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Euclid's ***Elements*** divides its ten premises into two groups of five.

The first five (*postulates*)—applying in geometry but nowhere else—are *specifically* geometrical. The first: "to draw a line from any point to any point"; the last: the parallel postulate.

The second five (*axioms*) apply in geometry *and* elsewhere. They are non-logical principles governing *magnitude types* both geometrical (e.g., lengths, areas) and non-geometrical (e.g., durations, weights). Euclid called axioms *koinai ennoia*: *koinai* ("shared", "communal", etc.), *ennoia* ("designs", "thoughts", etc.). The first axiom is:

Ta toi autoi isa kai allelois estin isa.
Things that equal the same thing equal one another.

One first-order translation in variable-enhanced English (cf. [2], p. 121) is:

(1) Given two things *x*, *y*, *if* for something *z*, *x* and *y* equal *z*,
then x equals *y*.

Translation (1) overlooks Euclid's plural construction not limited to two. Second-order translations avoid that objection.

(2) For any set *S*, *if* for something *z*, everything *x* in *S* equals *z*,
then anything *x* in *S* equals anything *y* in *S*.

Translations (1) and (2) are "too broad": they cover all magnitude types but by amalgamating them into a hodgepodge universe containing all magnitude types—a universe violating category restrictions and not itself a magnitude type.

Translation (3) is a *second-order axiom schema* (cf. [1]) having one instance for each magnitude type. 'MAG' is placeholder for magnitude words such as *length*, *area*, etc.

(3) For any set *S*, *if* for some MAG *z*, every MAG *x* in *S* equals *z*,
then any MAG *x* in *S* equals any MAG *y* in *S*.

We treat several other translations and formalizations.

[1] JOHN CORCORAN, *Schemata*, ***Bulletin of Symbolic Logic***, vol. 12 (2006), pp. 219–40.
[2] ALFRED TARSKI, ***Introduction to Logic***, Dover, New York, 1995.