The Time Flow Manifesto

Chapter 2. Time Symmetry in Physics.

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Contents

Chapter 2. Time Symmetry in Physics.

The following are rejected in this chapter as fallacies.

1. Conventional Fallacies of time symmetry and QM reversibility.

- *1** False Analytic Principle 1. The time reversal of a deterministic causal law like: $s_1(t) \rightarrow s_2(t+\Delta t)$ *is a law like:* $Ts_2(t) \rightarrow Ts_1(t+\Delta t)$.
- 2* False Analytic Principle 2. The time reversal of a probabilistic law like: $prob(s_2(t+\Delta t)| s_1(t)) = p$ is a law like: $prob(Ts_1(t+\Delta t)| Ts_2(t)) = p$
- 3* False Analytic Principle 3. The condition for time symmetry of a probabilistic theory is that: $prob(s_2(t+\Delta t)|s_1(t)) = prob(Ts_1(t+\Delta t)|Ts_2(t))$ *for all state transition laws.*
- 4* False Analytic Claim About Physics. Quantum mechanics is *time symmetric* (*reversible* or *symmetric under time reversal transformation.*)

I repeat what *time symmetry* means.

- *Time symmetry* means *invariance under the time reversal transformation,* a symmetry transformation based on the mapping: $\boldsymbol{T}: t \rightarrow -t$.
- A *symmetry transformation* is based on a 1-1 mapping of a fundamental variable (like time, space, charge, etc) back onto itself. This must logically induce transformations on all other complex constructions involving this quantity. E.g. the mapping $t \rightarrow -t$ *determines that* $d\boldsymbol{r}/dt \rightarrow d\boldsymbol{r}/d(-t)$ (velocity reversal follows from time reversal).
- Any kind of well-defined object or logical construction (e.g. variables, states, processes, laws, worlds) for which *the time reversal transformation* is defined may have the property of *time symmetry,* meaning that the object or construction is identical to its time-reversed image.
- The laws of physics are *time symmetric (reversible) just in case they are identical to their image under the time reversal transformation.*

the peculiar 'low entropy' state in which our universe began – not a matter of any intrinsic asymmetry in the fundamental laws of physics themselves. And this, we are told, is one of the most profound results in the history of science. It shows there is no scientific foundation for what we intuitively believe, viz. that there is an 'intrinsic flow of time', from past to future. The whole process of the universe could have happened in time-reversed order, as far as the laws of nature are concerned. And then we would all identify the opposite directions of time as 'past' and 'future'. This conclusion is the starting point for most modern writing on the naturalistic philosophy of time for the last 50 years or so.

But how do the physicists *prove* this result? Well of course we can't examine every possible process individually and check if it is 'reversible'. There are infinitely many possible processes. Instead, *we check the general laws of physics for the property of time symmetry*. These laws tell us what processes are possible at a fundamental level (according to present physics). These laws are written as equations ('fundamental equations'), and by doing some formal transformations on the equations, we can check whether they are time symmetric. This gives the second common method for explaining the meaning of time symmetry in physics.

The *time reversal transformation,* we are told, is simple and straightforward. It simply consists in *replacing the time variable, t, with its negative image, -t, throughout the equations of physics.* Oh, and replacing any state description, *s,* with its time-reversed image, *T(S).* If the laws are time symmetric, then the time reversal *TL* of any law *L* is also a law of physics. This seems easy enough to understand with examples. The simplest example of a process is a particle travelling in a straight line at a constant velocity:

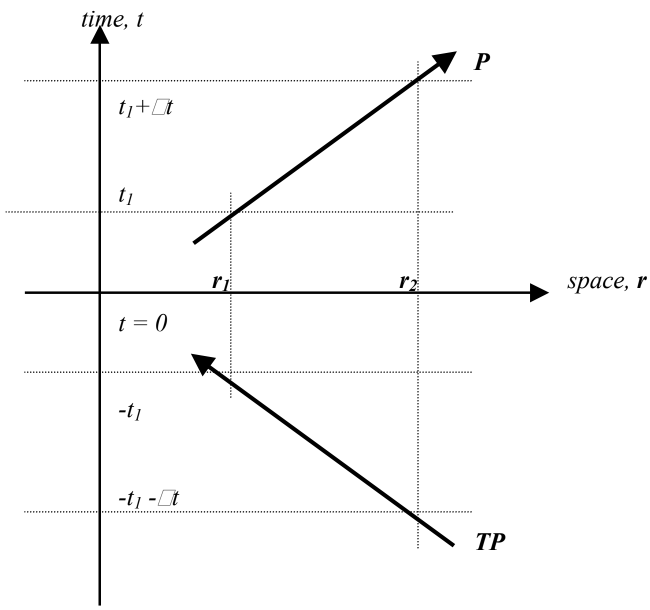

Figure 1. Space-time diagram illustrating a simple process (*P*) and its time reversal *(TP). TP* is the reflection of *P* through *t = 0.* In *P,* a particle moves from $\mathbf{r}_1$ *to* $\mathbf{r}_2$ in a period Δt*. In TP, the particle moves from* $\mathbf{r}_2$ *back to* $\mathbf{r}_1$ *in a period* Δt*.* But in *TP,* the *velocity* is reversed, because it is moving 'backwards'. Both of

these are possible processes for an isolated particle according to most theories of physics.

The intuitive line of thought goes like this. We take this first of all to be a deterministic process. For the process *P* to be physically possible (as in classical physics), there must be a law like:

L_D $\quad\quad$ $s_1(t) \rightarrow s_2\,(t+\Delta t)$

meaning that *an (isolated) system in a state s_1 at time t will develop, according to the laws of physics, into a later state s_2 at time $t+\Delta t$. Note that laws are assumed to be time translation invariant* - where we choose to assign the coordinate value: $t = 0$ is merely conventional - so this law applies to any time *t*. Logicians would say that the general laws have an implicit universal quantifier on *t,* meaning that "for all moments *t*, …".

We are interested in whether the reversed process *TP* is possible given that *P* is possible. Since *P* starts in state s_1 and ends in state s_2, the reversed process must start with Ts_2 and end with Ts_1. Given the law L_D that governs the process *P,* it seems that we then need a *time reversed* law like the following to allow the reversed process:

TL_D* $\quad\quad$ $Ts_2(t) \rightarrow Ts_1(t+\Delta t)$

I.e. *an (isolated) system in a state Ts_2 at time t will develop, according to the laws of physics, into a later state Ts_1 at time $t+\Delta t$. This is assumed to be the time reversal of the law L_D* in the conventional analysis. I have labelled it with an asterix, TL_D*, however, because actually it is not the time reversal of L_D at all! I will let the reader puzzle over this for a few moments, and see if they work out what the real time reversal of L_D is – it is obvious enough when you see it, but the conventional presentation, as above, conceals the correct answer under false intuition. Before revealing the answer I consider probabilistic laws.

Probabilistic Laws

The serious problems arise when we move on to *probabilistic laws.* Quantum mechanics is widely believed to be the fundamental theory of particle physics, and to require irreducibly probabilistic laws, and these laws are claimed to be time symmetric. Physicists take the time reversal of a *probabilistic transition law* of the following form:

$L \qquad Prob(\, s_1(t) \rightarrow s_2\,(t+\Delta t)\,) = p$

(The probability of a transition from s_1 *to* s_2 *after a period* Δt *equals* p, *with* p *a real number from 0 to 1*) to be a corresponding law of the form:

$TL^* \qquad Prob(Ts_2\,(t) \rightarrow Ts_1(t+\Delta t)) = p$

(The probability of a transition from Ts_2 *to* Ts_1 *after a period of* Δt *equals* p)

Again I have labelled *TL** with an asterix because it is not really the time reversal of *L*. The proof that quantum mechanics is time symmetric in its probabilistic laws then amounts to the claim that the following symmetry principle holds:

[QM cause-effect exchange symmetry]

$$Prob(\, s_1(t) \rightarrow s_2\,(t+\Delta t)\,) = Prob(Ts_2\,(t) \rightarrow Ts_1(t+\Delta t))$$

for all quantum state transitions – since this assures us that for every law *L* of the theory there is a corresponding law *TL**. Note also that if we take the transition probability to be p=1, then this reduces to the deterministic case above. However although this principle is generally true in quantum mechanics (with the exception of some meson decay processes), we will see that it *does not represent time symmetry at all.* This is why I have called it *QM cause-effect exchange symmetry,* instead of QM time reversal symmetry as stated in all the textbooks.

To illustrate let us consider another very simple example, of a clearly *time symmetric probabilistic process.* Imagine a system with just three possible states, call them s_0, s_1, s_2, which 'jumps' from state to state after every interval of time, Δt, *like this:*

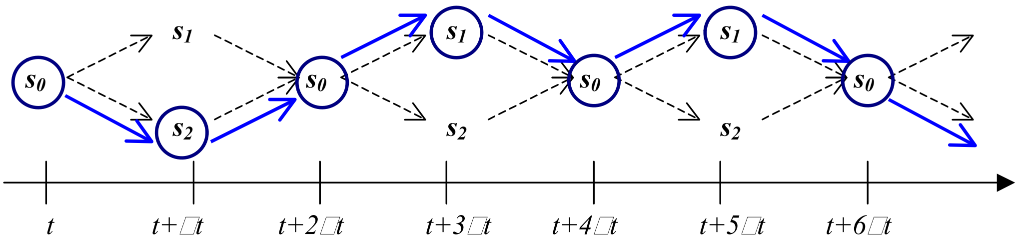

Figure 2. A simple probabilistic process. From state s_0 the system jumps randomly to either s_1 *or* s_2 , i.e. with probability 0.5 in each case. From state s_1 *or* s_2 the system always jumps back to s_0, i.e. with probability 1 in each case. The underlying probabilities are indicated in black, a series of actual events (actualised probabilities) is indicated in blue: …0201010…

There are four simple laws for the dynamics of this system:

L_{01} $\quad prob(s_1(t+\Delta t)|s_0(t)) = 0.5$

L_{02} $\quad prob(s_2(t+\Delta t)|s_0(t)) = 0.5$

L_{10} $\quad prob(s_0(t+\Delta t)|s_1(t)) = 1$

L_{20} $\quad prob(s_0(t+\Delta t)|s_2(t)) = 1$

To ensure the theory of this process as a whole is *time symmetric,* we also ensure that *there is no start or end to the process,* with an extra law that:

L_+ $\quad prob(s_0(t) \text{ or } s_1(t) \text{ or } s_2(t)) = 1$, *for all times, t.*

L_{01} means that *the probability of the state s_1 at time $t+\Delta t$ given the state s_0 at time t equals 0.5,* and so on. L_+ entails that system at any time always has an earlier and a later state. We could imagine this as an infinite coin-tossing process, where s_0 is the randomised state before each toss, s_1 is the outcome state *heads,* s_2 is the outcome state *tails,* and after each toss the coin is returned to its randomised state. (In quantum physics, this could be modelled as a series of spin-1/2 experiments, with ‘up’ and ‘down’ as outcomes, and the system returned to the superposition after each event.)

To keep the example simple, we define the states to be their own time reversals, i.e. $Ts_0 = s_0$, $Ts_1 = s_1$, $Ts_2 = s_2$. Hence when we play a sequence of states backwards, we see a sequence of the same kind of states again. E.g. suppose a process has a sub-sequence:

P …*020101010201020201020201020201010102010202010*…

Then the time reversed process has the sub-sequence:

TP …*010202010201010102020102020102020102010101020*…

Now it seems patently obvious that this process is *time symmetric,* and that the set of laws L_{01}, L_{02}, L_{10}, L_{20}, that govern it forms a *time symmetric set of laws*. It is impossible to tell a sequence and its time reversal apart statistically. Of course a directional pattern could occur, e.g.: …010101010101020202020202… But any directional pattern in an actual sequence is merely the result of coincidence, with the same probability of the reversed directional pattern occurring by coincidence, and this still doesn't help us determine any direction of time for the process from the stochastic laws.

The Physicists Reversal Fails

Let us now examine this set of laws using the physicist's criterion for finding the time reversal of probabilistic laws. According to that, the reversals of the laws for this system are:

Original Theory		***Physicists' Time Reversal***	
L_{01}	$prob(s_1(t+\Delta t)\|s_0(t)) = 0.5$	TL_{01} *	$prob(s_0(t+\Delta t)\|s_1(t)) = 0.5$
L_{02}	$prob(s_2(t+\Delta t)\|s_0(t)) = 0.5$	TL_{02} *	$prob(s_0(t+\Delta t)\|s_2(t)) = 0.5$
L_{10}	$prob(s_0(t+\Delta t)\|s_1(t)) = 1$	TL_{10} *	$prob(s_1(t+\Delta t)\|s_0(t)) = 1$
L_{20}	$prob(s_0(t+\Delta t)\|s_2(t)) = 1$	TL_{20} *	$prob(s_2(t+\Delta t)\|s_0(t)) = 1$

But there is something wrong here - the *physicists' time reversal of the theory contradicts the original theory!* E.g. in the original theory, $prob(s_1(t+\Delta t)|s_0(t)) = 0.5$, *but in the physicists' time reversal,* $prob(s_1(t+\Delta t)|s_0(t)) = 1$. *In fact the physicists'* time reversal of the theory gives a self-contradictory theory, stating that both: $prob(s_1(t+\Delta t)|s_0(t)) = 1$ *and* $prob(s_2(t+\Delta t)|s_0(t)) = 1$. *This requires that the state* $s_0(t)$ *develops deterministically to the state* $s_1(t+\Delta t)$ *and to the state* $s_2(t+\Delta t)$.

So this analysis using the physicist's principles would tell us that the theory is not time symmetric! But we know intuitively that the theory is perfectly time symmetric. The time reversal of the theory, if derived correctly, must be identical to the original theory. There is a fallacy in the physicists' derivation of time reversal.

The Correct Principle for Time Reversal

I now state the correct principle for deriving time reversal. First, for our original example of a deterministic law like:

L_D $\qquad s_1(t) \rightarrow s_2(t+\Delta t)$

The time reversal is actually:

TL_D $\qquad Ts_1(t) \rightarrow Ts_2(t-\Delta t)$

This means that the state Ts_1 *at t determines the earlier state,* Ts_2 *at* $t\text{-}\Delta t$. *That is to* say, the *future-directed deterministic law,* L_D, becomes a *past-directed deterministic law,* TL_D, when the law L_D is reversed.

More generally, the time reversal of a probabilistic law like:

L $\quad prob(s_2(t+\Delta t)|\, s_1(t)) = p$

Is actually:

TL $\quad prob(Ts_2(t-\Delta t)|\, Ts_1(t)) = p$

Again this is a *past directed law.* The requirement for time symmetry of a probabilistic theory, **T**, is then that:

[**T** is time symmetric] **T** entails that: $prob(s_2(t+\Delta t)|\, s_1(t)) = prob(Ts_2(t-\Delta t)|\, Ts_1(t))$, for all state transition laws of the theory.

Applying this to our example:

Original Theory		***True Time Reversal***	
L_{01}	$prob(s_1(t+\Delta t)\|s_0(t)) = 0.5$	TL_{01}	$prob(s_1(t-\Delta t)\|s_0(t)) = 0.5$
L_{02}	$prob(s_2(t+\Delta t)\|s_0(t)) = 0.5$	TL_{02}	$prob(s_2(t-\Delta t)\|s_0(t)) = 0.5$
L_{10}	$prob(s_0(t+\Delta t)\|s_1(t)) = 1$	TL_{10}	$prob(s_0(t-\Delta t)\|s_1(t)) = 1$
L_{20}	$prob(s_0(t+\Delta t)\|s_2(t)) = 1$	TL_{20}	$prob(s_0(t-\Delta t)\|s_2(t)) = 1$

L_+ $\quad prob(s_0(t)$ *or* $s_1(t)$ *or* $s_2(t)) = 1$, *for all times, t, is identical in both.*

And the time reversed theory indeed turns out to be exactly the same as the original theory. E.g. the original theory requires that the state s_1 *is always followed by* s_0 – and it equally entails that the state s_1 *is always preceded by* s_0. Similarly, the original theory requires that the state s_0 *is followed by* s_1 with 0.5 chance – and it equally entails that the state s_0 *is preceded by* s_1 with 0.5 chance. Without this symmetry between *future-directed transition statistics* and *past-directed transition statistics* the theory clearly could not be time symmetric, and this example matches all our intuitions.

The simplest way to assure yourself that *TL* is the time reversed image of *L* is simply to follow the 'formal recipe' recommended by physicists, and *substitute all time variables for their negatives in L (including substitution of Ts for each state s*). This first gives us: *prob*($Ts_2(-t-\Delta t)$| $Ts_1(-t)$) = *p*. Because *t* is universally quantified but -Δt is a specific constant, this is logically equivalent to: *prob*($Ts_2(t-\Delta t)$| $Ts_1(t)$) = *p*, which is *TL* as stated. Not that hard!

The Fallacy in the Physicists' Principle.

How did the physicists make this error? I think by using unanalysed intuition to formulate their 'reversal' principle, and then failing to check it. To obtain the physicists' *TL** we have to perform the substitution of *–t* for *t,* and *then also exchange the causal order of states.* This does not give the time reversed image of *L* at all – it sneaks in a 'double reversal', to satisfy our normal intuition that causal laws must go forward in time. In fact, this does not represent a symmetry transformation at all.

A *symmetry transformation* is based on a 1-1 mapping of a fundamental variable (like time, space, charge) back onto itself. This must logically induce transformations on all other complex constructions involving this quantity. But *TL** does not have any possible underlying transformation! A full proof of this is given in Holster 2003, where it is proved that the conventional criterion is neither a necessary nor sufficient condition for time symmetry. The physicists' time reversal principle is actually logically irrelevant to time symmetry!

What physicist have called *time reversal* is best called *cause-and-effect-reversal,* or *causal exchange* for short, because it involves exchanging the order of cause and effect, along with the time reversal of states. This is already seen in the deterministic case. The law L_D states that s_1 *at t will cause* s_2 *at* $t+\Delta t$. The physicists' reversal of this, *TL**, states that Ts_2 *at t will cause* Ts_1 *at* $t+\Delta t$. It may seem intuitive that this is time reversal, but that is a fallacy of intuition: it does not represent the *time reversal transformation,* as induced by the mapping: $t \rightarrow -t$, and it does not have any of the implications of time reversal that are critical to the philosopher's interpretation of what this means. Equally, what is called *time symmetry (or reversibility) of quantum* *mechanics* in textbooks should be called '*causal exchange symmetry of quantum mechanics'*.

I note that there is another problem with time reversal in both quantum theory and even classical electromagnetic theory, viz. the choice of the time reversal operator on states, i.e. the transformation: $s \rightarrow Ts$. *The literature on this reveals great confusion.*

Quantum Mechanics is Time Asymmetric.

The famous result that quantum mechanics is time symmetric is based on the fallacious principle we have just seen, and it is completely wrong. It is wrong in its method: it uses the wrong principle to analyse time symmetry, identifying TL^* instead of TL as the reversal of L. And it is wrong in its conclusion: when the analysis is done correctly, it is clear that quantum mechanics is *time asymmetric (irreversible).* The probabilistic laws of quantum mechanics *simply do not hold of time-reversed quantum processes.* This can be seen from a simple theorem to the effect that:

Theorem of QM Equilibrium. *Time symmetry* and *cause-effect exchange symmetry* jointly entail *thermodynamic equilibrium*, where absolute probabilities of all micro-states are equally likely.

This of course contradicts the observation of disequilibrium in our universe:

Observation of Disequilibrium. *The real universe is in a state of disequilibrium*.

A simple derivation of the previous theorem follows.

Derivation of the Theorem of QM Equilibrium.

The easiest way to demonstrate this is by combining the quantum principle of causal exchange:

$$prob(s_2(t+\Delta t)|\, s_1(t)) = prob(Ts_1(t+\Delta t)|\, Ts_2(t))$$

With the requirement for true time symmetry:

$$prob(s_2(t+\Delta t)|\, s_1(t)) = prob(Ts_2(t-\Delta t)|\, Ts_1(t))$$

If these both held generally, then equating the right hand sides:

$prob(Ts_1(t+\Delta t)|\ Ts_2(t)) = prob(Ts_2(t-\Delta t)|\ Ts_1(t))$

By substitution of Ts_1 and Ts_2 for s_1 and s_2 and using the identities: $TTs_1 = s_1$ and $TTs_2 = s_2$ and the general quantification of *t,* we then obtain:

$prob(s_1(t+\Delta t)|\ s_2(t)) = prob(s_2(t-\Delta t)|\ s_1(t)) = prob(s_2(t)|\ s_1(t+\Delta t))$

But this can only hold if the absolute probabilities for the two states, $s_2(t)$ and $s_1(t+\Delta t)$ are equal. This is seen by expanding into conditional probabilities:

$prob(s_1(t+\Delta t)|\ s_2(t)) = prob(s_1(t+\Delta t))/prob(s_1(t+\Delta t) \text{ and } s_2(t))$

$= prob(s_2(t)|\ s_1(t+\Delta t)) = prob(s_2(t))/prob(s_2(t) \text{ and } s_1(t+\Delta t))$

Hence equating the right hand sides:

$prob(s_2(t)) = prob(s_1(t+\Delta t))$ *(absolute probability law).*

And since the laws are universalised w.r.t. time, this requires that:

$prob(s_2(t)) = prob(s_1(t))$

This states that *the absolute probabilities of any two micro-states, $s_1(t)$ and $s_2(t)$, are equal.* But this is a condition for *thermodynamic equilibrium.* It is absolutely not a condition that is met by the real universe. See Holster (2003) for more detailed proofs. In summary:

- Time symmetry and cause-effect exchange symmetry can both hold only in a thermodynamic equilibrium. Our universe is not in equilibrium. Hence at least one symmetry must fail. Since cause-effect exchange symmetry holds in quantum mechanics, time symmetry must fail in quantum mechanics.

This shows that it is quite impossible for quantum theory to be time symmetric. As a result, quantum mechanics implies an intrinsic time direction. This is *the direction of actualisation of quantum probabilities.* In Holster, 1990 [PhD Thesis], I adapted McCall 1976 [9], in interpreting this as *the direction of time flow.*

The Error in Quantum Mechanics Textbooks.

This shows that the claims *1** to *4** are fallacies. This fallacy is perpetuated in philosophical accounts in a deeply misleading way, but also advanced in textbooks on quantum mechanics, in a relatively more harmless way, but needing correction. E.g.

> *"A system is said to exhibit symmetry under time reversal if, at least in principle, its time development may be reversed and all physical processes run backwards, with initial and final states interchanged. Symmetry between the two directions of motion in time implies that to every state Ψ there corresponds a time-reversed state $\Theta\Psi$ and that the transformation Θ preserves the values of all probabilities, thus leaving invariant the absolute value of any scalar product between the two states." Merzbacher, 1970, p.406-407.* [10].

To correct the fallacy, this might be modified to read (with alterations underlined):

> "A system is said to exhibit symmetry under *causal exchange* if, at least in principle, its time development may be reversed and all physical processes run backwards, with initial and final states interchanged. This symmetry implies that to every state Ψ there corresponds a time-reversed state $T\ \Psi$ and that the transformation T preserves the values of all probabilities, thus leaving invariant the absolute value of any scalar product between the two states. In quantum mechanics we normally identify the time reversal transformation, T, with the antiunitary operator, Θ.
>
> Note that this *causal exchange symmetry* is identified in older texts as *time reversal symmetry,* but it has been shown that it does not represent time reversal symmetry. True time reversal symmetry is not physically valid in quantum mechanics, and consequently of no interest in the technical development of the theory here. Implications of true time reversal symmetry cannot be inferred from the *causal exchange symmetry* which is explained here. There are currently no reliable textbooks treating time symmetry in quantum mechanics."

Along with similar replacement of the term *time reversal symmetry* with *causal exchange symmetry* at a few other places, this corrects the error represented in general physics textbooks. Of course this now leaves the concept of *time symmetry* unexplained, and leaves the rationale for choosing *Θ instead of T unclear, and leaves* the implications of *CPT* theorems for *time symmetry* unclear, but that goes beyond correcting the explicit error. The subsequent *mathematical derivations* in physics textbooks are usually reliable, the initial interpretation of what it *means* is incorrect. We can correct this by calling the symmetries by their proper names.

To forestall a common objection, I *insist* that this is not just a ‘semantic issue’ or ‘playing with definitions’. The meaning of the term ‘time reversal symmetry’ is not being conventionally defined or changed to our convenience – on the contrary *we are insisting on using it with its correct meaning*. The term has an objective meaning in physics. It means *symmetry under the time reversal transformation.* What is being corrected is a false identification, viz. of *causal exchange symmetry* as *time reversal symmetry*.

Conclusion. Fallacies 1* - 4*.

The fallacies of 1*- 4* have been demonstrated. This removes the present case for the conventional conclusion that *the conventional reversibility of physics means time has no intrinsic temporal direction.*

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Footnotes.