideration’ above, we sketched how Aristotle traces the oneness (indivisibility) of a man to the formal cause of man. Here, he shows that the arithmetician leaves being-a-man out of account and simply posits one indivisible. The arithmetician does not ask whether or why man is one (see VII.17.1041a21–b9); the arithmetician posits something as one in the way man is ordinarily taken to be one. Similarly, in his analysis of time (see *Physics* IV.12.220b18–24), Aristotle says that horses are counted by “the one horse,” which seems to mean that-in-respect-of-which each of the horses is the same, without regard to their several differences in size, color, and so on.<sup>14</sup> The sensible discreteness of a member of a familiar species is not Aristotle’s substantial form, but it is the effect of that form, and here it stands as sufficient for recognizing the indivisibility of horse as horse or man as man.<sup>15</sup> Thus, the

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<sup>13</sup> The final word in the passage, “them,” refers to being a man and being indivisible.

<sup>14</sup> This suggests that in ordinary counting of material beings, the measure for the count is the so-called composite universal mentioned in *Metaphysics* (see VII.11.1037a6–7).

<sup>15</sup> Robert Sokolowski calls this familiar sensible discreteness “the shape” of a thing. “The shape of a material substance is a property of it, but it is not just one property among many. It is the property that establishes the space where all the other properties will occur; they all ‘take their place’ within the shape of the thing. But the shape of the thing is still not the substance of the thing. As a property it points to something more elementary than itself; it points to the thing in its kind, in its essence or nature” (*Phenomenology of the Human Person* (Cambridge: Cambridge University Press, 2008), 109).

indivisibility of the monad — which is its essence insofar as it has one — is derived ultimately from the indivisibility of form, which is the cause of the composite substance's unity.

The sense in which the arithmetician studies the human being deserves more consideration. Aristotle treats mathematics as parasitic on the familiarity of sensible being. Within arithmetic, it is sufficient to conceive the monad as something one in the sense in which man is ordinarily recognized as one, even though Aristotle himself has questioned this apparent unity at the metaphysical level by asking: Why are these (material parts) this (one man)?<sup>16</sup> The complete answer to that question is expressed through Book VIII (and perhaps IX) of the *Metaphysics*, where one is shown to belong to form “straightaway” (εὐθύς VIII.6.1045a36). This metaphysical analysis of unity (incorporated into Book X) does not lead Aristotle to “correct” mathematics by re-grounding it in something more purely one, such as god (which is one in the highest sense). The oneness evident in natural beings suffices to account for mathematical monads. The arithmetician studies number, which is “incidental to man qua indivisible.”

Aristotle resorts to this ‘qua locution’ to identify the peculiar character of mathematical objects. In doing so, he makes clear that the determinate nature of man is irrelevant to the monad. The requisite indivisibility belongs straightaway to any nature, whether it is man or horse or what have you. The method of subtraction, to use John Cleary’s signature phrase, isolates indivisibility in a sensible substance, revealing it to be logically independent even of the determinateness of man’s nature itself.<sup>17</sup> In the arithmetician’s eyes, it matters not what the

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<sup>16</sup> Aristotle implies that we should ask this sort of question when he says that we miss what we seek when we do not distinguish “that *these* are *this* (ὅτι τὰδε τὸδε)” (VII.17.1041a32–b2). It is difficult to find a translation capturing the point of this formulation.

<sup>17</sup> “My claim is that the key to his answer lies in the ‘qua’ locution and in a related method of subtraction, both of which are to be found in the *Posterior Analytics* (I.5) where Aristotle explains the logical basis for a



determinate nature is. What matters is its oneness, its indivisibility, which is like the impenetrable hardness of a Democritean atom, except that this indivisibility is traced ultimately to formal rather than material causality. The indivisibility of form is a kind of positive determination. This oneness belongs to being as being, which Aristotle exploits in his treatment of the principle of non-contradiction in *Metaphysics* IV. The monad enjoys a *substantial* oneness, which it would not have if it were conceived as “a point without position” (see XIII.1084b23–32). A geometric point’s indivisibility arises from a kind of deficiency or privation. We conceive the point by having taken away the natural determinations that are divisible. Once we remove depth, breadth, and length, we are simply out of ways to divide. Aristotle establishes the monad not by reference to the point but sensible substance in its indivisibility. The arithmetician focuses on this indivisibility to the exclusion of every other feature and even of the determinate essence that carries this oneness with it.

For as there are many formulae about things considered only *qua* in motion, **apart from the essence of each such thing** (χωρὶς τοῦ τί ἕκαστόν ἐστι τῶν τοιούτων) and from their accidents, . . . so too in the case of moving things there will be formulae and sciences which treat them not *qua* moving but only *qua* bodies . . . or only *qua* indivisibles. (XIII.3.1077b22–30, trans. Ross, modified, with bolding added)

Aristotle’s metaphysical analysis of sensible substance shows that being and one belong to each determinate nature, but the logical analysis of the arithmetical monad drops the determinate what or essence from consideration and conceives only the indivisibility.<sup>18</sup>

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general theory of proportion” (Cleary, *Aristotle and Mathematics*, 312). Subtraction involves identifying a property that belongs to a sensible being and taking away features to find the proper subject to which the property belongs (ibid., 313). “So the method of subtraction is a comprehensive logical process for identifying the primary subject of any given attribute” (ibid.).

<sup>18</sup> Aristotle occasionally speaks as if mathematical beings also include intelligible matter, which seems to permit their multiplication. According to VII.10.1036a9–12, intelligible matter is “the matter that exists in perceptible objects but not as perceptible, for example, mathematical objects.” See also, VII.11.1036b32–

Sensible substances may also be considered as if this indivisibility did not belong to them. Being extended would have belonged to man even if he had not been a man and thus had not been indivisible. The geometer, then, also studies man or, rather, the properties belonging to him, leaving both being a man and being indivisible out of account. While the arithmetician approaches man from the side of form, the geometer does so from the side of matter. The object of each science depends on the thoughtful activities performed by the mathematician. Even given that man is one and undivided by nature, if there is to be arithmetical number, someone must still take man as indivisible and isolate this for arithmetical purposes; and if there is to be an actual or determinate geometrical magnitude, it is necessary to take man as an extended, divisible solid in which we can designate points, lines, and figures (see X.1.1052b3–20).<sup>19</sup> At this point we return to the question of the role of soul in establishing the objects of mathematics.

5. **Soul unifies mathematical beings.** Although there is no cause of the unification of the monads internal to the number (analogous to substantial form), the numbering soul, which

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1037a5, XI.1.1059b14–16, and XI.3.1061a28–35. See Cleary, *Aristotle and Mathematics*, 388, 428–29, 443–44, 453–55.

<sup>19</sup> See John Cleary: “So for Aristotle there is no doubt about the truth of arithmetic because a man qua man is an indivisible **form** and hence it is quite plausible to treat him as an instance of the unit as it is defined by the arithmetician” (*Aristotle and Mathematics*, 335). Cleary attaches a note to this sentence: “There is a difficulty about the truth of geometry, however, if one insists (as does Aristotle) that it is about sensible things like a man. In contrast to the procedure of the arithmetician, the geometer must ignore the reality of a man as a unified whole and treat him merely as a divisible solid or as a surface or as a line. Yet even with such exclusions, it is doubtful whether man as a solid corresponds exactly with the definition of a solid posited independently by the geometer. The precision of geometry comes from the sort of idealization that fits better with a Platonic account of the foundations of that science” (ibid.). Ursula Coope defends the necessity of soul in constituting time as a number (*Time for Aristotle: Physics IV.10–14* (Oxford: Clarendon Press, 2005), 169–72). Her analysis seems faithful to Aristotle, except that she oversimplifies the case of discrete quantity. There too soul must act to fix the measure for the count.

takes the measure and counts these to be just so many, is somehow the cause of the existence of the number. Aristotle says this about time, which he defines as a number.

Whether if soul did not exist time would exist or not, is a question that may fairly be asked; for if there cannot be some one to count there cannot be anything that can be counted either, so that evidently there cannot be number; for number is either what has been or what can be counted. But if nothing but soul, or in soul, reason, is qualified to count, it is impossible for there to be time unless there is soul, but only that of which time is an attribute, i.e. if *movement* can exist without soul. The before and after are attributes of movement, and time is these *qua* countable. (*Physics*, IV.14.223a21–29)<sup>20</sup>

In order for numbers to be, someone must count, i.e., add one to another or divide one from another (XIII.7.1082b16–18 and 34–36). Number exists as a multiplicity conceived at least as countable in its multiplicity. Although Aristotle does not explicitly identify the numbering or counting soul as the cause of the being of numbers in XIII and XIV, he does speak of numbers as being generated in counting (XIII.6.1080a30–33, XIII.7.1081a4–7, XIII.7.1081b12–20, and XIII.7.1082b11–19). To say that mathematical beings are constituted through human thinking does not commit us to a modern intuitionist understanding of mathematics.<sup>21</sup> Nor does it commit us to the claim that number or any geometrical being (such as a triangle) is a pure construct.<sup>22</sup> Some evidence for this claim can be found in Euclid. A geometrical circle is a disc

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<sup>20</sup> John Cleary comments, “Nothing can be numbered without soul; cf. *Phy.* 223a22–29” (*Aristotle and Mathematics*, 374). For an extended discussion of the *Physics* passage and some other interpretations of it, see Coope, *Time for Aristotle*, 159–72. See Meta. VIII.3.1044a2–9 and XII.10.1075b34–37.

<sup>21</sup> See Cleary, *Aristotle and Mathematics*, 495–504. For intuitionism, see L.E.J. Brouwer, “Intuitionism and Formalism,” in *Philosophy of Mathematics*, 2nd ed., edited by Paul Benacerraf and Hilary Putnam (Cambridge: Cambridge University Press, 1983), 77–89, and Arend Heyting, “The Intuitionist Foundations of Mathematics” in *ibid.*, 52–61. Heyting identifies it as essential to intuitionism that “we do not attribute an existence independent of our thought . . . to the integers or to any other mathematical objects” (*ibid.*, 53). Among other difficulties, to attribute intuitionism to Aristotle we would have to ignore his explicit statements that arithmetician and the geometer are thinking about man (XIII.3.1078a23–28) and that the mathematicians generally are thinking about being (XIII.3.1077b32–33).

<sup>22</sup> Annas says the mathematician’s activity is “genuinely constructive” but not a creation “*ex nihilo*” (*M and N*, 151). We suggest comparison with the poet. The mathematician posits what is not separate as separate (XIII.3.1078a21–23), and the poet “misrepresents the way things are by presenting the

rather than a hoop (*Elements* I, def. 15), and a triangle is not identical with its designated borders but with what is contained within its borders. Hence, even when we produce an equilateral triangle (*Elements* I, proposition 1), we do not construct it simply. We do not make its interior and borders, but we think its division from its background. We think it as one extended whole marked off from un-constructed surroundings.

Let us conclude by contrasting the oneness of the triangle, e.g., to the oneness of the monad. The triangle is thought as one being, but as a *divisible* being. When we draw a line from one vertex to the midpoint of the opposite side, whether what results is one, two, or three triangles depends upon how we think about the figure. Thought can divide it because thought made it one.<sup>23</sup> There is nothing in a continuous quantity to resist these divisions, which is to saying that any continuous quantity is indefinitely divisible. By contrast, the monad's oneness is not produced by thinking but recognized in the indivisibility of man as man. Forms provide

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impossible" (Thomas Prufer, "Providence and Imitation: Sophocles' *Oedipus Rex* and Aristotle's *Poetics*," in *Recapitulations* (Washington, D.C.: The Catholic University of America Press, 1993), 14; cf. *Poetics* 1460b23–28). Prufer says that the poet's work (plot) is the artificial *eidōs* of action, and it is known to be artificial. "The plot constructed by the poet is neither a slavish copy of the action it imitates, nor an autonomous structure, a structure set up on its own as a law unto itself, a structure in its own right, freed from its model: plot is not 'abstract' art," but an imitation, a likeness of action (*ibid.*, 18). "The original is enriched, not distorted, by its image," and "As the subject matter action (*praxis*) is to the artifice plot (*mythos*), so *hypokeimenon* is to *eidōs*" (*ibid.*, 19). Mathematical beings are necessarily based in natural being, but not reducible to it. In thinking mathematically, the soul may be said to enrich or idealize sensible substance (see note 19 above). For another approach, consider the discussion of the unity proper to a multiplicity according to Husserl as it is treated by Hopkins, "Authentic and Symbolic Numbers," esp. 49–50.

<sup>23</sup> Jonathan Lear quotes Aristotle's discussion of the discovery of a geometrical proof from within the potentiality of the diagram (see IX.9.1051a21–31), where Aristotle concludes, "Obviously, therefore, the potentially existing things are discovered by being brought to actuality; the explanation is that thinking is an actuality" (IX.9.1051a29–31, as quoted by Lear, "Aristotle's Philosophy of Mathematics," *Philosophical Review* 91 (1982): 179–80). In another suggestive formulation, Aristotle writes, "Thus, then, geometers speak correctly—they talk about existing things, and their subjects do exist; for being has two forms—it exists not only in fulfillment but also as matter (τὸ μὲν ἐντελεχείᾳ τὸ δ' ὑλικῶς)" (XIII.2.1078a28–31, trans. Ross). If sensible substances are potentially (ὑλικῶς) the objects of geometry, they must be brought to actuality by the geometer. See Prufer's remarks in preceding note.

the intelligible ground for discrete quantity. Numbers are pluralities, and when we collect units into a number, say, seven, the thinking of these seven monads in isolation from all others corresponds to the thinking of the triangle as one whole. Whatever unity we may think there is in a triangle (which, like any continuous quantity, admits countless divisions), there is even less unity in any discrete multitude, which is already determinately just so many.

### B. Contrast with Descartes

It is possible to appreciate Aristotle's account of number by drawing a contrast with Descartes. In his *Meditations*, Descartes presents material beings as knowable with certainty only in their quantitative features. They cease to be definable entities with an essence, and they become *res extensae*. For Descartes, the characteristic shape of a man in his sensible discreteness does not come to sight as the effect of substantial form. Beneath the human shape may be an automaton, and no necessary connection obtains between the essence of the wax and any of its sensible properties.<sup>24</sup>

Descartes does not consider the shape of things to be ultimate; it remains a property or an accident, but the only thing beneath it is extension. The shape is not a property of an entity with a definition, but a temporary condition of extended matter. Substances, with their form and definition, no longer come between sheer matter and the properties such as shape, color, and motion, and things no longer have ends apart from the uses to which we put them.<sup>25</sup>

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<sup>24</sup> "But then were I perchance to look out my window and observe men crossing the square, I would ordinarily say I see the men themselves just as I say I see the wax. But what do I see aside from hats and clothes, which could conceal automata? Yet I judge them to be men. Thus what I thought I had seen with my eyes, I actually grasped solely with the faculty of judgment, which is in my mind" (*Oeuvres de Descartes* VII, ed. Charles Adam and Paul Tannery (Paris: J. Vrin, 1996), 32; trans. Donald Cress, *Discourse on Method and Meditations on First Philosophy*, 4th ed. (Indianapolis and Cambridge: Hackett Publishing, 1998). It is worth noting Descartes's use of the term "judge" here. In the fourth meditation, judging in the absence of clear and distinct understanding is identified as the source of error.

<sup>25</sup> Sokolowski, *Phenomenology of the Human Person*, 113, note 14.

Absent a knowable principle of unity, material things become like geometrical solids, which are divisible or malleable as we please. Sokolowski writes, “Material things no longer show up as ‘ones,’ as beings that have an identity and a definition.”<sup>26</sup> The abandonment of the formal principle of indivisibility in nature leads to or supports the assimilation of discrete to continuous quantity. Number lines and Cartesian coordinates are familiar expressions of this assimilation. “One” is an arbitrarily chosen distance from zero; numbers like *two* and *seven* are related to the unit measure, but each is indefinitely divisible, just as one is.<sup>27</sup> When there is no natural unit or natural principle of indivisibility, any sensible being is indefinitely divisible, just as are geometric solids. To the extent that discrete quantity becomes just a special case of continuous quantity,<sup>28</sup> it is difficult if not impossible to preserve within mathematics the distinction between the two senses of one Aristotle identified at the beginning of Book X, namely, one as indivisible in itself and one as a measure. With the collapse of the distinction between discrete and continuous, one becomes a number, the same in kind as two only less.

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<sup>26</sup> Ibid., 114.

<sup>27</sup> More than a by-product of contemporary mathematical practices, this assimilation was deliberately sought by founders of modern mathematics. E.g., Simon Stevin: “La communauté et similitude de grandeur et nombre, est si universelle qu’il ressemble quasi identité” (*L’Arithmétique de Simon Stevin de Bruges*, Leiden, 1585, p. 3<sup>r</sup>; quoted in Klein, *Greek Mathematical Thought*, 194) and “Nombre n’est point quantité discontinue” (*L’Arithmétique*, 4<sup>v</sup>; in *ibid.*, 194); Descartes: “In geometry, to find required lines it is merely necessary to add or subtract other lines; or else, taking one line which I shall call unity in order to relate it as closely as possible to numbers, and which can in general be chosen arbitrarily (*a discretion*). . . .” (*La Géométrie* in *Oeuvres* VI, 369–70; trans. David Eugene Smith and Marcia L. Latham, *The Geometry of René Descartes* (New York: Dover Publications, 1954), 2). Also, consider Quine’s identification of number and distance at the origin of calculus: “So the founding fathers of the calculus assumed infinitesimal numbers, just barely distinct from zero and from one another. Going a mile a minute then meant going one of those infinitesimal *distances* in some infinitesimal time” (italics added) (“Foundations of Mathematics,” in *Ways of Paradox and Other Essays*, rev. ed. (Cambridge, Mass.: Harvard University Press, 1976), 23).

<sup>28</sup> “Although Eudoxus is almost always credited with articulating a theory of ratio that applies to commensurable and incommensurable magnitudes alike, there is not a single, unequivocal indication that either he or Euclid understood number as a species of magnitude” (David Lachterman, *The Ethics of Geometry* (New York and London: Routledge, 1989), 178).

What is called *one* has no formal density at the root of its oneness. It is taken as one *and* understood as indefinitely divisible. Insofar as we understand nature or material beings in this geometric manner, there are no longer knowable natural units or natural kinds. We fail to recognize any principle of unity in nature except the conventional ones or measures we introduce for handling continuous quantities conveniently. The price of denying or neglecting the natural kinds is the loss of natural ends, understood as the perfections of those natural kinds. Descartes, of course, sought and welcomed the exclusion of ends from the knowable part of nature,<sup>29</sup> and the wax experiment seems intended to convince us that we cannot know clearly and distinctly the nature or essence of wax (or that of any other material substance), but only some of the properties that temporarily attach to its infinitely malleable extension.

### C. Aristotle on Eidetic Number

Intermingled with his own account of number, Aristotle describes three ways to understand the so-called eidetic numbers. Roughly, these numbers were postulated by his Platonic predecessors or contemporaries as the objects of mathematical sciences. Either (1) there is a first number and a second, each number and each monad being different in kind (τῶ ἐϊδει), so that any monad is incomparable (ἀσύμβλητος) with any other, or (2) all the monads are successive and any one is comparable with any other,<sup>30</sup> or (3) the monads within a given

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<sup>29</sup> Descartes dismisses ends as unintelligible to us by presenting them as if they were not natural ends in things, but God's purposes in creating those things. See *Oeuvres* VII, 55. Descartes's conflation (presumably deliberate) of ends and purposes can be appreciated in light of recent articulations of the distinction between them. See, e.g., Francis Slade, "On the Ontological Priority of Ends and Its Relevance to the Narrative Arts," in *Beauty, Art, and the Polis*, ed. Alice Ramos (Washington, D.C.: The Catholic University of America Press, 2000), 58–69, and Sokolowski, *Phenomenology of the Human Person*, 186–89.

<sup>30</sup> Aristotle's understands mathematical number similarly, but his number differs at least in mode of being. "Successive" (ἐφεξῆς) does not mean that the monads are differentiated by being ordered as first, second, third, and so on (XIII.6.1080a22–23; cf. XIII.7.1081a5–7).

number are comparable with one another but not with the monads of another number.<sup>31</sup> On top of all the difficulties these views generate for an understanding of mathematical number, Aristotle sees the same defect in both the first and the last: both views make numbers one in ways that numbers are not (cf. XIII.8.1084b23–32). The last view regards the dyad as something over and above the two monads in it, and the triad and other numbers too are seen as something one (ἕν τι) apart from the monads (XIII.7.1082a15–16). In much the same way, the first view tries to understand the eidetic number both as one and as a number of monads (XIII.7.1082b24–26; cf. XIII.7.1081a8–17).

In another passage, Aristotle criticizes *any* attempt to make a number be both one and made of ones. On such a view, the several units are regarded like matter gathered into one by number functioning like form.

([A]nd the unity becomes the matter of numbers, and at the same time prior to two—though also subsequent, in fact, because the two is a whole and a unity and form). But because they were looking for the universal they treated the one that is predicated as also being a part even so. But it is impossible for both of these to apply simultaneously to the same thing. (XIII.8.1084b28–32, trans. Annas)

It is not possible that numbers composed of ones could also be one. “Just as two men are not something one apart from both” (XIII.7.1082a22–24), the dyad is nothing apart from the two monads because there is nothing to cause of their being one (XIII.7.1082a15–22; cf.

XIII.2.1077a20–24).

According to Aristotle, the ones in any number are not, *as such*, ordered to form some one whole (e.g., XIII.7.1082a22–26), which is especially clear with mathematical number

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<sup>31</sup> Aristotle goes on to discuss further variations or combinations of these senses of number (XIII.6.1080a33–b36). For detailed analyses see Annas, *M and N*, esp. 162–76, Cleary, *Aristotle and Mathematics*, 346–65, Klein, *Greek Mathematical Thought*, 79–99, and J. Cook Wilson, “On the Platonist Doctrine of the ἀσύμβλητοι ἀριθμοί,” *Classical Review* 18 (1904): 247–60.



(XIII.2.1077a20–24). Still, mathematical number is not the only number, and Aristotle does admit something like an eidetic number. “And we suppose generally that one and one, both whether [they are] equals or unequals, are two, such as the good and the bad, and horse and man” (XIII.7.1082b16–18). Clearly, these do not form arithmetical twos for Aristotle, since within each pair each one differs in kind from the other one.

The measure must always be some one and the same thing applying to all cases; for example, if there are horses the measure is horse, if men it is man. If there are a man, a horse, and a god, the measure will perhaps be living thing, and their number will be a number of living things. If there are a man, white, and walking, they will hardly have a number, because they all belong to the same thing which is numerically one. Still, they will have a number of categories or some such term. (XIV.1.1088a8–14; trans. Annas)

This extreme case—when the ones in the number happen to belong together as one being—is “hardly” or least of all (ἤκιστα) a number precisely because they are within one being. A number does not ordinarily form that kind of whole. Normally, only the common measure provides the basis for collecting the ones together in a number. Only because three horses or men can each be taken as one horse or man and only because a horse, a man, and a god can each be taken as one living being do they amount to three horses, men, or living beings.

Aristotle’s last example—man, white, and walking—is an interesting number because of his resistance to calling it a number. These three coincide in one being and thus “hardly” have a number, but still the multiplicity must be recognized. We must understand them as three classes or categories or senses of being. Despite being together, they are not one, but many. Their multiplicity appears in that each is *not* a being in the same sense. That each is a *different* sense of being (and thus one in a different sense) contributes to the difficulty of counting them

together. Man and white form a multitude by the distinction of one sense of being from another (XIII.7.1082b34–37), despite the fact that each one is one in its own way (XIII.7.1082b16–18).

#### D. Parmenides, Plato, and the Many Senses of Being

Aristotle's strange, eidetic number differs from the eidetic numbers he criticizes not only in mode of being but also by not being one. Aristotle's eidetic number is a multiplicity due to the oneness of each sense of being (e.g., man, white, and walking). That formal (eidetic) unity of each sense is the principle of discrete as opposed to continuous quantity. Aristotle sees nature—or being—as irreducibly multiple and punctuated by the presence of natural forms or essences. Descartes portrays nature as extension, and this essential sameness takes priority over any temporary division that occurs. Obviously, Aristotle was not trying to distance himself from a Cartesian conception of number. Aristotle's target was Parmenides. In his polemic in Books XIII and XIV against mistaken views of number and the principles of beings, Aristotle traces these errors especially to an inadequate response to Parmenides.

There are many reasons for their being led astray towards these causes, but the main one (μάλιστα) is their old-fashioned way of putting the problem. They thought that all existing things would be one, the original Being (αὐτὸ τὸ ὄν), unless one could refute and come to grips with Parmenides' words, 'Never shall this be forced through, that things that are not, are.' They thought it necessary to prove that what is not, is; for only in this way—from being and from something else—would it be possible for there to be many existing things. (XIV.2.1088b35–1089a6, trans. Annas)<sup>32</sup>

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<sup>32</sup> In another passage (XIII.6.1080b31–32), Aristotle seems to identify a different common cause of error, but John Cleary argues that these come to the same point. "Whether or not he is correct, Aristotle treats all of the surveyed opinions as sharing the assumption that the One is an element and a principle of things. This means that the whole inquiry into number comes under the guiding aporia about whether or not One and Being are the substances of things. But, as I have already shown, the aporia itself has a broader range of concerns than the mathematical cosmology of the Academy, since it takes up the Parmenidean issue about the uniqueness of Being that Aristotle resolves through his notion of its *pros hen* structure" (*Aristotle and Mathematics*, 357).

Whereas Plato tried to produce many beings from being and the false (understood as non-being),<sup>33</sup> Aristotle responds that this kind of non-being—for non-being is also said many ways—fails to explain the many senses of being (XIV.2.1089a12–16). According to Aristotle, part of the problem is that Plato did not try to account for the many senses (XIV.2.1089b20–24), but only for the many substances (XIV.2.1089a31–33; cf. XIV.2.1089b32–1090a2).

Aristotle’s own approach is not to generate plurality from principles but to begin with plurality. Immediately after mentioning Parmenides, Aristotle responds, just as he responded to Parmenides in *Physics*, by supposing that being is said in many ways (XIV.2.1089a7; cf. *Physics* I.2.185a20–26), and he asks *which* sense of being Parmenides means. As in *Physics*, that being is said many ways is taken as more evident than Parmenides’s claim that all the beings are one (XIV.2.1089a9). If Parmenides is wrong and the beings are not one, they must rather be many, i.e., a determinable multitude of ones, and Aristotle begins by distinguishing these many senses. Ultimately, a clear determination of the many senses of being would require us to *count* them.<sup>34</sup> It is worth noting that in Book I of *Physics*, after dismissing Parmenides, Aristotle *counts* the principles of movable being, and the result is an exceptionally clear presentation of a sort of eidetic number (*Physics* I.7.191a3–15).

### Conclusion

It is not necessary to pursue the details of Aristotle’s analysis further. The point is that the multiplicity of being is the manifestation of its formal or eidetic diversity over against a

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<sup>33</sup> See XIV.2.1089a20–21: “He [Plato] means falsity and that kind of thing by not-being, from which, together with being, there come to be many things” (trans. Annas).

<sup>34</sup> “And since he was asking how there can be many existing things, it was even more necessary, as we said, not only to ask about things in the same category—how there can be many objects (οὐσίαι) or many qualities—but to ask how existing things in general can be many (ἀλλὰ πῶς πολλὰ τὰ ὄντα), some of them being objects (οὐσίαι), some characteristics, some relatives” (XIV.1089b20–24, trans. Annas).

Parmenidean (or Cartesian) denial of formal differences. Whether we agree with Aristotle's identification of the natural ontological units as (primarily) the forms of biological substances is less important than that we consider what is at stake in his claim that some such units divide or punctuate being. Aristotle's thesis, "being is said many ways," asserts diversity rather than uniformity of being. We can understand or clarify Aristotle's assertion primarily by determining the multiplicity, i.e., by counting. Aristotle's claim is that some beings are essentially or formally distinct from others and that we must recognize these distinctions. The counting of the causes in Book I of *Metaphysics* is a paradigmatic case. We could also appeal to the differences between kinds of friendship or of political regime, the difference between memory and imagination, and the difference between moral weakness and vice. To say that these distinctions are recognized rather than created is to conceive being as diverse and plural after the fashion of Aristotle.