Anything goes*

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"Anything goes in; anything goes out./Fish, bananas, old pajamas, mutton, beef, and trout." (Cole Porter, "Anything goes")

1 Intro

In this paper I'm going to consider mainly Prior's `tonk` connective [Prior, 1960]. I think this has implications for paradoxes of various sorts (I have in mind semantic paradox—both truth- and validity-based—as well as set-theoretic paradoxes and perhaps most of all sorites paradox), but I'm not going to draw that out very much at all here, except for some very brief comments at the end.[1] Instead, I want to take a step back from the particularly paradoxy stuff, and focus on some problems raised by `tonk` for certain ways of thinking about meaning. These problems are, I think, completely parallel to the sorts of problems raised by truth predicates, vague predicates, and the like—this is why I find the problems interesting. But that parallel is not my topic here. `Tonk`, structural rules, and the nature of consequence will be the order of the day. The goal is to sketch a new way of looking at some old problems: one on which they turn out not to be problems after all.

1.1 Generalities

The sort of perspective I'll work from is familiar enough, but I'll still take the first half or so of the paper to lay it out in a particular way—that way we can see how naturally the final resolution flows from this starting point.

Begin with an *inferentialist* or *proof-theoretic* theory of meaning; although there are a lot of specific theories in this broad area, we don't yet need to fuss about the differences. (I'll be narrowing down in a little bit.) On this kind of view, things mean what they do in virtue of the roles that they play in valid arguments. More specifically, since arguments have both premises and

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[1] In fact, I came to this topic through its potential applications to paradoxes in the first place. See eg [Cobreros et al., 2012, Ripley, 2013a, Cobreros et al., 2013, Ripley, 2013b]. Others have explored related avenues for addressing these paradoxes as well; see for example [Zardini, 2008, Schroeder-Heister, 2012, Weir, 2005].

$$\neg R: \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \qquad \neg L: \frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta}$$

$$\wedge R: \frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta} \qquad \wedge L: \frac{\Gamma, A/B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$$

Figure 1: Sequent rules for classical $\neg$ and $\wedge$

conclusions, meaning on this kind of picture is a matter of: 1) when something can occur in the *premises* of valid arguments, and 2) when it can occur in the *conclusions*. This kind of idea has been explored by a great number of authors in a great number of settings,[2] and it's this kind of approach that Prior's `tonk` is meant to shed some light on.

Here, I'll work in a setting closest to that of [Kremer, 1988, Restall, 2005]: with left and right rules in a sequent calculus. (For example, see Figure 1, which includes some standard rules for classical negation and conjunction. I'll be referring back to these rules frequently as we go.[3]) Every rule tells you how to *bring something in* to the sequent. To see how to conclude something, we have rules that introduce it on the *right* side of the sequent, where the conclusions go; and to see how to use something as a premise, we have rules that introduce it on the *left* side of the sequent, where the premises go.

The difference with usual natural deduction rules is going to turn out to matter for my purposes. For example, we might consider 'sequent-style natural deduction' rules for $\wedge$ like those in Figure 2. Here, the introduction rule corresponds exactly to the *right* introduction rule in the familiar sequent setting, but the elimination rule is a different kind of critter altogether. Instead of telling us how to use $A \wedge B$ as a premise or a conclusion, it tells us that once we have $A \wedge B$ as a conclusion, we can go on to conclude A or B in its place (our choice). In fact, the rules in Figure 2 are entirely conclusion-focussed; they tell us *nothing at all*—at least not directly—about how to use $A \wedge B$ (or anything else) as a *premise*. The only guidance we could get would have to come by way of making assumptions about $\vdash$ itself. This is totally fine when those assumptions are secure and backgrounded, of course; but here they will be brought in for interrogation. Better, then, to use rules that don't depend on these assumptions: left and right introduction rules, which tell us *explicitly* how to use our vocabulary in premises and conclusions alike.[4]

[2] See, for just a few examples, [Dummett, 1983, Prawitz, 1965, Tennant, 1987, Restall, 2009, Schroeder-Heister, 2006].

[3] Notation, etc: capital Roman letters are individual formulas; capital Greek letters (those that aren't also capital Roman letters!) are structures, with comma as sole structural connective. For much of the paper, you can think of these structures as multisets; I'll only look for a moment at any finer distinctions between structures. A/B can be either A or B.

[4] On the other hand, natural-deduction elimination rules like those of [Tennant, 2012], where the major premise is required to 'stand proud', can be seen as telling us something *directly* about premises. See also [Schroeder-Heister, 2004].

$$\wedge\text{I}: \quad \frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta} \qquad\qquad \wedge\text{E}: \quad \frac{\Gamma \vdash A \wedge B, \Delta}{\Gamma \vdash A/B, \Delta}$$

Figure 2: Sequent-style natural deduction rules for $\wedge$ (boo hiss!)

$$\texttt{tonk}\text{R}: \quad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash A \,\texttt{tonk}\, B, \Delta} \qquad\qquad \texttt{tonk}\text{L}: \quad \frac{\Gamma, B \vdash \Delta}{\Gamma, A \,\texttt{tonk}\, B \vdash \Delta}$$

Figure 3: The `tonk` rules

Of course, we can argy-bargy about whether this is the right theory of negation or conjunction. My main point for now is just that we can see these rules themselves as specifying something that negation and conjunction *might* mean; we've got the right sort of thing here *to* argy-bargy about. (In fact, this whole paper is going to work in a particular classical setting—I'm not going to consider other theories of negation, conjunction, etc, and I'll stick to a multiple-conclusion sequent framework. That's just for concreteness, though; the main ideas of the paper translate straightforwardly to at least some other settings.)

1.2 The threat of `tonk`

There's a well-known issue that this kind of approach—the kind that specifies meanings using introduction and elimination rules, or right and left rules—faces. In sequentland the issue arises like this: consider a binary connective `tonk` that obeys the rules in Figure 3.

The rule `tonk`R tells us that whenever you can conclude A in a certain context, then you can conclude $A\,\texttt{tonk}\,B$ in that same context; and `tonk`L tells us that if you can use B as a premise somewhere, then you can use $A\,\texttt{tonk}\,B$ as a premise in that same place. You might think "Here's a thing that I'll specify: I'll specify `tonk`, and I'll specify it in this way". And why not? After all, these rules certainly succeed in specifying particular ways to use `tonk`-sentences as premises and as conclusions. (This is the sequent version; Prior's original presentation [Prior, 1960] uses corresponding natural deduction rules.)

There's a potential (serious) problem with this, though; you can see it in Figure 4. From the assumption that A and B each entails itself (marked Ref), plus one application of each of the `tonk` rules, plus one application of transitivity (marked Trans), we can conclude that A entails B.

Depending on what A and B are, that might not be too much of a problem—some things do entail some other things, after all. But here, A and B here are *completely arbitrary*, and that's bad, because at least some things ought to *fail* to entail some other things.

There are only three things going on here, though. We've assumed that consequence is reflexive, we've assumed the `tonk` rules, and we've assumed tran-

$$\text{Trans: }\dfrac{\text{tonkR: }\dfrac{\text{Ref: }\dfrac{}{A \vdash A}}{A \vdash A\,\texttt{tonk}\,B} \qquad \text{tonkL: }\dfrac{\text{Ref: }\dfrac{}{B \vdash B}}{A\,\texttt{tonk}\,B \vdash B}}{A \vdash B}$$

Figure 4: A problem

sitivity. (At least we've assumed the particular instances of these involved.) So there's the problem. Something has to go. Reflexivity, the `tonk` rules, or transitivity. There's just no way to have all of them together.[5]

There's an obvious kneejerk response here: it's the `tonk` rules' fault! Reflexivity and transitivity are really plausible and intuitive, and they're the sorts of things we want around. Then there's the `tonk` rules, and they're completely useless and strange, and nobody cares about `tonk` and it's not good for anything, and it gets in the way of these very natural principles of reflexivity and transitivity. `Tonk` has to be ruled out; nothing can obey the `tonk` rules.

That's the usual way of responding to this, and *if you just look at Figure 4*, it pretty obviously seems like the right response. But I think it's the *wrong* response, that we need to take a broader view of things, and that when we do take a broader view of things, there's a way of seeing this situation that says not only is `tonk` fine, but that *any way at all* of introducing vocabulary using the right-and-left-rule strategy is fine. Anything goes.

Now that means at least one of reflexivity or transitivity has to go, since we certainly don't want to conclude that everything entails everything else. But let's not go straight to that quite yet.

2 The need for structural rules

The negation rules in Figure 1 seem to capture something important about classical negation. In fact, they seem to me to capture, in some sense, the *full story* about classical negation. And similarly for classical conjunction: in some sense the full story about classical conjunction is in those rules, in Figure 1.

There's an immediate objection to this claim that's got to be dealt with; the point of this section is seeing just what that's going to take. The problem is this: one core thing we know about classical negation and conjunction is that $A \wedge \neg A$ entails B, for any A and B. Explosion is classically valid. So if our story about classical negation and conjunction can't yield this conclusion—if it can't tell us that explosion is valid—then it can't be the whole story.

But try to take just the rules from Figure 1 and derive that. It won't happen; you can't get started. Those rules are just *conditional* rules. They tell you how to move from something you've already derived on to something new, but they don't give you anyplace to *begin* at all. So, the objection has it, the rules in

[5] I promised I wouldn't say too much about paradox, but here I can't resist: note how similar this is to the situation created by your favorite paradox.

$$
\begin{array}{rc}
\text{Ref:} & \overline{\quad A \vdash A \quad} \\
\wedge\text{L:} & \overline{\quad A \wedge \neg A \vdash A \quad} \\
\neg\text{L:} & \overline{\quad A \wedge \neg A, \neg A \vdash \quad} \\
\wedge\text{L:} & \overline{\quad A \wedge \neg A, A \wedge \neg A \vdash \quad} \\
\text{WL:} & \overline{\quad A \wedge \neg A \vdash \quad} \\
\text{KR:} & \overline{\quad A \wedge \neg A \vdash B \quad}
\end{array}
$$

Figure 5: Deriving explosion

Figure 1 *don't* give the whole story about classical negation and conjunction, since those rules alone can't show that explosion is valid—indeed, they can't show that *anything* is valid!

To answer this objection, let's first look at what it *would* take to show that explosion is valid, given the rules in Figure 1. We know we need someplace to start. Here's one place to start: from reflexivity. A entails A. We'll just help ourselves to that. Note that it's nothing about negation or conjunction; we're just assuming reflexivity is in play. Once we've got that, now we can do some work. Figure 5 gives a sample derivation of the sequent we're after—there are others that would work too, but they will all make the same point.

After the start, the derivation here gets to work with the negation and conjunction rules. Starting from $A \vdash A$, we first apply a $\wedge$L rule: if A entails A, then $A \wedge \neg A$ entails A. Now we pop on a $\neg$L rule, and we get a step closer; we can follow it with another $\wedge$L to get to $A \wedge \neg A, A \wedge \neg A \vdash$, which is a lot like our eventual target.

But there's more manipulation needed to get there. First we need to contract the two occurrences of $A \wedge \neg A$ down to one (here marked WL for contraction (W) on the left), and then we need to weaken in B as a conclusion (here marked KR for weakening (K) on the right). Contraction and weakening, like reflexivity, are *structural* rules: they don't involve any particular vocabulary. They apply across the board. In particular, they—again—have nothing to do with either negation or conjunction.

So overall, there's three big steps in that derivation that have nothing to do with the negation or the conjunction rules. We need reflexivity to start off, we need contraction to collapse the two $A \wedge \neg A$s together, and we need weakening to get the B involved. The rules in Figure 1 don't give us any of those; where do they come from?

They come from $\vdash$.

Recall the original motivation: to give the meanings of various bits of vocabulary by specifying how they work in valid arguments. This moves from validity to meanings; what validity is must be *antecedently given.* It's only with a conception of validity *already in hand* that the rules in Figure 1 ever purported to give the full story about classical negation and conjunction. But the objection under consideration fails to take this into account; it supposes that the only thing we know about validity itself is what's covered in Figure 1.

We should reject that supposition. Given an appropriate theory of validity, the rules in Figure 1 *do* give the full story about classical $\neg$ and $\wedge$. This is a response that comes with a price: to see the rules in Figure 1 as giving the full story of classical negation and conjunction, we must give some story about $\vdash$ that justifies all of reflexivity, contraction, and weakening.[6]

3 A conception of consequence

I'm going to give a certain story of what that turnstile amounts to, so that I can address the structural rules. It's not a conception that's original with me; I'm lifting it from Greg Restall, who's presented and defended it in a number of places, eg [Restall, 2005, Restall, 2009, Restall, 2013]. It's a conception that fits nicely into my overall plans, which is why I'm lifting it. I won't argue for it here; I just want to see what we can do with it.

Here's the rough idea. What it is for a bunch of premises to entail a bunch of conclusions is that if you assert the premises and deny the conclusions, then you're out of bounds. That's the story I want to work with. It's a story that starts with three moving parts: assertion, denial, and out of bounds. Using those three notions, it gives an understanding of consequence.

For purposes of this paper, I don't have much to say about assertion or denial. Let's take those theories as they come: everybody needs some theory or other of assertion and denial, and I'm happy not to look at the details for now. But I do want to say a little bit at least about the notion of a collection of speech acts being *out of bounds*, which might be less familiar.

The role this is playing is as a constraint on what kinds of things people can get away with in the conversational positions that they adopt. For example, if you think about what reflexivity amounts to on this conception, reflexivity is the claim that asserting and denying the same thing is out of bounds. Whatever assertion and denial are like, you're conflicting with yourself if you assert and deny the same thing. Not that you *can't* do it; go ahead. But what you've done *clashes* in some way. It's to be ruled out by some coherence-based norms on assertion and denial.

There are *lots* of potential norms on asserting and denying: assertions should be true, or known, or justified, or kind, or relevant, or whatever kinds of norms you like, and similarly for denials, mutatis mutandis.[7] Among these norms, it seems clear, is something like a coherence constraint. A particular assertion might tick all the boxes that apply to it *in isolation*—it might be true, known,

[6] There are ways to formulate classical logic without some structural rules, by giving different operational rules that build in the needed structural effects. See for example the system G3c in [Troelstra and Schwichtenberg, 2000, p. 77]. The negation and conjunction rules of G3c can determine classical negation and conjunction somewhat more directly; rules of contraction and weakening are dispensed with. But the 'weakened reflexivity' axioms of G3c still need to be justified before we can get full classical negation and conjunction—so the situation is not really very different. Other possible ways to divide up the structural/operational work would also have to be dealt with case by case.

[7] Although see [Ripley, 2014c] for worries about pulling off the mutation in certain cases.

justified, etc—but it might still go wrong because it doesn't fit with other assertions or denials that've been made. This norm of fit applies in the first place to *groups* of assertions or denials rather than individual ones.

This arises in conversations frequently enough. Consider what's going on when someone makes a conversational move like 'But wait a minute, no no no, because a minute ago, you said . . . '. They're spotting, or at least claiming to spot, that the person they're talking to has said things that don't fit together. Someone's gone out of bounds; they've adopted an overall position that conflicts with itself in some way.[8]

3.1 Justifying structural rules

The exact details of this kind of fit remain to be worked out; for now, I think this is enough to work with a bit. Returning to the example of reflexivity, this amounts to the claim that asserting and denying the very same thing is no good, out of bounds, that an assertion and a denial of the very same thing don't fit together. I think that's plausible enough, for what it's worth. It might be worth considering what things would be like without that constraint, but I'll work within it going forward.

How about the other rules used in Figure 5? There's contraction. On this conception contraction on the left amounts to: if asserting some thing twice puts you out of bounds, then asserting it once puts you out of bounds already. (Contraction on the right amounts to the corresponding claim about denials.) Is this so? This is a question essentially about what it is to go out of bounds, about which norms of fit actually govern our conversational practice. When we're spotting clashes, when we're spotting self-conflict, is it the sort of thing that's sensitive to how many times someone has asserted something, or when they say it once is that enough to hold them to it across the board?

I think it's the latter. I think if they say it once then that's enough to hold them to it across the board, that when we're sensitive to conflicts in someone's position like this, what we're sensitive to is *which things* they've asserted, not the *number of times* they went about asserting them, and similarly for denials. As such, $\vdash$ is closed under contraction *come what may*; no matter what vocabulary is in play, consequence in this sense is closed under contraction (on both sides).[9] One's as good (or as bad) as a hundred, as far as fit goes.

But here's a really nice thing about adopting a concrete conception like this: even if you disagree with me on that, we now have a useful way to proceed. We have a conception of what consequence is, and differences over contraction are now differences about substantive issues, in this case norms of coherence on

[8] I discuss this in more detail in [Ripley, 2014b]. [Dutilh Novaes, 2011] is also relevant.

[9] Compare the 'Venusian connective' $\mathbb{V}$ in [Humberstone, 2012, p. 592–3]. As Humberstone points out, if the Venusians allege that they have a connective $\mathbb{V}$ such that $\mathbb{V}B$ doesn't follow from $A \wedge \mathbb{V}B$, the right thing to conclude is not that $\mathbb{V}$ is very strange—it's that the Venusians don't understand how our $\wedge$ works. Similarly, if the Venusians allege that $\mathbb{V}B, \mathbb{V}B \vdash A$ but $\mathbb{V}B \not\vdash A$, it can't just be that $\mathbb{V}$ is very strange—instead, they must not understand how $\vdash$ works.

assertion and denial. So not only does this conception allow for a defense of contraction, but the way it does this is by making clearer *just what contraction amounts to* in this context.[10]

So anyway we've got contraction on board, for the reading I've given of the turnstile. How about weakening? Weakening is the claim that, if you've already gone out of bounds, and you just keep asserting or denying things, then you can't bring yourself back in bounds. That's plausible enough: if you've said things that clash with themselves, if what you've already said doesn't fit together, then what you have to do to undo the clash is *take some of those things back*. Adding more things on top won't help. Because adding more assertions won't help, we have weakening on the left. Because adding more denials won't help, we have weakening on the right. Like contraction, weakening is justified *come what may*; it's simply part of what fit is, no matter what vocabulary we're considering.

And again, even if you reject weakening, this now gives us a *substantive issue* for weakening to be about, rather than just: can I add this kind of squiggle over here? It becomes a concrete issue in the norms of assertion and denial.

So, given this kind of understanding of the turnstile, I think that all of reflexivity, contraction, and weakening are justified, so I reckon the derivation in Figure 5 goes through just fine. Its structural moves are justified by features that assertion and denial have with absolute generality, no matter what's being asserted or denied, and its operational moves are justified by the rules in Figure 1, which give $\neg$ and $\wedge$ their meaning by relating them to norms on assertions and denials in a particular way.

I haven't mentioned structural rules of exchange or permutation, where you shift the order of different premises or different conclusions, and I haven't mentioned structural rules of associativity. I think those are going to follow from this conception as well: when we're sensitive to a clash in someone's position, we don't really care what order they laid out their position in. That might matter for some things, but it doesn't matter for this thing, and that'll justify exchange. Associative structure goes similarly.

So this gives you a conception of consequence on which these structural rules—reflexivity, weakening, exchange, contraction, associativity—are all justifiable in complete generality, they're to be expected, because of these features of the norms we're looking at. And it turns out, if you add all these structural rules to the operational rules of Figure 1, pow, you get classical propositional logic: a version where you define everything from $\neg$ and $\wedge$. These rules are enough. It's a cut-free sequent calculus for classical propositional logic.

If you'd prefer (and I certainly would prefer) not to define everything out of $\neg$ and $\wedge$, but instead to deal with things like disjunction and the material

[10]You might, of course, offer a different conception of the turnstile, and hopefully that would make it clear what contraction would amount to in *that* context. For some purposes, contraction is clearly no good (see [Restall, 2000] for a bunch of examples). Crucially, though, it's only with *some* conception of what the turnstile amounts to that we can have any idea of what we're differing about, if we're differing about contraction. And just saying 'consequence' isn't enough to get this benefit—there are too many candidates for what that might amount to.

$$\text{Cut:}\quad \frac{\Gamma \vdash A, \Delta \qquad \Gamma', A \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

Figure 6: The cut rule

conditional on their own terms, directly, in terms of operational rules tailored to them, then just whack those rules in too, and it will turn out, well, you still have classical logic. You like quantifiers? Add them too.[11]

These structural rules, the ones that we have here, are the ones that you need to get from the operational rules under consideration to the full classical consequence relation. Note that all of these rules, structural and operational alike, govern the full language, whatever else the language turns out to include: all the rules are in schematic form, applying to As and Bs whatever they are. Because of this, what you get is the logic extended to the full language *whatever* it turns out that the language includes. No matter what A is—even if it turns out to have, say, a `tonk` in it—its negation obeys the $\neg$ rules, and it contracts, etc.

3.2 Cut

I haven't mentioned the rule of cut yet, but it's where a great deal of the action is. Cut is the rule in Figure 6. It's a particular sort of generalization of transitivity.[12] But *in terms of the going conception of the turnstile*, what does cut mean? The situation is just as it was for the other structural rules: the relevant question is *not*: 'When I think consequence, does transitivity pop into my head?' (It probably does. It certainly pops into *my* head, anyway.) Instead, we have a concrete issue about the norms on assertion and denial. Which potential norms are encoded by the rule of cut? Do they in fact apply?

The norms encoded by this rule require a certain sort of *extensibility.* Cut says that if asserting the Γs and denying the Δs blocks you from denying A without going out of bounds, and asserting the Γ's and denying the Δ's blocks you from asserting A without going out of bounds, then if you do all those things together, asserting the Γs and the Γ's and denying the Δs and the Δ's, then you're *already* out of bounds. You don't have to say anything about A (A doesn't appear in the conclusion-sequent of the cut); you're already clashing before A even comes up.

[11]For an example such formulation of classical logic, see the calculus G1c of [Troelstra and Schwichtenberg, 2000, p. 61]. There are plenty of different formulations, though, that vary in ways that don't matter for my purposes here.

[12]There are a few different versions of it out there, but with both contraction and weakening in play, some of them turn out equivalent. Since we haven't assumed anything like compactness, some dimensions of variation still remain (see [Shoesmith and Smiley, 1978, Humberstone, 2012]); the version of cut I'm using is a particularly weak version, and even it will have to go, so that tells you what I think of stronger versions.

That is, cut says that if you've done some things that rule out your denying A, and you've done some other things that rule out your asserting A, then it's already too late. What you've already done doesn't fit together. You've got to leave yourself a path open to take on A, either leave open the option of asserting it or the option of denying it, *for any A always.*

I don't see any particular reason why we should expect that. It certainly seems very straightforward, very coherent, to reject that kind of constraint, and say no, there are some sorts of things that the problem is with *them.* They can't be coherently asserted, and they can't be coherently denied, but that's not *my* fault. I'm perfectly well in bounds. It's the thing itself that's the source of the trouble with either asserting or denying it.

That isn't yet an argument that there are such things. It doesn't need to be. It's just saying that there's nothing in the nature of assertion and denial that rules them out. So we shouldn't impose cut come what may; it would be unjustified. Cut is unlike the other structural rules I've considered here: it is not part of the basic norms on assertion and denial. (I'll consider an argument to the contrary, due to Restall, in §4.1.) If entailment happens to be closed under cut, it is not because it had to be; it will be because the things we have available to assert and deny turn out to be reasonably well-behaved.

3.3 Admissibility

Even though we've seen no reason to *impose* a rule of cut for $\vdash$, cut's still *admissible* over the kind of formulation of classical logic I've gestured at.[13] (Let's keep `tonk` off the table just a bit longer.) Any sequent we could derive by helping ourselves to cut is in fact already derivable, even though we haven't imposed cut at all. So I want to look at the status of admissible rules, particularly admissible structural rules, because that will clarify what's going on with cut more generally.

The idea is this: admissibility is fine and dandy as far as it goes, but it only goes so far. When a rule is admissible in a system, that may make it (among other things) a useful shortcut for finding out what's derivable in that system; but it doesn't do anything to *justify* the rule in the way I've attempted to justify reflexivity, weakening, contraction, and so on. And without such justification, it's a mistake to impose the rule *come what may.* Admissibility is too weak a property to justify that kind of imposition.

A toy example will help make my point. Consider a sequent system for $\vdash$ before we add on any rules governing *any vocabulary at all.* Just take it with the structural rules that I've argued it obeys. So we've got a turnstile, it's reflexive, it contracts, it weakens, it obeys permutation and associativity, but there are no rules governing any vocabulary at all. All we have is the basic structure of assertions and denials. Call this the *bare calculus.*

In the bare calculus, what can we demonstrate? What arguments must be valid if we make no assumptions about any vocabulary? Not much, it turns out:

[13] A rule is *admissible* in a sequent calculus iff whenever the rule's premises are derivable in the calculus, then so is its conclusion.

$$\text{Swap:} \quad \frac{\Gamma \vdash \Delta}{\Delta \vdash \Gamma}$$

Figure 7: The swap rule

$\Gamma \vdash \Delta$ is derivable in the bare calculus exactly when Γ and Δ overlap, just when there's something that comes up on both sides.

Now consider the rule of *swap*, given in Figure 7. This rule, given the going reading of $\vdash$, says that if one collection of assertions and denials is out of bounds, then the swapped collection—the one that denies everything the first collection asserts and asserts everything the first collection denies—is out of bounds too. As with cut, there is no reason to expect this to be the case in any generality; it certainly is not built into the norms governing fit between assertions and denials. But it does have at least this to recommend it: it's *admissible* in the bare calculus. After all, if Γ overlaps with Δ, then Δ overlaps with Γ.

If we found ourselves concerned (for some reason) with which arguments can be shown valid in the bare calculus, swap could turn out to be a handy sort of rule to have around. After all, it would cut our work in half; like duality principles in lattice or category theory, it would mean every one of our derivations turns out to prove two things. I don't know about you, but I could get used to that kind of shortcut.

As we expand the bare calculus by adding rules for certain vocabulary, we can see that there are two kinds of vocabulary we can add. Certain vocabulary, like $\neg$, plays nice with swap; we can add the Figure 1 rules for $\neg$ to the bare calculus without disturbing swap at all. Other vocabulary, like $\wedge$, breaks this admissibility. Once we include rules for $\wedge$, there's a counterexample to swap: given $\wedge$L (together with reflexivity), we can derive that $p \wedge q$ entails p, but p certainly does not entail $p \wedge q$, and we can't derive that it does.

Although swap is admissible for some restricted systems, like the bare calculus and its $\neg$-only extension, there's nothing at all in the nature of the turnstile that's guaranteeing swap. And so, as soon as we take account of vocabulary like $\wedge$, swap fails. This isn't any problem with $\wedge$; it's just one of those things. Although swap is convenient where it happens to be admissible, to expect it to govern our full language would be a mistake: it would prevent us from giving the kind of theory of things like conjunction that we'd otherwise like.

Cut's like *that*. The only difference is that cut turns out to be so convenient for certain purposes that the rules we study are often designed around preserving cut-admissibility. There's a whole industry producing rules for various pieces of vocabulary in various settings, showing that those rules leave cut admissible; and that's all well and good. Indeed, it's vital for a number of purposes. But cut doesn't have to be—and shouldn't be—wired into our theory of meaning.[14]

[14] This is not the first time cut's been questioned, of course. In addition to the citations in footnote 1, see also [Cook, 2005, Tennant, 1987], as well as [Paoli, 2002] and citations therein. Of these, only [Cook, 2005] takes `tonk` seriously.

Because cut isn't really part of the nature of consequence, since it's just a pattern that some collections of rules exhibit (again—perhaps for good reason!), we might well find some vocabulary that does to cut exactly what conjunction did to swap. Once we take account of that extra vocabulary, the things we can derive are no longer closed under cut—because they never *had* to be; they only ever *happened* to be. Mistaking cut for a constraint that consequence *has* to be closed under would block us from giving a good theory of such vocabulary, just as mistaking swap for such a constraint would block a good theory of conjunction.

As it happens, `tonk` is a piece of vocabulary that stands to cut as conjunction stands to swap. Once the `tonk` rules are added, cut is no longer admissible. Given reflexivity, which is still not in question, you can derive that the argument from A to $A\,\texttt{tonk}\,B$ is valid, and that the argument from $A\,\texttt{tonk}\,B$ to B is valid, but there is no way to derive that A entails B in full generality—and a good thing, too![15]

Think again of the claim cut makes requiring extensibility. Given the `tonk` rules, if I assert A and deny B, I can't remain in bounds while asserting $A\,\texttt{tonk}\,B$, and I can't remain in bounds while denying $A\,\texttt{tonk}\,B$. For all that, asserting A while denying B may perfectly well be in bounds; after all, A and B are arbitrary here. The trouble with asserting or denying $A\,\texttt{tonk}\,B$ doesn't cause any trouble in my position itself; it simply prevents extending my position in various ways. This is just the kind of case where we should expect counterexamples to cut.

3.4 Conservativeness

In the neighborhood here, there's just about the world's easiest general conservative extension result, which I'll pause to sketch. One of the reasons we bother to prove cut elimination for various sequent systems is in order to prove conservative extension results. You whack some rules on for the new vocabulary, you assume that the output is closed under cut, and then you eliminate cut so you know you don't get anything new that actually relies on cut. Why do you do that? Because conservative extension results are *really easy* to prove if you're in a system that doesn't have any cut in it, at least if the system is based on left and right rules of the kind I've been working with here.

Here's how it works for `tonk`: when you apply a `tonk` rule, `tonk` itself appears in the conclusion-sequent, and there's no way to get rid of it. Any applications of other rules (besides cut) might embed it more deeply, but cut is the only rule

[15] Here an anonymous referee objects: 'So what? If multiplicative conjunction is added to the vocabulary, contraction is no longer admissible, and if relevant implication is added, thinning [weakening] is no longer admissible.' But this is not the case. In section 3.1, I argued that contraction and weakening govern $\vdash$ (on the particular reading in play here!) *come what may*. A full theory of $\vdash$, then, will *impose* contraction and weakening (along with reflexivity, permutation, associativity).

If multiplicative conjunction is added to such a theory, contraction remains admissible (because derivable!). Similarly for relevant implication and weakening. Only rules that are not independently justified—like swap and cut—are at risk from new vocabulary. See also footnote 9.

that gets rid of anything once it appears. (Contraction can get rid of *occurrences* of things, which is why it buggers up proof search, but it can't get rid of the things themselves.) So anything new that the `tonk` rules allow you to derive must be something that *actually contains* `tonk`. That's conservative extension.

Given right and left rules for a piece of vocabulary, rules that tell you how to use the thing as a conclusion and that tell you how to use the thing as a premise, you can get a conservative extension result along these lines. That's an important sense in which anything goes: anything defined in this way is *conservative*.

The only way this can go wrong is if there is a rule present *before* the new vocabulary is introduced that can get rid of the new vocabulary. In this case, cut isn't the *only* thing that can hide uses of the new vocabulary, and so it is not the only threat to conservativity. But so long as this situation doesn't occur, rules that introduce new vocabulary, no matter what they are, will result in a conservative extension.[16]

Conservativeness is sometimes put forward as an important requirement on giving rules for new vocabulary, most famously in [Belnap, 1962]. Belnap's discussion of conservativeness assumes that a particular form of transitivity is in force, but makes plain that this is merely part of an example chosen for definiteness; the idea of conservativeness applies easily even in the absence of cut. In fact, so long as we do not impose cut, conservativeness is a very easy hurdle to clear; we can have a much freer hand with our defining rules than you might have suspected, while still remaining conservative.

[16]To answer an anonymous referee's query: this thus requires no restriction on the number of times a new piece of vocabulary can occur in its rules, or on how deeply embedded it might occur, or on whether it occurs on both sides of the turnstile in the conclusion-sequent. As long as cut is not present, as long as no old rules eliminate the new vocabulary, and as long as the new vocabulary itself occurs in the conclusion-sequent of its rules, these rules will be conservative. See also [Kremer, 1988] for discussion.

This referee also asks whether it is true in general that a connective definable by an arbitrary set of conservative rules (in the present setting) has an equivalent definition in terms of left and right sequent rules, adding 'A positive answer to this general question would be a strong reason to restrict one's attention to left and right rules'.

On a certain understanding, the answer is indeed positive—but the understanding may well be unsatisfying. Suppose we read 'equivalent' as 'determines the same consequence relation'. Consider a calculus C determining a consequence relation $\vdash_C$, conservatively extended with rules for a new piece of vocabulary to yield a calculus C' determining a consequence relation $\vdash_{C'}$. Consider the set X of sequents valid in C' but not in C; since the extension to C' was conservative, the new vocabulary occurs in each sequent in X. Now, add to C, as the rule for the new vocabulary, a rule allowing us to conclude any sequent in X from no premise-sequents—or, what amounts to the same, add a new rule for each sequent in X, allowing us to conclude that sequent from no premise-sequents. Nothing besides sequents already in X will follow from this addition, since C' was conservative.

One reason this may be unsatisfying is this: there may be no schematic formulation of the addition. Another is that there may be members of X involving the new vocabulary on both the left and right, or deeply embedded. Another—and this seems to me the most important—is that this is a very weak notion of equivalence between sequent systems. I don't know to what extent these features are avoidable.

4 Objections

4.1 Restall's argument for cut

There's an argument due to Restall to the effect that $\vdash$, even on the present reading, must be closed under cut. He's given variants of this argument in [Restall, 2005, Restall, 2009, Restall, 2013]; here I'll address it as presented in [Restall, 2013].[17] Restall says (notation changed):

> [I]f there is no clash in asserting every member of Γ and denying every member of Δ, we can see that together with asserting each member of Γ and denying each member of Δ either there is no clash in asserting A or there is no clash in denying A. In other words, if asserting A is ruled out by means of the rules of the game alone, then since A is unassertible, its denial is implicit in the assertion of every member of Γ and the denial of every member of Δ, so its explicit denial involves no clash. Contraposing this, if there is a clash in denying A (together with asserting every member of Γ and denying every member of Δ) and there is also a clash in asserting A (together with asserting every member of every member of Γ and denying every member of Δ), then there is a clash in asserting every member of Γ and denying every member of Δ alone. In other words, we have [cut].

Of the four sentences here, the first, third, and fourth simply state what is to be shown. The core of this argument is in its second sentence: according to Restall, if asserting A is ruled out by having made some other acts (asserting everything in Γ and denying everything in Δ), then the denial of A is implicit in having made those acts. Call this conditional RP, for 'Restall's principle'. By RP, if the acts themselves are in bounds, and if they rule out asserting A, then adding a denial of A to the acts must also be in bounds (assuming, what seems plausible, that the difference between implicit and explicit denial doesn't matter for inboundsness).

But this begs the question, as a quick formalization of the situation will show. Following Restall, let a *position* be a pair $\langle \Gamma, \Delta \rangle$ of sets of sentences; each position *asserts* each member of Γ and *denies* each member of Δ. A position $\langle \Gamma, \Delta \rangle$ is *out of bounds* iff $\Gamma \vdash \Delta$.[18] Let two positions $\langle \Gamma, \Delta \rangle$ and $\langle \Gamma', \Delta' \rangle$ be *equivalent* iff for every position $\langle \Gamma'', \Delta'' \rangle$, either both $\langle \Gamma \cup \Gamma'', \Delta \cup \Delta'' \rangle$ and $\langle \Gamma' \cup \Gamma'', \Delta' \cup \Delta'' \rangle$ are in bounds or else both are out of bounds. In other words, two positions are equivalent when they have exactly the same possibilities of being extended by further assertions and denials, where possibility is delimited

[17] In [Ripley, 2013a], I consider the argument as presented in [Restall, 2005]. In all cases, Restall considers so-called 'additive cut' rather than the 'multiplicative' version I've used here, but since I agree with Restall that contraction and weakening are in force for $\vdash$, this makes no difference.

[18] Enough structural rules are uncontroversial here that this is well-defined even though positions use sets.

by staying in bounds. Equivalent positions are the same, as far as the bounds can tell.

The most plausible interpretation of 'implicit denial', as it occurs above, is as follows: a position $\langle \Gamma, \Delta \rangle$ *implicitly denies* A iff it is equivalent to $\langle \Gamma, \Delta \cup \{A\} \rangle$. According to this definition, a position implicitly denies A iff going on to *actually* deny A makes no difference to it; adding an explicit denial of A leaves the position in just the same situation it was already in. This fits neatly with the role implicit denial plays in Restall's argument: implicit denial, on this construal, is anything that's just like explicit denial as far as inboundsness goes.

Now we have a precise content for RP: if $\langle \Gamma \cup \{A\}, \Delta \rangle$ is out of bounds, then $\langle \Gamma, \Delta \rangle$ implicitly denies A. This is the principle at the heart of Restall's argument for cut. But this principle is *equivalent* to cut; it's a mere restatement of what's at issue.[19]

So Restall's argument does not provide support for cut. It's circular, invoking RP as its main premise, when RP is equivalent to the conclusion. If we have no reason to accept cut for this reading of $\vdash$ in the first place—and I maintain that we don't—then Restall's argument gives us no new reason.

4.2 Harmony

At the end of section 3.4, I briefly mentioned demands for *conservativeness* in giving rules for new vocabulary, to point out that `tonk` meets these demands if we don't impose cut. But these are not the only demands in the literature; demands for *harmony* are at least as common. Authors in the broad tradition I'm engaging with here seem widely agreed: it's important that the left and right rules for a piece of vocabulary (or its introduction and elimination rules in a natural deduction setting) be *harmonious* with each other. Rules for `tonk` and the like *don't* meet this demand, even without cut, so it's worth pausing on this for a moment.

One thing to note about the demand for harmony: there is no single thing that it is a demand for. As examples, [Read, 2010, Francez, 2014, Dummett, 1991, Poggiolesi, 2011, Humberstone, 2012, Prawitz, 1965, Schroeder-Heister, 2013, Sambin et al., 2000, Tennant, 1997] all offer understandings of harmony or something closely related, but these accounts differ from each other in various respects. Although the accounts are certainly related, there are classes of rules that count as harmonious on some accounts

[19] To get from cut to RP, assume that $\langle \Gamma \cup \{A\}, \Delta \rangle$ is out of bounds; that is, $\Gamma, A \vdash \Delta$. Now we must show, using cut, that $\langle \Gamma, \Delta \rangle$ is equivalent to $\langle \Gamma, \Delta \cup \{A\} \rangle$; that is, that $\Gamma, \Gamma' \vdash \Delta, \Delta'$ iff $\Gamma, \Gamma' \vdash \Delta, \Delta', A$, for any Γ', Δ'. LTR: $\Gamma, \Gamma' \vdash \Delta, \Delta', A$ follows from $\Gamma, \Gamma' \vdash \Delta, \Delta'$ by weakening on the right. RTL: $\Gamma, \Gamma' \vdash \Delta, \Delta'$ follows from $\Gamma, \Gamma' \vdash \Delta, \Delta', A$ together with $\Gamma, A \vdash \Delta$ (which we assumed) by cut and contraction on both sides.

To get from RP to cut, assume the premises of a cut: $\Gamma \vdash A, \Delta$ and $\Gamma', A \vdash \Delta'$. By Restall's principle applied to the second premise, $\langle \Gamma', \Delta' \rangle$ is equivalent to $\langle \Gamma', \Delta' \cup \{A\} \rangle$. Thus, position $X = \langle \Gamma \cup \Gamma', \Delta \cup \Delta' \rangle$ is out of bounds iff position $Y = \langle \Gamma \cup \Gamma', \Delta \cup \Delta' \cup \{A\} \rangle$ is out of bounds. But we have assumed as our first premise that $\langle \Gamma, \Delta \cup \{A\} \rangle$ is out of bounds; by weakening, it follows that position Y is out of bounds; and so position X is out of bounds too. That is, $\Gamma, \Gamma' \vdash \Delta, \Delta'$; this is the conclusion of the cut.

but not others. The details of these accounts are interesting, but they cannot be my topic here. However, this doesn't scupper the discussion; there are some points of clear commonality, and those are enough to steer by: the rules given above for $\wedge$ and $\neg$ are harmonious. The rules given above for `tonk` are not.[20]

The common core to these understandings is something like: harmonious left and right rules *fit together* in a certain way. Perhaps it must be possible to determine one from the other, or perhaps they must interact in a certain pleasant way in cut-elimination proofs (or normalization proofs, in natural deduction presentations), things along these lines. `tonk` is disharmonious because the two `tonk` rules *don't* fit together in these ways; `tonk` is a mongrel connective, coming from an asymmetric-disjunction-like right rule together with an asymmetric-conjunction-like left rule. There would have been no way to predict either of these rules from the other, nor do they allow for cut-elimination (as we've seen, this is because they provide counterexamples to cut). So the `tonk` rules are certainly disharmonious.

This is not yet a problem for the present view. There are at least two importantly distinct stories about what harmony is a requirement *for*.[21] On one more modest story, taken up in eg [Tennant, 2014], rules must be harmonious in order to succeed in specifying the meaning of *a logical constant*. On another, taken up by eg [Francez, 2014, Poggiolesi, 2011], rules must be harmonious in order to succeed in specifying the meaning of *anything at all*. I have no quarrel with any account of the former modest stripe, because I have nothing at all to say about which things are or are not logical constants.[22] `tonk` is perfectly meaningful, I say, but I make no claims about its logicality.[23] So it is only the latter, more restrictive, accounts, that I must (and do) reject.

What reason is there, then, to think that harmony *does* set limits on possible meanings? The most common argument in the literature is based on `tonk` and its ilk. (This is the motivation given in [Poggiolesi, 2011, Francez, 2014] for the strong harmony restriction.) The worry is that, without a harmony restriction or something like it, we will be forced to the conclusion of Figure 4: everything entails everything else! And of course this would be a bad conclusion to be

[20] I ignore in this discussion [Dummett, 1991, p. 250]'s notion of 'total harmony'. This is conservativeness, which I've already discussed. His 'instrinsic harmony' is closer to the stream of literature I'm pointing to.

[21] If it is a requirement at all. Consider for example [Read, 2008, p. 294]: '[P]erfectly decent connectives can be governed by inharmonious rules. Such cases are unhelpful, in obscuring the meaning of the connective, but the lack of harmony is not itself a source of incoherence, nor does it mean that the rules do not define the meaning of the connectives as logical.'

[22] There are other theories too about what harmony is required for. For example, there is the claim in [Dummett, 1991, p. 247] that rules must be harmonious in order to be *self-justifying*. This claim too does not intersect my topic here, since I make no claims about self-justification.

[23] An anonymous referee wonders just what `tonk`'s meaning *is*. I reckon the question is misplaced, just as the corresponding question about, say, conjunction would be. We have a full description of the way `tonk` interacts with norms governing assertions and denials; this is enough—just as it is for conjunction. If we want to find some *thing* to be the meaning (whether of `tonk` or of $\wedge$), there are multiple candidates: the rules themselves, the constraints the rules put on us, some abstract object determined by the rules, the patterns of conversational norms the rules encode, etc. I doubt it is productive to fix on a single answer.

$$\text{super-tonk 1:} \quad \frac{\Gamma \vdash \Delta}{\Sigma \vdash \texttt{super-tonk}} \qquad \text{super-tonk 2:} \quad \frac{\Sigma \vdash \texttt{super-tonk}}{\Sigma \vdash \Theta}$$

Figure 8: `Super-tonk` rules

forced to.

But as we've seen, there is no need to fear. Given an appropriate conception of $\vdash$, `tonk` will not trivialize our language. More restricted threats are also not realized: as discussed above, we can achieve conservativeness quite easily in the absence of cut. So we don't need harmony at all to block bad conclusions that would otherwise follow.

Sometimes, on the other hand, harmony is pushed by a different route: by supposing that, say, the right rules for a piece of vocabulary must completely specify its meaning, so that the left rules then must be justifiable in light of that independently-given meaning. (Or vice versa.) Harmony then provides the route for this justification to travel. The consequences of this supposition—that one side's worth of rules *must* be enough—have been explored with great sensitivity and in great detail, from [Gentzen, 1969] onwards, including in many of the above-cited works. But the supposition itself is an optional add-on to the claim that meanings are given by roles in valid argument; we are not forced to it. (See again [Kremer, 1988], especially p. 58.)

Aside from arguments based on `tonk` itself, or arguments based on nonconservativeness (the first dialectically out of place and the second already dealt with), direct arguments that one side's worth of rules must be enough are in fact pretty thin on the ground. What we tend to get instead are frameworks (eg that of [Schroeder-Heister, 2013]) that *start* from the idea that one side's worth of rules is enough. The illumination we get from such frameworks then provides at best an indirect argument that this assumption was correct. But this illumination—which I do not deny—is just as compatible with the following idea: these frameworks are great for understanding *harmonious* vocabulary. It does not show that disharmonious vocabulary is thereby meaningless.

5 *Anything* anything?

Alas, it's maybe not quite *anything* anything that goes. For example, there's a connective that Heinrich Wansing discusses called `super-tonk` [Wansing, 2006]. And `super-tonk`, unlike `tonk`, is genuinely no good; it is not a legit piece of vocabulary. So I'll close by reflecting on `super-tonk`.

`Super-tonk` is a zero-place connective, and it obeys the rules in Figure 8. Given these rules, if anything at all entails anything at all, then everything whatsoever entails everything else. We're forced to either the empty or the universal consequence relation. This is clearly a bad result, and it does not depend at all on cut; the `super-tonk` rules alone are enough to derive it.

However, note that the rules, in particular `super-tonk` 2, don't have the form of right and left rules. The rule `super-tonk` 1 tells us how to conclude `super-tonk`, but `super-tonk` 2 doesn't tell us how to conclude `super-tonk`, or how to use it as a premise. Instead it gives us another kind of constraint. And this sort of constraint is not the kind that we can safely assume is going to work out ok.

Completely arbitrary rules are not a good way of specifying what's going on in a piece of vocabulary; they run the risk of leading us to falsehoods when we simply want to introduce a new bit of vocabulary. What *is* a good way is the basic setup of right and left rules, rules that show us how we can actually conclude things involving the vocabulary in question, and how we can draw on them as premises. Rules of this sort *can* be completely arbitrary; this is what the general conservativeness that obtains in the absence of cut can give us. But it is not arbitrary that they be of this sort.[24]

6 Conclusion

Although in the end I do need to rule out `super-tonk`, the approach outlined here is still a distinctively permissive one. Even if `super-tonk` isn't invited to the party, `tonk` still is. Importantly, this isn't just an attempt to sneak `tonk` in—`tonk` still seems, at best, pretty useless. The point is that some of the strategies we use to keep `tonk` locked out are also locking out interesting and valuable theories of certain important phenomena.

For example, consider the rules in Figure 9. These are potential rules governing the interaction between a truth predicate T and a naming device $\langle\ \rangle$; in many settings, they result in a transparent truth predicate, and combining them with usual rules for $\supset$ and $\wedge$ gives all the instances of the T-schema. But they are rules of the appropriate form; they tell us how to use $T\langle\ \rangle$ claims as premises and as conclusions. So we can add them conservatively to full classical logic, and get an interesting theory of the truthy paradoxes (as in [Ripley, 2013a]). Similarly, I reckon there's a good theory of vague predicates available that follows this kind of strategy as well (as in [Ripley, 2013b]). I also think there are interesting approaches to naive set theory and to confused concepts that work similarly.

[24] An anonymous referee gives the following rules for a unary connective $\star$, suggesting that it shows a problem with the present approach.

$$\star\text{L:}\quad \frac{\Gamma, A \vdash \Delta}{\Gamma, \star A \vdash \Delta} \qquad \star\text{R:}\quad \frac{}{\Gamma \vdash \star A, \Delta}$$

But this is no more of a problem than `tonk` was; it's just a new connective, perfectly well-specified. It blocks admissibility of cut in any sensible system, but that's fine.

The referee points out, by way of objecting, that $\star A \vdash \star B$ is derivable given these rules (in fact, given just $\star$R), for any A, B. That, though, just *can't* be a problem: that much holds as well of uncontroversially legitimate bits of vocabulary, for example $\Box$ in the modal logic **Ver** (see [Hughes and Cresswell, 1996, p. 66]), or classically where $\star A$ is interpreted as $A \supset A$, or as $A \wedge \neg A$.

$$T\langle\,\rangle\text{R:}\quad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash T\langle A\rangle, \Delta} \qquad\qquad T\langle\,\rangle\text{L:}\quad \frac{\Gamma, A \vdash \Delta}{\Gamma, T\langle A\rangle \vdash \Delta}$$

Figure 9: Some rules for truth

You might worry (as did an anonymous referee) that truth is not relevantly like `tonk`, since although both lead to failures of cut in the present setting, they do so for very different reasons—truth because attempts to eliminate cut in its presence never end (see eg [Tennant, 1982]), `tonk` because attempts to eliminate cut in its presence can't get started. But this is just to point to some of the diversity we rule out when we insist on cut. Truth and `tonk` are indeed very different, in just this way. Yet insisting on cut prevents us from engaging fully with either. (Vague predicates are more closely related to `tonk`; see [Ripley, 2013b].)

Trying to understand a way to keep full classical logic in the sense I specified, while opening up the option of giving these kinds of theories that we couldn't have given before, I think is really worthwhile. This is where the application to paradoxes comes in, and it's what's motivating me to do all this stuff in the first place. For now, though, that's all.[25]

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