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[This is a draft of a companion piece to G.C. Field's (1932) "The Place of Definition in Ethics", *Proceedings of the Aristotelian Society*, 32: 79-94, for a virtual issue of the PAS edited by Ben Colburn]

Objectivity in Ethics and Mathematics[1]

Suppose that ethical and mathematical claims are truth-apt. Field [1931] raises an interesting question. How do axioms, or first principles, in ethics compare to those in mathematics? In this note, I argue that there are similarities between the cases. However, these are premised on an assumption which can be questioned, and which highlights the peculiarity of normative inquiry.

I. Objectivity in Mathematics

Which is the true geometry? Field sometimes writes as if this is a serious question [Field 1931, 82]. But most philosophers and mathematicians today would disagree. There are various geometries – e.g., Euclidean and hyperbolic – each of which is consistent if the others are. Rather than privileging any one geometry, it is natural to hold that all consistent geometries are true (under a face-value Tarskian truth definition). They are simply true of different structures.[2]

[1] Thanks to Ben Colburn, Hartry Field, Haim Gaifman, Joel David Hamkins, Brian Leiter, Colin Marshall, Ian Rumfitt, and Katja Vogt for helpful comments.

[2] By "geometry", I mean a branch of pure mathematics. Obviously not all consistent geometries are true of physical spacetime. (I also assume that at least one such geometry is true, and that no one geometry, in addition to being true, is "metaphysically distinguished" or "carves at the joints" in the sense of Sider [2011]. I make a similar assumption in Section III.)

By contrast, it is commonly supposed that a "foundational" theory, such as some formulation of set theory, can be false without being inconsistent. ZF + the Axiom of Choice (AC) and ZF + the negation of AC are not generally thought to both be true – like geometry with the Parallel Postulate and geometry with its negation. But they are no less consistent if ZF is consistent. There is supposed to be an "objective" fact as to whether every set has a choice function.

II. Ethics and Set Theory

It is a familiar point that in in both ethics and mathematics we seem to "have no observable facts…to which we can turn, as the [empirical scientist] does, for the real subject of our investigation [Field 1931, 85]." But if set theory is "objective", in the sense in which geometry is not, then the analogy between ethics and set theory, in particular, can be carried further.

First, if set theory is objective, then there is a gap between consistency and truth in set theory, just as there is supposed to be a gap between (logical) consistency and truth in ethics. The overwhelming majority of consistent set theories are false, just as the overwhelming majority of consistent ethical theories are false.

Second, if set theory is objective, then set-theoretic axioms seem to be scarcely more "self-evident" than ethical "axioms".[3] Consider the Axiom of Infinity. This says that there is an infinite (inductive) set. *Given that consistency does not suffice for truth* in set theory any more than it does in ethics, how could this be self-evident? Even if it is "metaphysically" necessary that there is an infinite set, it certainly seems *intelligible* that there is not. As Mayberry writes,

[3] Of course, an ethical particularist may regard the search for ethical "axioms" as misguided. But for the purposes of this article, I assume, with Field [1931], that it is not.

> The set-theoretical axioms that sustain modern mathematics are self-evident in differing degrees. One of them – indeed, the most important of them, namely Cantor's axiom, the so-called axiom of infinity – has scarcely any claim to self-evidence at all [2000, 10].[4]

Finally, given that consistency does not suffice for truth, and that few axioms of interest are self-evident, the proper method of inquiry in set theory seems to resemble the proper method of inquiry in ethics – "reflective equilibrium" [Rawls 1971]. We identify plausible propositions, and seek general principles – axioms – which systematize them. The latter may pressure us to reject some of the propositions with which we began as we seek harmony between the two. Of course, this process requires determining what follows from what. It is unsurprising that ethics and set theory might proceed via proof *in some sense*. However, if both areas are objective, then we are not just trying to determine what follows from various axioms. We are also trying to determine what axioms are true – i.e., "the facts that we must suppose…in order to account for the way in which we think about matters [Field 1931, 83]". As Whitehead and Russell write,

> The reason for accepting an axiom, as for accepting any other proposition, is always largely inductive, namely that many propositions which are nearly indubitable can be deduced from it, and that no equally plausible way is known by which these propositions

[4] See also Boolos [1998, 130]. A related point is that set-theoretic reductions may be vulnerable to Moore's Open Question Argument, at least if its premise is that "we can never be quite sure of the correctness of any definition that we offer [Field 1931, 93]." Consider the various set-theoretic reductions of the natural numbers, such as Zermelo's or von Neumann's. Benacerraf [1965] noted that more than one is formally adequate, and that there is no obvious reason to privilege any one formally adequate reduction over all others. He took himself to have thereby showed that the numbers were irreducible. But whereas Moore [2004] concluded that since moral properties are irreducible, they must be *sui generis*, Benacerraf [1965] concluded that since the numbers are irreducible, they must not exist at all [Clarke-Doane 2008, 246, fn. 5]. Of course, an objectivist about set theory can be an anti-objectivist about questions on which alternative reductions of the numbers differ.

> could be true if the axiom were false, and nothing which is probably false can be deduced from it [1997, 59].

III. Objectivity in Set Theory

I have argued that if set theory is objective, then there are similarities between ethical "axioms" and set theoretic axioms beyond the familiar one that both seem to be non-empirically justified. But contrary to the assumption of Section I, set theory, and foundational mathematical theories generally, may not be objective. They may be relevantly like geometry. As Hamkins writes,

> [G]eometers have a deep understanding of the alternative geometries, which are regarded as fully real...The situation with set theory is the same….[S]et theory is saturated with [alternative universes]….[S]et theorists [make] the same step…that geometers…made long ago, namely, to accept the alternative worlds as fully real [Hamkins 2012, 426].[5]

How should we understand this view? It is uncontroversial that every consistent set of axioms – set-theoretic or otherwise – has a model. That is the Completeness Theorem, which is itself a theorem of standard set theory. But in claiming that ZF + AC and ZF + ~AC are both true, the anti-objectivist is presumably advocating more than the Completeness Theorem.[6] The view is not that every consistent formulation of set theory has a model built out of some background set theory, but that it has an *intended* model – i.e., that every consistent such formulation is satisfied under a face-value Tarskian satisfaction relation [Field 1998, 333]. The intuition is that, just as

[5] See also Balaguer [1998].
[6] Though Burgess seems to interpret Balaguer as merely advocating the Completeness Theorem in his [2001], p. 80.

no one concept of point or line should be metaphysically privileged, no one concept of set should be. (Of course, some such concepts may be more interesting, fruitful, and intuitive than others.)

IV. Ethics and Set Theory Again

If such a view of set theory is correct, then the analogies of Section II break down. First, if set theory is not objective, then no matter what set-theoretic beliefs we had had, so long as they were consistent, they would have been true. If one could argue that we could not have easily had inconsistent set-theoretic beliefs, and that the set-theoretic truths could not have easily been different, then one could argue that our set-theoretic beliefs are *safe* – i.e., that we could not have easily had false ones [Clarke-Doane Forthcoming].[7] However, in light of apparently pervasive (non-logical) ethical disagreement, it is hard to see how to argue that our ethical beliefs are safe.

Second, given knowledge of set-theoretic anti-objectivism, the truth of set-theoretic axioms may be more self-evident than the truth of ethical "axioms", because the consistency of set-theoretic axioms may be more self-evident than the truth (as opposed to consistency) of ethical axioms. If anti-objectivism is true of set theory (but not of ethics), then the fact that it is "impossible in ethics to start, as [set theory] does, with [axioms] which will be generally and immediately accepted" is less *prima facie* puzzling than it might otherwise be [Field 1931, 84].

[7] This presupposes the radical formulation of anti-objectivism above. One could instead advocate a less radical formulation of the view according to which, while both of ZF+AC and ZF+~AC are true, only one of, e.g., ZF+Con(ZF) and ZF+~Con(ZF) is (despite both being consistent if ZF is). More conservative formulations of the anti-objectivism are also possible (Gaifman [2012], Sec. 2.4), as are more radical formulations (Priest [2013]). For the purposes of arguing that our set-theoretic beliefs are safe, it seems sufficient to argue for a conservative formulation of anti-objectivism (since, presumably, we could not have easily believed the likes of ZF+~Con(ZF)).

Finally, assuming that consistency suffices for truth in set theory, the proper method of inquiry in set theory does not seem to resemble the proper method of inquiry in ethics. The question of whether AC is true is like that of whether the Parallel Postulate is true. Given a determinate use of "is a member of", the question has an answer – and, for all that has been said, it may depend entirely on the way the mind-and-language independent sets are. But in learning it we are really just learning whether we are talking about this universe of sets or that, rather than learning what universes of sets there are.[8] If set-theoretic anti-objectivism is true, then we *already know* that ZF+AC is true of some universe of sets (assuming that we already know that ZF+AC is consistent). The interesting question is what follows from it and other consistent sets of axioms. In this sense, the proper method of inquiry in set theory may approximate the "Euclidean ideal."

By contrast, since there is supposed to be a speculative distance between (logical) consistency and truth in ethics, it is a considerable challenge to find ethical "axioms" whose truth is remotely uniquely determined by the data points with which the process of reflective equilibrium begins.

V. Truth and Normativity

If set theory is not objective, then set theory is *in a sense* trivialized. If logic is objective, then the question of what follows from set-theoretic axioms remains genuine.[9] But no peculiarly set-theoretic questions seem to remain genuine. One can ask which set theory regiments *our* concept

[8] Strictly speaking, we are learning whether we are talking about this universe of set-like-things or that. Given a determine use of "is a member of", nothing failing to satisfy the axioms true of the corresponding relation will count as a set.

[9] How to spell out the claim that logic is objective is not straightforward. (Obviously, we cannot say that logic is objective if not every consistent set of logical axioms is true, since the claim that a set of sentences is consistent is itself relative to a logic.) For relevant discussion, see Beall and Restall [2005] and Field [2009].

of set, or satisfies some theoretical or aesthetic desiderata. But given set-theoretic anti-objectivism, there is no question of which "consistent" such concept is satisfied. All are.

Could ethics be trivialized similarly? Imagine that a philosopher convinces us that, contrary to all appearances, ethics too is like geometry – that every consistent ethical theory is true, albeit true of different entities. In addition to goodness, obligation, and so on, there is shgoodness, shobligation, and so on. Indeed, for every logically consistent ethical theory, there are corresponding properties, and all of them are instantiated "side by side".[10] Knowing that there are logically (even if not Kantian) consistent formulations of both deontological and consequentialist ethical theories, we conclude that each is true (albeit of different entities). Is our deliberation as to whether we ought to lie when utility would be maximized thereby short-circuited (and likewise for every question on which logically consistent ethical theories diverge)?

It is hard to see how it could be. A general – even if not universal – rule is that if we conclude that we ought to X, then we cannot continue to regard the view that we ought to not-X as on a par. But given that that view *is* on a par with respect to truth, learning that "we ought to X" is true seems insufficient to resolve our deliberation. While knowledge that any consistent set theory is true, and knowledge that ZF+AC and ZF+~AC are both consistent, frees us of the question of whether AC, something similar would not seem to hold in the ethical – and, more generally, normative – case. The fact-value gap appears to be even wider than Hume and Moore suggested. Even knowledge of the *normative* facts may fail to resolve a normative deliberation.

[10] Field uses this locution to describe Balaguer's view of sets in his [1998].

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