# The Principle of Indifference and Inductive Skepticism

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## 1 Introduction

The inductive skeptic claims that we have no epistemic grounds for accepting the conclusions of inductive inferences. But many theorists (see, e.g., Huemer (2009), Laplace (1995), Bayes (1763)) have claimed that we can use the Principle of Indifference (PI) to defend inductive inferences. In this paper, I will show that, in fact, the opposite is true: if anything, the PI supports the rationality of inductive skepticism. Huemer (2009) provides the most well-developed and sophisticated attempt to use the PI to defeat the inductive skeptic. So in the remarks ahead, I'll focus on Huemer's proposal. But the discussion should generalize to any approach that uses the PI to defend induction.

In section 2, I'll explain how Huemer uses the Principle of Indifference to solve the problem of induction. In sections 3-5, I'll give two objections that confront Huemer's proposal and any other solutions to the problem of induction that appeal to the PI.

## 2 The probabilistic formulation of the problem of induction

Since the premises of inductive inferences do not guarantee the truth of the conclusion, it is natural to adopt a probabilistic formulation of the problem of induction. To defeat the skeptic, the inductivist must show why the premises of an inductive argument render the conclusion more probable.[1] Huemer (2009) offers the following example to illustrate the probabilistic formulation of the problem of induction:[2]

> **Example 1**. A physical process X has been discovered, the laws governing which are as yet unknown, except that the process must produce exactly one of two outcomes, A or B, on every occasion. No relevant further information is known about X, A, or B. We plan an experiment in which X will occur n times, and we will observe on each occasion whether A or B results. (p. 347)

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[1]Throughout this paper, I'll assume an epistemic/logical interpretation of probability. I'll also restrict attention to "next case" inductive inferences (All observed A's have been B. Therefore: it is probable that the next observed A will be B.)

[2]All references to Huemer will be from Huemer (2009).

Let $A_i =$ [Outcome A occurs on the $i$th trial] and let $U_i =$ [Outcome A occurs on all of the first $i$ trials]. Does $U_i$ make $A_{i+1}$ more probable? Here are three possible positions:

**Inductivism**: $P(A_{i+1}|U_i) > P(A_{i+1})$
**Skepticism**: $P(A_{i+1}|U_i) = P(A_{i+1})$
**Counter-inductivism**: $P(A_{i+1}|U_i) < P(A_{i+1})$

According to Huemer, we can think of the inductivist, the skeptic, and the counter-inductivist as offering competing constraints on the probability distribution for the phenomenon in question. To defeat the skeptic, the inductivist needs to show why the skeptic's probability distribution is rationally inferior to the inductivist's probability distribution.[3]

Huemer claims that the objective Bayesian can defeat the skeptic by appealing to the Principle of Indifference (PI): whenever one lacks reason for favoring one hypothesis over another, one should assign equal probabilities to each hypothesis. The PI can be motivated with a simple example. Suppose a coin is flipped. Intuitively, if all you know about the coin is that the two possible outcomes are heads and tails, you should assign the same prior probability to each of the outcomes. If you were to instead assign a greater prior probability to the heads outcome, it would imply that you have more reason to expect heads than tails.

I'll now explain how Huemer thinks the PI can be used to support inductivism in example 1. To defeat the skeptic, we need to show that $P(A_{i+1}|U_i) > P(A_{i+1})$. Given that, prior to any observations, we don't know anything about physical process X besides the fact that it can produce only two outcomes, it seems that the prior epistemic probability $P(A_{i+1})$ should be $1/2$. To compute the value for $P(A_{i+1}|U_i)$, we use the PI. Before we know anything about physical process X, it seems that we have no reason to expect any particular proportion of A outcomes in a given $i$-membered sequence. There are $i+1$ possible proportions of As in any such sequence, given that the proportion of As is either $0/i$ or $1/i$ or . . . or $i/i$. So to apply the PI, we assign each of these possibilities an initial probability of $1/(i + 1)$. We can then use the definition of conditional probability to calculate a value for $P(A_{i+1}|U_i)$ as follows:

$$P(A_{i+1}|U_i) = \frac{P(A_{i+1}\&U_i)}{P(U_i)} = \frac{P(U_{i+1})}{P(U_i)} = \frac{1/(i+2)}{1/(i+1)} = \frac{i+1}{i+2} \tag{1}$$

One can see that, for any $i$ greater than 0, $P(A_{i+1}|U_i) > P(A_{i+1})$. So this probability distribution, which I will call the Laplacean distribution, supports inductivism. In fact, many theorists have appealed to this distribution to support induction.[4]

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[3]Following Lange (2004), I think it's better to think of the skeptic as remaining *agnostic* about the correct probability distribution. But I agree with Huemer that, in order to defeat the skeptic, the objective Bayesian must show that there is something rationally inferior about a distribution where $P(A_{i+1}|U_i) = P(A_{i+1})$. So there is no harm in continuing to refer to such a distribution as a skeptical distribution.

[4]See, e.g., Bayes (1763), Carnap (1962), and Laplace (1995). Huemer observes that this distribution is equivalent to Carnap's (1962) m* measure. (p. 352)

## 2.1 The inconsistency objection

But now consider the following alternative path of reasoning. Before we know anything about physical process X, it seems that we have no reason to expect that any particular sequence of $i$ A/B outcomes is more likely than any other. There are $2^i$ ways to distribute A and B among the $i$ members of a sequence. So to apply the PI, we should assign a prior probability of $(1/2)^i$ to each of these sequences. Since $U_i$ has the same probability as every other possible $i$-membered sequence, $P(U_i) = (1/2)^i$. Similarly, since $(U_i$ & $A_{i+1})$ has the same probability as every other possible $i+1$-membered sequence, $P(U_i$ & $A_{i+1}) = (1/2)^{i+1}$. So we calculate $P(A_{i+1}|U_i)$ as follows:

$$P(A_{i+1}|U_i) = \frac{P(A_{i+1}\&U_i)}{P(U_i)} = \frac{(1/2)^{i+1}}{(1/2)^i} = 1/2 \tag{2}$$

We see that $P(A_{i+1}|U_i) = P(A_{i+1})$, so this probability distribution supports skepticism. The conflicting results derived above are an example of a well known objection to the PI: the PI delivers inconsistent results depending on how we partition the space of possibilities. One well-known version of this objection is van Fraassen's (1989) cube factory example (a version of Bertrand's Paradox). Suppose a factory produces cubes with side-length between 0 and 1 foot. If asked for the probability that a randomly chosen cube has a side-length between 0 and 1/2 foot, the natural first thought is to apply the PI to possible side-lengths in order to obtain the answer of 1/2. But we could have equivalently asked for the probability that a randomly chosen cube has face-area between 0 and 1/4 square-feet. Now the natural first thought is to apply the PI to possible face-areas. But if we do, we obtain the conflicting answer of 1/4.

In both the cube factory case and the induction case, the objective Bayesian faces a challenge. If she is to claim that the PI is legitimately applicable in these cases, the objective Bayesian must give some principled method for choosing the "correct" partition.

## 2.2 The explanationist defense

To respond to the inconsistency objection, Huemer offers the Explanatory Priority Proviso (EPP) to the Principle of Indifference: when applying the PI, one ought to assign equal probabilities (or a uniform probability density) across the partition at the most explanatorily basic level.[5] Huemer motivates the EPP with several examples.

> **Example 3**[6]. You are informed that a certain lamp is either on or off, and also that a single marble was recently drawn from a bag containing only red, blue, and/or green marbles. If a red marble was drawn from the bag, then

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[5]Huemer does not provide an analysis of "explanatorily basic levels", but he gives many examples: a cause is prior to an effect, an earlier state is prior to a later state, a part is prior to a whole, basic facts are prior to supervening facts, etc. From this list, it is clear that Huemer's notion of explanation subsumes both causal and metaphysical explanation.

[6]Here, I'm following Huemer's numbering.

the person drawing the marble made sure the lamp would be on (turning it on if necessary). If either a blue or a green marble was drawn, then he made sure the lamp would be off. Given just this information, what is the probability that the lamp is on? What is the probability that a red marble was drawn? (p. 355)

*Solution 1*: Apply the PI to the possible lamp states, assigning 1/2 probability to each state. The lamp is in the on state iff the marble is red, so the red marble is drawn with probability 1/2. If the lamp is in the off-state, the marble is either blue or green, so we apply the PI a second time across this sub-partition, yielding a 1/4 probability for blue and a 1/4 probability for green.

*Solution 2*: Apply the PI to the possible marble drawings, assigning 1/3 probability to each state. The lamp is on iff a red marble was drawn, so the probability that the lamp is on is 1/3.

*Verdict*: Since the marble states are causally and temporally prior to the lamp states, the EPP suggests that we should first apply the PI to the marble states, as in solution 2.

> **Example 4**. You are informed that a conscious brain has recently been artificially created. (This supposition is meant to neutralize your background knowledge of the sorts of states that brains are typically in.) The brain has been put in one of the 4 million possible states recognized by modern brain science. Assume that mental states supervene on physical states, and that 100,000 of the 4 million possible brain states realize overall painful mental states, 50,000 realize pleasurable mental states, and the remainder realize hedonically neutral mental states (or states that are between pleasure and pain). What is the probability, on this information, that the brain is in pain? (p. 356)

*Solution 1*: The brain is either in a pain state, a pleasure state, or a neutral state. Applying the PI to this partition yields a probability of 1/3 for the hypothesis that the brain is in pain.

*Solution 2*: Apply the PI to each of the 4 million possible neuro-functional states. Since 100,000 of these states realize pain, the probability that the brain is in a pain state is 100,000/4,000,000 = 0.025.

*Verdict:* Since phenomenal states supervene on neuro-functional states, the EPP suggests that we should apply the PI to the neuro-functional states, as in solution 2.

In each of the above examples, the EPP seems to generate the intuitively correct verdict about the initial probabilities. Having motivated the EPP with these simple cases, Huemer next applies the EPP to the original example:

> **Example 1**. Process X is to be repeated n times, producing either A or B on each occasion. Where $A_i$ is the proposition that outcome A occurs on the $i$th trial and $U_i$ is the proposition that A occurs on all of the first $i$ trials, what is $P(A_{i+1}|U_i)$? (p. 358)

Huemer notes that the physical process X has some objective chance of producing A on any given occasion. According to Huemer, this objective chance is explanatorily prior to the actual A/B outcomes generated by the process, so Huemer suggests we apply the PI at the level of possible objective chances.[7] Let c be the objective chance and $\rho$(c) be the probability density function for c. Huemer assigns a uniform probability density over the possible values for objective chance:

$$
\begin{aligned}
\rho(c) & = & 1, \text{ for } 0 \leq c \leq 1 \\
& = & 0 \text{ otherwise}
\end{aligned}
\tag{3}
$$

Then, by applying the definition of conditional probability and performing some simple integration, Huemer is able to derive the Laplacean Distribution[8]:

$$
P(A_{i+1}|U_i) = \frac{P(U_{i+1})}{P(U_i)} = \frac{1/(i+2)}{1/(i+1)} = \frac{i+1}{i+2}
\tag{4}
$$

This is the same probability distribution derived when applying the PI to possible *proportions* of A and B (see Eq. 1). This probability distribution supports inductivism over skepticism. For example, if we observe 98 As in a row, the probability that the $99^{\text{th}}$ outcome will be an A is 0.99.[9]

## 2.3   The EPP and apriority

Later in his paper, Huemer considers an important objection. In order to apply the EPP, we need to be able to identify the most explanatorily basic level. But then it is unclear how the proposal is supposed to get off the ground: in order to determine explanatory priority, one needs to engage in empirical reasoning, but in order to engage in empirical reasoning, one needs to have already determined explanatory priority. I'll present Huemer's response to this objection because I think it offers crucial insight into how the EPP strategy is supposed to work.

Huemer acknowledges that his approach requires that at least *some* facts about explanatory priority can be known a priori. But he claims that the EPP doesn't require that *all* such facts are knowable a priori. He illustrates this claim with the following simplified case:

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[7]Earlier, we were considering whether the objective Bayesian should apply the PI to possible *sequences* of A/B outcomes or to possible *proportions* of A and B. Since sequences seem explanatorily prior to proportions, it may seem like the EPP should recommend applying the PI to possible sequences. But presumably Huemer must be thinking that the level of possible objective chances is explanatorily prior to both the sequences *and* the proportions.

[8]For simplicity, I haven't included the details of Huemer's calculation. The important point is that Huemer derives the same probability distribution as in Eq. 1.

[9]What about van Fraassen's cube factory example? It is not obvious to me how the EPP would apply in this case. So perhaps it is better to think of the EPP as providing guidance in some but not all cases where it is unclear how to apply the PI.

**Example 5**. Psychologists have discovered a set of correlations between people's political beliefs and people's emotional states. For instance, subjects with emotional attitude $E_1$ are more likely to have belief $B_1$, and vice versa. The psychologists consider three theories: (i) the relevant emotions causally influence the relevant beliefs; (ii) the relevant beliefs influence the relevant emotions; and (iii) there are no relevant causal relations, so any correlations are purely coincidental.[10] It is not known which of these theories is true. (p. 364)

Suppose that we don't have any direct evidence about the prevalence of various emotional states in the population, and we want to know: what is the probability that the next individual sampled from the population will be in state $E_1$? The EPP tells us to apply the PI to a partition at the most explanatorily basic level. The problem is that there are three competing theories on what the most explanatorily basic level is. But we cannot know which of these theories is correct without first engaging in empirical reasoning.

According to Huemer, the solution is to move to a even more basic explanatory level. Huemer notes that psychological laws are plausibly prior to a subject's emotional and cognitive states. One can think of theories (i)-(iii) as describing three hypotheses about the nature of psychological laws. So Huemer suggests that we apply the PI to (i)-(iii), assigning each a prior probability of 1/3. One can then reapply the PI across emotional states contingent on theory (i), reapply the PI across cognitive states contingent on theory (ii), etc., in order to generate the prior probabilities that are relevant to our original question.

To summarize: Huemer acknowledges that, if the objective Bayesian's proposal is to succeed, we have to start with an *a priori* prior probability distribution.[11] The worry is that, in a case like example 5, there doesn't seem to be any way we could know a priori whether cognitive states or emotional states are explanatorily basic. Huemer's response, which I've just presented, is that if we really go down to the most fundamental explanatory level, we can apply the PI *without* engaging in empirical reasoning. I've presented Huemer's response because I think that it crucially clarifies how the EPP proposal is supposed to work.

## 3   The problem of underdetermination

While Huemer's proposal is very interesting, I think the PI actually supports *skepticism* rather than inductivism. I'll raise two independent challenges to the proposal: the *problem of underdetermination* and the *problem of asymmetric partitions*. This section will present the problem of underdetermination.

In order to support inductivism in the case of example 1, Huemer assumes that the objective chance that X causes A is stable across time. But why should the inductive

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[10]As Huemer notes, this case is simplified; for example, it doesn't canvas the possibility that $E_1$ and $B_1$ have a common cause. But since Huemer is merely using this case to clarify how the EPP is supposed to work, this omission isn't important.

[11]Huemer underscores this major point in section 4.5 of his paper (pp. 370-373).

skeptic grant this assumption? Why not instead think that objective chances change across time, perhaps unpredictably?[12] The inductive skeptic acknowledges that there have been regularities in the past; she wants to know why we should expect these regularities to continue into the future. The fact that the PI supports inductivism may not seem so impressive or surprising when one recognizes that the proof *assumes* the existence of stable objective chances.

Huemer acknowledges the force of this objection, but thinks that he can give a generalized argument that does not require the assumption of stable objective chances. The generalized argument employs the same strategy as in the psychological laws case (see 2.3). In that case, Huemer identified a new partition at the level of possible psychological laws that is explanatorily prior to the partitions across emotional states, cognitive states, etc. Similarly, to respond to the current objection, Huemer identifies a new explanatory level that he claims is explanatorily prior to the level of possible objective chances. I will call this new level: *the level of possible probabilistic laws.* Huemer offers $\{S, \sim S\}$ as a partition at this new level, where S is the hypothesis that X produces A with a stable objective chance that is a some fixed number on the interval [0,1] (p. 368). In 3.1, I will present Huemer's generalized argument. In 3.2, I will show that it falls prey to the problem of underdetermination.

## 3.1 Huemer's generalized argument

We are interested in the overall probability of $A_{i+1}$ given $U_i$:

$$P(A_{i+1}|U_i) = P(S|U_i) \cdot P(A_{i+1}|U_i, S) + P(\sim S|U_i) \cdot P(A_{i+1}|U_i, \sim S) \tag{5}$$

From section 2.2 (when we were assuming stable objective chances), we already have one component of the right hand side of this equation: $P(A_{i+1}|U_i, S) = (i+1)/(i+2)$. The probability of $A_{i+1}$ given $U_i$ on the assumption that $\sim S$ is not as clear. So Huemer suggests that we take "the most pessimistic, skeptical assumption ... [that] inductive evidence is completely irrelevant to predictions about the future" (p. 369). On this assumption: $P(A_{i+1}|U_i, \sim S) = P(A_{i+1}) = 1/2$.[13]

To calculate the respective probabilities of S and of $\sim S$ given $U_i$, we use Bayes' Theorem:

$$P(S|U_i) = \frac{P(S) \cdot P(U_i|S)}{P(S) \cdot P(U_i|S) + P(\sim S) \cdot P(U_i|\sim S)} \tag{6}$$

$$P(\sim S|U_i) = \frac{P(\sim S) \cdot P(U_i|\sim S)}{P(S) \cdot P(U_i|S) + P(\sim S) \cdot P(U_i|\sim S)} \tag{7}$$

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[12]Indeed, why should the skeptic accept objective chances at all? I will consider this more basic issue in section 5.

[13]I think that there is a good reason why $P(A_{i+1}|U_i, \sim S)$ is unclear: $\sim S$ is a catch-all hypothesis. We can think of $\sim S$ as a (possibly infinite) disjunction of all hypotheses $S_i$ that are incompatible with S. But there's no reason to think that the probability of $A_{i+1}$ given $U_i$ is the same for all $S_i$ that are incompatible with S. I will soon object that Huemer's generalized argument is only successful when one makes certain assumptions about how to partition the level of possible probabilistic laws (assumptions that the skeptic has no reason to endorse).

Huemer appeals to the "pessimistic" assumption a second time in order to assess $P(U_i|\sim S)$ as $(1/2)^i$.[14] From his earlier derivation of the Laplacean distribution (see Eq. 4) which *assumed* S, Huemer assesses $P(U_i|S)$ as $1/(i+1)$. We can now let $s$ be the initial probability assigned to the hypothesis S.[15] So (1-$s$) is the initial probability assigned to the hypothesis $\sim$S. With these values and the values for $P(U_i|\sim S)$ and $P(U_i|S)$, we can evaluate the probabilities in (6) and (7). We then have all the materials we need to fill in Eq. 5, thereby describing the probability of $A_i + 1$ given $U_i$ as a function of $s$ and $i$. Omitting the intermediate steps, Huemer derives the following:

$$P(A_{i+1}|U_i) = \left[\frac{i+1}{i+2}\right]\left[\frac{(1-s)(i+2)+s2^{i+1}}{(1-s)(2i+2)+s2^{i+1}}\right] \tag{8}$$

$P(A_{i+1}|U_i)$ takes a minimum value of $1/2$ when $i = 0$ (so long as $s$ is positive). As $i$ increases with additional observations of A, $P(A_{i+1}|U_i)$ increases monotonically with $i$. This is because the term $2^{i+1}$ dominates both the numerator and the denominator as $i$ increases. So even if a small prior probability $s$ is assigned to hypothesis S, enough observed instances of A will support inductivism. So, Huemer claims, the EPP defense need not assume stable objective chances.

## 3.2 A skeptic's underdetermining hypothesis

In this section, I'll argue that Huemer's generalized argument is only successful when one makes certain assumptions about how to partition the level of possible probabilistic laws. The simplest way to make my argument will be to give an example of a partition that does *not* support inductivism. Let $t_i$ be the time at which the $i$th observation has been made. Consider the following partition:

> **Underdetermining partition**
> $S_1$: X produces A with a stable objective chance that is some fixed number on the interval [0,1]
> $S_2$: X produces A with a stable objective chance that is some fixed number on the interval [0,1] for all times $t < t_i$ and X produces A with an objective chance of 0 for all times $t \geq t_i$
> $S_3$: $\sim(S_1 \vee S_2)$

We are interested in the overall probability of $A_{i+1}$ given $U_i$:

$$P(A_{i+1}|U_i) = P(S_1|U_i) \cdot P(A_{i+1}|U_i, S_1) + P(S_2|U_i) \cdot P(A_{i+1}|U_i, S_2) + P(S_3|U_i) \cdot P(A_{i+1}|U_i, S_3) \tag{9}$$

We are given $P(A_{i+1}|U_i, S_2) = 0$ in $S_2$ itself, so we don't need to worry about the the middle expression on the right hand side. Since $S_1$ is just the same as S from

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[14]In the next sub-section, I will show that this is the move that ought to be resisted by the skeptic.

[15]Why is Huemer assigning probability $s$ to S instead of a probability $1/2$, given that the partition $\{S, \sim S\}$ involves two hypotheses? Huemer doesn't bother appealing to the PI because he thinks that his argument is successful even on the assumption that we assign a prior probability $s < 1/2$ to S. In the present case, it turns out that it doesn't matter what prior probability one assigns to S, so long as it isn't 0. Despite this disanology between the present case and example 5, the overall strategy is the same in each case. I note in every other case in the paper, Huemer uses the PI to assign prior probabilities.

Huemer's proof, we can assess $P(A_{i+1}|U_i, S_1)$ as $(i + 1)/(i + 2)$. If we adopt Huemer's "pessimistic" assumption from above, then $P(A_{i+1}|U_i, S_3)$ will be 1/2. To calculate $P(S_1|U_i)$ and $P(S_3|U_i)$, we use Bayes' Theorem just as before:

$$P(S_1|U_i) = \frac{P(S_1) \cdot P(U_i|S_1)}{P(S_1) \cdot P(U_i|S_1) + P(S_2) \cdot P(U_i|S_2) + P(S_3) \cdot P(U_i|S_3)} \quad (10)$$

$$P(S_3|U_i) = \frac{P(S_3) \cdot P(U_i|S_3)}{P(S_1) \cdot P(U_i|S_1) + P(S_2) \cdot P(U_i|S_2) + P(S_3) \cdot P(U_i|S_3)} \quad (11)$$

To evaluate Eqs. 10 and 11, we can follow almost exactly the same steps that Huemer followed to evaluate Eqs. 6 and 7. Since $S_1$ is the same as S from Huemer's proof, $P(U_i|S_1)$ will be $1/(i+1)$. Since $S_1$ and $S_2$ are observationally equivalent up until $t_i$, $P(U_i|S_2)$ will be $1/(i+1)$ as well. Adopting Huemer's pessimistic assumption from above, we can assess $P(U_i|S_3)$ as $(1/2)^i$ (the same as $P(U_i|\sim S)$ in Huemer's proof). Since there are now three competing hypotheses, we can let $P(S_1) = s$, $P(S_2) = t$, and $P(S_3) = (1 - s - t)$. With these values, we can determine the probabilities in Eqs. 10 and 11. Plugging back into (9), we obtain the following[16]:

$$P(A_{i+1}|U_i) = \left[\frac{i + 1}{i + 2}\right] \left[\frac{(1 - s - t)(i + 2) + s2^{i+1}}{(1 - s - t)(2i + 2) + (s + t)2^{i+1}}\right] \quad (12)$$

When $s$ and $t$ are nonzero, the $2^{i+1}$ term dominates both the numerator and the denominator, just like in equation 8 in Huemer's generalized proof. But while the limit as $i$ approaches infinity on Huemer's partition is 1, the limit on the new partition is $(s/s + t)$. How should we assign the initial probabilities $s$ and $t$? Because $s$ doesn't matter on Huemer's partition, he allowed $s$ to be arbitrarily small in order to prove the robustness of his argument. But on the underdetermining partition, $s$ matters. Using the strategy Huemer employs throughout his paper, we can appeal to the PI and let $s = t$. We can then calculate that in the limit, $P(A_{i+1}|U_i) = (1/2)$. So this partition supports skepticism.

It's now easy to see why Huemer's partition delivers the appropriate inductivist conclusions. In Huemer's argument, we assumed that $P(A_{i+1}|U_i, \sim S) = 1/2$. Huemer claims that, using this same assumption, we should also assess $P(U_i|\sim S)$ as $(1/2)^i$. But why should we use $P(A_{i+1}|U_i, \sim S)$ to calculate $P(U_i|\sim S)$? If we do, we saddle the skeptic with a hypothesis on which the observed sequence of As is immensely improbable. No properly-thinking skeptic would ever endorse such a hypothesis. The skeptic is perfectly willing to concede that there has been a regularity in the past; she wants to know why we should expect this regularity to continue into the future.

In conclusion: Huemer's generalized argument fails as it stands because the skeptic need not and should not accept a partition on which $P(U_i)$ is tremendously improbable. The skeptic has rational grounds to ask: why not assign prior probabilities according to a partition (like $\{S_1, S_2, S_3\}$) on which $P(A_{i+1}|U_i) = P(A_{i+1})$? One can see that the underdetermination problem is very general: the existence of an alternative skeptical

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[16]See appendix A for the intermediate algebra.

partition hinges on the existence of a skeptical hypothesis (in this case, $S_2$) that is empirically equivalent to the inductivist hypothesis (in this case, $S_1$) up until the $i$th observation.

## 4   Aid from symmetry considerations?

Of course, the mere existence of a "skeptical" partition would not threaten Huemer's generalized argument if the objective Bayesian could give a principled reason for why skeptical partitions like $\{S_1,S_2,S_3\}$ are rationally inferior to partitions like $\{S,\sim S\}$. Huemer has appealed to the EPP to sort out partitions before; might the EPP support $\{S,\sim S\}$ over $\{S_1,S_2,S_3\}$ in the current case? This doesn't seem promising: it seems most plausible to say that $\{S,\sim S\}$ and $\{S_1,S_2,S_3\}$ are just two ways of carving up the *same* explanatory level (the level of possible probabilistic laws).[17] So in this section, I will develop (on the behalf of the objective Bayesian) a constraint on how one can rationally partition epistemic space across a *single* explanatory level.

Suppose I have a playing card. What is the probability that the card is the three of clubs? Here are two competing partitions across the level of possible card types:

| **Partition A** | **Partition B** |
|---|---|
| $A_1$: The card is a 2 of clubs. | $A_2$: The card is a 3 of clubs. |
| $A_2$: The card is a 3 of clubs. | $\sim A_2$: $A_1 \vee A_3 \vee A_4 \ldots A_{52}$ |
| $A_3$: The card is a 4 of clubs. | |
| ... | |
| $A_{52}$: The card is an ace of spades | |

If you apply the PI to Partition A, we give $A_2$ a prior probability of 1/52; if we apply the PI to Partition B, we give $A_2$ a prior probability of 1/2. But something seems clearly wrong with Partition B. Applying equal probabilities to $A_2$ and $\sim A_2$ may uphold the letter of the PI, but it violates its spirit. This isn't just an intuition that the inductivist has and which the skeptic may lack. There is an important symmetry between the hypotheses on the partition $\{A_1, \ldots, A_{52}\}$ that is missing on the partition $\{A_2,\sim A_2\}$. Intuitively, $\{A_2,\sim A_2\}$ is an asymmetric partition because $\sim A_2$ is a disjunction of 51 more narrow epistemic hypotheses, each of which are on an epistemic par with the hypothesis $A_2$.

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[17]Alternatively, one might claim that $\{S_1,S_2,S_3\}$ is explanatorily prior to $\{S,\sim S\}$: perhaps the fact that there is no stable objective chance is explained by the fact that the objective chance has one value at some times and another value at other times. If one takes this line, then in general one will one think that more (relevantly) specific partitions will be explanatorily prior to less specific partitions. As it happens, I'll soon give an argument for why, even on the assumption that $\{S,\sim S\}$ and $\{S_1,S_2,S_3\}$ carve up the same explanatory level, we should apply the PI to the most (relevantly) specific partition. So nothing will turn on which assumption one makes about explanatory priority.

Another possibility worth considering: could the inductivist try to apply the PI to some yet more basic explanatory level? I don't think this would affect the present choice between $\{S,\sim S\}$ and $\{S_1,S_2,S_3\}$. On this strategy, we would still need to know how to apply the PI across the sub-partition of probabilistic laws in order to generate prior probabilities.

Let's suppose that the EPP has helped us identify an appropriate explanatory level over which to apply the PI. On the behalf of the objective Bayesian, I'll now suggest a constraint on how we can appropriately partition the explanatory level in question. Let's say that a hypothesis $H_i$ is a sub-hypothesis of a hypothesis $H_j$ when (i) $H_i$ and $H_j$ are hypotheses on the same explanatory level[18], (ii) $H_i$ entails $H_j$ and (iii) $H_j$ does not entail $H_i$ (for example: $A_1$ is a sub-hypothesis of $\sim\mathbf{A_2}$). Let's say that $H_i$ can be decomposed into a sub-partition $H_j$, $H_k$, ... $H_n$ when $H_i$ is expressible as the disjunction $H_j \vee H_k \vee \dots \vee H_n$ (for example: $\sim\mathbf{A_2}$ can be decomposed into the sub-partition $\{A_1, A_3, A_4,$ ... $A_{52}\}$). Let's say that a hypothesis is *level-specific* when it has no sub-hypotheses and that a partition is *level-specific* when it contains only level-specific hypotheses (for example: $A_1$ is a level-specific hypothesis and $\{A_1, \dots, A_{52}\}$ is a level-specific partition). Finally, let's say that a partition $\{H_i, \dots, H_n\}$ is *symmetric* iff (a) it is level-specific or (b) its hypotheses can be decomposed into level-specific sub-partitions with the same number of hypotheses as members.[19]

With the above terminology, we can offer the following symmetry constraint on the behalf of the objective Bayesian: *the PI can only be appropriately applied to symmetric partitions.* With this constraint, it is permissible to apply the PI to $\{A_1, A_2, A_4, \dots A_{52}\}$ or a partition consisting of the four suits of cards, while it is impermissible to apply the PI to partitions like $\{A_2, \sim A_2\}$.

Some observations: (1) With condition (i), level-specific partitions need not be partitions across the absolute most basic explanatory level (whatever that level may be). So the fact that a four of clubs could be realized by many different microphysical states does not prevent it from qualifying as a level-specific hypothesis.[20] (2) Condition (b) is needed because, intuitively, it would be appropriate to apply the PI across a partition consisting of the four card suits, even though the hypothesis [The card is a club] is not level-specific. (3) The symmetry constraint is clearly not intended to be a stand-alone criterion for when we can apply the PI. For example, the symmetry constaint itself doesn't say anything about whether or not it is actually appropriate to apply the PI across the level of probabilistic laws.[21] The constraint is merely supposed to a guide to

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[18]Note: this means that sub-hypotheses need not be hypotheses across the absolute most basic explanatory level (whatever that level may be). I explain this condition below.

[19]Analogously, we can say that a partition across a continuous probability space is symmetric iff (i) it is level-specific or (ii) its hypotheses can be decomposed into level-specific sub-partitions with the same measure.

[20]Of course, one might still worry about the vagueness of the notion of an "explanatory level". But for present purposes, I'm granting the objective Bayesian some intuitive notion of an explanatory level, since this notion is directly built into the EPP proposal. As an aside, this example raises interesting questions about how to interpret the EPP. Since microphysical states are explanatorily prior to card-type states, does the EPP recommend applying the PI across microphysical states instead of a partition like $\{A_1, A_2, A_3, \dots A_{52}\}$? My own view is that this is not the most plausible way to interpret the EPP. In the present case, we presumably have background knowledge that standard decks of cards contain one member of each card type. With this background knowledge, the different possible microphysical states no longer seem explanatorily relevant. This interpretation helps avoid the implausible result that, *whenever* we apply the PI, it must be across across a partition at the level of fundamental physics.

[21]Nor, for example, could we simply appeal to the symmetry constraint in order to give a solution to van Fraassen's cube factory example (see 2.1).

how we should apply the PI on the assumption that we've already identified an appropriate explanatory level.

## 4.1   Symmetry at the level of probalistic laws

If symmetry considerations can show that a partition like $\{S_1, S_2, S_3\}$ is rationally inferior to a partition like $\{S, \sim S\}$, the objective Bayesian will have a response to the underdetermination problem developed in section 3. What would a symmetric partition across the level of possible probabilistic laws look like? I want to suggest that the following partition is symmetric:

> **Partition 2 (P2)**: The set of hypotheses of the form:
>     *X produces A according to the time-to-objective chance function $F_i$*
>     (where $F_i$ is a function from $[0, \infty)$ to $[0, 1]$)

P2 is plausibly symmetric because the hypotheses of P2 are plausibly level-specific: they are the most specific hypotheses at the level of possible probabilistic laws.[22] In order to apply the PI to P2, we apply a uniform probability density across the space of all functions from $[0, \infty)$ to $[0, 1]$. To consider whether the PI supports inductivism, we can let $t_{i+1}$ be the time of the $i + 1$th observation. Let F be an arbitrary function from $[0, t_{i+1})$ to $[0, 1]$. Now consider the functions from $[0, t_{i+1}]$ to $[0, 1]$ specified below, where $r_1$, $r_2$, ... are reals:

$$
\begin{aligned}
F_{r1}(x) &= Fx \quad \text{for all x } [0, t_{i+1}) \\
&= r_1 \quad\quad \text{for x} = t_{i+1} \\
F_{r2}(x) &= Fx \quad \text{for all x } [0, t_{i+1}) \\
&= r_2 \quad\quad \text{for x} = t_{i+1} \\
&\qquad\qquad ...
\end{aligned}
\tag{13}
$$

On any natural measure, the subset of $\{F_{r1}, F_{r2}, ... \}$ that take a value $r_i \leq 0.5$ at $t_{i+1}$ will have the same measure of the subset that take a value $r_i > 0.5$ at $t_{i+1}$. From this we see that applying the PI to P2 actually supports *skepticism*, not inductivism. It doesn't matter how we have updated probabilities across P2 after observing $U_i$: for any way that the function from time to objective chance has behaved up until the ith observation, there is a 0.5 probability that the function will yield an objective chance less than 0.5 at the time of the $i$th observation and a 0.5 probability that the function will yield a chance greater than 0.5.

What about Huemer's partition: is $\{S, \sim S\}$ symmetric? S will be decomposable into those level-specific hypotheses with a constant time-to-objective chance function,

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[22]This assumes that probabilistic laws yield time-to-chance conditionals (for example: "If it is time t, then X produces A with objective chance r"). One might prefer a more complex account according to which probabilistic laws yield history-to-chance conditionals (see, e.g., Lewis (1994, 478)). None of the arguments I give in this section hinge on this choice.

while $\sim S$ will be decomposable into those level-specific hypotheses where the function is not constant. On any natural measure, the latter set of hypotheses will have a greater measure than the former. So Huemer's partition is not symmetric (see footnote 19). So I conclude that it was inappropriate for Huemer to apply the PI to $\{S, \sim S\}$.[23]

But as we saw from considering P2, the problem is actually worse. When we consider a partition that *is* symmetric, it looks like the PI can actually be used to give a positive argument for skepticism.

## 5   The problem of asymmetric partitions

Huemer acknowledges that the level of possible probabilistic laws is not the most fundamental explanatory level. This can be seen in the fact the partitions at this level (such as P2) do not canvas all of epistemic space *simpliciter*. It is epistemically possible that the laws of our world are deterministic rather than probabilistic. It is also epistemically possible that a Humean account of laws is correct, according to which there are no natural necessities at all. (Note: here, I am following Huemer's taxonomy. One potentially misleading issue is that Humean theorists don't typically deny that laws of nature are probabilistic or deterministic. But Huemer is restricting the use of these terms to non-Humean laws.)

How can the objective Bayesian settle the question of whether the laws are deterministic, probabilistic, or Humean? Huemer's response is familiar: one should settle the question by applying the PI at a yet more basic explanatory level, which I will call *the level of possible law types*. One can then decide the question with Bayesian conditionalization on the evidence.

Before considering this proposal, let me note up front that moving to the level of possible law types doesn't help the objective Bayesian avoid the problem of underdetermination. In sections 3 and 4, we saw that applying the PI across the level of possible probabilistic laws supports the view that $P(A_{i+1}|U_i) = P(A_{i+1})$. But if we assumed deterministic or Humean laws, we could derive the same result. So matter how we partition the level of possible law types, the PI supports skepticism.

Putting aside the underdetermination issue, I'll now raise a second, independent objection to using the PI to support inductivism: the problem of asymmetric partitions. In this section, I'll consider how one might decide on a correct partition at the level of possible law types. I will argue that a new type of asymmetry emerges in partitions at this level which poses a serious problem for the objective Bayesian's task of assigning

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[23]One might worry: isn't the hypothesis of stable objective chances qualitatively different from all the hypotheses where the objective chance changes? So, in treating these hypotheses as on a par, isn't the symmetry constraint giving the wrong result? *Response*: Here's the dialectical situation. In 3.2, I first provided an underdetermining partition that threatens Huemer's argument to support inductivism. In section 4, I offered the symmetry constraint *on the inductivist's behalf* in order to see if the underdetermination problem could be avoided. I think the symmetry constraint helps uphold the spirit of the PI, but of course nothing forces the inductivist to take this suggestion. If the inductivist *rejects* the symmetry constraint, she still is left with the task of explaining why a partition like $\{S, \sim S\}$ is rationally superior to a "skeptical" partition like $\{S_1, S_2, S_3\}$.

objective prior probabilities.

In order to decide between deterministic, probabilistic, and Humean laws, Huemer's suggestion is that we apply the PI to the partition P3: (p. 368)

**Partition 3 (P3)**

$H_1$: Laws are deterministic.
$H_2$: Laws are probalilistic.
$H_3$: Laws are Humean.

**Partition 4 (P4)**

$H_3$: Laws are Humean.
**H\***: Laws are non-Humean.
  $H_1$: Laws are deterministic.
  $H_2$: Laws are probabilistic.

For simplicity, I will grant Huemer the major assumption that these three alternatives exhaust epistemic space at the level of possible law types. There are three hypotheses within P3, so Huemer suggests that we assign 1/3 prior probability to each (p. 368). But why not instead apply the PI to partition P4? $H_3$ now receives 1/2 prior probability, while $H_1$ and $H_2$ receive only 1/4. So P3 and P4 generate inconsistent probabilities. Of course, one could generate further partitions. For example, there are different types of non-Humean accounts of laws; one could form a partition where $H_3$ is decomposed into more specific hypotheses. Which partition is correct? This is another case where the EPP provides no useful guidance: P3 and P4 are two different ways of partitioning epistemic space at the same explanatory level. So instead, we should examine whether symmetry considerations support one partition over the others. We've already developed P2 as a symmetric sub-partition for $H_2$. I now suggest the following sub-partitions for $H_1$ and $H_3$, respectively:

> **Sub-partition 5 (S5)**: The set of hypotheses of the form:
>   *X deterministically produces A according to the time-outcome function $F_i$*
>   (where $F_i$ is a function from $[0, \infty)$ to $\{0, 1\}$)

> **Sub-partition 6 (S6)**: The set of hypotheses of the form:
>   *X produces A according to the time-outcome function $F_i$*
>   (where $F_i$ is a function from $[0, \infty)$ to $\{0, 1\}$)

Taking P2, S5, and S6 together, we have the following candidate as a partition at the level of possible law types:

> **Partition 7 (P7)**: The union of P2, S5, and S6

All of the hypotheses under P7 are level-specific. But I will argue that, even though P7 satisfies the symmetry constraint given in section 4, it is still inappropriate to apply the PI to P7. An easy way to see the problem is to compare the Lesbesgue measure of the image of the functions corresponding P2, S5, and S6. The Lesbegue measure of the image of the functions under S5 and S6 (the space of functions from $[0, \infty)$ to $\{0, 1\}$) is 0. In contrast, the Lesbesgue measure of the image of the functions in the space defined by P2 (the space of functions from $[0, \infty)$ to $[0, 1]$) is 1.

On this measure, P2 has a measure of 1 while S5 and S6 have measure 0. It follows that, if one were to apply a uniform probability density across the level-specific hypotheses of P7, one would assign a probability of 1 to P2 and a probability of 0 to S5 and S6. But this can't be a legitimate way for the objective Bayesian to support the conclusion that there are probabilistic laws of nature. On this proposal, probabilistic laws win out over deterministic laws and non-Humean laws only because there are more epistemically possible probabilistic laws. But surely this is no way to settle a dispute over the nature of laws.

Intuitively, the reason we shouldn't apply the PI to P7 is that there is again an important asymmetry between the hypotheses of P7 even though they are level-specific. The hypotheses of P2 vary across the space of functions from $[0, \infty)$ to $[0, 1]$ while the hypotheses of S5 and S6 vary across the space of functions from $[0, \infty)$ to $\{0, 1\}$. So I'll amend the analysis of symmetry given in section 4. I'll say that a partition $\{H_i, \ldots, H_n\}$ is *symmetric* iff (i) it is either level-specific or its hypotheses can be decomposed into level-specific sub-partitions with the same number of hypotheses as members and (ii) there is a uniformly varying parameter across the set $H_i, \ldots, H_n$. With this modified notion of symmetry, we can reaffirm the symmetry constraint: the PI can only appropriately be applied across symmetric partitions. (Once again, the symmetry constraint is not intended to be a stand-alone criterion for applying the PI. The symmetry constraint assumes that we've already identified an explanatory level to which we want to apply the PI. The symmetry constraint then gives two necessary conditions a partition across that explanatory level must meet for an application of the PI to be appropriate.)

If P7 isn't symmetric, it doesn't seem like there is *any* symmetric partition across the level of possible law types. But if she cannot appeal to symmetry, I don't see any principled reason the objective Bayesian can give to support the application of the PI to one partition over another at this level. This is a serious problem because the objective Bayesian's strategy cannot get off the ground without a priori prior probabilities at the fundamental explanatory level. This problem is a problem for any objective Bayesian, independent of whether she is a skeptic, inductivist, etc.[24]

## 6 Conclusion

In this paper, I've raised two independent problems confronting attempts to use the PI to support inductivism. Since Huemer offers a particularly sophisticated attempt, the focus of this paper has been Huemer's EPP proposal. That being said, the two objections raised should generalize to other proposals using the PI to support inductivism.

First, I raised an underdetermination problem: while applying the PI to certain partitions may seem to support inductivism, the skeptic can always propose applying

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[24]One way to evade the difficulties would be to provide a conclusive a priori argument for non-Humean laws, probabilistic laws, etc. But even if such an argument was available, it seems like analogous problems would arise at less fundamental levels. For example, the space of possible psychological laws considered in example 5 isn't a space with a uniformly varying parameter across hypotheses. So the problem of asymmetric partitions is a general worry for objective Bayesianism and the PI.

the PI to an alternative partition that supports skepticism. On behalf of the objective Bayesian, I proposed the symmetry constraint as a principled way to decide between the relevant partitions. But as it turns out, when we have a partition that satisfies this constraint, the PI actually supports skepticism.

My second objection applies generally to any objective Bayesian who wants to use the PI to assign a priori probabilities. On some explanatory levels, there is not a uniformly varying parameter across the set of relevant hypotheses. In these cases, it does not seem legitimate for the objective Bayesian to use the PI to assign prior probabilities. This is a serious problem because objective Bayesianism requires prior probabilities to get off the ground.

## A    Appendix: Derivation of Eq. 11

In this appendix, I provide the derivation of Eq. 11. We are interested in the overall probability of $A_{i+1}$ given $U_i$:

$$P(A_{i+1}|U_i) = P(S_1|U_i) \cdot P(A_{i+1}|U_i, S_1) + P(S_2|U_i) \cdot P(A_{i+1}|U_i, S_2) + P(S_3|U_i) \cdot P(A_{i+1}|U_i, S_3) \quad (14)$$

As explained in the 3.2, $P(A_{i+1}|U_i, S_1) = (i + 1)/(i + 2)$, $P(A_{i+1}|U_i, S_2) = 0$, and $P(A_{i+1}|U_i, S_3) = 1/2$. Since $P(A_{i+1}|U_i, S_2) = 0$, we don't need to evaluate $P(S_2|U_i)$. The probabilities of $P(S_1|U_i)$ and $P(S_3|U_i)$ are evaluated using Bayes' Theorem as follows (in 3.2, I explain how we assess the various terms in these equations):

$$P(S_1|U_i) = \frac{s \cdot \left(\frac{1}{i+1}\right)}{s \cdot \left(\frac{1}{i+1}\right) + t \cdot \left(\frac{1}{i+1}\right) + (1 - s - t)(\frac{1}{2})^i} \quad (15)$$

$$P(S_3|U_i) = \frac{(1 - s - t)(\frac{1}{2})^i}{s \cdot \left(\frac{1}{i+1}\right) + t \cdot \left(\frac{1}{i+1}\right) + (1 - s - t)(\frac{1}{2})^i} \quad (16)$$

Plugging the above results back into Eq. 14, we obtain:

$$P(A_{i+1}|U_i) = \frac{\left(\frac{s}{i+1}\right) \cdot \left(\frac{i+1}{i+2}\right) + (0) + (1 - s - t)(\frac{1}{2})^i \cdot (\frac{1}{2})}{\frac{s}{i+1} + \frac{t}{i+1} + (1 - s - t)(\frac{1}{2})^i} \quad (17)$$

$$= \frac{\left(\frac{s}{i+2}\right) + (1 - s - t)(\frac{1}{2})^{i+1}}{\frac{s}{i+1} + \frac{t}{i+1} + (1 - s - t)(\frac{1}{2})^i} \quad (18)$$

$$= \left(\frac{i+1}{i+2}\right)\left(\frac{s + (i + 2)(1 - s - t)(\frac{1}{2})^{i+1}}{s + t + (i + 1)(1 - s - t)(\frac{1}{2})^{i+1} \cdot (2)}\right) \quad (19)$$

$$= \left(\frac{i+1}{i+2}\right)\left(\frac{s2^{i+1} + (i + 2)(1 - s - t)}{(s + t)2^{i+1} + (2i + 2)(1 - s - t)}\right) \quad (20)$$

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