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Investigating Knowledge and Opinion

John Corcoran and Idris Samawi Hamid

Philosophy, University at Buffalo, Buffalo, NY 14260-4150 USA

E-mail: corcoran@buffalo.edu

Philosophy, Colorado State University, Fort Collins, CO 80523-1781 USA

E-mail: ishamid@colostate.edu

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Abstract

This work treats the correlative concepts knowledge and opinion, in various senses. In all senses of ‘knowledge’ and ‘opinion’, a belief known to be true is *knowledge*; a belief not known to be true is *opinion*. In this sense of ‘belief’, a *belief* is a proposition thought to be true—perhaps, but not necessarily, known to be true. All knowledge is truth. Some but not all opinion is truth. Every proposition known to be true is believed to be true. Some but not every proposition believed to be true is known to be true. Our focus is thus on *propositional* belief (“belief-that”): the combination of propositional knowledge (“knowledge-that”) and propositional opinion (“opinion-that”). Each of a person’s beliefs, whether knowledge or opinion, is the end result of a particular thought process that continued during a particular time interval and ended at a particular time with a conclusive act—a *judgment* that something is the case. This work is mainly about beliefs in substantive informative propositions—not empty tautologies.

We also treat *objectual* knowledge (knowledge of objects in the broadest sense, or “knowledge-of”), *operational* knowledge (abilities and skills, “knowledge-how-to”, or “know-how”), and *expert* knowledge (expertise). Most points made in this work have been made by previous writers, but to the best of our knowledge, they have never before been collected into a coherent work accessible to a wide audience.

Key words:

belief, knowledge/opinion, propositional, operational, objectual, cognition, Gettier

There was a time when I *believed* that you belonged to me. But, now I *know* your heart is shackled to a memory. – Hank Williams

Preliminaries

It is only with respect to propositions¹ that the distinction between [sc. propositional] knowledge and opinion applies—for example, a person’s abilities cannot be said to be opinion or knowledge in the correlative sense. In fact, it is only with respect to propositions that belief applies: every belief is a proposition that is or was believed to be true by some person or persons, and conversely, of course.

The last sentence is lexically ambiguous² in virtue of its one occurrence of the ambiguous common noun ‘belief’: in the more basic sense, a belief is someone’s attitude of believing of a proposition; in the more derivative sense used above, a belief is a proposition someone believes. In the basic *attitudinal* sense, a person’s belief that Plato admired Socrates is inseparable from the person. In the derivative *propositional* sense, a person’s belief that Plato admired Socrates is independent of the person.

In the basic attitudinal sense, no two persons can have the same belief and every belief comes into being when its believer starts believing its propositional content and it perishes no later than its believer’s demise. Moreover, in the attitudinal sense, no belief per se has a truth-value: the belief’s propositional content might be either true or false.

In the derivative propositional sense, two persons can have the same belief. Moreover, it would be wrong to say that every belief comes into being when its believer starts believing its propositional content and it perishes no later than its believer’s demise. Rather, being a belief is an extrinsic³ property of a proposition: it is not that the belief comes into being and then perishes but that the proposition becomes and then ceases to be a certain person’s belief. More explicitly, the proposition comes to be believed and then ceases to be believed. Moreover, in the propositional sense, every belief has a truth-value. Having a truth-value is an intrinsic property of propositions—as explained further in Corcoran 2009.

In the basic attitudinal sense, having a certain believer is an intrinsic property of a belief. In the derivative propositional sense, having a certain believer is an extrinsic property of a belief.

Besides *propositional* knowledge, each of us has objectual knowledge—of the objects that propositions are about—and operational knowledge, or “know-how”. Propositional knowledge that five plus seven is twelve might be held to presuppose objectual knowledge—of five, seven, and twelve—as well as operational knowledge—

knowing how to add, or at least knowing how to add five to seven. As said above, being a belief applies to propositions alone: all belief is propositional.⁴

The distinction between propositional and objectual knowledge is routinely illustrated by reference to two interpretations of the ancient Greek injunction “Know thyself” [Preus 2007, p. 240]. Taking ‘know’ in the propositional sense, “Know thyself” can be understood as “Know *for* thyself”, i.e., take the responsibility to verify your own beliefs; do not rely on the word of others. This is related to the contemporary saying, “Trust but verify”. Taking ‘know’ in the objectual sense, “Know thyself” can be understood as “Know *of* or *about* thyself” i.e., learn your own nature; determine your abilities and limitations; do not allow others to define your identity for you.

There is no corresponding “objectual belief” and no corresponding “operational belief”. Objectual knowledge does not contrast with “objectual opinion”, nor does operational knowledge contrast with “operational opinion”. The expression ‘my knowledge of [a certain entity]’ can be used to refer to propositional knowledge or to objectual knowledge or even in some cases to operational knowledge. However, the expressions ‘my belief about [a certain entity]’ and ‘my opinion about [a certain entity]’ can be used coherently to refer only to propositional belief and propositional opinion, respectively.⁵ In a given context, the appropriate senses of the word ‘knowledge’ are correlated with corresponding appropriate senses of the word ‘opinion’. In appropriately correlated senses, a belief is knowledge if and only if it is not opinion.

Nevertheless, strictly speaking, it would be wrong to say that no knowledge is opinion—if ‘knowledge’ refers to everyone’s propositional knowledge and ‘opinion’ refers to everyone’s opinions. As a rule, we have the following situation for a given person, say A: Some of A’s opinions are known to be true by others, and some of other people’s opinions are known to be true by A. What is true is that no proposition is knowledge and opinion for the same person at the same time.

Another basic point is that every proposition that can be knowledge can be opinion: every subject matter that can be the subject matter of a proposition known to be true can be the subject matter of a proposition believed but not known to be true. Moreover, a fundamental assumption of this investigation is that no subject matter is beyond the scope of belief.⁶

The words ‘knowledge’ and ‘opinion’ have complementary senses determined by the propositional sense of ‘know’ being used: we will consider the strict traditional sense as well as other widely used senses. In effect, ‘knowledge’ means “belief that is not opinion” and ‘opinion’ means “belief that is not knowledge”.⁷ In any given context, the senses of the words ‘know’, ‘knowledge’, and ‘opinion’ are interdependent.

In every sense considered, propositional knowledge implies truth and belief. If a person *knows* that something is the case, it *is* the case and the knower *believes* that it is. However, neither truth nor belief, nor even the combination of truth and belief, implies knowledge. In contrast, propositional opinion implies belief but does not imply truth. As suggested above, no proposition known to be true is false; but many opinions, which are propositions believed to be true, are false. Each of a person’s beliefs, whether knowledge or opinion, is the end result—the culmination, so to speak—of a particular thought process that ended at a particular time with a judgment: the process that produces knowledge has distinctive characteristics.

As said, this work treats four forms of knowledge: propositional, objectual, operational, and expert. Each instance of the first three (propositional, objectual, and operational knowledge) can involve instances of one or both of the other two. The paradigm case of the fourth—expert knowledge—is the knowledge possessed by an active qualified practitioner—the medical knowledge possessed by a physician or the mathematical knowledge possessed by a mathematician. A person’s expertise includes their experience—practical and theoretical. Moreover, it involves all of the other three kinds of knowledge. Perhaps most importantly, experts know the limitations of their own expertise. Typically, the expert has a stock of unsolved problems and unsettled hypotheses to be investigated. In fact, the hallmark of the expert is the ability to call to mind open questions, or hypotheses, propositions not known to be true and not known to be false.⁸ The expert has no hesitation and no embarrassment saying, “I do not know”—when appropriate. Experts do not just share their knowledge: Experts unashamedly share their “ignorance”. Being an expert excludes being a know-it-all. In the traditional university, the central concern is propositional knowledge; in the traditional professional school, the central concern is expert knowledge.

One who does not embrace the fundamental principles of Logic or any other Science, whatever he may have taken on authority and learned by rote, knows, properly speaking, nothing of [Logic or] that Science.—Whately 1826, xvii.

Introduction

The first sense of the word ‘knowledge’ considered in this paper is the traditional strict sense. In this sense, a given person knows that a given proposition is true only if they have judged that the proposition is true by means of a cognitive judgment. A cognitive judgment is one that was the culmination of a process including the following: understanding the proposition, gathering sufficient evidence based on personal experience of the facts the proposition is about, and bringing that evidence to bear on the issue of whether the proposition is true.⁹ In this work, belief that is knowledge in the strict sense is called *certain knowledge* or *cognition*.

The strict sense of ‘knowledge’ is what is referred to in the ancient Greek injunction “Know thyself” in at least one of its interpretations that take ‘know’ in a propositional sense. Moreover, in the strict sense of ‘knowledge’, knowledge is personal, e.g. reliable testimony does not produce knowledge: one person cannot gain knowledge of a proposition simply by being told that it is true by another person, even if the other person knows in the strict sense. Cognitions are non-transferable. Although a proposition, whether true or false, is communicable, knowledge of the truth of a proposition is not communicable [Corcoran 1999, pp.113f].

Other belief merits being called knowledge only to the extent that its acquisition approximates that of cognition, knowledge in the strict sense. Cognition fulfills an ideal only approximated by other knowledge—just as mathematical circles fulfill an ideal only approximated by certain visible shapes, we nevertheless also call such visible shapes circles. Once we leave the realm of certain knowledge, the border between what is called knowledge and what in contrast is called opinion shifts according to the degree of certainty required. Moreover, it loses its sharpness. More and more beliefs that had been “opinions” become “knowledge”. This is explained more fully below.¹⁰

The process that results in cognition deserves the name *the cognitive process*. It is important to emphasize that the cognitive process begins with the process of grasping, understanding, a proposition—a process that has its own component processes and its own antecedents. Besides the grasping of a proposition, which is also called apprehension,¹¹ the cognitive process also involves evidence gathering and evidence marshalling—the bringing of the evidence to bear on the proposition to be decided. To be explicit, the cognitive process has four components: proposition grasping, evidence gathering, evidence marshalling, and proposition judging. The four are more succinctly but less explicitly called respectively: apprehension, observation, marshalling, and judging. The positions of the first and last are fixed. The first is required to set the goal of the whole process and the last achieves that goal. Cognition is a goal-directed activity.

In a way, it was somewhat arbitrary to begin the cognitive process with apprehension, grasping a proposition. For some purposes, it is important to include whatever prompts or inspires the grasping. Philosophers, including Aristotle, Peirce, Dewey, and Kuhn, have started the cognitive process before the grasping stage with experience of an obstacle, an “*aporia*”, a blockage, a difficulty, frustration, confusion, failure, disappointment, or some other sort of discontentment or unpleasantness.¹² For reasons that will become clear, we emphatically exclude doubt from the list.

Above we used the word ‘cognition’ as an epistemic mass noun, which—like ‘knowledge’—does not have a plural form (‘knowledges’ is not even an English word) and which does not follow grammatical indefinite articles (‘a knowledge is true’ is not well formed). In fact, ‘cognition’ was an exact synonym for ‘knowledge’ in the strict sense. Below, it is convenient to follow J. S. Mill (1843/1879, vol. I, 317) and others in using the word ‘cognition’ as a common or count noun that does pluralize and that does take article. A *cognition* is a proposition known to be true: cognitions are propositions known to be true. Although knowledge is cognition and cognition is knowledge, because the word ‘knowledge’ does not pluralize or take articles, it thus cannot be substituted in the previous sentence. The following is ungrammatical: “A knowledge is a proposition known to be true: knowledges are propositions known to be true”.

The word ‘truth’ is used as a count noun as in ‘truths are propositions that are true’; it is also used as a mass noun as in ‘some truth is knowledge’.

In every sense considered, every proposition known to be true is true. Moreover, knowledge in every such sense is personal; it represents a cognitive accomplishment by the knower. There is no way to buy knowledge or even to impart it. A teacher can assist in various ways, e.g., by directing students' attention and by encouraging students to become more autonomous by thinking and doing for and by themselves. Not all 'learning' is gaining knowledge. Very little of what is learned in school is knowledge. The acquisition of knowledge is autonomous, self-affirming, disciplined, courageous, and dignified. It presupposes intellectual autonomy, intellectual freedom, and responsibility.

Propositional knowledge in the broadest sense spans a spectrum between two limiting cases.¹³ One limiting case is included, the other excluded. At the included end, we have cognition—which is thought to be more common in logic and mathematics than in other fields. On the excluded end, we have groundless true belief—which is not knowledge at all. Such belief is called *credence* here. *Credence* is groundless belief in true propositions. In this broad sense, knowledge includes all cognition, at one extreme; but it excludes all credence, at the other extreme. Between the two extremes, we have non-certain knowledge—also called *probable knowledge*.¹⁴ This might include most true beliefs. Our essential daily decisions are based more often on probable knowledge than on knowledge in the strict sense.

Absolute, apodictic, or mathematical certainty is the state of having knowledge in the strict sense. As we move away from mathematical certainty along the spectrum of probable knowledge, we come to *scientific certainty*, which has a slightly lower level of warranted assertibility. In some cases, a belief held with scientific certainty is close to cognition. We can have scientific certainty that smoking is deleterious to health. A person's decision to give up smoking is often based on probable knowledge. As we move further along, we come to *moral certainty*, the state of holding true belief that is sufficiently grounded to serve as basis for responsible action and warranted assertion. In many cases, a member of a jury should vote to acquit unless moral certainty has been achieved.¹⁵ After moral certainty, the level of warranted assertibility continues to decline.

Eventually we find true beliefs that were acquired through processes that hardly qualify them to be called probable knowledge. It is worth emphasizing that in the sense of 'probable' used here, probable knowledge is actually knowledge, non-certain knowledge,

and thus is actually true. We need to be clear that this sense of ‘probable’ is not one of the senses used in connection with probabilities of events. A belief that is probable knowledge might be almost a cognition, known with certainty, or it might be little more than mere credence, a true belief not known at all. Being probable knowledge is not the same as probably being knowledge. With every level of warranted assertibility, there is a corresponding level of opinion. Some members of a jury who do not have moral certainty of the guilt of the accused might nevertheless have a somewhat grounded, true belief, which would then be opinion in comparison with moral certainty.

It seems pointless to try to coin terminology for the levels of opinion that complement scientific certainty and moral certainty. It is clear that in these contexts the word ‘scientific’ carries epistemically honorific connotations not warranted by many opinions lacking scientific certainty. Thus, the expression ‘scientific opinion’ is often inappropriate. Likewise, the word ‘moral’ carries epistemically honorific connotations not warranted by many opinions lacking moral certainty. Thus, the expression ‘moral opinion’ is often inappropriate.

The expressions ‘absolute certainty’, ‘mathematical certainty’, ‘scientific certainty’, and ‘moral certainty’ are peculiarly ambiguous constructions. In the sense used here, ‘absolute certainty’ does not mean “certainty about absolutes”, and ‘mathematical certainty’ does not mean “certainty about mathematics”. Likewise, for ‘scientific certainty’ and ‘moral certainty’. Moreover, ‘certain knowledge’ does not mean “knowledge about certainties” and ‘probable knowledge’ does not mean “knowledge about probabilities”.

We read fine things but never feel them to
the full until we have gone the same steps as
the author.— John Keats

The Spectrum of True Beliefs

As said above, the distinction between knowledge and opinion applies only to propositional knowledge (or knowledge-that) and not to either objectual knowledge (knowledge-of) or operational knowledge (knowledge-how-to).¹⁶ It was also said that in many cases, each of the three requires the other two. To see how propositional knowledge

involves the other two, consider propositional knowledge that no square number is twice a square number. This involves among other things objectual knowledge of the system of numbers (positive integers) and operational knowledge how to square numbers. It also involves further operational knowledge: ability to count and to perform other arithmetic operations, ability to understand propositions, ability to make judgments, and ability to deduce conclusions from premises—to mention a few of the skills used acquiring arithmetic knowledge.

As already emphasized, in every sense of ‘know’ used in this essay, every proposition known to be true actually is true. Truth is a precondition of knowledge. Moreover, except in rare cases, every such proposition was true before it was known. Fact is prior to knowledge. As Frege explicitly noted, the process of achieving knowledge in no way *intrinsically* alters the proposition known.¹⁷ Consider the proposition that no square number is twice a square number. This very proposition, which Corcoran came to know only a few years ago, was unknown until first known to be true by one of our predecessors thousands of years ago, perhaps in ancient Greece or ancient China. Later, it came to be known by countless others, one after another, down through the years in Asia Minor, North Africa, and in many other places. One and the same proposition that was at one time not known at all became known by one person and then by more and more people, speaking in different languages, at different times, and in different places—each time by one person acting autonomously. Although knowledge cannot be transmitted, a person who knows a proposition to be true can sometimes help others to come to know the same proposition.¹⁸

However, by becoming a cognition of a new person the proposition is changed *extrinsically*: more people know it than knew it earlier. A proposition not widely known becomes widely known. However, everything people relate to changes extrinsically every time someone relates to it: thinking of the number one changes it extrinsically.

Knowledge is *objective* in that it is of objective reality. Moreover, there is no such thing as disembodied knowledge. Every proposition known to be true is known to be true by a person. Knowledge in all senses is personal: it is *subjective* in the sense of being achieved by a knowing *subject*. The objectivity of knowledge is prior but no less essential

than its subjectivity. The knowing subject willingly defers to the object known, so to speak.

Thus, knowledge *per se* is both objective and subjective, in appropriate senses of these troublingly ambiguous words. In contrast, truth *per se* is objective but not subjective. If everyone who knows a given true proposition were to forget it, there would no longer be knowledge of it, but its truth would not be altered. Being known to be true is an extrinsic property of propositions; being true is an intrinsic property.

Truth is necessary for knowledge, but knowledge is not necessary for truth. As said above, strictly speaking, knowledge is sufficient for truth. However, this should not be taken to mean that knowledge produces truth, which is a common fallacy. Rather, knowledge is sufficient for truth because knowledge has truth as a precondition.

To indicate that ‘know’ is being used in the strict sense emphasized in this work, words such as ‘with certainty’, ‘categorically’, or ‘conclusively’ can be added—as long as this is meant to refer to what the knower did objectively as opposed to what the knower felt subjectively. Below we will distinguish knowing with certainty from believing with certitude. Strict knowing requires that the knower accurately judge based on conclusive evidence. Philosophers can agree to use the words ‘know’ and ‘knowledge’ in the strict sense and yet disagree on whether a given proposition is known with certainty to be true by a given person at a given time or whether there is any such knowledge outside mathematics or even whether anyone has ever known any proposition with certainty. To some philosophers it seems amazing, implausible, or even incredible to think that people might have certain knowledge.

As already noted, true opinion complementing strict knowledge spans the spectrum *starting* after cognition or absolute certainty—the limiting case of entirely adequate, evidence-based deliberation included in knowledge in the broad sense. The spectrum of true belief continues through scientific certainty and through moral certainty. It is important to recall that scientific certainty and moral certainty are only opinion in relation to knowledge in the strict sense: scientific certainty is knowledge in a less strict sense—a sense in which moral certainty is in the realm of opinion. The spectrum of true opinion finally ends with true credence or groundless opinion that happens to be true—the limiting case of total lack of evidence-based deliberation.

As an analogy, compare the spectrum of true belief to the fractions between one and zero inclusive of end-points. Imagine that the sizes somehow represent amounts of “evidentiality”. The spectrum of probable knowledge excludes both one (the cognitions) and zero (the credences), but the spectrum of knowledge in the broad sense excludes only zero.

All humans [*anthropoi*] by nature
desire knowledge.—Aristotle,
Metaphysics A.1

The Spectra of Certainty

In a given person’s lifetime or even in a relatively short interval of years or days, their degree of certainty in a given known belief may vary from near credence to absolute or near absolute certainty though increasing degrees as evidence accumulates. In fact, sometimes credence taken on trust is transformed by the cognitive process into certain knowledge. Of course, in another case, the process might reverse and a true proposition, one time known with certainty, through deteriorating memory or other factors, may over time end up being taken purely on faith.

Thus, for every person, and for each of their true beliefs, there is a spectrum of degrees of certainty within which their actual degree of certainty at a given time is located. Of course, each of these many spectra, or spectrums, is similar to what we called the spectrum of true belief above.

In appropriate orthogonal senses of ‘objective’ and ‘subjective’, there is no contradiction in saying that one and the same thing is both objective and subjective, e. g., perception, inference, and cognition.—Albert Hammond (paraphrase).

Senses of ‘Objective’ and ‘Subjective’

Above we used the words ‘objective’ and ‘subjective’ as non-opposing adjectives in the domain of cognitions.

As opposing adjectives, the words ‘objective’ and ‘subjective’ are applied in different senses in different domains or ranges of applicability (Corcoran 1989, 38f). A

judgment is objective to the extent that it is based on logic and evidence or other factors pertaining to the objects that the judgment concerns; a judgment is subjective to the extent that it is based on loyalties and feelings or other factors pertaining to the subject making the judgment. People are objective to the extent they make objective judgments; people are subjective to the extent they make subjective judgments. Advice, testimony, journalism, and the like are objective to the extent that they reflect objective judgments, and subjective to the extent that they reflect subjective judgments. A person is objective to the extent that their judgments are objective, and they are subjective to the extent that their judgments are subjective. These are all interrelated but distinguishable uses of ‘objective’ and ‘subjective’.

A distinction drawn between two objects in a given domain is objective insofar as it is based on intrinsic features of those objects, and it is subjective insofar as it is based on the tastes, loyalties, feelings, etc. of the person who drew it. For example, in the domain of integers, the distinction between even and odd integers is objective but any distinction between large and small is subjective. Again, in the domain of propositions about integers, the distinction between true and false is objective but any distinction between interesting and uninteresting or between simple and complicated is subjective. Of course, we are not denying any author’s right to stipulate objective usage to words whose normal use is to mark subjective distinctions. For example, certain numbers are called perfect without any suggestion that they are better than the non-perfect numbers: the perfect versus non-perfect distinction in mathematics is objective even though the same words are applied elsewhere for subjective value judgments.

Church, Tarski, and other mathematically oriented logicians use cognates of the words *object* and *subject*—such as *objective*, *subjective*, *objectively*, *subjectively*, *objectivity*, and *subjectivity*—in senses related to those in traditional subject-object epistemology where thinking *subjects* make judgments about factual *objects*. In Corcoran-Hamid 2014, we survey use and conspicuous non-use of such words in logic.

For example, modern formalizations of geometry—and other historically established disciplines—require distinguishing between the *underlying logic* and the *overlying science*. Church said such distinctions are “subjective and essentially arbitrary”[Church 1956, pp. 58ff]. Tarski implied there is no “objective” basis for them

[2, esp. pp. 412 and 418f]. Neither gives grounds for his claim—and neither explains his crucial term.

Distinguishing between the underlying logic and the overlying science requires distinguishing between the *logical* and the *scientific* concepts or—what amounts to the same thing with respect to interpreted formalized languages—between the *logical* and the *scientific* constants. Tarski explicitly stated that he knew of no “objective” basis for the latter distinction [2, esp. pp. 412, 418f].¹⁹

Remarkably, Church [1956] uses *subjective* frequently while completely avoiding *objective* whereas in Tarski [1983] the exact opposite holds: there *objective* occurs frequently while *subjective* is completely absent. The above epistemic uses of cognates of *subject* and *object* contrast with other essentially unrelated uses. For example, in logical syntax the word *zero* is called the subject of the sentence *zero precedes two* and the word *two* its object. Moreover, the goal or aim of a work is often called its *objective*. There seems to be no parallel contrasting usage of ‘subjective’. Curiously, although the objectives of Quine and Ullian’s 1970 book *The Web of Belief* overlap with those of this work, it uses ‘objective’ only twice, both in the sense of “aim”, and it uses ‘subjective’ only twice, both in the sense of “not objectively grounded”.

In primary senses, *objective* and *subjective* are correlative adjectives like *old* and *young*. It is difficult to determine what is being conveyed by calling something *objective* or *subjective* unless writers give 1), for each, examples where they would apply one as opposed to the other and 2) some sense of their criteria for applying each word. Moreover, when a writer asserts something without giving any objective grounds, the reader is justified in suspecting that the assertion was empty rhetoric or that it was based on a subjective judgment. In the case of the distinction between logical and scientific concepts, there is a history of such distinctions being made or implicitly used going back through modern, renaissance, medieval, and ancient logic. If all those logicians were mistaken, we need some kind of explanation of how they went wrong. The apparent fact that Tarski could not think of an objective basis for distinguishing between the *logical* and the *scientific* concepts does not warrant his conclusion that no such basis exists. Likewise, the apparent fact that Church could not think of an objective basis for

distinguishing between the *underlying logic* and the *overlying science* does not warrant *his* conclusion that there is none.

In the broad sense of ‘animal’, every human is an animal; in the narrow sense, no human is an animal.—Frango Nabrasa, 2001 (personal communication).

Broad and Narrow Senses of ‘Believe’

The verb ‘believe’ is used in several senses even when its direct object is the proposition. In all such senses, belief can manifest itself in action. If the right circumstances occur, a person who believes something will react in one way and a person not believing it will react in another way—even if in some cases the action is entirely private and not observable by others. One broad inclusive sense and two narrow exclusive senses of ‘believe’ are pertinent. In the broad and inclusive sense used above and throughout this work, believing is accepting as true regardless of whether done with absolute certainty, with a lower level of certainty, or with no certainty. A person believes every one of his or her own cognitions and every one of his or her own opinions.

In both of the narrow and exclusive senses, discussed but not used here—sometimes signaled by a word such as ‘merely’—no proposition is known to be true *and* believed to be true by the same person: “beliefs exclude knowledge”. A person aware of merely believing will sometimes answer, “I believe so” where a person who knows would answer simply “Yes”.

Of course, it makes a difference whether only certain knowledge is excluded or whether all knowledge including probable knowledge is excluded. The less exclusive narrow sense in which only certain knowledge is excluded is used mainly in mathematics and philosophy. In this sense, most scientific and medical knowledge is mere belief. In the more exclusive narrow sense of ‘belief’, all knowledge, certain and probable, is being excluded. In this sense, the mere beliefs are only the credences. In this sense all mere belief is entirely subjective in the sense of coming from within the believing subject: the process leading to it includes no component, however small, based on objective evidence and deliberation. For example, in some cases, all of what was taken as evidence was in

some sense constructed by the believer. In other conceivable cases, the believers made no effort to square their judgments with objective reality: loyalties, preconceptions, fears, hopes, or other subjective factors were dominant.

Many mere beliefs can be expected to be false, regardless of whether ‘mere belief’ is taken in the less or the more exclusive sense. However, nothing precludes subjective beliefs from being true—in some cases by accident, so to speak. True mere beliefs in the more exclusive sense are what we called true credences above. By contrast, in the broad and inclusive sense of ‘believe’ used in this work—sometimes signaled by suffixing words such as ‘in the broad sense’—every proposition known to be true is believed to be true: “beliefs include knowledge”. More explicitly, for a given person, the set of their beliefs includes their cognitions. In order to speak of belief that a proposition is true, we sometimes speak of *belief in* the proposition. Sometimes, “I think that” is interchangeable with “I believe that”.

Certitude is not the test of certainty.

—O. W. Holmes, Jr.

Certitude and Certainty

Certitude is the subjective feeling of assurance of the truth of a proposition.²⁰ Certitude can be the result of thorough objective investigation which started from a suspension of belief or even from doubt or disbelief, and it can also arise without investigation or be the result of deception, rationalization, indoctrination, error, or hallucination, to mention a few. O. W. Holmes, Jr., reminded us that “Certitude is not the test of certainty”.

Certainty, as used by Holmes and in this work, is not a feeling at all; it is the state of having knowledge in the broad sense. The expression ‘false certitude’ is sometimes used for misdirected certitude: a feeling of assurance toward a false proposition. It is also used for improperly derived certitude: certitude arrived at by persuasion, enthusiasm, illusion, fallacious reasoning, and the like.

False certitude is analogous to false trust, false distrust, false security, false danger, false guilt, false righteousness, false pride, false shame, and the rest. Holmes could have added, for example, that guilt is not the test of immorality: the world has

known people who had done nothing wrong but who were consumed with guilt. Freud and Tarski lamented these cases.

Aristotle said that every person by nature desires knowledge. Peirce disagreed; he said that every person by nature desires belief. According to Aristotle, the goal of inquiry is the possession of truth. According to Peirce, the goal of inquiry is the cessation of doubt. Aristotle sought certainty, the possession of truth.²¹ Peirce sought certitude, the subjective feeling of certainty.²² Aristotle's view relates more to the spectra of certainty; Peirce's relates more to spectra of certitude. Of course, it is a matter for scholars to decide whether, for the respective thinker, either view is a "mature" one.

Absolute, apodictic, or mathematical certainty is the state of having knowledge in the strict sense. *Scientific certainty* and *moral certainty* are two states of having probable knowledge. It is unfortunate that the words "I am certain" often indicate certitude, not certainty. Certitude is a feeling of confidence in a belief. Some authors such as Tarski write of "intuitive certainty" and of being "intuitively certain" to refer to certitude. It is also unfortunate that certitude, the feeling, is often not distinguished from the scientific or moral certainty it sometimes reflects. It is even more unfortunate that the two words 'certitude' and 'certainty' are sometimes used interchangeably with one meaning. It is confusing that sometimes when the two meanings are distinguished, the words are used with the meanings reversed.

As Holmes said, certitude is not always based on certainty. Moreover, certainty does not always give rise to certitude; and when it does, time might pass between the achieving of certainty and the feeling of certitude. Even absolute certainty is not always or not immediately accompanied by certitude—especially in cases where the knower is at first surprised, delighted, or dismayed to find out that the proposition is true. In fact, for modest and objective investigators into important issues, sometimes the more certainty they achieve, the more they grasp the complexities and the less certitude they might feel. As implied above, philosophers disagree on whether absolute certainty is achievable. John Stuart Mill had certitude that "There is no such thing as absolute certainty".

Many of a person's beliefs, including all those based on testimony, are not their certain knowledge, and some, even some that are true, are probably not certain knowledge for most. An example is the famous Fermat Theorem: given any three

numbers that are all the same power exceeding two, no one is the sum of the other two. This implies that no cube is a sum of two cubes, that no fourth power is a sum of two fourth powers, and so on. Corcoran, for example, thinks that he is fully justified in believing this. Part of his justification is based on his knowledge that mathematicians he has reason to respect have testified in print that it has been proved to be true by a proof that has been carefully studied and found to be cogent by qualified experts. Here is a justified and true belief of Corcoran's that is not his certain knowledge, i.e. not his cognition; it is of course one of his scientific certainties. In contrast, for the other author, Hamid—whose expertise does not extend as far into mathematics as Corcoran's—the Fermat Theorem is not a scientific certainty: it is merely a moral certainty. Probably most people who believe Fermat's Last Theorem have less evidence than Corcoran or even Hamid.

But the proposition in question is a justified true belief of dozens of mathematicians for whom it *is* knowledge. With regard to the Fermat Theorem, while Corcoran has certitude but not absolute certainty, some mathematicians have certitude and absolute certainty. Corcoran has probable knowledge that the Fermat Theorem is true. In fact, he thinks he has moral certainty, even scientific certainty.

There is no such thing as absolute
certainty.— John Stuart Mill.

The Spectra of Certitude

In a given person's lifetime, or even a relatively short interval of years or days, their degree of certitude in a given belief may vary from absolute or near absolute certitude though lesser degrees until there is no certitude whatever, when the person's belief might be said to lack all conviction. At such a point, doubt may start to accumulate until a maximum is approached.

Of course, a similar observation applies to each disbelief. Moreover, as we all know, one day's belief might be replaced over time by the diametrically opposed disbelief. However, let us focus on a given belief belonging to a given person. Analogous to the spectra of certainty there is a spectra of certitude, which range from a maximum of

certitude through ever more diminishing level until a state of subjective neutrality is passed and doubt begins to increase to its maximum.

To picture a spectrum of [objective] certainty with the corresponding spectrum of [subjective] certitude, we may represent the former spectrum as a horizontal axis that extends from credence (starting point) on the left to cognition on the right. We may then represent the certitude spectrum as a vertical line on the left whose neutral midpoint lies at the point of credence. The high point of the vertical axis above the horizontal axis represents maximal certitude; the low point of the vertical axis below the horizontal axis represents maximal doubt.

As a given cognitive process moves the knower's level of certainty from credence to cognition, certitude sometimes may increase, and the curve rises above the axis of the certainty spectrum. However, in some cases, as evidence increases doubt increases and the curve goes below the axis of the certainty spectrum.

An extreme skeptic may experience doubt when they have certainty, even when there is cognition. An extreme *paraskeptic*—to coin a word—may experience certitude even when there is only credence. Such extreme skeptic's subjective feeling of doubt is disconnected from their objective state of certainty in one direction; such an extreme paraskeptic's subjective feeling of certitude is disconnected from their objective state of certainty in the other direction.

Philosophers counsel thinkers to adjust their level of certitude to be proportional to their level of certainty without suggesting how this can be accomplished.

The certitude-doubt axis is independent of the certainty axis. Besides, a person can have unbounded certitude in a false proposition and one unbounded doubt in a true proposition.

Certain Knowledge

How does a person go about arriving at certain knowledge of the truth of a proposition even if, in at least some cases, knowledge in the strict sense is the ideal limit of a process that can never be completed—except perhaps in mathematics? Let us use the word 'hypothesis' for a proposition not known to be true and not known to be false by a given knower. In the first place it is necessary to understand the hypothesis to be

investigated. Next, it is necessary to connect with the reality that the proposition is about in order to acquire from it evidence sufficient to ground a judgment that the proposition is true. Third, it is necessary to marshal the evidence, to bring the evidence to bear on the hypothesis. Finally, it is necessary to see that the evidence is conclusive and to accurately judge on the basis of the understanding and marshalling that the proposition is true. A belief that resulted from successful completion of this process is said to be *cognitively* grounded or justified. Husserl was referring to knowledge in this strict sense when he said that in having knowledge, “we possess truth as the object of a correct judgment” (Giorgi 2002, 130).

In the case of the proposition that no square number is twice a square number, which was probably known to be true by Socrates, Plato, Aristotle, Leibniz, Pascal and many others, the evidence phase included reviewing previously known arithmetic propositions and the marshalling phase included inferring the hypothesis from them by logical deduction.

Certain knowledge is cognitively justified true belief. In this context the word ‘true’ is redundant in the sense that every cognitively justified belief is true. A belief that a given proposition is true is cognitively justified only if there was a successfully completed four-step method or its equivalent. Moreover, a proposition that is a certain person’s true belief but not now their knowledge can become cognitively justified and thus become knowledge if the believer successfully completes the four-step method.

Notice that in regard to cognitive justification, praise and blame are often beside the point. A person might flawlessly make every effort to apply the four-step method to a proposition that they believe and yet fail because of circumstances beyond their control such as the necessary evidence having been destroyed.

The verb ‘justify’ is ambiguous. In several other senses of ‘justified’, the word ‘true’ is not redundant in the sentence ‘knowledge is justified true belief’. Unless ‘true’ is redundant, the sentence expresses a false and misleading proposition, as I show below—in agreement with Plato, who criticized such formulations toward the end of the *Theaetetus*. Preus (2007, 93) wrote: “In the *Theaetetus*, the hypotheses that knowledge might be “true belief” (*alēthē doxa*) or “true belief plus an account” (*logos*) are discussed and refuted”. Other qualified scholars concur: see for example (Peters 1967, 59).

In some of the other senses, justifying a belief involves explaining something to others: perhaps why I should not be blamed for having the belief or why other people in my circumstances would have come to the same conclusion. No matter how these explanatory senses of justification are spelled out, it is clear that knowledge is not justified true belief. Gaining knowledge that a proposition is true does not require explaining anything to anyone.

As explained above, acquisition of knowledge comes about through a personal process that begins with apprehending a proposition and ends with a judgement. There is no need for the knower to explain anything to anyone. There is no room in the process for accounting to others. In fact, the suggestion that knowledge requires explanation is contrary to the principle of the autonomy of knowing—a principle emphasized in this paper.

More generally, in any sense of ‘justify’ in which a false belief is justified, it is not the case that justified true belief is knowledge in the strict sense—there are indefinitely many propositions that could become justified true beliefs without thereby becoming my certain knowledge. The reason is based on the fact that from any false proposition indefinitely many true propositions are deducible. If we justifiably believe that you own a new pen when in fact your pens are all old, then we could justifiably believe without knowing the true proposition that you own a pen—if we were to deduce the latter truth from the former falsehood. Our belief that you own a pen would have been formed by a flawed process—in this case inferring from a false premise, one form of begging-the-question (Corcoran 1989, 22f).

In the cognitive sense of ‘justify’, every justified belief is true and, in the strict sense of ‘knowledge’, justified belief is knowledge; knowledge is justified belief. To be even more explicit, cognition is cognitively justified belief. The formula ‘knowledge is justified true belief’ is dangerously flawed and should never be used without adequate qualification. In statements using the formula without qualification, ‘justify’ can be interpreted either cognitively or non-cognitively: if the former ‘true’ is misleadingly redundant; if the latter, it expresses a falsehood.

Speaking of Propositional Knowledge

Previous paragraphs present one of many ways of organizing propositional knowledge for purposes of discussion. Every item of propositional knowledge is someone's true belief that something is the case. These beliefs have all been established with some degree of certainty. Each was derived by a process involving some objective considerations.

We started with knowledge in the strict sense, the most firmly and objectively established beliefs, which we called cognitions.²³ These beliefs are known with certainty and are thus also known as certain knowledge. If the word 'knowledge' is used in the strict sense, 'certain knowledge' is a redundancy, a mere rhetorical flourish, and 'probable knowledge' is an oxymoron. When 'knowledge' is used in a less strict sense, we can speak of certain knowledge without redundancy and of probable knowledge without contradiction. Even then, putting the adverb 'absolutely' in front of 'certain knowledge' is a mere rhetorical flourish exactly analogous to other uses of the adverb in expressions such as 'absolutely true' and 'absolutely perpendicular'.

The established true beliefs that are not cognitions are all called probable knowledge. The degree of firmness starts very high but then shades off imperceptibly through the less and less probable, tending toward but excluding true credences, which are not established at all and not knowledge. The most firmly established of true beliefs that are not cognitions have scientific certainty. Moving on from there, we reach those with moral certainty, less firmly established but still sufficiently so to serve as the basis for responsible action and warranted assertion. After that we eventually come to beliefs that hardly merit being called probable knowledge but are still awkwardly (but strictly) so-called. In this usage, putting 'scientifically' and 'morally' in front of 'certain' weakens the adjective in the same way that 'partly' and 'lightly' weakens 'cooked', 'scrubbed', 'seeded', and other such expressions.

As observed above, in the sense of 'probable' used here, all probable knowledge is knowledge. There are other senses of 'probable' recognized by dictionaries. In another sense, saying that a given proposition is probable knowledge might mean that it is likely to have been investigated and found to be true. In the latter sense, probable knowledge is *probably* knowledge, but it might not actually be knowledge; in this sense, then, not all

probable knowledge is knowledge. In fact, in this sense, not all probable knowledge is true. In the sense of ‘probable’ used in this work, all probable knowledge is true.

Cognitions are said to be known with mathematical certainty, not because such cognitions are necessarily about mathematics, but only because mathematical cognitions are often taken as paradigm examples. The mathematical community is known to aspire to the highest standards of clarity, transparency, rigor, and cogency. Likewise, beliefs said to be known with scientific certainty are not necessarily about science; they are so-called only because this level of certainty is often aspired to or achieved by the scientific community. Finally, beliefs said to be known with moral certainty are hardly ever about morality; they are so-called only because this level of certainty is often required for beliefs on which moral action is to be based.

People might have cognitions that they just locked up their house. Shortly after locking up, they might have scientific certainty that the house is locked. But after enough time has passed, perhaps they can have only moral certainty that the house is locked. And after even more time has passed they might not be able to have even moral certainty.

Notice that this organization applies only to propositional knowledge, not to objectual knowledge or operational knowledge. Moreover, it also does not apply to knowledge in the sense of scientific competence or expertise. As explained above, by ‘expertise’ we refer to the result of years of observation, experimentation, investigation, and deliberation that fine-tunes instincts of the dedicated scientist. For example, Newton probably had more scientific competence and expertise in mechanics than any person who came before him, and yet, after Einstein, many of Newton’s beliefs about mechanics have been shown to be false, even though they are close enough for many applications. In fact, many scientists who have extensive knowledge in their respective fields expect that future researchers will find to be false many if not all of the propositions they currently believe to be true. If scientific knowledge were to be measured by propositional knowledge, the propositions currently regarded as established truths, then there would be little or no progress in science.

Focus on propositional knowledge to the exclusion of expertise is unfortunate, even in mathematics or philosophy. Studies of propositional knowledge, including this work, will probably be valued less for specific achievements than for preparing the way

for a broader and more inclusive study of knowledge and for underlining the need for more attention to expertise, the fourth kind of knowledge mentioned. The fact that contemporary epistemology focuses almost exclusively on propositional knowledge is especially regrettable.

It is true that traditional philosophy often extolled the seeking of wisdom, which is inseparable from expertise. But the fact that expertise and the accompanying wisdom require practice, experience, learning from mistakes, and other mundane activity is rarely mentioned. Even more unfortunate from our perspective is that in many discussions of wisdom-seeking no mention is made of propositional, operational, and objectual knowledge.

Cognitivism, Probabilism, and Skepticism

Among the tensions that pervade discussions of knowledge and belief is the perennial issue of whether knowledge in the strict sense is ever achievable or whether it is merely an ideal to which objective people can only strive but never fully reach. Our personal opinion is that it is achievable—but often only with great difficulty and often not at all. We believe that the *main* role of the concept of cognition in our lives is serving as an ideal standard by which to measure our performances: an ideal goal to strive for and a constant reminder of the shakiness of many of our beliefs. It would serve these important purposes even were there no cognitions, no knowledge in the strict sense. When Aristotle wrote that every human by nature desires to know, he was thinking of knowing in the strict sense.

With respect to any given proposition, we can distinguish three philosophic viewpoints a given person might have: cognitivism, probabilism, and skepticism. *Cognitivism* holds that the proposition is or can be known in the strict sense, i.e. that it or its negation is or can become certain knowledge or cognition. *Probabilism* holds that the proposition cannot be known in the strict sense, but that it or its negation is or can become morally certain or even scientifically certain. *Skepticism* holds that the proposition cannot be known in the strict sense, and that neither it nor its negation is or can become even morally certain, much less scientifically certain.²⁴

Both authors agree with cognitivism with respect to many but not all mathematical propositions. They also agree with probabilism with respect to many but not all propositions about the material world. For example, they think that they have scientific certainty that smoking is deleterious to health. However, they differ on philosophical propositions: Corcoran agrees with skepticism but Hamid agrees with probabilism with respect to most but not all philosophical propositions. For example, Corcoran thinks that it is impossible to know even with moral certainty whether there is cosmic justice, i.e. whether each good act will be rewarded in proportion to its degree of goodness and each bad act will be punished in accord with its degree of badness (Corcoran-Frank, 2014). However, Hamid thinks that this “cosmic justice hypothesis” can be settled with moral certainty, perhaps even with scientific certainty.

Understanding Propositions

Understanding a proposition is grasping its truth-condition, knowing what its being true would be and what its being false would be. A person who understands a given proposition will often be able to imagine what it would be like to affirm it, or to deny it. A person who understands a given proposition can wonder whether it is true or false. Such a person can recognize in simple cases its implications—what it implies, what follows from it. They can also recognize in simple cases its implicants—what implies it, what it is implied by. In addition, again in simple cases, they can recognize what it contradicts, what is inconsistent with it. In some cases, a person who understands a proposition can look for methods to settle it affirmatively or negatively.

Before a person can begin to marshal or even acquire evidence by which to judge that a proposition is true, it is necessary to understand the proposition. In fact, as Frege said in several places, it is necessary to understand a proposition before one can make a judgment concerning it.²⁵ He suggested without explicitly saying that understanding is necessary before we can wonder whether it is true or false, before one can become aware that it is a hypothesis. There are many propositions that are widely understood but are not known to be true and not known to be false by anyone—or so it is said. Clearly, in most if not all cases, it would be impossible to know with absolute certainty, to have a cognition, that a given proposition is neither known to be true nor known to be false by

anyone. This would require exhaustive knowledge of the mental states of every person now alive.

The Goldbach Hypothesis is that every even number exceeding two is the sum of two prime numbers. This proposition is easy to understand but difficult to settle. Despite the fact that many able mathematicians have spent long years trying to determine whether it is true, none have succeeded.

It is possible to understand all of the concepts in a proposition without understanding the proposition—just as it is possible to understand each step in a proof without understanding the proof and to grasp each note in a melody without grasping the melody. In all cases, even adding to apprehension of the constituents a grasping of their order does not account for an understanding of the whole. One of the most perplexing problems in the theory of propositional knowledge is that of how a proposition is understood—assuming that its constituent concepts are understood. However, the problem is even more challenging to philosophers such as Frege who believed, or at least said, that we apprehend a proposition's constituent concepts only after apprehending the proposition, that the concept is grasped by analyzing a proposition containing it.

In order to understand a proposition it is normally not necessary to know what understanding a proposition is. In order for a person, a young student for instance, to understand the proposition that there are four single-digit square numbers it is not necessary for the student to know what it is to have understanding of a proposition. However, in order for a person, Hamid for instance, to understand the proposition that a proposition must be understood before it can be known to be true it is necessary for him to know what it is to have understanding of a proposition. However, this is not a typical case. A child can come to know that the family dogs each have four legs without it understanding the proposition that it understood the proposition that the family dogs each have four legs. Our assumption in this article is that understanding propositions precedes having knowledge of what understanding propositions is.

At the 1990 Buffalo Church Symposium, Hartley Rogers said that John Myhill once called him up to ask whether a certain hypothesis that had just occurred to him had

been settled. Rogers replied that it was known to the field as Myhill's Theorem.—
Sriram Nambiar

Knowing That One Knows

Let us start with an example of a cognition and then consider what else would be required to know that it is a cognition. Corcoran *believes* that he knows that every [*sc.* geometric] square is equal in area to the sum of two smaller squares of different sizes. He also believes he knows that each square is equal in area to the sum of two smaller squares that are equal to each other. *Every* square is equal to twice a square. If you draw the two diagonals, you make four isosceles right triangles, any two of which make a square.

He remembers discovering this and proving it in connection with one of his many readings of Plato's *Meno*. This geometric proposition is related to the Pythagorean Theorem, which might have been discovered and proved by Pythagoras.

In order for Corcoran to know that this proposition is true, it was not necessary for him to recall the details of his own thought process. But for him to *know that he knows* this proposition to be true, it is necessary for him to be able to recall the processes by which he gained that knowledge and to verify that the steps were properly carried out and completed. For example, he must verify that, *at the exact time of the process*, he succeeded in understanding the proposition, he succeeded in gathering sufficient evidence, and he succeeded in properly bringing the evidence to bear on the issue of the truth of the proposition. We doubt whether this is even possible.

The proposition itself is about geometric squares; it is not about Corcoran, and it is not about a proposition. The proposition that he knows that it is true is about him (his past cognitive history) *and* about the proposition; it is not about geometric squares. Knowing that one knows is different than knowing, and far more complicated and problematic. Corcoran believes that he knows the geometrical proposition with mathematical certainty; but he thinks he has at best moral certainty that he knows that he knows.

Similar points apply in the case of belief. We imagine that there are propositions that we believe but concerning which the issue of *whether* we actually do believe them has never come up. In such cases, we believe that it is true, but we do not even believe,

much less know, *that* we believe it. There are difficult issues about knowing that one knows even though it is often very easy to know that one does *not* know. In order to know that one knows it is necessary to know that one believes. But, by what criterion do we determine that we believe a given proposition? We have made mistakes about what we thought we believed. For example, Corcoran now knows that he was wrong in believing *that* he believed certain propositions. He now knows that there are propositions that he did not even understand but that he believed that he believed. People he trusted led him to believe that he believed propositions that, as he now knows, he did not believe. As he reflects on his youthful views, he is frequently surprised at what in his youth he thought were his beliefs.

One of the most confusing mistakes that we can make regarding cognition or even knowledge in the broad sense is thinking that in order to know it is necessary to know that we know. There are many people who have cognitions who do not even understand, much less know to be true, the proposition that they know the cognition to be true. A person who knows that five plus seven is twelve need not understand the proposition that the proposition that five plus seven is twelve is known to be true. The idea that knowledge that a proposition is true requires knowledge that the proposition is known to be true leads to an infinite regress and thus to skepticism, the view that knowledge is impossible. Persons who know that we are writing about knowledge and opinion need not know that they know that they know that they know that we are writing about knowledge and opinion. Why anyone should have thought otherwise has baffled us for years.²⁶ We should never forget that the skeptics do not feel bound by logic or by the requirement of testifying in accord with their own knowledge—which they deny they have. But, it is not just skeptics who held that knowledge requires knowledge of itself. Aristotle held that having knowledge of an axiom (*arche*) required knowledge of the knowledge (Anagnostopoulos 2013, 68).

The above treats knowing with absolute certainty that one knows with absolute certainty. Knowing with scientific certainty that one knows with scientific certainty is equally problematic. Knowing with scientific certainty that one knows a given proposition with scientific certainty requires knowing that the proposition is true, but it does not require knowing with absolute certainty that the proposition is true. Thus, it

might be possible to know with scientific certainty, given a proposition that cannot be known with absolute certainty but is known with scientific certainty, that it is known with scientific certainty. However, it is impossible to know with absolute certainty, given a proposition that cannot be known with absolute certainty but is known with scientific certainty, that it is known with scientific certainty.

The problem of knowing *that* one knows is closely related to the problem of knowing *how* one knew. For example, can we determine of a given cognition whether it was inferred from previous cognitions or whether it was achieved by some other means?

Beliefs formed by logical deduction from previous beliefs were called *inferences* and those formed without deduction were called *intuitions*. In this sense, not every intuition is known to be true and, in fact, not every intuition is true. Intuitions that are known with absolute certainty, i.e. that are cognitions, are called *cognitive intuitions* or *intuitive cognitions*. Cognitions formed by logical deduction from previous cognitions can be called *cognitive inferences* or *inferential cognitions*.

The word ‘intuition’ has other meanings, of course. One relatively common usage is in a way broader and in a way narrower. It is broader in that it applies to objectual knowledge as well. It is narrower in that it does not apply to beliefs that are not cognitions. The common usage just mentioned is in Bruce Russell’s entry “Intuition” in the 1999 *Cambridge Dictionary of Philosophy*. It also agrees with the 1868 Peirce article “Questions Concerning Certain Faculties Claimed for Man”, which has a long and interesting footnote on the history of the word. Peirce wrote (1868, 11): “*Intuition* here will be nearly the same as ‘premise not itself a conclusion’; the only difference being that the premises and conclusions are judgments, whereas an intuition may be any kind of cognition whatever”.

Some philosophers from previous centuries believed that it is possible to determine with certainty whether a given cognition was an inference or an intuition. In other words, it was held that in every case a person who has a given cognition can have a cognition about that given cognition, viz. either a cognition that the given cognition is an inference or a cognition that it is an intuition. Moreover, Aristotle, Frege (1884, §3, §4), and others have been interpreted as holding that, in case of a cognitive inference, knowers have the capacity to trace their chains of reasoning back and back until they come upon

cognitive intuitions, propositions that they knew to be true without inference. The cognitive intuitions were called axioms, principles, first principles, primitive truths, or something similar. Even though the overwhelming majority of mathematical cognitions were held to be the results of inference from cognitive intuitions, not one example of such backwards tracing has been presented and no one has ever proposed a criterion for determining of a given belief whether it is an axiom or an inference. Needless to say, we are skeptical concerning the hypothesis that knowers have the capacity to trace each of their cognition-producing chains of reasoning back to cognitive intuitions.

Each of the four steps in the method of cognition admits of slippage. Coming to know that a proposition is true is like securing a house having four complicated locks. If a person's attention wavers at any lock in the process, a person might have locked the house without knowing so. And a person might be mistakenly certain of having locked the house regardless of whether they actually had locked it.

Results, Intuitions, Inductions, and Inferences

We need a word for a belief that is the result of a cognitive process however complete or incomplete, successful or unsuccessful, it may have been. In other words, we need a word for beliefs that are not entirely subjective. We propose *result*. Every cognition is a result, and so is every moral certainty and every scientific certainty. Every result that is not the conclusion of a chain of deduction is believed on the basis of experience, whether sense-based, intuition-based, or mixed, whether reliable or unreliable. Using this terminology, we can say that every belief is either a result or a credence.

We propose *induction* for a belief arrived at through experience and not through deduction. We are not the first to adopt this usage. Aristotle's word for this kind of belief, *epagoge*, is routinely translated 'induction'. Further, we can say that every result is either a deduction or an induction, using the word 'deduction' in the broad sense in which 'fallacious deduction' is not oxymoronic.

One famous induction is Aristotle's belief that all swans are white (*Prior Analytics* A4.26b7-14). Another example of an induction is Archimedes' belief in his Law of Buoyancy that an immersed body is buoyed by a force equal to the weight of the

displaced fluid. Corcoran's initial belief in Archimedes' Law was also an induction in this sense, but unlike Archimedes' belief, his involved very little experience: it was largely based on his teacher's testimony and the experiment he showed the class how to do.

There is an important difference between inductions involving sense experience such as the Archimedes example and those based on abstract experience such as those traditionally attributed to Thales, Pythagoras, Euclid and other mathematicians. Those involving sense experience are not normally given a special term, but they may be called *empirical inductions*. Those involving abstract experience are often called *mathematical intuitions*. Mathematical intuitions are sometimes misleadingly said to be self-evident even though nothing is evident to anyone unless they have gone to the trouble to understand it, to experience the relevant reality, and to complete the cognitive process. The ambiguous words 'apriori' and 'aposteriori', which are not *used* in this investigation, have been used to distinguish mathematical from empirical inductions—and in several other ways as well (Lachs and Talisse 2007, 1-3).

Every mathematical cognition is either an inference or a mathematical intuition. Of course, no cognition is both an inference and a mathematical intuition. But there is no reason we can see for not thinking that one and the same proposition that is the content of one person's cognitive inference might also be the content of another person's cognitive intuition. In fact, we would say that one and the same proposition which is the content of one person's cognitive intuition might also be deducible from one or more of the same person's cognitive intuitions.

When is a statement true? There is a temptation to answer, 'When it corresponds to the facts'. And, as a piece of standard English, this can hardly be wrong. Indeed, I must confess I do not really think it is wrong at all.—Austin 1950/1961, 89.

Truth and Knowledge

In this work, the word 'true' is used in the traditional classical sense traced by Tarski back to Aristotle. If something is the case, then it is *true* that it is the case. And

conversely, if it is true that something is the case, then it *is* the case. To separate the classical sense of ‘true’ from its near neighbors, we can observe that in the classical sense the following are obvious to everyone who understands them.

The proposition that Aristotle read Plato is true if and only if Aristotle read Plato.

The proposition that Aristotle read Euclid is true if and only if Aristotle read Euclid.

The proposition that Euclid read Aristotle is true if and only if Euclid read Aristotle.

The proposition that Euclid didn’t read Aristotle is true if and only if Euclid didn’t read Aristotle.

These are obvious even though the four proposition they are about are not obvious to many. The proposition that Aristotle read Plato is known with moral certainty to be true by persons who have read Plato and Aristotle. The proposition that Aristotle read Euclid is known with moral certainty to be false: Aristotle died before Euclid wrote anything. The proposition that Euclid read Aristotle is a hypothesis—highly improbable to many given what the two wrote, but far from settled.

Every proposition is either true or false. But not every proposition is either known to be true or known to be false, by either of us or by any other person. In fact, not every true proposition is known to be true. Corcoran knows that there is a true proposition not known by him to be true. He can say this even though he knows that he cannot give an example. It is logically impossible for Corcoran to give a proposition that he knows to be a true proposition not known by him to be true. There are many counterexamples for the proposition that every true proposition is known to be true. And, for all we know, “every perfect number is even” might be one of them. But we can never give a counterexample and know that it is a counterexample (Corcoran 2005).

Truth includes knowledge and goes well beyond knowledge: the set of true propositions includes as a relatively small subset of the set of propositions known to be true. And it includes much more—namely the vast expanse of true propositions not known to be true.

No proposition is both true and false. And no proposition is both known to be true and known to be false, by either of us or by any given person. In fact, no proposition is both known to be true by either of us and known to be false by some other person.

Nevertheless, there are many propositions believed by one of us to be true and believed by others to be false.

There are many modern philosophers who believe that Corcoran was wrong to say that he has certain knowledge of the fact that no square number is twice a square number. It is not that they have any doubt of the fact, rather they disbelieve that Corcoran has absolute certainty of it. We are glad to discuss the issue with them. Without them this work would be less interesting.

Above we explained what we mean by saying that a certain proposition is true. For example, the proposition “No square is twice a square” is true. Better, the proposition “No number which is the sum of two numbers of the same power exceeding two is also of that same power” is true. Even better, the proposition “No perfect number is odd” is true. The reason we moved through these examples has to do with the following facts: we think we know the first to be true, we think that the second is true but that it is not known to be true by us, and we neither believe nor disbelieve the third. Moreover, we think that the third is not known to be true or known to be false by anyone. Nevertheless, as Tarski taught, we are fully warranted in stating the following:

The proposition “No perfect number is odd” is true
if and only if no perfect number is odd.

The proposition “Every perfect number is even” is true
if and only if every perfect number is even.

These contrast with the following:

The proposition “No perfect number is odd” is known to be true
if and only if some person knows that no perfect number is odd.
The proposition “Every perfect number is even” is known to be true
if and only if some person knows that every perfect number is even.

In order for either of the two arithmetic propositions to be true it is not necessary for anyone to do anything. In order for these propositions to be *known* to be true it is

necessary for someone to do something—something very difficult that no one has yet managed to accomplish. In addition, for these propositions to be *known* to be true it is necessary for them to be true. It is not that the knowing would be what would make them true, as some philosophers and mathematicians have held.

In many cases, in order for a person to responsibly *state* that a proposition is true, it is necessary for that person to know that it is true. But this should not be taken as evidence that knowledge is necessary for truth. What was just said of the two arithmetic propositions could be said with equal warrant of the four propositions about the two arithmetic propositions.

A cerebral habit of the highest kind, which will determine what we do in fancy as well as what we do in action, is called a *belief*. The representation to ourselves that we have a specified habit of this kind is called a *judgment*. Peirce 1880, §1.

Beliefs and Disbeliefs

For a given person saying, “I *believe* a certain proposition” amounts to saying “I believe it to be true”, or “it is one of my beliefs”. Saying “I *disbelieve* a certain proposition” amounts to saying “I believe it to be false”, or “it is one of my disbeliefs”. It is important to be explicit about some elementary points. Although every proposition that is not true is false, it is not the case that every proposition not one of a person’s beliefs is one of their disbeliefs. There are many propositions a certain person has never thought of, and among those they have thought of, there are many they have no belief concerning. One way this point is missed is that the words “I do not believe it” are used to say the same as “I disbelieve it”, not simply “it is not one of my beliefs”. It would be better either to say “I do not believe it; I need to see how the evidence available is sufficient for concluding it” or else to say “I disbelieve it; I have sufficient evidence to the contrary”. The atheist disbelieves what the theist believes; the agnostic does not believe or disbelieve what the theist believes—and therefore does not believe or disbelieve what the atheist believes.

In judging, a fresh belief is formed, often a belief in the truth of a proposition not previously believed by the person judging, but people often form a *new* belief in (the truth of) a proposition they previously believed. Judgment creates belief and thus

contrary to Peirce's 1880 usage, belief begins as judgment ends. Of course, once beliefs are formed, it can happen that the believers perceive their having those beliefs and then form judgments to that effect. This creates beliefs about beliefs, which we have called *belief-beliefs*. However, as a general point, this article's usage conflicts with Peirce's 1880 usage quoted above.

As Frege, Husserl, and others taught, propositions are timeless, "beyond time"—to use Husserl's phrase. In contrast, statements, judgments, and beliefs are dated (Corcoran 2009). The belief Corcoran formed years ago when he first learned that 10° Celsius is 50° Fahrenheit started years before the new belief he formed of the same proposition today. We imagine that many readers had to do a calculation to achieve belief that 10° Celsius is 50° Fahrenheit and could substitute their own name for Corcoran's above.

In contrast, the belief Corcoran formed years ago when he first learned that seven plus five is twelve might have persisted uninterrupted to this day even though weeks may pass without him reflecting on the fact or on the circumstances of his learning it. Often, our beliefs persist for years even when not put to use, so to speak.

Each belief comes into existence sometime during the life of a person who believes it and perishes no later than the death of the believer—earlier if memory fades. Each belief depends for its existence on its believer. No two persons have the same belief although in many cases two persons have different beliefs with the same propositional content—as emphasized earlier in this article.

The word 'belief' is frequently used elliptically for 'the content of a belief', the proposition believed. In this sense, there are beliefs that no one still believes: there are beliefs in the *non-temporal* sense that are not beliefs in the *temporal* sense. In the temporal sense, none of a person's mere beliefs can ever become anything else; a mere belief can never become a cognition—even if the person subjects the propositional content to a cognitive process that produces knowledge. In such a case, the belief in the non-temporal sense became a cognition, but the cognition is a belief with a later starting date. Of course, the words 'statement' and 'judgment' both have temporal and non-temporal senses.

In contrast, the act of stating per se does not form new beliefs, although there are simple cases in which judging and stating are simultaneous, or very nearly so. Usually the date of a statement is not the same as that of the belief stated. Sometimes the statement is made before the belief is formed. Of course, propositions are not dated at all even though, as just mentioned, the word ‘belief’ is sometimes used non-temporally to refer not to the dated belief but to the undated proposition believed. This is the case when we say that Euclid and Pythagoras had the same belief about right triangles even though one was born centuries after the other died.

As said above, propositions do not change in the process of becoming belief or knowledge. The proposition about right triangles was not changed as Pythagoras came to know it. The proposition about right triangles known by Pythagoras was not changed as Euclid came to know it. John Dewey is sometimes interpreted as saying the opposite, that every proposition is transformed by becoming knowledge.

Moreover, just as propositions are undated, propositions that are true did not *become* true. A proposition known to be a truth became a *belief*, but it did not become a *truth*. A proposition known to be a falsehood became a *disbelief*, but it did not become a *falsehood*. Every proposition either *is* a truth or *is* a falsehood, but not every proposition either is a belief or is a disbelief. William James is sometimes interpreted as disputing these points.

Consensus among objective investigators researching the same hypothesis from different points of view is taken to be a mark of truth or probable truth. We might wish that in the fullness of time the community of investigators will come to share common beliefs in the central true hypotheses under current investigation; we might wish that consensus will be achieved—and also that it will be correct. But the idea that in the long run every true proposition will become a belief, much less a consensus belief, is absurd—most true propositions will never even be understood much less believed. The view, sometimes attributed to Peirce, that being true is coextensive with being believed to be true in the long run by the community of investigators must be mistaken. Perhaps it should be attributed to wishful thinking. Besides, in the fullness of time there might be no investigators.

Others have attributed to Peirce the even more irresponsible view that ‘being true’ means “being believed in the long run by the community of investigators”. This view implies that every attribution of truth to a proposition is a statement about the future attitudes or mental states of people who have not yet been born (Nolt 2008, Corcoran 2008n).

Lying and Telling the Truth

This section is about deliberate statements. It excludes inadvertent remarks, misstatements, statements made under distracting conditions, and the like. The two expressions ‘telling a falsehood’ and ‘telling the truth’ can be misleading. The first does not mean “saying something false”, and the second does not mean “saying something true”. Telling a falsehood is lying, and that is not necessarily saying something false. And a person can say something false without lying. Likewise, telling the truth is stating what one believes to be true and beliefs are not necessarily true. And a person can say something true without telling the truth. Lying and telling the truth are forms of statement-making: they are human actions called *speech-acts*. Some philosophers have misunderstood the nature of the lie. A lie is a speech-act, not merely a sentence, or a proposition. A lie is a statement of a proposition that is not a belief of the speaker. Speakers who state their false beliefs are not lying. Likewise, speakers who state true propositions that they do not believe are lying—regardless of whether the non-belief is disbelief. Persons who state propositions on which they have no opinion are lying as much as those who state propositions they believe to be false.

Lies of ignorance are statements that are neither believed nor disbelieved by the speaker. *Lies of knowledge* are statements contrary to the speaker’s beliefs or disbeliefs. Lies of ignorance can be just as harmful as lies of knowledge and just as effective in promoting the aims of the liar. Moreover, because of confusion about the nature of lying, it is often easier to get away with lies of ignorance. As a matter of terminology, it might be better to call lies of ignorance *lies of unbelief* and to call lies of knowledge *lies of belief or disbelief*.

Perhaps paradoxically, there is often no way to tell a lie of ignorance without indirectly telling a lie of knowledge. The reason is that any given statement that

something is the case carries with it the indirect statement that the speaker believes the given statement. When speakers *know* that they do not believe their direct statements, their indirect statements are lies of knowledge.

If a person states that the house is locked up, there are at least two propositions that they have stated: a *primary* or direct statement about the house—that it is locked up—and a *secondary* or indirect statement about themselves—that they believe that the house is locked up. If they have no such belief, the primary statement is a lie of ignorance. If they know that they have no such belief, the secondary statement is a lie of knowledge.

Objectual Knowledge and Operational Knowledge

In order to have propositional knowledge, e.g. knowledge that two is the only even prime number, it is necessary to have objectual knowledge of several objects including but by no means limited to the following: the number two, the properties of being even and of being prime, the system of numbers, and the *exemplification* relation, i.e. the logical relation of a number to a numerical property that belongs to it. The number two exemplifies the property of being even. The word ‘object’ is being used in a very broad sense. Objectual knowledge is knowing of objects, including properties, relations, concepts, and anything else. We have objectual knowledge of everything we are acquainted with directly or indirectly as well as everything we know *of* by inference or reflection, such as the concept of truth. Some philosophers have implied that it is possible to know of an object by describing it, but this has always seemed to us to have the facts reversed. We do not see how we could describe something and thereby acquire knowledge of it unless we did not previously know of it. At this point it is not necessary to be more precise about the limits of objectual knowledge.

In the process of acquiring propositional knowledge that two is the only even prime number, Corcoran used various know-how, skills, or abilities that he had acquired previously. The skills and abilities that he has are what we have been calling his operational knowledge or knowledge-how-to. In the case being discussed, several items of operational knowledge might have come into play: the ability to factor a number, the ability to survey the progression of numbers starting with one, the ability to deduce

consequences of propositions, the ability to understand propositions, the ability to make judgments.

It might well be that some or all of his objectual knowledge derives from exercising operational knowledge, for example that he learns of numbers by counting, or conversely that some or all of his operational knowledge somehow derives from or depends on reflecting on his objectual knowledge, for example, that he learns how to count through reflecting on numbers. It is beyond the scope of this work to reflect on such issues. However, to forestall possible confusion, it is important to notice that he has objectual knowledge of propositions and of skills, which of course are objects in the broad sense. As is evident from the above, we *know of* propositions concerning which we do not have propositional knowledge, for example the Goldbach Hypothesis. In addition, we know of skills that we have not acquired. For example, Corcoran does not have operational knowledge of playing the violin or touch-typing but he has objectual knowledge of those skills.

My first act of free will shall be to believe in free will.—William James, 1870.

Choosing Beliefs and Disbeliefs

A person presented with several beliefs in the propositional sense might want to choose one for any number of purposes, e.g., to discuss first or to think about first. There is no problem here: it is always possible to choose randomly. However, when beliefs in the attitudinal sense are considered, the situation has changed.

Forming or shedding a belief and holding or lacking a belief are not acts like turning a switch on or turning it off. Forming and shedding beliefs are more like waking up and falling asleep. Holding and lacking a belief are more like staying awake and staying asleep. Turning a switch is voluntary and arbitrary in a way that belief formation is not. I can never form a belief in a given proposition by deciding to believe it and then throwing some sort of switch. This point has been denied by William James, who believed in “the will’s primacy, even in choosing what to believe”—to use Rebecca Goldstein’s formulation. According to Goldstein (2006, 25), James wrote in his journal in 1870, “My first act of free will shall be to believe in free will”.

Once a proposition has been understood and the evidence gathered and marshalled, the judgment is almost automatic—if it happens. Sometimes no result is reached. When a result is reached, it could be contrary to the desires of the believer. However, it is important to realize that although judgment is voluntary, it is not arbitrary at least when knowledge is achieved. An attitude or state of mind that was caused and not autonomously achieved through judgment is not even belief in the sense of this investigation.

Likewise, once there is awareness of a serious deficiency in the process that led to a belief, say that the source of testimony has been discredited or the instruments used found flawed, often the belief is lost or at least its certitude is diminished regardless of how attached to it the believer is. Charles Sanders Peirce agreed here, but he might have gone a little too far with it when he wrote (1877, 119-120): “Now, there are some people [...] who, when they see that any belief of theirs is determined by any circumstance extraneous to the facts, will from that moment [...], experience real doubt of it, so that it ceases to be a belief”.

If a coin is flipped and the outcome covered, as long as the evidence is unavailable there is no way for a person to form the belief that it is heads, say, no matter how much they might want it to be heads. More generally, there is no way to form a belief in a proposition that is now a hypothesis simply by deciding to believe it. The statement that some people believe what they want to believe is a misleading half-truth. To the extent that it would be fair to say it, it would be just as fair to say that some people believe what they want to disbelieve. People sometimes confuse hopes and fears with evidence. The expressions “too awful to be true” and “too good to be true” are common enough.

Forming a belief is not like deciding to purchase a given item and then putting it in the shopping cart. Some philosophers have disagreed, holding that a credence such as a religious belief is an exception, that a belief can be freely chosen when there is no hope of finding evidence. However, some religious thinkers have disputed that conclusion, saying that humans are powerless to construct such beliefs, that humans must await divine intervention, and that “the gift of faith” cannot be chosen but is freely bestowed.

Persons who find that two or more of their beliefs are inconsistent have all of those beliefs undermined. They are not free to decide which they prefer to keep and which to drop. They cannot arbitrarily decide, contrary to what some logicians seem to say. There is no switch to pull that reinstates some as beliefs while rendering others as disbeliefs. The issue here is not whether it is morally acceptable to adopt arbitrarily a belief and it is not whether a person who does this risks losing intellectual integrity. The issue concerns how beliefs are formed.

Some logicians recommend that we adjust our degrees of certitude so that they are proportional to our degrees of certainty. They say that the more certainty we have of a belief the more certitude we should feel and, accordingly, the less certainty we have the less certitude we should feel. However, we have no more control over the intensity of our feelings of certitude than we have over our judgments. Moreover, no way of measuring relative certitude or relative certainty has been devised. Those logicians might as well have recommended that we adjust our level of fear in proportion to the level of danger or that we adjust our level of happiness in proportion to our level of well-being.

The above should not be taken to deny that we often choose *to try* to understand one proposition while choosing *to try* not to understand another. Selective attention and willful ignorance are common enough. Nor should it be taken to deny that we can choose to seek evidence or arguments for one proposition while choosing to ignore evidence and arguments for another. The reality of partisanship, rationalization, and self-deception must be acknowledged, and it is just to hold people responsible in such cases. There is something unsavory about trying to choose a belief; it seems to violate intellectual integrity. Choosing to adopt or shed a belief seems to be a kind of self-deception, a kind of lie of ignorance.

The selfless autonomy of the objective judgment is counterpoised against the selfish arbitrariness of the subjective decision. Selfishness, laziness, impatience, lack of discipline, and other character defects that interfere with the successful completion of the cognitive process are open to blame.

However, with exceptions such as those just noted, it is absurd to blame people for holding, lacking, adopting, or shedding belief in a given proposition regardless of how deleterious or beneficial we might think it would be. It is even more absurd to try to

require someone to adopt or shed a belief in a given proposition. This would be like trying to require people to enjoy something they find repulsive or requiring them to be repulsed by something they enjoy. Attempting to coerce belief or disbelief compounds absurdity with injustice.

Background

It might be supposed that discussion of knowledge versus opinion took center stage in American philosophy with the 1877 publication by Charles Sanders Peirce of his seminal paper “The Fixation of Belief”, which has justly become somewhat a *locus classicus* for the issue. It is evident to scholars that the above discussion is heavily indebted to the Peirce paper both in spirit and in particular views. The above resonates with several of Peirce’s points including his “method of science”, his supposition that “there is some one thing to which a proposition should conform”, and his view that what is believed is in no way determined or changed by our thinking—to mention only three. In terms of the present discussion, roughly speaking, “The Fixation of Belief” presents four methods that can be used to increase *certitude* in a proposition already believed. One of those methods, the method of science, increases *certainty*.

Nevertheless, Peirce does not explicitly raise the issue of distinguishing “knowledge” from “opinion”. Perhaps surprisingly, he does not even use the word ‘knowledge’ or a synonym, and he does not make a distinction analogous to *certitude/certainty*.

However, the above discussion does not relate to certain “pragmatic themes” in later Peirce writings, in which focus on the *nature* of truth as “conforming to facts” gives way to focus on the *criterion* of truth as leading us to fulfilling our aims. *A fortiori*, the above does not relate to other classic American philosophers such as William James and John Dewey who worked in paradigms that might even be incommensurable with those currently flourishing in the United States. James and Dewey would dispute that “truth is a precondition to knowledge” and that “the proposition known is not changed by becoming known”.

The present discussion of the knowledge/opinion distinction addresses the more analytic side of post-World War II American philosophy. One of the most accessible of

relevant texts is the 1978 monograph *The Web of Belief* by Quine and Ullian. An excellent contemporary American treatment of propositional knowledge, which explicitly treats the knowledge/opinion distinction, is the 1991 treatise *Knowledge and Evidence* by Paul Moser.

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Many of the scholars mentioned disagree with some, many, or most of the conclusions of this work. We should also acknowledge many previous writers to whom we are indebted for most of the points made in this work. Our main contribution was to have collected their thoughts into a coherent work accessible to a wide audience. This paper is a substantial reworking of “An Essay on Knowledge and Belief” [Corcoran 2006]. The word ‘essay’ in its title was carefully chosen: the essay has no footnotes or references and it is much shorter than this investigation. The essay was inspired by responses to early drafts of the encyclopedia article “Knowledge and Belief” [Corcoran 2007k].²⁷

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¹ We use the word 'proposition' in the abstract sense in which one proposition might be expressed by many different sentences. Our usage—which follows Church 1956 and 1956(2) where 'proposition' is a near synonym for 'thought' in the sense of Frege's 1918 "The Thought"—is explained more fully in Corcoran 2009. Also, see Frege 1997, 325.

² Here, ambiguity is having multiple normal meanings and a sentence is lexically ambiguous if it contains an ambiguous word (lexical item). See Corcoran 2009a.

³ Being an even square is an intrinsic property of the number four, being the number of Evangelists is an extrinsic property of four. Having four letters is an intrinsic property of the word 'four', being a name of the number of Evangelists is an extrinsic property of 'four'. Changes in intrinsic properties are known as ordinary changes whereas changes in extrinsic properties are variously called "Cambridge changes",

“Pickwickian changes”, “relative changes”, and others. For further discussion and examples, see Corcoran 1995.

⁴ In the sense of ‘belief’ used in this work, propositions are exclusively the objects of believing: what a person believes is a proposition. Thus, a person’s acceptance as true of an incoherency such as ‘Socrates is equal’ is not belief in the sense of this paper.

⁵ An expression ‘my belief of [a certain entity]’ is questionable English if English at all unless the entity is something special such as a person’s statement: ‘my belief of the number one’ is ungrammatical. An expression ‘my opinion of [a certain entity]’ would be taken to refer to a propositional opinion. For example, our opinion of knowledge is that people’s lives are improved by it.

⁶ For Corcoran, this is a working hypothesis; for Hamid, it is firmly held belief.

⁷ Writing in 1846, Augustus De Morgan (1966, 1-3) thought that that negative substantive expressions such as ‘non-human’ were logically defective and would not occur in a logically perfect language. Instead, each substantive would have its own equally “positive” complementary substantive—as ‘knowledge’ has its complementary ‘opinion’.

⁸ The terminology for propositions not known to be true and not known to be false is awkward and unsettled. What is called an open question is often not a question in any of the more usual senses. Moreover, what is called a hypothesis was never hypothesized by anyone. See the article “Conjecture” in Audi 2014.

⁹ The late Dr. Ray Lucas asked whether having made a cognitive judgment is sufficient for having a cognition. Unfortunately, the answer is no, people can lose cognitions. People can lose belief in a proposition they once knew because of a later mistake or because of memory decay. We suspect that there are propositions we once knew but no longer believe and thus no longer know.

¹⁰ Nothing said above should be interpreted as suggesting that any given cognition is more meritorious or more worthy than any non-cognitive item of knowledge. The relative worth of two items of knowledge is beyond the scope of this essay. However, there are clearly cases in which knowledge of a certain mathematical theorem is less valuable than knowledge that a certain pill stops a certain pain.

¹¹ Applied to propositions, apprehension is an action while comprehension is an attitude or state. Normally, after someone apprehends a given proposition, they comprehend it for a time—often a long time during which they are only occasionally aware of the fact that they comprehend it. Apprehending a proposition takes place in a time interval—often relatively short—whose end coincides with the onset of comprehension. In a way, apprehension is to comprehension as judging is to believing: judging is an act not an attitude, believing is an attitude not an act [Corcoran 2009].

¹² Of course, all or at least many such experiences require the subject to be pursuing a goal prior to the event. Thus, some thinkers might prefer to start the process with a goal, or even with the pursuit of the goal, or even with the desire that prompted the pursuit.

¹³ See “Limiting case” and “Borderline case” in Audi 2014.

¹⁴ The word ‘probable’ is used in the original sense going back to around 1600 before the invention of “probability theory” gave it another meaning. In this sense, it applies to beliefs and contrasts with ‘certain’. See any dictionary that dates senses, for example, definition 1 in the 2008 *Merriam-Webster Collegiate Dictionary*. It is still widely used in the original sense where there is no question of assigning numbers to “events”. Frege uses it in the original sense in his classic 1918 paper “The Thought” (p. 306).

¹⁵ We do not know the history of the expression ‘moral certainty’. Antoine Arnauld (1612–1694) and Pierre Nicole (1625–1695) use the expression without comment as if it were a common locution in their 1662 masterpiece *The Art of Thinking* known as *The Port-Royal Logic* (Arnauld and Nicole, 1662/1996, 264, 270). Dessi (1988, xvii) traces it to John Locke (1632–1704). Dessi (loc. cit.) and Whately (1826, 243) both use it in the sense just explained.

¹⁶ The complementarity and interconnectedness of objectual, operational, and propositional knowledge has been a cornerstone of our thinking for many years. See Corcoran 1971.

¹⁷ Frege makes this point several places. However, without qualification, it is misleading to say that he said propositions do not change *intrinsically*—he did not emphasize the intrinsic/extrinsic distinction explained below.

¹⁸ In fact, even belief cannot be transmitted in this sense. One person’s blood can be transfused into another’s body but one person’s belief cannot not be moved to another person or copied on another person’s mind. Beliefs are formed by judging and each judging must be autonomous. Of course, magicians, for example, trick people into making judgments they would not otherwise make.

¹⁹ However, in the posthumously published “What are logical notions? (Tarski 1986), he proposed a condition for distinguishing logical from non-logical objects: individuals, sets, relations, functions, etc. This proposal does not imply a condition for distinguishing logical from non-logical concepts (senses) and thus does not yield a condition for distinguishing logical from non-logical terms (expressions).

²⁰ The expression ‘the truth of a proposition’ should not be detached from its context. By ‘assurance of the truth of a proposition’, I mean assurance that the proposition is true. It is a mistake to think that the truth of a proposition is an entity separate from the proposition itself.

²¹ There are two difficult issues here for the Aristotle scholar. The easier is whether our indirect quotation of Aristotle—that every person by nature desires knowledge—is a fair interpretation of the famous first line of *Metaphysics*. The other issue is whether Aristotle held the general view that we attribute to him. Scholars we consulted do not all agree. After a nuanced discussion, David Hitchcock (per. comm.) concluded: “So I think that it is fair to say that Aristotle took one cognitive goal of all human beings to be the possession of truth.” Very recent scholarship concurs. See Anagnostopoulous 2013, 102f.

²² Perhaps the classic expression of Peirce’s views is in sections III, IV, and V of his famous 1877 article “The Fixation of Belief” reprinted in the 1992 Houser and C. Kloesel volume (Peirce 1992, 109-123). The interpretation putting Peirce in diametrical opposition to Aristotle is almost universally shared by Peirce scholars as being a view that Peirce actually held at the time of the article. However, Peirce’s writings are replete with subtlety, irony, and scathing sarcasm. So much so, that it is hard to be certain that he was not actually expressing the opposite of what he wrote. Moreover, it might well be that he later came to embrace in a nuanced form a view he had formerly ridiculed in a naïve and exaggerated form.

²³ A person’s most firmly established beliefs are rarely those they believe most firmly, i.e. those of which they are most strongly convinced. This point relates to the contrast, dealt with elsewhere in this work, between the objective state of certainty and the subjective feeling of certitude.

²⁴ The word ‘skepticism’, or ‘scepticism’, derives from the Greek verb meaning “to consider carefully”, which was taken by some to mean “to consider so carefully that no conclusion is reached”. The Greek skeptic (*skeptikos*) did not subscribe to the view called ‘scepticism’ above; the skeptic meticulously avoided subscribing to any view at all. Today, the word ‘skepticism’, or ‘scepticism’, is used in various senses, often as above, rarely if ever in the etymological sense. See Preus 2007, 237-8.

²⁵ Frege (1918, 62) speaks of understanding as “grasping” the proposition and he speaks of judging as “acknowledging” its truth. See Frege 1997, 329. He seems oblivious of the fact that understanding is an act-process that takes time to complete. He seems likewise oblivious of the process intervening between understanding and judging. Moreover, he is vague about the nature of judging.

²⁶ We are aware of the literature on “knowing that one knows” centered on or stemming from “epistemic logic”, for example Hilpinen 1970. The definitions of knowing used in that literature are so alien to those used here that little written there is relevant.

²⁷ Corcoran’s encyclopedia article 2007 “Knowledge and Belief” was written earlier than the 2006 “An Essay on Knowledge and Belief”, but assigned a later publication date.