Russell on Plurality

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1 Russell's theory of quantification before "On Denoting"

Russell's famous paper of 1905 "On Denoting" is a document which shows that he finally arrived at the conception of quantificational logic which is still standard today. Before that paper there had been several attempts at the treatment of quantification; the most well-known among them is contained in a chapter called "Denoting" of *The Principles of Mathematics*, which was published in 1903 (this work will be referred as *The Principles* in the following). In moving from this chapter to the 1905 paper, Russell had changed some of the explanatory agenda and discarded many ideas found in the earlier treatment.

Today I would like to look at one aspect of the earlier treatment which shows the striking contrast to the later one, namely the attitudes to the distinction between plural and singular expressions.

There are at least two reasons why it is worthwhile to look at the earlier Russell's attitudes to the singular-plural distinction. One of them is historical; for any concept, it is always both interesting and instructive to see in detail how the present conception of it was prepared in the past; in the history of a concept, sometimes we can discern the possibilities which were felt only vaguely and left unexplored as well as some deadends. The talk about the unexplored possibilities brings us to another reason why we should look again at Russell's 1903 treatment of quantification; there is now an intensive research done in the logic of plurality; we have now several systems of plural logic and many theories in formal semantics whose aim is to describe and

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explain the linguistic phenomena concerning plurality. It has been said that Russell's treatment of quantification in *The Principles* was full of various mistakes and obscurities, and it cannot be denied that such a ciriticism has no grounds; but, we can find there an awareness of some data and problems which became invisible later, because these data and problems came to be regarded peripheral after the standard conception of quantificational logic took roots; and, some of those are concerned with plurality. Now it has become more and more apparent that we should be free from the long-held belief in philosophy that first-order predicate logic is all there is to logic, some of Russell's remarks in *The Principles* can be seen in a new and positive light, namely, as one of the early attempts at a more comprehensive treatment of quantification.

2 "All" and "every": plural vs. singular quantification

The chapter on denoting in *The Principles* is concerned with a class of expressions Russell calls "a denoting phrase"; it is characterized as a common noun preceded by "all," "every," "any," "a," "some," or "the" (and some synonym of it) [1] Hence, the following are all denoting phrases;

all men, every man, any man, a man, some man, the man

Turning to the 1905 paper, we find it begins with the sentence

By a 'denoting phrase' I mean a phrase such as any one of the following: a man, some man, any man, every man, all men, the present King of England, the present King of France, the centre of mass of the Solar System at the first instant of the twentieth century, the revolution of the earth round the sun, the revolution of the sun round the earth [2] .

We see that a "denoting phrase" covers the same range of English expressions in both accounts; however, the phrases whose semantics were sharply

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[1] B.Russell, *The Principles of Mathematics*, p.56. Russell's terminology for what we call here a "common noun" is a "class-concept"; according to him, "human" is a predicate and "man" is a class-concept, although "the distinction is perhaps verbal." What Russell counts as a synonym among the expressions which does not have the form specified here is not very clear. Probably "each N" might be a synonym of "every N."

[2] B.Russell, *Essays in Analysis*, edited by D.Lackey, 1973, p.103.

distinguished from each other in the earlier account, became assimilated in the later account: whereas "all men," "every man," and "any man" were all given a different semantic explanation in 1903, two years later in 1905 these three were all considered to be semantically equivalent. For example, after stating that

'C(all men)' means ' "If x is human, then C(x) is true" is always true'.

he says

'C(every man)' will mean the same as 'C(all men)'. [3]

This statement should be surprising to a reader of *The Principles*, for "every man" and "all men" were treated as a contrasting pair and given entirely different semantic accounts there; whereas "all" was explained in terms of what Russell called a "numerical conjunction," the sentence containing "every" was explained by an equivalence to some propositional conjunction of its instances. For example, if our domain were to include only two men, namely, Brown and Jones, then "all men laughed" would be equivalent to

(1) Brown and Jones laughed

and "every man laughed" would be equivalent to

(2) Brown laughed and Jones laughed.

In this particular case, there seems to be no difference in truth condition between "all men laughed" and "every man laughed" because (1) and (2) are equivalent to each other. However, if we consider a pair of sentences like Russell's own examples

(3) Brown and Jones are two of Miss Smith's suitors.
(4) Brown and Jones are paying court to Miss Smith.

we notice immediately that, whereas (4) is equivalent to

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[3] *Essays in Analysis*, p.106. Although there is no explicit statement to the effect that 'C(any man)' means the same as 'C(all men)' (or 'C(every man)'), the very lack of any statement in that matter suggests that it might be so.

(5) Brown is paying court to Miss Smith and Jones is paying court to Miss Smith.

we will not find any such equivalent sentence easily for (3) [4] ; as Russell wrote, "it is Brown *and* Jones who are two, and this is not true of either separately." [5] He described the difference between (3) and (4) in the following way:

> the first case [=(3)] concerned *all* of them [= Brown and Jones] collectively, while the second [=(4)] concerns *all* distributively, *i.e.* each or every one of them. [6]

It is interesting to note that Russell's use of "collective" and "distributive" is just the same as the one that is now standard in the literature. As this quotation from Russell itself shows, "all" can be used either collectively or distributively. Whereas "all" in (3) is collective, "all" in (1) is distributive. What distinguishes the distributive use of "all" is that the sentence in which it occurs is equivalent to a conjunction of some singular sentences; remember that "all men laughed" would be equivalent to "Brown laughed and Jones laughed" if there were no men except Brown and Jones.

Although the Russell of 1903 did not express the matter in this way, it can be said that in general there are two kinds of quantification, namely, a singular quantification and a plural quantification; to the former belong "every" (and "each" and "any") and "all" used distributively, and to the latter belongs the collective "all." A sentence with singular quantification is either a grammatically singular sentence such as "Every man laughed" or equivalent to a conjunction of singular sentences just as it is the case with "All men laughed" when the domain of "men" is finite. In contrast to this, a sentence with the collective "all" like "All men met in an assembly" cannot be reduced to any singular sentences; the plurality is essential to it [7] .

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[4] Later we will see that there is such a sentence after all.

[5] *The Principles*, p.57.

[6] *Ibid.*

[7] Can we find the two kinds of quantification even in a language like Japanese which does not have any systematic distinction between singular and plural? It might be said that, of the following pair of sentences, the former involves collective quantification, whereas the latter involves distributive quantification:

(a) *Gakusei-wa minna waratta.*

In the 1905 paper we find no mention of the distinction between the collective and distributive "all"; the essentially plural constructions like "Brown and Jones are two of Miss Smith's suitors" or "All men met in an assembly" made no appearance in it either. But, back in 1903, Russell had many things to say about the collective "all" and plurality in general. Let us turn to them.

3 Plurality and classes as many

In *The Principles*, the chapter on classes comes just after the one on denoting. In the summary of this chapter, Russell characterized a class thus:

> All classes, whether finite or infinite, can be obtained as the objects denoted by the plurals of class-concepts—men, numbers, points, etc. [8]

In the scheme of *The Principles* there is nothing linguistic in the denoting relation; it holds between some concepts called "denoting concepts" and objects; moreover, Russell's "object" surprisingly covers both singular and plural, and hence it is the word with the widest extension, though it is very doubtful such a usage is at all coherent, as Russell himself admits [9] . With these terminological matters in mind, we should note that (i) a class need not be a single thing, even though its linguistic representation inevitably makes it a single thing, and that (ii) "the plurals of class-concepts" does not refer to any linguistic items but the essentially non-linguistic items; but, it is not hard to find the linguistic items corresponding to these non-linguistic items: they are the common nouns in plural, such as "men," "numbers," "points" and the like, just as the above quotaion shows.

(b) *Dono gakusei mo waratta.*

However, we should consider more data before we will be able to get any conclusion, in particular, we should ask whether there is any good argument for the singularity of "gakusei" in (b).

[8] *The Principles*, p.80.

[9] "I shall use the word *object* in a wider sense than *term*, to cover both singular and plural, and also cases of ambiguity as "a man." The fact a word can be framed with a wider meaning than *term* raises grave logical problems." (*The Principles*, p.55, footnote.) However, in a place about 10 pages earlier (p.43), Russell had said that "term" was the widest word in the philosophical vocaburary.

Classes can be also characterized as the objects denoted by the denoting concepts indicated by the denoting phrases of the form "all u's," since "all u's" is synonymous with "u's" according to Russell [10] . Therefore, the concept of plurality is involved both in the analysis of classes and that of quantification with "all"; Russell thought that both raised the same problem, namely, "the meaning of the plural" [11] .

What he wanted to maintain in the chapter on classes of *The Principles* was that classes were essentially plural objects, provided that "object" is understood as having the very peculiar (and doubutfully coherent) sense of his. Within *The Principles* such plural objects were already introduced in the preceding chapter on denoting: they are the objects denoted by the denoting concept *all a's*, and called the numeral conjunctions of terms. We encountered one example of them before in the sentence (3) [= Brown and Jones are two of Miss Smith's suitors], where "Brown and Jones" refers to (or, indicates, as Russell says) a class and it is a numerical conjunction of Brown and Jones. Although this is an example of a finite class, Russell did not think the matter was logically different in an infinite class [12] , and concentrated on the finite cases.

Suppose there are only three students A, B, and C, and all of them came together to the party; then, we can describe this situation by a sentence

(6) All students came together to the party.

According to Russell's theory, "all students" in this situation denotes (or, indicates a denoting concept that denotes, to be more exact—in the following, I also use "denote" in such a loose way) a numerical conjunction of A, B, and C. Thus, the same situation can be also described by the following sentence:

(7) A, B, and C came together to the party.

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[10] *The Principles*, pp.67, 72.

[11] *The Principles*, p.72.

[12] Characteristically, he writes thus:

> I believe this distinction [= define a class either by enumerating its members or by specifying its defining property] to be purely psychological: logically, the extensional definition appears to be equally applicable to infinite classes, but practically, if we were to attempt it, Death would cut short our laudable endeavour before it had attained its goal. (*The Principles*, p.69.)

In Russell's theory, a numerical conjunction of A, B, and C is nothing but the class of A, B, and C. His question is whether such a class is to be regarded as one or many. The following quotation shows very well the dilemma Russell found himself

> Taking the class as equivalent simply to the numerical conjunction "A and B and C and etc.," it seems plain that it is many; yet it is quite necessary that we should be able to count classes as one each, and we do habitually speak of *a* class. Thus classes would seem to be one in one sense and many in another. [13]

It might be thought that there is one possibility Russell failed to consider, namely, the possibility that "A, B and C" in (7) might not be a semantical unit. In that case, (7) would express a 3-term relation holding between A, B and C, and the "and" between B and C would function as a punctuation mark. But I think it is not hard to see why Russell did not bother to consider this possibility; firstly, he thought that, in the described situation, "A, B and C" refers to the object (in Russell's sense) denoted by "all students"; secondly it seems obvious that any expression which refers to some object constitutes a semantic unit [14].

On the assumptions that "A, B and C" in (7) consitutes a semantic unit, and that what this semantic unit does is referring to a Russelian "object," the question is whether "A, B and C" refers to many things or not. Russell thought that a positive answer to this question led one to the idea of a class as many, and a negative answer led to the idea of a class as one. We can also reformulate this question as the one asking whether "A, B and C" is a plural term which has plural reference or it is a singular term which has singular reference. I believe Russell's struggles to make sense of the idea of a class as many in *The Principles* can be interpreted as the struggles to make sense of the idea of plural reference.

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[13] *The Principles*, p.76

[14] One may think, even if we drop the assumption that "all students" denotes an object just as Russell himself did in 1905, there is no future for this option. For, if (7) contained a 3-place predicate such as "x, y and z came together to the party," then we would need a countably infinite number of primitive predicates, as there is no limit in the number of people who can come together. This is obviously absurd. However, this is rather a short-sighted reaction, because there is now an option of admitting multigrade predicates. See A.Oliver and T.Smiley, "Multigrade Predicates" *Mind* 113 (2004).

4 The discovery of logical form and the disappearance of plurality

There is a remark in *The Principles* which could be regarded as a methodological principle guiding Russell's thinking in this period.

> On the whole, grammar seems to me to bring us much nearer to
> a correct logic than the current opinions of philosophers; and in
> what follows, grammar, though not our master, will yet be taken
> as our guide. [15]

As is well known, after "On Denoting" Russell ceased to have such a high opinion of grammar. Instead, he came to the conclusion that grammar stood almost always as an obstacle for a philosopher to discover the truth. Thus, the logical form of a sentence had to be distinguished sharply from its grammatical form.

However, what is called "the logical form of a sentence" is nothing but a translation of the original sentence into some logical language; for Russell, it was the language of *Principia Mathematica*, and for later philosophers, especially those influenced by Quine, it is the language of first-order predicate logic. So, the grammatical constructions which have no counterparts in such a logical language were either paraphrased away or just ignored. We can see the working of such a strategy in the case of plurals. On one hand, some plural constructions like distributive "all" are paraphrased in terms of singular constructions as a matter of course. Even Russell's "Brown and Jones are two of Miss Smith's suitors" (=(3)) can be paraphrased into a sentence which has no plural constructions. It is equivalent to the following.

> Brown is Miss Smith's suitor, and Jones is Miss Smith's suitor,
> and Brown is not identical with Jones

On the other hand, the fact that we never meet such a sentence like "All students met at an assembly" in a standard textbook of logic, shows that such plural constructions which cannot be easily paraphrased away are just ignored.

We can see the early signs of such a tendency in Russell's writings after "On Denoting"; in them, a plural "all" quantification was always assimilated to a singular "every" quantification, and it was not mentioned that there

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[15] *The Principles*, p.42.

were cases of plural quantification that could not be so assimilated; it is a significant fact that a sentence such as "Brown and Jones are two" did not make appearance in Russell's later writings.

There is one striking example of such a disregard of plural in *Introduction to Mathematical Philosophy* of 1919; in a passage where Russell was explaining how traditional term logic could be reformulated in terms of propositional functions, he wrote "all S is P" instead of "all S are P" or "all S's are P," even though he gave "*men*" as an example of S [16].

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[16] *Introduction to Mathematical Philosophy*, p.161.