# Indirect Passives and Relational Nouns (III) 

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#### Abstract

In the previous installment of this paper [Iida 2013], it was claimed that a relational noun is a noun which takes an argument. Now we start looking at the various ways of such argument-taking. The argument place of a relational noun is filled by either explicit binding or implicit binding. In the following sections, we will discuss explicit argument binding by means of a definite or bare indefinite noun phrase. The topics that will be discussed are: the interaction between the conjoining particle "to" and the possessive (genitive) particle "no", the comparison of scopal difference with distributive/non-distributive distinction, and the interplay between a relational noun and plurality in general.


## 8 Explicit argument binding by a definite or bare indefinite noun phrase that results in an indefinite noun phrase

In a Fregean terminology, a relational noun is an unsaturated expression; it has an argument place which should be filled in if it is to be used in a statement. We claim that the "saturation" of a relational noun always occurs through binding its argument. Binding is either explicit or implicit. Explicit binding is always accompanied with an occurrence of a genitive particle "no", while the particle "no" does not occur in implicit binding.

The simplest way of filling in the argument place of a relational noun is seen in the following examples.

| (57) | Taro | no | seito | ga |
| :--- | :--- | :--- | :--- | :--- | atsumatta.

(Taro's pupils got together, or some pupils of Taro got together.)

[^0]| (58) | Watashi | no | seito | ga | atsumatta. |
| :--- | :--- | :--- | :--- | :--- | :--- |
| I | GEN | pupil(s) | NOM | got together |  |

(My pupils got together, or some pupils of mine got together.)
A proper name like "Taro" and a personal pronoun like "watashi" are nouns which typically refer to some definite person or persons ${ }^{1}$. In general, a noun phrase that refers to some definite object or objects will be called "definite noun phrase" or "definite NP".

A question that naturally arises here is whether the complex noun phrase consisting of a definite noun and a relational noun is definite or not. In particular, is "Taro no seito" in (57), or "watashi no seito" in (58) definite or not?

It is easily seen that both (57) and (58) have two readings. For example, as the English translations attached above show, (57) has two readings, namely
(a) Taro's pupils got together.
(b) Some pupils of Taro got together.

According to a reading that corresponds to (a), "Taro no seito" is a definite NP, while it is an indefinite NP according to the other reading that corresponds to (b).

Although we can know whether a particular occurrence of a noun phrase is definite or not only from the conversational context, this knowledge is vital to understand any particular utterance in Japanese. Hence, it will be a great help for a systematic study of Japanese syntax and semantics if we have some device to mark an occurrence of a noun phrase as definite. We will do this in the following way.
(a) $\langle\langle$ Taro $\rangle$ no seito $\rangle$ ga atsumatta.
(b) $\langle$ Taro $\rangle$ no seito ga atsumatta.

As "Taro" in (57) is a proper name, and hence, a definite NP, its occurrence should be enclosed with angle brackets. In the reading (a), the complex noun phrase "Taro no seito" also occurs as a definite noun phrase in contrast to its indefinite occurrence in the reading (b).

Let $\alpha$ be a noun phrase and $R$ a relational noun. If $\alpha$ occurs as a definite NP, then the combination of $\alpha$ and $R$ results in either a definite NP or indefinite NP.

[^1](a) definite NP: $\langle\langle\alpha\rangle$ no $R\rangle$
(b) indefinite NP: $\langle\alpha\rangle$ no $R$

How can we find out, however, whether a particular occurrence of "Taro no seito" is definite or not? There is a test whether a noun phrase in a sentence occurs as indefinite or not. It is to transform the original sentence into an existential sentence by a Japanese counterpart of there-insertion and to see whether there is a reading of the new sentence that preserves the truth condition of the original ${ }^{2}$.

From (57), applying such an operation, we get a sentence
(59) Atsumatta Taro no seito ga iru. got together GEN pupil(s) NOM exist

If "Taro no seito" in (57) is indefinite and refers to some pupil or pupils of Taro, then the truth condition of (57) should be the same as that of (59) uttered in the same context. In contrast, if "Taro no seito" in (57) refers to some particular pupil or pupils of Taro and its occurrence in (59) is also understood in the same manner, then (59) means that such particular pupil or pupils of Taro are present now.

An indefinite noun phrase may be complex like "Taro no seito" or simple like "seito". An indefinite noun phrase may be a quantified one like "san-nin no seito" (three pupils). For want of a better term, I call a non-quantified indefinite noun phrase a "bare" indefinite noun phrase. Any of the following can occur as an indefinite noun phrase as well as a definite noun phrase:
(i) seito (pupil(s))
(ii) Taro no seito (pupil(s))
(iii) wakai seito (young pupil(s))
(iv) san-nin no seito (three pupils)
(v) ta-suu no seito (many pupils)

Among them, (iv) and (v) are quantified noun phrases, and hence, they are not bare indefinite noun phrases if they occur as indefinites, while any of (i)(iii) can occur as a bare indefinite noun phrase in a sentence. The cases in which an indefinite NP like (iv) and (v) is combined with a relational noun will be discussed later with those other cases in which the argument place of a relational noun is bound by a quantified noun phrase. In the present chapter, we discuss the combination of a bare indefinite noun phrase with a relational noun as well as that of a definite noun phrase with it.

Suppose that $\alpha$ is a non-quantified noun phrase that occurs as indefinite. Does the combination of $\alpha$ and a relational noun $R$ result in a definite NP or indefinite NP? It seems that the result will be always an indefinite NP; it

[^2]is inconceivable that some definite things are determined by having a relation with some indefinite things. Thus, the combination of a bare indefinite noun phrase $\alpha$ and a relational noun $R$ will be always like
(c) indefinite NP: $\alpha$ no $R$,
and never like
definite NP: $\langle\alpha$ no $R\rangle$.
We have a noun phrase of the pattern (c) in the following sentence.
\[

$$
\begin{array}{lllll}
\text { (60) } & \begin{array}{lll}
\text { Otroko-no-ko } & \text { no } & \text { oya } \\
\text { male-child(ren) } & \text { GEN } & \text { ga } \\
\text { parent(s) } & \text { NOM } & \text { atsumatta. } \\
\text { got together } \\
\text { (Parents of a boy/boys got together.) } & &
\end{array} .
\end{array}
$$
\]

### 8.1 Plurality as a relational noun argument

Let me start with the cases (b) and (c), namely, the cases in which the combination of a noun phrase and a relational noun forms an indefinite noun phrase. First, suppose $\alpha$ is a noun phrase that occurs as definite and that $R$ is a relational noun. How should the semantic values of " $\langle\alpha\rangle$ no $R$ " be given in terms of those of $\alpha$ and $R$ ? What will be a semantic account of "〈Taro〉 no seito" (pupil(s) of Taro)?

There seems to be no difficulty in giving such an account. Isn't the following the necessary and sufficient condition for some things $X$ to be among the semantic values of " $\langle$ Taro $\rangle$ no seito"?

$$
\text { (i) } \exists Y[\operatorname{Val}(Y, "\langle\text { Taro }\rangle ") \wedge \operatorname{Val}(\langle X, Y\rangle \text {, "seito" })]
$$

As we know that

$$
\text { for any } Y, \operatorname{Val}(Y, "\langle\text { Taro }\rangle ") \leftrightarrow Y \equiv \text { Taro }^{3}
$$

and that
for any $X$ and $Y, \operatorname{Val}(\langle X, Y\rangle$, "seito" $) \leftrightarrow X$ are pupils of $Y$,
we know that (i) comes to the same as
${ }^{3}$ If "Taro" is indefinite as in a sentence
(ii) Futari no Taro ga atsumatta. two (persons) GEN NOM got together. (Two Taros got together.)
then the semantic clause for it will be like this.
for any $X, \operatorname{Val}(X$, "Taro") $\leftrightarrow X$ are named "Taro".
(ii) $X$ are pupils of Taro.

There is no doubt that this is the right result.
As for the case in which $\alpha$ occurs as a bare indefinite noun phrase, again there seems to be no difficulty. The necessary and sufficient condition for some things $X$ to be among the semantic values of "otoko-no-ko no oya" (parents of a boy/boys) seems to be given in the following, which is almost the same as the previous one.
(iii) $\exists Y[\operatorname{Val}(Y$, "otoko-no-ko") $\wedge \operatorname{Val}(\langle X, Y\rangle$, "oya" $)]$

This is equivalent to
(iv) $\exists Y[Y$ are boys $\wedge X$ are parents of $Y)$.

Namely,
(v) $X$ are parents of some boys.

Again everything seems to be all right.
There is some reason, however, to suspect that such an impression might be premature, at least for the cases in which a definite noun phrase is involved. Can we generalize a semantic account like (i) to other cases like the following?
(61) Taro to Hanako no seito ga atsumatta. and GEN pupil(s) NOM got together
(Pupils of Taro and Hanako's got together.)

| Kodomo-tachi | no | sensei | ga | atsumatta. |
| :--- | :--- | :--- | :--- | :--- |
| child-PL | GEN | teacher(s) | NOM | got together |

(Teachers of children got together.)
The noun phrase "Taro to Hanako no seito" is a definite NP. There are two readings for (62); according to one, "kodomo-tachi" is a definite NP and refers to a certain group of children given in the context, while it is an indefinite NP according to the other. Unlike (57) and (58), these noun phrases are explicitly plural in their form and meaning ${ }^{4}$.

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\({ }^{4}\) Semantics of plural suffix "-tachi" may be generally given by the following.
    Let \(\alpha\) be a common noun used for people (and some other animals), then
\[
\operatorname{Val}(X, " \alpha \text {-tachi" }) \leftrightarrow[\operatorname{Val}(X, \alpha) \wedge \neg \mathrm{I} X] .
\]
```

Here " $\mathrm{I} X$ " means that $X$ is an individual, which is a concept definable by the among relation $\eta$, namely,

$$
\mathrm{I} X \leftrightarrow \forall Y[Y \eta X \rightarrow X \eta Y] .
$$

See [McKay 2006], p. 120.
I wish to add that if $\alpha$ is not a common noun but a proper noun for people like "Taro" then the semantic values of " $\alpha$-tachi" are a number of people that include the semantic value of $\alpha$.

It might be argued that a semantical account like (i) should be all right even for the cases like (61) and (62) in the sense that there will be a match between the original sentences and the sentences that state their truth conditions which can be derived from the above account. The problem, however, is that these sentences are ambiguous and that the source of the ambiguity is in the compound noun phrases "Taro to Hanako no seito" and "kodomo-tachi no sensei" ${ }^{5}$. A semantic account of these noun phrases should explain why there is such an ambiguity and show how to derive the different readings in a principled way.

Take (61). It is ambiguous between two readings, which may be roughly expressed in the following English sentences.
(A) Pupil(s) of Taro and pupil(s) of Hanako got together.
(B) Pupil(s) whom Taro and Hanako both teach got together.

Suppose that John and Mary are pupils of Taro and that Mary and Susan are pupils of Hanako. If both John and Susan came but Mary did not come, then (61) is true in the first reading but it is false in the second.

According to the reading (A), (61) is equivalent to

| (61A) | Taro | to | Hanako | sore-zore | no |
| :---: | :--- | :--- | :--- | :--- | :--- |
| ga | and | seito |  |  |  |
| NOM | atsumatta. |  |  | get together | pupil(s) |

According to the reading (B), on the other hand, (61) is equivalent to


Similarly, (62) has two readings. In one, each of the children referred to by "kodomo-tachi" may have different teachers and these teachers got together, while in the other reading each of the teachers who got together teach all of these children.

What could be the source of such an ambiguity? It is very difficult to think that it comes from some ambiguity of a word that occurs in (61) or (62). If it is not a lexical ambiguity, then it is reasonable to suspect that it is a structural ambiguity. At least in the case of (61), there is a good candidate for a source of such a structural ambiguity; we might think that it contains two operators

[^3]which operate on noun phrases and return a more complex noun phrase, namely, the noun conjoining particle "to" and the genitive particle "no". Whenever an expression contains a number of operators, there arises an issue of relative scope of different occurrences of operators, which is one of the typical sources of structural ambiguity. Thus, let us try to see whether we can show this is the real source of the ambiguity that is found in a sentence like (61).

### 8.2 Wide scope and narrow scope readings of a noun phrase with the conjoining particle "to"

Take the reading (A) of "Taro to Hanako no seito". As it was remarked above, in this reading, this noun phrase is equivalent to

| Taro | no | seito | to | Hanako | no | seito |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | GEN | pupil(s) | and |  | GEN | pupil(s) |

Here the conjoining particle "to" is the principal operator in whose scope the two occurrences of the genitive particle "no" lie. We can represent this fact in the following way.
(A) to (Taro no seito, Hanako no seito)

If we think the combination of the genitive particle "no" and a relational noun $R$ can operate on a list of noun phrases, this can be rewritten.
(A) to ((Taro, Hanako) no seito).

This shows that the relative scope of "to" is wider than the argument filling operation that is indicated by the genitive particle.

In the reading (B), on the other hand, the relative scope of "to" is narrower than "no", and hence, it would be reasonable to represent the reading (B) of "Taro to Hanako no seito" in this way.
(B) (to(Taro, Hanako)) no seito.

From now on, let us call a reading like (A) a "wide scope reading" and a reading like (B) a "narrow scope reading". If we wish to give a formal treatment of a noun phrase like "Taro to Hanako no seito", we should have some way of representing the different readings of it. One way of doing this is to mark the scope of "to" in the following way.
wide scope reading: [Taro to Hanako no seito $]_{\text {to }}$ narrow scope reading: [Taro to Hanako] to no seito

In general, given noun phrases $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ conjoined by "to" and a relational noun $R$, there are two readings of a noun phrase

$$
\alpha_{1} \text { to } \alpha_{2} \text { to } \ldots \alpha_{n} \text { no } R,
$$

namely,
wide scope reading: $\left[\alpha_{1} \text { to } \alpha_{2} \text { to } \ldots \text { to } \alpha_{n} \text { no } R\right]_{\text {to }}$, and narrow scope reading: $\left[\alpha_{1} \text { to } \alpha_{2} \text { to } \ldots \text { to } \alpha_{n}\right]_{\text {to }}$ no $R$.

These two readings can be expressed in a semantically more perspicuous notation, namely,
wide scope reading: $\mathbf{t o}\left(\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right)\right.$ no $\left.R\right)$, and
narrow scope reading: ( $\left.\mathbf{t o}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right)\right)$ no $R$.
Wide scope reading can be further transformed into

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to ( }\mp@subsup{\alpha}{1}{}\mathrm{ no }R,\mp@subsup{\alpha}{2}{}\mathrm{ no }R,\ldots,\mp@subsup{\alpha}{n}{}\mathrm{ no }R\mathrm{ ).
```

If the ambiguity in a sentence like (61) comes from the presence of two term operators "to" and "no $R$ " and the difference in relative scope of them, then the semantical accounts of these two operators should explain this ambiguity as their natural consequence. In particular, our account of the operation of filling in the argument place of a relational noun should be combined with that of the conjoining particle "to" in such a way that the difference between a wide scope reading and a narrow scope reading is derived from it.

We have not yet stated the syntax and semantics of the conjoining particle "to", however. For its syntax, the following will be enough.

Suppose that each of $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ is a definite NP or a bare indefinite NP.

If all of $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ are definite NPs, then " $\mathbf{t o}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right)$ " is a definite NP; otherwise, it is a bare indefinite NP.

Although we have also a more familiar notation

$$
\alpha_{1} \text { to } \alpha_{2} \text { to } \ldots \text { to } \alpha_{n}
$$

in our formalism, it will have to be always accompanied a pair of brackets and a scope indicator in it.

For a semantic account of the conjoining particle, we can use the concept of sum that can be defined in plural logic ${ }^{6}$. Let

$$
\Sigma\left(X_{1}, X_{2}, \ldots, X_{n}\right)
$$

be the sum of $X_{1}, X_{2}, \ldots, X_{n}$. Then, the semantic axiom of the conjoining particle "to" is given in the following.

## Axiom 8.1

${ }^{6}$ [McKay 2006], p. 131.

Let $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ be definite or bare indefinite noun phrases, then

$$
\begin{aligned}
& \operatorname{Val}\left(X, " \operatorname{to}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right) "\right) \leftrightarrow \exists X_{1} \exists X_{2} \ldots \exists X_{n}\left[\operatorname{Val}\left(X_{1}, \alpha_{1}\right)\right. \\
& \left.\quad \wedge \operatorname{Val}\left(X_{2}, \alpha_{2}\right) \wedge \ldots \wedge \operatorname{Val}\left(X_{n}, \alpha_{n}\right) \wedge X \equiv \Sigma\left(X_{1}, X_{2}, \ldots, X_{n}\right)\right]
\end{aligned}
$$

It is easily verified, for example, that the semantic values of "to(Taro,
Hanako)" is $\Sigma$ (Taro, Hanako). This last will be also written as " $\lfloor$ Taro, Hanako $]$ ", which is a plural term that refers to Taro and Hanako ${ }^{7}$.

Let us adopt the following axiom as an account of filling in the argument of a relational noun by a definite noun phrase. This is a generalization of our account of "Taro no seito" (pupil(s) of Taro) and "otoko-no-ko no oya" (parent(s) of boy(s)) above.

## Axiom 8.2

Suppose that $\alpha$ is a definite or bare indefinite noun phrase and $R$ a relational noun. Then,
$\operatorname{Val}(X, "(\alpha$ no $) R ") \leftrightarrow \exists Y[\operatorname{Val}(Y, \alpha) \wedge \operatorname{Val}(\langle X, Y\rangle, R)]$.

We need two more axioms which tell us what a scope indicator for "to" does.

## Axiom 8.3

Let $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ be definite or bare indefinite noun phrases, and $R$ a relational noun, then
(i) $\operatorname{Val}\left(X, "\left[\alpha_{1} \text { to } \alpha_{2} \text { to } \ldots \text { to } \alpha_{n} \text { no } R\right]_{\mathbf{t o}} "\right) \leftrightarrow$
$\operatorname{Val}\left(X\right.$, "to ( $\alpha_{1}$ no $R, \alpha_{2}$ no $R, \ldots, \alpha_{n}$ no $R$ )").
(ii) $\operatorname{Val}\left(X, "\left[\alpha_{1} \text { to } \alpha_{2} \text { to } \ldots \text { to } \alpha_{n}\right]_{\text {to }} "\right) \leftrightarrow$ $\operatorname{Val}\left(X, " \operatorname{to}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right) "\right)$.

We can verify that these axioms allow us to derive the two readings of "Taro and Hanako no seito".

The wide scope reading is easily derived; it will run like the following.
$(60-\mathrm{W}) \operatorname{Val}\left(X, "[\text { Taro to Hanako no seito }]_{\text {to" }}\right) \leftrightarrow$ $\exists X_{1} \exists X_{2}\left[X_{1}\right.$ are pupils of Taro $\wedge X_{2}$ are pupils of Hanako $\wedge$ $\left.X \equiv \Sigma\left(X_{1}, X_{2}\right)\right]$.

[^4]Its right hand side says that the semantical values of the wide scope reading of "Taro to Hanako no seito" consist of pupils of Taro on one hand and pupils of Hanako on the other. It should be noted that this is not the same as saying that $X$ are either pupils of Taro or pupils of Hanako. The last is what is expressed by the following sentence.

| (63) Taro | ka Hanako | no | seito | ga | atsumatta. |
| :---: | :--- | :--- | :--- | :--- | :--- |
| or | GEN | pupil(s) | NOM | got together |  |

(63) can be true when no pupils of, say, Taro are among those who got together, while both pupils of Taro and those of Hanako should be among them for (60) to be true.

This might be a good place to see what will be a semantic acccount of a noun phrase like

$$
\begin{array}{ccccl}
\text { Taro } & \text { ka } & \text { Hanako } & \text { no } & \text { seito } \\
& \text { or } & & \text { GEN } & \text { pupil(s) } \\
\\
\text { (pupil(s) of Taro or } & \text { Hanako) }
\end{array}
$$

Unlike "Taro to Hanako no seito" this noun phrase does not strike us as ambiguous, and I think this impression must be right. What needs explaining is why this is so.

We can set up an axiom for the conjoining particle "ka" mimicking that for "to".

## Axiom (conjoining particle "ka")

Let $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ be definite or bare indefinite noun phrases, then

$$
\begin{aligned}
& \operatorname{Val}\left(X, " \operatorname{ka}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right) "\right) \leftrightarrow \operatorname{Val}\left(X, \alpha_{1}\right) \vee \operatorname{Val}\left(X, \alpha_{2}\right) \vee \ldots \\
& \quad \vee \operatorname{Val}\left(X, \alpha_{n}\right)
\end{aligned}
$$

As there are two operators "ka" and "no", the issue of relative scope should arise here, too. Thus, we have a wide scope reading and a narrow scope reading of "Taro ka Hanako no seito".
wide scope: [Taro ka Hanako no seito] ${ }_{k a}$
narrow scope: [Taro ka Hanako] ${ }_{\text {ka }}$ no seito
Just as we had an axiom for scope indicator, we have

## Axiom (scope indicator for "ka")

Let $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ be definite or bare indefinite noun phrases, and $R$ a relational noun, then
(i) $\operatorname{Val}\left(X, "\left[\alpha_{1} \mathbf{k a} \alpha_{2} \mathbf{~ k a} \ldots \mathbf{k a} \alpha_{n} \text { no } R\right]_{\mathbf{k a}}\right.$ ") $\leftrightarrow$
$\operatorname{Val}\left(X\right.$, "ka ( $\alpha_{1}$ no $R, \alpha_{2}$ no $R, \ldots, \alpha_{n}$ no $\left.R\right) "$ ).
(ii) $\operatorname{Val}\left(X, "\left[\alpha_{1} \mathbf{k a} \alpha_{2} \mathbf{k a} \ldots \mathbf{k a} \alpha_{n}\right]_{\mathbf{k a}}\right.$ " $) \leftrightarrow$
$\operatorname{Val}\left(X, " \operatorname{ka}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right) "\right)$.

From these axioms, we can prove that with "ka" conjoined noun phrases the wide scope reading and narrow scope one are logically equivalent. Here we prove this for the simplest case, but it is not difficult to generalize this.

Consider the noun phrase " $\alpha_{1}$ ka $\alpha_{2}$ no $R$ ". For its narrow scope reading, we have the following from the above axioms.
(n) $\operatorname{Val}\left(X, "\left[\alpha_{1} \text { ka } \alpha_{2}\right]_{\mathbf{k a}}\right)$ no $\left.R "\right) \leftrightarrow \exists Y\left[\left[\operatorname{Val}\left(Y, \alpha_{1}\right) \vee \operatorname{Val}\left(Y, \alpha_{2}\right)\right] \wedge \operatorname{Val}(\langle X, Y\rangle, R)\right]$.

On the other hand, for its wide scope reading, what we have is this.
$\left.(\mathrm{w}) \operatorname{Val}\left(X, "\left[\alpha_{1} \text { ka } \alpha_{2} \text { no } R\right]_{\mathbf{k a}}\right) "\right) \leftrightarrow \exists Y\left[\operatorname{Val}\left(Y, \alpha_{1}\right) \wedge \operatorname{Val}(\langle X, Y\rangle, R)\right] \vee$
$\exists Y\left[\operatorname{Val}\left(Y, \alpha_{2}\right) \wedge \operatorname{Val}(\langle X, Y\rangle, R)\right]$.
The right hand side of (n) is of the form

$$
\exists Y[[F(Y) \vee G(Y)] \wedge H(Y)]
$$

while the right hand side of $(\mathrm{w})$ is of the form

$$
\exists Y[F(Y) \wedge H(Y)] \vee \exists Y[G(Y) \wedge H(Y)]
$$

But they are logically equivalent. Thus, the narrow scope reading of " $\alpha_{1}$ ka $\alpha_{2}$ no $R$ " and the wide scope one are logically equivalent.

Let us go back to our main business and see how the narrow scope reading of (60) can be derived. First of all, we must remind ourselves that " $\left[\alpha_{1}\right.$ to ... to $\left.\alpha_{n}\right]_{\text {to }} "$ is also a definite noun phrase, if $\alpha_{1}, \ldots, \alpha_{n}$ are all definite noun phrases. The derived condition will be the following.

```
(60-N) Val( }X,\mp@code{"[Taro to Hanako] to no seito") }\leftrightarrowXX\mathrm{ are pupils
of \Sigma(Taro, Hanako).
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As will be explained in the following subsection, a relational noun "seito" (pupil) is distributive in its argument, and hence, the right hand side of $(60-\mathrm{N})$ amounts to
$X$ are pupils of Taro and pupils of Hanako.
Thus, the narrow scope reading of "Taro to Hanako no seito" refers to those who are taught by both Taro and Hanako, as we claimed above

### 8.3 Scopal difference and distributivity

There is a well-known distinction between distributive and non-distributive predication. Consider the following pair of sentences.

| (65) | Taro | to and | Hanako | to and | Jyon | ga <br> NOM | kita. came |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (Taro, Hanako and John came.) |  |  |  |  |  |  |  |
| (66) | Taro | to and | Hanako | to and | Jyon | ga <br> NOM | atsumatta. got together |

(Taro, Hanako and John got together.)

The predication in (65) is said to be distributive, because it follows from (65) that each of Taro, Hanako, and John came. In contrast, the predication in (66) is said to be non-distributive, because no such consequence follows from (66); in the first place, it does not make sense to say of one person that she got together.

If we construe the verbal phrases like "kita" (came) and "atsumatta" (got together) as monadic predicates, the contrast between them can be seen in the following. (" $x \eta X$ " means that an individual $x$ is among $X$.)

$$
\begin{aligned}
& \text { True: } \operatorname{Val}(X, " k i t a) \rightarrow \forall x[x \eta X \rightarrow \operatorname{Val}(x, " k i t a ")] . \\
& \text { False: } \operatorname{Val}(X, \text { "atsumatta }) \rightarrow \forall x[x \eta X \rightarrow \operatorname{Val}(x, " \text { atsumatta" })] .
\end{aligned}
$$

We may make a similar distinction among noun phrases, because from the standpoint of logic a noun phrase can be regarded as a predicate that applies to things. Let $\alpha$ be a noun phrase. $\alpha$ is distributive if and only if

$$
\operatorname{Val}(X, \alpha) \rightarrow \forall x[x \eta X \rightarrow \operatorname{Val}(x, \alpha)]
$$

Many of Japanese common nouns like "neko" (cat, cats) are distributive ${ }^{8}$.

$$
\operatorname{Val}(X, \text { "neko" }) \rightarrow \forall x[x \eta X \rightarrow \operatorname{Val}(x, \text { "neko" })] .
$$

There are also non-distributive nouns in Japanese, however. In $\S 7.1$ of [Iida 2013], we have discussed non-relational nouns derived from relational nouns such as "oyako" (parent-child) and "tomodachi" (friends) in its non-relational use $^{9}$. They are non-distributive nouns. A complex noun phrase like "inu to neko" ( $\operatorname{dog}(\mathrm{s})$ and $\operatorname{cat}(\mathrm{s})$ ) gives us another example of non-distributive noun phrases. Still another example is a noun phrase with a quantity word like "sannin no kodomo" (three children).

As a relational noun has ordered pairs of things as its semantic values, it can be distributive or not in two different ways; it can be distributive in its arguments, and it can be so in its values.

$$
\begin{aligned}
(\text { Dist-A) }) & \operatorname{Val}(\langle X, Y\rangle, R) \rightarrow \forall y[y \eta Y \rightarrow \operatorname{Val}(\langle X, y\rangle, R)] \\
(\text { Dist-V) } & \operatorname{Val}(\langle X, Y\rangle, R) \rightarrow \forall x[x \eta X \rightarrow \operatorname{Val}(\langle x, Y\rangle, R)]
\end{aligned}
$$

If (Dist-A) holds, then a relational noun $R$ is distributive in its arguments, while $R$ is distributive in its values if (Dist-V) holds. Both kinds of distributivity hold with the relational nouns we have considered here like "seito" (pupil(s)), "sensei" (teacher(s)) and "oya" (parent(s)). There seem to be very few examples of relational nouns which do not satisfy either version of distributivity, but there

[^5]are some. To (Dist-V), "ryō-shin" (both parents) is a counterexample, and "kyōyū-zaisan" (joint estate) is one to (Dist-A).

An example like a noun phrase "inu to neko" ( $\operatorname{dog}(\mathrm{s})$ and cat(s)) shows that a complex noun phrase may not be distributive even though its constituent nouns are all distributive. In this connection, it may be interesting to know that the following fact holds.

## Proposition 8.4

If $\alpha$ is a definite or bare indefinite noun phrase, and $R$ is a relational noun which is distributive in its value, then " $\alpha$ no $R$ " is distributive.

Proof. This can be easily proved by Axiom 8.2 and the definition of distributivity.

This also shows that the distributivity of " $\alpha$ no $R$ " does not depend on that of $\alpha$. Though a noun phrase "Hanako to Taro" is not distributive, "Hanako to Taro no seito" (pupils of Hanako and Taro) is distributive. It must be emphasized here that an occurrence of a noun phrase like "Hanako to Taro no seito" is not necessarily of the form " $\alpha$ no $R$ ". For, it has two readings, wide scope and narrow scope, and, in its wide scope reading, "Hanako to Taro no seito" is, in reality, of the form "(Hanako no seito) to (Taro no seito)". In contrast, "Hanako to Taro no seito" in its narrow reading is truly of the form " $\alpha$ no $R$ " with "Hanako to Taro" as $\alpha$.

If we have this in mind, then we can see that the following holds.

## Proposition 8.5

Let each of $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ is either a definite noun phrase or a bare indefinite noun phrase, and $R$ be a relational noun. Let " $\vec{\alpha}$ " be an abbreviation of

$$
\alpha_{1} \text { to } \alpha_{2} \text { to } \ldots \text { to } \alpha_{n} .
$$

Then,
(i) a noun phrase " $[\vec{\alpha} \text { no } R]_{\text {to }}$ ", namely the wide scope reading of " $\vec{\alpha}$ no $R$ ", is generally non-distributive.
(ii) a noun phrase " $[\vec{\alpha}]_{\text {to }}$ no $R$ ", namely the narrow scope reading of " $\vec{\alpha}$ no $R$ ", is distributive if $R$ is distributive in its value.

Proof. (i) Suppose that $X$ are among the semantic values of " $[\vec{\alpha} \text { no } R]_{\text {to }}$ " and that $x \eta X$. Then, $x$ must be among the semantic values of " $\alpha_{k}$ no $R$ " for
some $\alpha_{k}$ which is one of $\alpha_{i}$ s. But $x$ need not be among the semantic values for any other $\alpha_{i}$ s. Hence, $x$ may not be among the semantic values of " $[\vec{\alpha} \text { no } R]_{\text {to }}$ ".
(ii) As " $[\vec{\alpha}]_{\text {to }}$ " is either a definite noun phrase or a bare indefinite noun phrase, Proposition 8.4 above applies to the noun phrase " $[\vec{\alpha}]_{\text {to }}$ no $R$ ".

Next we consider another proposition that shows how the wide/narrow readings relate to distributivity. For this proposition, we need the concept of cumulativity as well distritubitivity.

The two predicates "kita" (came) and "atsumatta" (got together) are also different in that one is cumulative while the other is not, namely,

```
True: [Val(X,"kita") }\wedge\operatorname{Val}(Y,"kita")] -> Val( (\Sigma(X,Y),"kita")
False: [Val(X,"atsumatta") }\wedge\operatorname{Val}(Y,"atsumatta")] -> Val(\Sigma(X,Y)
"atsumatta").
```

A similar distinction can be made among noun phrases, namely, a noun phrase $\alpha$ is cumulative if and only if

$$
\left.\forall X \forall X^{\prime}\left[\operatorname{Val}(X, \alpha) \wedge \operatorname{Val}\left(X^{\prime}, \alpha\right)\right] \rightarrow \operatorname{Val}\left(\Sigma\left(X, X^{\prime}\right), \alpha\right)\right]
$$

Many common nouns like "neko" (cat(s)) and "kuruma" (car(s)) in Japanese are cumulative. Examples of non-cumulative nouns in Japanese are a noun derived from a relational noun such as "tomodachi" (friends) and a quantified noun phrase like "syou-sū no seito" (a few pupils).

We may make a similar distinction with a relational noun. Clearly there must be two versions of cumulativity with them, one in argument and the other in value.

$$
\begin{aligned}
(\mathrm{Cum}-\mathrm{A}) & {\left.\left[\operatorname{Val}(\langle X, Y\rangle, R) \wedge \operatorname{Val}\left(\left\langle X, Y^{\prime}\right\rangle, R\right)\right] \rightarrow \operatorname{Val}\left(\left\langle X, \Sigma\left(Y, Y^{\prime}\right)\right\rangle, R\right)\right) } \\
(\mathrm{Cum}-\mathrm{V}) & {\left.\left[\operatorname{Val}(\langle X, Y\rangle, R) \wedge \operatorname{Val}\left(\left\langle X^{\prime}, Y\right\rangle, R\right)\right] \rightarrow \operatorname{Val}\left(\left\langle\Sigma\left(X, X^{\prime}\right), Y\right\rangle, R\right)\right) . }
\end{aligned}
$$

A relational noun "seito" is cumulative in both its arguments and values. Just as most relational nouns are distributive in both argument and value, most relational nouns are cumulative in both senses.

Now we can state the following proposition.

## Proposition 8.6

Let $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}, R$ and " $\vec{\alpha}$ " be as they were in the previous Propostion. If $R$ is distributive and cumulative in its argument, then, for each $i(1 \leq i \leq n)$,

$$
\left.\operatorname{Val}\left(X, "[\vec{\alpha}]_{\text {to }} \text { no } R "\right) \rightarrow \operatorname{Val}\left(X, " \alpha_{i} \text { no } R\right) "\right)
$$

Proof. Suppose that $1 \leq i \leq n$ and that $\operatorname{Val}\left(X, "[\vec{\alpha}]_{\text {to }}\right.$ no $\left.R "\right)$. Then, by Axiom 8.2, for some $Y$,

1. $\operatorname{Val}\left(Y, "[\vec{\alpha}]{ }_{\mathbf{t o}} "\right)$ and $\operatorname{Val}(\langle X, Y\rangle, R)$.

By Axiom 8.1, there are some $Y^{\prime}$ such that $\operatorname{Val}\left(Y^{\prime}, \alpha_{i}\right)$ and $Y^{\prime} \eta Y$. As $R$ is distributive in its argument, for each $y \eta Y^{\prime}$,
2. $\operatorname{Val}(\langle X, y\rangle, R)$.

As $R$ is also cumulative in its argument, from this
3. $\operatorname{Val}\left(\left\langle X, Y^{\prime}\right\rangle, R\right)$

By Axiom 8.2, we get from 1. and 3.
$\operatorname{Val}\left(X, " \alpha_{i}\right.$ no $\left.R "\right)$.

Next proposition shows that the narrow scope reading "entails" the wide scope reading.

## Proposition 8.7

Let $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}, R$ and " $\vec{\alpha}$ " be as they were in the previous two Propostions. Further suppose that $R$ is both distributive and cumulative in its argument. Then,

$$
\left.\operatorname{Val}\left(\mathrm{X}, "[\vec{\alpha}]_{\mathbf{t o}} \text { no } R "\right) \rightarrow \operatorname{Val}(\mathrm{X},[\overrightarrow{[\alpha}] \text { no } R]_{\mathbf{t o}} "\right)
$$

Proof. Suppose that $\operatorname{Val}\left(\mathrm{X}, "[\vec{\alpha}]_{\text {to }}\right.$ no $\left.R "\right)$. Then, by Axiom 8.2, there exists $Y$ such that

1. $\operatorname{Val}\left(Y, "[\vec{\alpha}]_{\text {to }} "\right)$, and
2. $\operatorname{Val}(\langle X, Y\rangle, R)$.

By Axiom 8.1, from 1. we have, for each $i$ such that $1 \leq i \leq n$, there exist $Y_{i}$ such that
3. $\operatorname{Val}\left(Y_{i}, " \alpha_{i} "\right)$.
and
4. $Y \equiv \Sigma\left(Y_{1}, Y_{2}, \ldots, Y_{n}\right)$.

As $Y_{i} \eta Y$ for each $i$ from 4., and from 2. and the argument distributivity and cumulativity of $R$, for each $i$, we have
5. $\operatorname{Val}\left(\left\langle X, Y_{i}\right\rangle, R\right)$

From 3. and 5. by Axiom 8.2, we have for each $i$,
6. $\operatorname{Val}\left(X, " \alpha_{i}\right.$ no $\left.R "\right)$.

Then, by Axiom 8.1, we can conclude that
$\operatorname{Val}\left(X, " \operatorname{to}\left(\alpha_{1}\right.\right.$ no $R, \alpha_{2}$ no $R, \ldots, \alpha_{n}$ no $\left.\left.R\right) "\right)$,
which is equivalent to
$\left.\operatorname{Val}(\mathrm{X}, "[\mid \alpha] \text { no } R]_{\text {to }} "\right)$,
by Axiom 8.3.
As we have noted above, a relational noun is mostly both distributive and cumulative in its argument. Hence, we may expect such an entailment holds between narrow scope reading and wide scope reading. It must be obvious that the converse of Proposition 8.7 is not valid. For example, from the fact that there are a group of teachers who teach at least one of a group of students, it does not follow that there are teachers who teach all of them.

### 8.4 Plurality operator PL

How can we account for the ambiguity of (62), which does not contain the conjoining particle "to"? I claim that this is also a case of scopal ambiguity. It is a question of relative scope between "no" and what I call "plurality operator PL". There are two readings of (62) which differ in relative scope between them.
(62A) [Kodomo-tachi no sensei $]_{\text {PL }}$ ga atsumatta.
(62B) $[\text { Kodomo-tachi }]_{\text {PL }}$ no sensei ga atsumatta.
Naturally, Japanese has resources for marking whether the intended reading is wide scope or narrow scope without appealing to such an artificial device as bracketing. We can rewrite ( 62 A ) and (62B) respectively in this way.

| Kodomo-tachi sore-zore no | sensei | ga |  |  |
| :--- | :--- | :--- | :--- | :--- |
| child-PL | each | GEN | teacher(s) | NOM |
| atsumatta. |  |  |  |  |
| got together |  |  |  |  |
| achers of each child got together.) |  |  |  |  |

(62B) Kodomo-tachi ni kyōtsū no sensei ga child-PL OBL in common GEN teacher(s) NOM atsumatta. got together

## (Teachers common to children got together.)

It should be noted that even a sentence without plural marker "tachi" can be ambiguous just as (62) is. Consider the sentence which is only slightly different from (62).

| (67) | Kodomo <br> child(ren) | no | sensei | ga | atsumatta. |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | teacher(s) | NOM | got together |  |  |

As "kodomo" may refer to a number of children as well as a single child, there can be the same ambiguity as (62). The two different readings of (67) may be expressed explicitly like this.


Instead of the phrases like "sore-zore" or "hitori-hiroti" and "ni kyoōtsū no", we use bracketing in our formal representation for disambiguation. In (62A) and ( 67 A ), the operator "no" lies inside the scope of the plurality operator PL, while PL lies inside "no" in (62B) and (67B). If "kodomo" in (67) refers to a single child, then (67A) and (67B) are equivalent to each other, but (67) has in principle two different readings.

Before giving a semantic account of PL, I should warn that the account I present here is a very limited one. I believe that the plurality operator PL occurs with respect to other sorts of indefinite noun phrases besides relational noun compounds. Ideally what should be given is an account of PL that applies to indefinite noun phrases in general, but I am not yet prepared to provide one; at present I can give only a rough sketch for such a general treatment, which I will present later on.

First, we have to consider the case in which the plurality operator is applied to a common noun like "seito" and "kodomo" (used as classificatory labels). As a Japanese common noun usually does not refer to a single thing but a number of things, its semantic values will not be affected by PL. We record this fact by the next axiom.

## Axiom 8.8

Let $\alpha$ be a common noun. Then
(i) $\operatorname{Val}(X, " \operatorname{PL}(\alpha) ") \leftrightarrow \operatorname{Val}(X, \alpha)$, and
(ii) $\operatorname{Val}(X, " \operatorname{PL}(\alpha$-tachi $) ") \leftrightarrow \operatorname{Val}(X, " \alpha$-tachi" $)$.

Next axiom tells us how PL works for a relational compound " $\alpha$ no $R$ ".

## Axiom 8.9

Let $\alpha$ be a definite or bare indefinite noun and $R$ a relational noun.
Then,
$\operatorname{Val}(X, " \operatorname{PL}(\alpha$ no $R) ") \leftrightarrow \exists Y[\operatorname{Val}(Y, \alpha) \wedge$
$\forall y[y \eta Y \rightarrow \exists Z \exists W[Z \eta X \wedge W \eta Y \wedge y \eta W \wedge \operatorname{Val}(\langle Z, W\rangle, R)] \wedge$
$\forall x[x \eta X \rightarrow \exists Z \exists W[Z \eta X \wedge W \eta Y \wedge x \eta Z \wedge \operatorname{Val}(\langle Z, W\rangle, R)]]]$
This is a little complicated, but it should not be too difficult to understand. Let us try to see what should be the semantic values of "kodomo no sensei" (teachers of children) according to this axiom.

It tells us that for $X$ to be the semantic values of "PL(kodomo no sensei)" it is necessary and sufficient that

1. for each child $y$, there are some among $X$ who are teachers of some children including $y$, that is, each child must be one of the pupils who are taught by a certain group of teachers, and that
2. for each teacher $x$ among $X$, there are some children who are taught by some teachers including $x$, that is, each teacher must belong to some group of teachers who teach a certain group of children.

For most of relational nouns which are distributive in both argument and value, Axiom 8.9 can be much simplified.

## Proposition 8.10

Let $\alpha$ be a definite or bare indefinite noun and $R$ a relational noun which is distributive and cumulative in both its argument and value. Then,

$$
\begin{gathered}
\operatorname{Val}(X, " \operatorname{PL}(\alpha \text { no } R) ") \leftrightarrow \exists Y[\operatorname{Val}(Y, \alpha) \wedge \\
\forall y[y \eta Y \rightarrow \exists x[x \eta X \wedge \operatorname{Val}(\langle x, y\rangle, R)]] \wedge \\
\forall x[x \eta X \rightarrow \exists y[y \eta Y \wedge \operatorname{Val}(\langle x, y\rangle, R)]]]
\end{gathered}
$$

Proof. Let (I) be the right-hand side of the equivalence that appears in Axiom 8.9 and similarly (II) be the left-hand side of the equivalence that appears in the statement above. It is enough to show the equivalence of (I) and (II) on the assumption that $R$ is distributive and cumulative in both its argument and value. But it is obvious that (II) implies (I), and (II) follows from (I) by the assumed distributivity.

Thus, the conditions for $X$ to be the semantic values of " PL (kodomo no sensei)" can be now stated much simply, namely, they are that

1. each child is taught by some teacher among $X$.
2. each teacher among $X$ teaches some child.

Furthermore, when the extension of $\alpha$ is finite, we can give an account of PL which is similar to that of the conjoining particle "to" with the concept of sum $\Sigma$.

## Proposition 8.11

Let $\alpha$ be a definite or bare indefinite noun and $R$ a relational noun which is distributive and cumulative in both argument and value. Suppose that the extension of $\alpha$ consists of $n$ individuals. Then,

$$
\begin{aligned}
& \operatorname{Val}(X, " \operatorname{PL}(\alpha \text { no } R) ") \leftrightarrow \\
& \quad \exists Y\left[\operatorname { V a l } ( Y , \alpha ) \wedge \exists y _ { 1 } \exists y _ { 2 } \ldots \exists y _ { n } \left[Y \equiv \Sigma\left(y_{1}, y_{2}, \ldots, y_{n}\right) \wedge\right.\right. \\
& \quad \exists X_{1} \exists X_{2} \ldots \exists X_{n}\left[\operatorname{Val}\left(\left\langle X_{1}, y_{1}\right\rangle, R\right) \wedge \operatorname{Val}\left(\left\langle X_{2}, y_{2}\right\rangle, R\right) \wedge \ldots\right. \\
& \left.\left.\quad \wedge \operatorname{Val}\left(\left\langle X_{n}, y_{n}\right\rangle, R\right) \wedge X \equiv \Sigma\left(X_{1}, X_{2}, \ldots, X_{n}\right)\right]\right] .
\end{aligned}
$$

Proof. As in the previous proof, let (II) the left-hand side of the equivalence that appeared in Proposition 8.10 and let (III) be the right-hand side of the equivalence in Proposition 8.11.
(II) $\rightarrow$ (III): As it is assumed that the extension of $\alpha$ consists of $n$ individuals, let these be $y_{1}, y_{2}, \ldots, y_{n}$. Then, there exist $Y$ such that $\operatorname{Val}(Y, \alpha)$ and

$$
Y \equiv \Sigma\left(y_{1}, y_{2}, \ldots, y_{n}\right)
$$

By (II), for each $y_{i}(1 \leq i \leq n)$, there exists $x_{i}$ such that $x_{i} \eta X$ and $\operatorname{Val}\left(\left\langle x_{i}, y_{i}\right\rangle\right.$, $R$ ). Let $X_{i}$ be these $x_{i} \mathrm{~s}$, that is,

$$
x_{i} \eta X_{i} \leftrightarrow x_{i} \eta X \wedge \operatorname{Val}\left(\left\langle x_{i}, y_{i}\right\rangle, R\right) .
$$

As $R$ is assumed to be cumulative in value, $\operatorname{Val}\left(\left\langle X_{i}, y_{i}\right\rangle, R\right)$.

Now we show that

$$
X \equiv \Sigma\left(X_{1}, X_{2}, \ldots, X_{n}\right)
$$

Suppose that there is some $x_{0}$ that is among $X$ but not among $X_{i}$ for any $i(1 \leq i \leq n)$. As $x_{0} \eta X$, there exists some $y_{k}$ such that $y_{k} \eta Y$ and $\operatorname{Val}\left(\left\langle x_{0}, y_{k}\right\rangle, R\right)$ by (II). Then, $x_{0}$ should be among $X_{k}$ which is one of $X_{k}(1 \leq k \leq n)$, which is contrary to the assumption.
(III) $\rightarrow$ (II): Suppose that $\operatorname{Val}(Y, \alpha)$. By (III), for each $y \eta Y$, there exist some $X_{i}$ such that $X_{i} \eta X$ and $\operatorname{Val}\left(\left\langle X_{i}, y\right\rangle, R\right)$. As $R$ is distributive in its value, for $x \eta X_{i}, \operatorname{Val}(\langle x, y\rangle, R)$. On the other hand, for each $x \eta X$, it is among one of $X_{i} \mathrm{~s}$ such that $\operatorname{Val}\left(\left\langle X_{i}, y\right\rangle, R\right)$ for some $y \eta Y$ by (III). Again, by $R$ 's distributivity in value, for this $y, \operatorname{Val}(\langle x, y\rangle, R)$.

Proposition 8.11 gives us a more intuitive condition for the wide scope reading of a relational noun compound. It explains the wide scope reading of "kodomo no sensei" (teachers of children) in this way: let $a_{1}, a_{2}, \ldots, a_{n}$ be all the children to whom "kodomo" refers; for each child $a_{i}$, pick up some of its teachers and let them be $A_{i}$; if you gather up all the $A_{i}$ s, they are semantic values of "kodomo no sensei" in its wide scope reading.

### 8.5 Towards a general account of a plurality denoting noun phrase

Now we relaize that our PL axiom, Axiom 8.9, automatically takes care of the difference between wide scope and narrow scope readings of the form " $\alpha_{1}$ to $\alpha_{2}$ to ...to $\alpha_{n}$ no $R$ ". First, noun phrases conjoined by "to" give rise to a definite or bare indefinite NP, if each of conjoined NPs is either definite or bare indefinite. Secondly, such an NP formed by "to" is recognized to be a noun phrase whose reference is plural. This means that Axiom 8.9 applies to a noun phrase of the form " $\alpha_{1}$ to $\alpha_{2}$ to $\ldots$ to $\alpha_{n}$ no $R$ ".

This means that we can dispense with Axiom 8.3, which is specially designed to get both wide scope and narrow scope readings for a relational noun compound with an NP formed by "to". If we put one additional clause to Axiom 8.8, then Axiom 8.1, which gives us semantics of such an NP, together with PL axioms makes it possible to derive wide scope and narrow scope readings for relational noun compounds with an NP formed by "to". An additional clause to Axiom 8.1 that is necessary is this:

Let $\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$ be definite or bare indefinite noun phrases. Then
(iii) $\operatorname{Val}\left(X, " \operatorname{PL}\left(\mathbf{t o}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right)\right) "\right) \leftrightarrow \operatorname{Val}\left(X, " \operatorname{to}\left(\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}\right) "\right)$

Now Propositions 8.5-8.7 are seen to be special cases of more general facts that hold with a relational noun compound having a plurality denoting NP.

They are respectively consequences of the following Propositions, the proofs of which are more or less obvious.

## Proposition 8.12

Let $\alpha$ be a definite or bare indefinite noun phrase, and $R$ a relational noun. Then,
(i) a noun phrase " $[\alpha \text { no } R]_{\text {PL }}$ ", namely the wide scope reading of " $\alpha$ no $R$ ", is generally non-distributive.
(ii) a noun phrase " $[\alpha]_{\text {PL }}$ no $R$ ", namely the narrow scope reading of " $\alpha$ no $R$ ", is distributive if and only if $R$ is distributive in its value.

## Proposition 8.13

Let $\alpha$ and $R$ as it was in the previous Proposition. Let $\nu$ be a definite noun phrase that refers to some things which are among semantic values of $\alpha$. If $R$ is distributive and cumulative in its argument, then,

$$
\operatorname{Val}\left(X, "[\alpha]_{\mathrm{PL}} \text { no } R "\right) \rightarrow \operatorname{Val}(X, " \nu \text { no } R) ",
$$

For an example, consider the noun phrase "kodomo no sensei" (teachers of children). What Proposition 8.13 says is that if $X$ are teachers of the children which "kodomo" refers to, and, for example, Taro and Hanako are among them, then $X$ are also teachers of both Taro and Hanako. Proposition 8.13 expresses such facts in terms of semantic values of expressions.

## Proposition 8.14

Let $\alpha$ and $R$ as they were in the previous two Propositions. Further suppose that $R$ is distributive in both its argument and value. Then,

$$
\operatorname{Val}\left(X, "[\alpha]_{\mathrm{PL}} \text { no } R "\right) \rightarrow \operatorname{Val}\left(X, "[\alpha \text { no } R]_{\mathrm{PL}} "\right)
$$

Now that we know that the scopal difference which was thought to be caused by "to" is caused by PL operator, we might wonder whether such a difference should be really that of scope. For, as the previous Axiom 8.8 (with its added clause with "to" noun phrases) shows, PL operator has no semantic effects in its narrow scope use.

Still, I think that the talk of scope is useful. If we consider how the wide scope reading of a complex NP arises, we notice that there must exist some NP which is a constituent of the complex NP and may have plural reference. Such a possibly plural NP triggers the plurality of the entire complex NP. This may be seen from the fact that we recognize the ambiguity of a relational noun compound "kodomo-tachi no sensei" (teachers of a child/children) with explicitly plural "kodomo-tachi" much more easily than a similar NP "kodomo no sensei". Yet, as is usual the case with a Japanese noun, "kodomo" may refer to a number of children, and hence, "kodomo no sensei" has both of wide scope and narrow scope readings.

As we remarked just now, a Japanese noun phrase should be regarded as possibly plural as default. Thus, the narrow scope reading of "kodomo no sensei" should be represented as

> (kodomopl no ) oyapl.

In contrast, its wide scope reading is represented as

$$
((\text { kodomopl no }) \text { oya })_{\text {PL }} .
$$

But default plural markings can be omitted just because they are default. Then, the representations of narrow scope and wide scope readings become the following.

> narrow scope: (kodomo no) oya.
> wide scope: $\quad\left((\text { kodomo no oya })_{\mathrm{PL}}\right.$.

Thus, the difference between wide scope and narrow readings now coincides with the presence and absence of PL operator.

Let us try this way of scope marking with a case of a complex relational noun compound with multiple ambigutity. As a definite or bare indefinite noun phrase constructed with a relational noun can be an argument for another occurrence of a relational noun, an ambiguity may become multiple.

Here is an example.

| (68) | to | Taro | no | oya | no |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | and |  | GEN | parent(s) | GEN |
| shiri-ai | ga | atsumatta. |  |  |  |
| acquaintance(s) | NOM | got together |  |  |  |
| (Acquaintances of parent(s) of Hanako and | Taro got together.) |  |  |  |  |

If we suppose that a noun phrase "Hanako to Taro no oya no shiri-ai" is indefinite here, it is fourfould ambiguous, namely, they are ambiguous between
(i) ((Hanako to Taro) no oya) no shiriai
(ii) ((Hanako to Taro) no oya) $)_{\text {PL }}$ no shiriai
(iii) $\left(\left((\text { Hanako to Taro) no oya) no shiriai })_{\text {PL }}\right.\right.$
(iv) $\left(((\text { Hanako to Taro }) \text { no oya })_{\text {PL }} \text { no shiriai }\right)_{\text {PL }}$

These different readings can be disambiguated in Japanese by using phrases like "sore-zore" (each) and "ni kyōtsū" as it was mentioned above. Here are how (i)-(iv) are distinguished from each other in this way.
(i) Hanako to Taro ni kyōtsū no oya ni kyōtsū no shiriai
(ii) Hanako to Taro sore-zore no oya ni kyōtsū no shiriai
(iii) Hanako to Taro ni kyōts $\bar{u}$ no oya sore-zore no shiriai
(iv) Hanako to Taro sore-zore no oya sore-zore no shiriai

Or, we can express the difference between (i)-(iv) in a more perspicuous way, if we realize that this double relational noun compound is of the form

$$
(\alpha \text { no } R) \text { no } S,
$$

where $\alpha$ is a noun phrase, and $R$ and $S$ are relational nouns. Then, the difference between (i)-(iv) can be displayed in this way.
(i) $(\alpha$ no $R)$ no $S$
(ii) $(\operatorname{PL}(\alpha$ no $R))$ no $S$
(iii) $\mathrm{PL}((\alpha$ no $R)$ no $S)$
(iv) $\operatorname{PL}((\operatorname{PL}(\alpha$ no $R))$ no $S)$

Thus, our axioms for PL operator, in particular, Axiom 8.9 cover a fairly wide class of complex noun phrases. It is obvious, however, that PL operator should occur in a wide variety of complex noun phrases containing noun phrases which possibly refer to a number of things. In the case of a relational noun compound

$$
\alpha \text { no } R,
$$

where $\alpha$ possibly refers to a number of things, this possible plurality may affect the entire noun phrase in which $\alpha$ is only a part. This gives rise to a wide scope reading, which is indicated by PL-operator:

$$
[\alpha \text { no } R]_{\mathrm{PL}} .
$$

Although we may put PL-operator to $\alpha$ itself for a narrow reading of the complex noun phrase like

$$
[\alpha]_{\mathrm{PL}} \text { no } R,
$$

this does not require any extra account, because we are working in the framework of plural logic and it generally handles well such plurality. That is also a reason for deciding not to explicitly put PL-operator in such cases.

What we need is a much more extended version of Axiom 8.9 which applies to (definite or bare indefinite) complex noun phrases in general. For that, we need a general syntactic account of complex noun phrases in Japanese. Until such an account is available, we cannot hope to have a truly general account of PL operator. But, we may consider at least in a rough outline what a satisfactory account of PL operator should be like.

There are two major ways to construct a complex noun phrase in Japanese ${ }^{10}$. One is to modify a noun $N$ by a noun phrase $\alpha$ with the particle "no" (and sometimes together with some other case particle). A complex noun phrase which is constructed in this way has the form

$$
\alpha(\mathrm{cp}) \mathbf{n o} N,
$$

where "(cp)" indicates a possible presence of a case particle.
As you can see, a relational noun compound is a special case in which $N$ is a relational noun. Here are some examples of this pattern in which a modified noun is not relational.

| (69) Hanako | to |
| :--- | :--- | :--- | :--- | :--- |
|  | and |$\quad$ Taro | no |
| :--- |
| GEN | hon $\quad$ book(s)

(A book/Books of Taro and Hanako)

| kodomo-tachi | kara | no | tegami |
| :--- | :--- | :--- | :--- |
| child-PL | from | GEN | letter(s) |

(A letter/Letters from children)
Note that there exists the same ambiguity in these examples as there was in a relational noun compound. For example, (70) may mean a letter or letters from children together, or it may mean several different letters from different children. This ambiguity could be explained by the different scope of PL operator.

Another major way of constructing a complex noun phrase is to modify a noun by a predicate clause which has typically a "gap" for the modified noun. Examples are these.

$$
\begin{array}{lllll}
\text { (71) } & \begin{array}{l}
\text { kodomo } \\
\text { child(ren) }
\end{array} & \text { o } & \text { ACC } & \text { oshieta } \\
\text { taught }
\end{array} \quad \begin{aligned}
& \text { sensei } \\
& \text { teacher(s) }
\end{aligned}
$$

(A teacher/Teachers who taught a child/children)

| sensei | ga | kodomo | ni | ageta | hon |
| :--- | :--- | :--- | :--- | :--- | :--- |
| teacher(s) | NOM | child(ren) | OBL | gave | book(s) |

(A book/Books that a teacher/teachers gave to a child/children)

[^6]Again these noun phrases are ambiguous, if at least one of the nouns that occur in their modifying clauses is thought to refer to plurality. It should be easy to know what the different readings are for (71) and (72). In this case also, the different scope of PL operator causes different readings.

A general account of PL operator should apply to a definite or bare indefinite noun phrase no matter how it is constructed. Though we cannot expect to have such an account without any satisfactory account of the syntax of Japanese noun phrases, here is a sketch for it, which may be hopefully developed into a more satisfactory account.

What we need to do is to generalize our Axiom 8.9 in such a way that it covers complex noun phrases other than relational noun compounds. They are NPs formed by either (I) noun phrase modification accompanying "no", or (II) predicate clause modification. In both cases, the presence of the wide scope PL operator is caused by some noun or noun phrase that occurs in a modifying part.
(I) As remarked above, noun phrases that belong to class (I) has the form

$$
\alpha(\mathrm{cp}) \text { no } N .
$$

When a case particle (cp) is absent and $N$ is not a relational noun, we interpret this NP as implicitly containing some relation $\pi$. Usually the context of the utterance determines which relation is implicitly there. Take (69). This noun phrase may be interpreted in many ways: a book/books owned by Taro and Hanako, a book/books written by Taro and Hanako, a book/books about Taro and Hanako, and so on. When a case particle is present as in (70), the relation in question is more or less obvious. In any case, this class of complex noun phrases have the same meaning as
those $N$ s which have relation $\pi$ to $\alpha$
Then, it is not difficult to give a semantic account of such noun phrases. Axiom 8.2 for relational noun compounds gives us a model. It might be something like the following.

## Axiom 8.15

Let $\alpha$ be a definite or bare indefinite noun phrase and $N$ a noun. Suppose that $\pi$ is a relation that is determined by context. Then,

$$
\operatorname{Val}(X, " \alpha \text { no } N ") \leftrightarrow \exists[\operatorname{Val}(Y, \alpha) \wedge \pi(X, Y) \wedge \operatorname{Val}(X, N)]
$$

Then, it will not be difficult to extend Axiom 8.9 to the class of these noun phrases.
(II) Consider (71). This noun phrase can be interpreted in two ways. Let ":" express the modifying relation that holds between a predicate and a noun. It may have the form "(kodomo o oshieta: sensei)", in which a noun "sensei" (teacher(s)) is modified by a clause "kodomo o oshieta" (taught a child/children). It may also has the form "(kodomo o)(oshieta: sensei)", in which an "unsaturated" noun phrase "(oshieta: sensei)" is "saturated" by "(kodomo o)". In the former interpretation, possible plurality of "kodomo" cannot influence the outside of the modifying clause, and hence it results in a narrow scope reading. In contrast to this, in the latter interpretation possible plurality of "kodomo" can influence the phrase "(oshieta: sensei)", and this results in a wide scope reading.

In general, noun phrases belonging to (II) may be interpreted either as

$$
((\alpha \operatorname{cp} \phi): N),
$$

or

$$
(\alpha \mathrm{cp})(\phi: N)
$$

The former gives rise to a narrow scope reading, while a wide scope reading is only possible with the latter. Suppose that $\alpha$ has plural reference. In the former, PL operator should attach to $\alpha$, and hence, lie in the scope of the modifying operator ":". It is the other way round with the latter; ":" must lie in the scope of PL operator.

Consider the case in which $\phi$ is a binary predicate like "oshieta" (taught). If we form a compound "(oshieta: sensei)" (taught: teacher(s)), then this becomes a noun-like expression which needs to be "saturated" by some expression which denotes those that were taught by teacher(s). Logically it is like a relational noun, which should be "saturated" by its argument. This suggests that we can again use Axiom 8.2 as a model for providing a semantic account of this sort of noun phrases. Thus we have this.

## Axiom 8.16

Let $\alpha$ be a definite or bare indefinite noun phrase, $\phi$ a binary predicate, and $N$ a noun. Then,

$$
\begin{aligned}
& \operatorname{Val}(X, "(\alpha \operatorname{cp})(\phi: N) ") \leftrightarrow \exists Y[\operatorname{Val}(Y, \alpha) \wedge \operatorname{Val}(\langle X, Y\rangle, \phi) \\
& \quad \wedge \operatorname{Val}(X, N)] .
\end{aligned}
$$

Then we can extend Axiom 8.9 to noun phrases of this form.
Of course, $\phi$ may be a predicate of more than two arguments. (72) provides an example. It has three readings, namely,
(i) (sensei ga kodomo ni ageta: hon)
(ii) (sensei ga)(kodomo ni ageta: hon)
(iii) (sensei ga)((kodomo ni)(ageta: hon))

In (i) both "sensei" and "kodomo" are inside the modifying clause, and their possible plurality remains local, that is, they do not give rise to a wide scope reading. In (ii), although "kodomo" is inside the modifying clause, "sensei" is outside it, and may cause a wide scope reading, according to which (ii) denotes possibly different books which each teacher gave to a group of children. In (iii), both "sensei" and "kodomo" are outside the modifying clause, and it denotes possibly different books which each teacher gave to each child.

For (iii) we have to consider an expression of the form

$$
(\alpha \mathrm{cp})(\phi: N) .
$$

where $\phi$ is a predicate with three arguments. As $N$ provides one of them, " $(\phi: N)$ " is a noun-like expression which has two arguments. Hence, a natural suggestion is to have an ordered pair version of Axiom 8.16.

## Axiom 8.17

Let $\alpha$ be a definite or bare indefinite noun phrase, $\phi$ a ternary predicate, and $N$ a noun. Then,

$$
\begin{aligned}
& \operatorname{Val}(\langle X, Y\rangle, "(\alpha \operatorname{cp})(\phi: N) ") \leftrightarrow \exists Z[\operatorname{Val}(Z, \alpha) \wedge \operatorname{Val}(\langle X, Y, Z\rangle, \phi) \\
& \quad \wedge \operatorname{Val}(X, N)] .
\end{aligned}
$$

Then, although it may be a rather tedious task, it will not be difficult to extend Axiom 8.9 to this case.

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[^1]:    1 "Taro" sometimes denotes a number of people with that name. In such cases, "Taro" must be regarded as a general term, and hence, an indefinite noun phrase. Even "watashi" can be a general term, as when we discuss some problems in the philosophy of mind like asymmetry between watashi (I) and tanin (others). Such uses of a proper name and a personal pronoun as a general term can be distinguished from its standard use as a definite noun phrase by our convention of using brackets distinguishing a definite occurrence of a noun phrase from an indefinite one, which is introduced in this section.

[^2]:    ${ }^{2}$ See [Iida 2007].

[^3]:    ${ }^{5}$ (61) has also a reading that Taro and pupil(s) of Hanako got together. As this reading has no interest in our present concern, it will be ignored in the following.

[^4]:    ${ }^{7}$ I borrowed this notation from [McKay 2006] (p.58).

[^5]:    ${ }^{8}$ It is interesting to note that English noun phrases in plural form are not distributive. For, though "cats" applies to a number of cats $X$ and a cat $x$ is among them, "cats" does not apply to $x$.

    9 As it was observed there, "tomodachi" has also a use as a relational noun.

[^6]:    ${ }^{10}$ See [Masuoka and Takubo 1993], pp.157ff.

