

WHERE WORDS FAIL

But if someone were to say “So logic too is an empirical science” he would be wrong. Yet this is right: the same proposition may get treated at one time as something to be tested by experience, at another as a rule governing such testing. (*On Certainty*, §98)

A thought makes truth turn on how things are; a given thought on whether things are a given way *for* them to be. A (singly) singular thought, such as that Sid smokes, is decomposable into two elements: one making truth turn on how some object is; the other making it turn on which objects are some particular way for an object to be. From 1891 on Frege speaks of thoughts, and thought-elements, as having what he calls *Bedeutung*. The *Bedeutung* of a predicative element, he writes, is what, from 1891 on, he calls a *concept*; of an element which names (makes truth turn on how some given object is) an object. Concepts and objects (he *seems* to say) are fundamentally different: a concept depends for existence on objects to fall under it or not; an object is in no such need of completion. No object is a concept; hence no concept an object. So he *seems* to say. But, he also (sometimes) finds, something prevents our saying so.

Sometimes Frege sees language as the obstacle:

Granted, a peculiar obstacle stands in the way of communication with the reader, namely, that, with a peculiar necessity of language, my expression, taken strictly literally, sometimes misses the thought, in that where a concept is meant an object is named. I am fully aware of being directed in such cases to the accommodating good will of the reader who will not be sparing with a pinch of salt. (1892: 204)

Sometimes the obstacle appears as ‘the nature of a concept’ itself:

The nature of a concept is now a great obstacle for proper expression, and for communication. Where I want to speak of a concept, language forces me with nearly inescapable force to use an unfitting expression, by which the thought is obscured—I could almost say falsified. If I say, ‘the concept *equilateral triangle*’, one would assume, following the linguistic analogy, that I thereby designate a concept, just as, without doubt, I name a planet if I say ‘the planet Neptune’. But this is not the case; for predicative nature is missing. Accordingly, the *Bedeutung* of the expression ‘The concept *equilateral triangle*’ (insofar as there is one) is an object. We cannot avoid words such as ‘the concept’, but must then always be conscious of their unsuitability. (1892-1895: 130)

The *Bedeutung* of ‘The concept *equilateral triangle*’ is an object, *hence* not a concept. A concept could *only* be *bedeutet* by a certain sort of expression (‘unsaturated’) which, in turn, could *only* function to predicate of something being what that concept was a concept *of* being (e.g., equilateral). Such an expression would yield no predication of a concept. Nor would

any other which might predicate things of objects. So goes the idea. It *looks* as if we can say what the *Bedeutung* of a predicative expression is. We would like to say: it is *such-and-such* concept. If Frege is right there is no saying this. Is this insight or confusion?

In a wonderfully provocative essay, “In Search of Logically Alien Thought” (1991), James Conant answers ‘Yes’—on behalf of himself and young Wittgenstein (henceforth YW). YW, he observes, was inspired by the idea. He writes:

Another cousin of our problem can be seen in Frege’s treatment of the Kerry paradox, when he insists that the words he himself must resort to (“the concept horse is not a concept”) in order to illuminate what is confused in Kerry’s talk about concepts do not themselves express coherent thoughts—any more than Kerry’s own formulations do. ...

Frege’s discussion of these two cousins of our problem are viewed by many contemporary commentators as among the most embarrassing moments in all of his work—sudden signs of an otherwise uncharacteristic softening of the mind. Yet they are precisely the moments in Frege’s work from which Wittgenstein takes himself to learn the most. (1991: 140-141)

In that same paragraph, Conant also observes that YW was inspired, too, by another idea of Frege’s, this time to a negative response. This other idea concerns the nature of logic. Conant expresses it as the idea that logic is a science. YW expresses his rejection of it in the idea that laws of logic ‘say nothing’.

Here is the plan of the present essay. I will argue that YW’s reactions were off target both times. They are, I will suggest, among what mature Wittgenstein (henceforth MW) came to see as among the *Tractatus*’ ‘grave mistakes’. Further, to see how YW’s critique of Frege’s view of logic misses the mark, it will emerge, is also to see what is wrong with Frege’s ‘insight’ about concepts and objects. It is the same thing which goes wrong in each. The essay divides in three parts. Part I scouts some notions which will be needed for discussing Frege’s obstacle. Part II discusses it. Part III takes up Frege’s, and YW’s, conceptions of logic. It ends with MW’s replacement for these.

If Frege is right about the obstacle, I may, from time to time in what follows, try to say the unsayable (or say something about concepts which is not really about them). I am betting on his being wrong. *Pro tem* a case that I have succumbed to some such fate should not just *assume* that he is not.

I

1.1. Thoughts First: *What* blocks saying what it *seems* there is to say about concepts (and objects)? And *how* does it? For answering these questions some distinctions need to be observed. Part I aims to draw them. To begin I follow Frege’s strategy: whole thoughts come first. In 1919 he put this thus:

What is distinctive in my view of logic is made recognisable, first of all,

by the fact that I put the content of the word 'true' at the forefront, and then by the fact that I let thoughts follow immediately as that by which truth can come into question at all. Thus I do not begin with concepts and build thoughts, or judgements, out of them, but I arrive at thought-elements by the decomposition of thoughts. (1919a: 273)

In the same vein, in 1882 he wrote,

I do not believe that the formation of concepts could precede judgements, because that presupposes an autonomous existence of concepts, but I think that concepts arise through the decomposition (*Zerfallen*) of a judgeable content. (1882: 118)

'Judgeable content' is Frege's early term for that by which truth can come into question at all. His first try at capturing *just* this, he came to see, built more into it than really belonged there. So, by 1891, he split off what did no work in bringing truth into question, leaving behind *Sinn*, and dubbing what was split off *Bedeutung*. *Sinn* is a species with two genera: *whole* thoughts (ways for truth to come into question); and thought-elements.

A whole thought decomposes (not *verrotten*, but *zerfallen*), or is decomposable (*zerlegbar*) into elements. An element (*on* a decomposition) is whatever one thus comes to. To decompose a thought is to decompose a given way for truth to come into question; a given question of truth. A whole thought represents *things* as some given way there is for things to be. 'Things', so used, blocks the question, 'Which ones?'. It is the way things are, full stop, which is, or is not, as represented.

A general point about decomposition: a *Zerfallung*, so a *Zerlegung*, of something is just that thing, decomposed; the whole thing, presented as dividing into given parts. So one *has* a decomposition only, but also just, where the elements on it are, jointly, the whole being decomposed.

A thought as here conceived has a defining mission, or task: to make truth turn in such-and-such way on how things are. Decomposing a thought can be thought of as decomposing its mission into subtasks. An element of a thought on a decomposition is such a subtask, or, if one wishes to reify, precisely that which performs it. Any task can be decomposed into subtasks. So far, there is nothing special about thoughts. The task might be whitewashing a *casita* in a *Pueblo Blanco*. One might decompose this (in one of countless ways) into whitewashing the front *façade*, whitewashing the rear *façade*, whitewashing the sidewall. A thought's task is making truth turn in a particular way on how things are. So an element in it (*on* a decomposition) would be making truth turn in part in such-and-such way on how things are; or, more fully, making the thought's way of making truth turn on how things are in part its turning in such-and-such way on how things are. Making truth turn on whether Sid smokes might, for example, be making truth turn in part on who smokes, or, more fully, on whether relevant things smoke (those made so relevant, that is, by other elements in the thought).

'Whole thoughts first' thus loses philosophy a problem. Where a thought is decomposed into elements, no intelligible question remains as to what might join those elements into 'the unity' of a whole thought. Similarly, if whitewashing a *casita* has been

decomposed into whitewashing its parts, there is no intelligible question as to what makes whitewashing those parts whitewashing the *casita*. For a thought's elements to constitute a decomposition just *is* for them, jointly, to be the whole; for those subtasks, each performed in performing the others, to be that thought's identifying task. Losing a problem, though, has a price. It is essential for thought-elements to be what *do* add up to the whole thought. Where a thought makes truth turn on whether Sid smokes, making truth turn on whether things which matter smoke might be an element in it—where there is another which makes truth turn on whether Sid is ways which matter. Such elements are in the same business as the whole: making truth turn on how things are. They are also in the thought's same business, representing-as. One represents relevant things as smokers, the other Sid as the one who is relevant ways. By contrast (a familiar point) a way for a thing to be—say, such as to smoke—is not in any of these lines of work. It makes truth turn, even in part, on nothing; represents nothing as any given way. On this conception of a decomposition it is thus ineligible to be a thought-element.

Thus Frege's insistence that Sid, e.g., can be no thought-element. A thought's truth might turn on how Sid is. But *Sid* cannot make himself, or anything else, that on which the truth of any given thought turns; nor make truth turn on whether anything is any given way. Sid, as opposed to whether things that matter smoke, is not even a partial question of truth. (Similarly for Mont Blanc and The True.) Sid and being such as to smoke are equally, and for the same reasons, ineligible to be thought-elements.

The second point about thought-elements is that the same thought may be decomposed in many ways, where what is an element on one decomposition may be absent from another. What a thought's elements are is thus relative to how it is decomposed. To *be* such an element is, more properly, to be an element of some decomposition(s) of the thought. (See, e.g., (1892: 199-200.)) *Casitas* follow suit here too. There are many ways of decomposing a *casita's* outer surface into surface-parts; correspondingly many (at least) of dividing up a task of whitewashing it. None can claim serious priority *per se*. So, too, Frege tells us, for different decompositions of the same thought. (See 1882: 118.)

Where a thought is decomposed into, *inter alia*, an element which makes truth turn (somehow) on which things are some given way, or which way relevant objects must be, I will call the element *predicative*. Where an element makes such-and-such object the one which must be relevant ways, I will call it a *naming* element.

A third point. A thought has a mission, decomposable into subtasks, just as whitewashing a *casita* is decomposable into subtasks. But there is a difference. A subtask in whitewashing might be done on its own. Pia whitewashes the front façade. Sid, charged with the rear, instead sneaks off for a nap. The *casita* has not been whitewashed. But at least something has been done. It has been *partially* whitewashed. There is no parallel in a task of making truth turn on how things are. Subtasks in doing so cannot be parcelled out. More crucially, there is no such thing as doing just *one* such subtask (and then, as it were, stopping). There is no making truth turn in part on how Sid is independent of making truth turn on how things are; no such thing as taking satisfaction in a job part done. Here to stop short of the whole job would be to do nothing. Such is one form of Frege's context principle.

1.2. *Bedeutung*: 1891 marks a watershed in Frege's thought. It is the point by which he had abandoned the notion *judgeable content* as unfit for purpose. He broke off from this what had

no role in identifying questions of truth. What remained was *Sinn*, in the varieties *whole thought* and *thought-element*. What was broken off he called 'Bedeutung'. What was left behind retained this feature: the whole comes first; parts of it are arrived at by decomposition. A key question: Does the same apply to what was broken off?

Frege describes the breakup as follows:

When I wrote my *Foundations of Arithmetic* I had not yet made the distinction between *Sinn* and *Bedeutung*, and thus still brought together in the notion *judgeable content* what I now designate differentially with the words 'thought' and 'truth value'. (1892: 198)

Truth value appears here as the prime case of what needed breaking off. It thus appears as what is most central to Frege's notion of *Bedeutung* that a truth-value is the *Bedeutung* of a whole thought. The problem of what the *Bedeutungen* of *Sinne* of various kinds are is often approached as follows: we *know* that the *Bedeutung* of a naming thought-element is an object. In that case, what should the *Bedeutungen* of other sorts of *Sinne* be? This makes for particular problems when it comes to the *Bedeutung* of a whole thought. Frege himself sometimes encourages this way of thinking. One *might* be further encouraged if he reads 'bedeuten' in Frege's mouth as roughly 'denote', or 'refer', and then consults his own intuitions about denoting to see what, if anything, one might say a whole thought to name. But the problems thus raised are needless, or idle. To see what *Frege's* notion of *Bedeutung* is, we ought to proceed the other way around. For *this* notion, it is a given that the *Bedeutung* of a whole thought is a truth-value. What, then, ought the *Bedeutungen* of other elements to be?

To begin with, *why* does truth-value need to be excised from *judgeable content*? A simple answer: there are no two thoughts which are distinguished from each other precisely in the one being true, the other false. There are not, e.g., two thoughts, the true one that Sid smokes and the false one that he does. Being true, or false, is thus not part of precisely that by which truth can come into question at all.

A more complicated answer: having the truth-value it does *cannot* be part of what identifies a thought (or its question of truth) as the one it is. Truth cannot belong to what brings itself in question. Otherwise it would not be fixed what thought was at issue until it was fixed whether what was in question was a *true* thought or a false one. In a thought, something is represented *as* being something: the way things are as being some way there *is* for things to be (e.g., such that Sid smokes). Truth now enters the picture with the question whether things *are* as thus represented. A thought is identified by what is thus variable: the second factor. Where truth belonged to bringing itself into question, *how* a thought represented things as being would not be fixed until it was fixed whether such was representing truly. But *such* reduces the behaviour of a thought, and of representing-as, to the behaviour of something else entirely: *factive* meaning, or (where falsity plays the identifying role) *factive counter-meaning*. Factive meaning leaves no room for the question whether things are as represented: if not, then they simply were not so represented. Representing-as, so truth, would thus be abolished.

Still, truth matters. "Laws of logic", Frege tells us, "are in the first place laws in the realm of *Bedeutung*, and only mediately pertain to *Sinn*." (1892-1895: 133) At the least, logic's central concern is truth-preservation. But there is talk of truth *being* preserved or not only

where whatever it is that would fix a truth-value can be supposed to have done its work. For a thought to fix what it is meant to, some determinate question of truth, is for it to make truth turn in some determinate way on how things are (normally, *perhaps* not necessarily, a way which makes how things are *matter*. To assign a thought a truth-value is to suppose the way things are to have delivered an answer to the question thus posed; for truth *to have* turned in that way, how things are then delivering a given outcome. So, we might say, for a given thought to have a given truth-value is for the way things are to have delivered *that* answer to the question posed. So, substituting 'the world' for 'how things are', for the truth-value to be the *Bedeutung* of the thought is for it to be the world's answer to the question posed.

One *might* thus think of the *Bedeutung* of a thought as being 'The True', or 'The False' in the same way one thinks of 'Yes' as being the answer to a yes-no question. As Frege points out, the content of such a 'Yes' is really the content of the question: both express, or contain, the same thought. We *could* thus think of the *Bedeutung* of the thought that Sid smokes as the answer to that question being 'True'.

From this starting point we now approach the question what the *Bedeutungen* of thought-elements would be. The obvious answer: a thought-element makes the thought's truth turn *in part* in such-and-such way on how things are; the *Bedeutung* of such an element should then be the answer to the thought's question of truth, insofar as this is settled by that element (so by that partial way of turning) and things being as they are. Suppose, e.g., that a predicative element in the thought that Sid smokes makes truth turn on whether relevant things smoke. Speaking loosely, objects divide into those which smoke and those which do not. *How* the objects there are thus divide depends, of course, on how things are. It is part of the world's answer to a question which makes truth turn in the way just mentioned on how things are. This partial answer can be registered as follows. Consider that function which (as it happens) maps an object onto The True just in case that object is a smoker; onto The False otherwise. Then the answer to the thought's question is: the truth-value of the whole is the values of that function for relevant arguments. The relevant *Bedeutung*-element would thus be: (the answer) being those values of that function. Similarly, a naming element in that thought which made truth turn on how *Sid* is would have as its *Bedeutung* (the answer) being the value of the relevant function (as determined by other elements) for the argument Sid.

We thus arrive at a notion, *Bedeutung*-element. For each thought-element (on a decomposition) there is a corresponding *Bedeutung*-element. The sort of element this is is determined by the sort of thought-element to which it corresponds. A whole *Bedeutung* (a truth-value, considered as answer) decomposes into *Bedeutung*-elements. For each decomposition of the whole thought, it does this in a corresponding way. So, as with thoughts, the basic notion is decomposing *on a decomposition*.

Our general principle about decompositions (of white-washings, of pecan pies, of whatever)—that principle which lost us the problem of forming 'unities' out of elements—now applies: one has a decomposition of a whole *Bedeutung* just where the *Bedeutung*-elements are, jointly, the whole answer to the question posed. Because of this, a function from objects to truth-values is ineligible for being a *Bedeutung*-element of (or fixed by) a predicative thought-element (and the world). Such functions, and some object they map into a truth value, simply form a list. The problem of unity arises.

We can, though, introduce a new notion; that of what I will call an *ingredient* (of an element). An ingredient of a thought-element is to be that on which it makes truth turn, and

by which it is distinguished from other elements of its type as the particular element of that type which it is. For example, a predicative element makes truth turn on which things are some particular way for a thing to be. It is identified as the *predicative* element it is by the way in question for a thing to be—e.g., that that way is, being such as to smoke. That way for things to be is thus an ingredient in that element. Similarly, a naming element makes truth turn, say, on how *Sid* is, thus distinguishing itself from other naming elements (though, if there are many ways of making truth turn on how *Sid* is, then not from *all* such). *Sid* might thus count as an ingredient of that naming element. Similarly, a predicative *Bedeutung*-element makes it a particular function from objects to truth-values which is the one whose value for relevant arguments is the answer to the whole thought's question. So such a function might be counted as an ingredient of this *Bedeutung*-element. (Here, too, different predicative elements might each fix a *Bedeutung*-element with the same ingredient.)

This notion of *ingredient* does not fix a *unique* ingredient for every element. Choices between candidates, were such arise, are to be made as is then convenient. The immediate point is just to point to something it might mean to call a function from objects to truth-values the *Bedeutung* of a predicative element. *Bedeutung*-elements need not be what Frege meant by *Bedeutungen*. Nor, given the choices now on the table, need Frege have been perfectly clear as to what he did mean by 'Bedeutung'. The primary object of the present exercise is to identify various different things for which Frege's obstacle might, or might not, arise. We are now halfway.

1.3. Concepts (First Notion): What Frege's obstacle concerns, he tells us, is 'concepts', or talk thereof. What, though, might a concept be? Two notions of a concept emerge from the roles to which Frege puts the term in identifying those structures to which logic must be sensitive. This section develops one of these. The need to be served is this: there must be a way for a thought to make truth turn on how relevant objects are other than by specifying *which* objects these are to be. Frege illustrates this need, and its being served, as follows:

One must not think that I say something about an inner-African chieftain, entirely unknown to me, if I say, "All men are mortal." I speak neither of this one, or of that one; but I subordinate the concept *man* to the concept *mortal*. ... Nor must one think that the *Sinn* of the sentence 'Cato is mortal' is contained in the sentence 'All men are mortal', so that should I utter [this last] I will have, in the same stroke, expressed the thought content of [this first]. The matter is much more as follows. In the sentence 'All men are mortal', I say, 'If something is a man, it is mortal.' By an inferential move from the general to the particular, I get from this the sentence 'If Cato is a man, then Cato is mortal.' Now I still need a second premise, namely, 'Cato is a man.' From these two premises I conclude, 'Cato is mortal.' (1914: 231)

Suppose that 'All men', or 'men' in it, functioned as a name (albeit, perhaps, a highly polysemous one): it specified on the mortality or not of which objects the truth of the thought expressed depended. Then it would name, *inter alia*, Cato. That is, it would make truth turn, *inter alia*, on how Cato was, specifically, whether mortal. The truth of the whole

thought would thus require Cato to be. Suppose, then, that we assume (or assert) its truth. It follows immediately from our assumption that Cato is mortal. So the inference from 'All men are mortal' to 'Cato is' should be immediate. It should depend on no further premiss. No more is required than the truth of our assumption for that Cato is mortal to be true too. Such is obviously not the case. For it to follow that Cato is mortal, something else must be so: that Cato is a man. Such demonstrates the need for a different way for a thought's truth to depend on who is mortal.

Frege also sketches what this other way would have to be. It would have to be a way of involving all objects meeting a certain condition. A concept, on our first notion, is what plays a certain role in achieving this. The crucial things about a concept so conceived is expressed by Frege thus:

What I see as the essential thing for a concept is that the question whether something falls under it has a sense. (1882: 118)

As one might think of a concept, the core of the notion is a way for a thing to be. (Such is the simplest case. The core generalises to a way for an n -tuple of objects to be, $n \geq 1$. It also generalises to a way for things (read catholically): a way which, indifferently, any n -tuple of objects is as much as any other (e.g., such that Sid smokes). One might think of this as the case $n = 0$.) For any way for a thing to be, let there be the concept of a thing so being. For an object to fall under such a concept can just be for it to be the relevant way. Such gives a sense to the question whether an object falls under it, asked of a concept in present sense.

Such a concept figures as follows in the envisioned non-naming way of making truth turn on how relevant objects are. It (or, equally, that which it is of) may be put to work as an ingredient in a predicative thought element. But, in the context of the whole thought that predicative element would be working in a special way: not such that for things to be as represented would be for the right things to be that way; but rather such that *whatever* was that way was also relevant other ways (ingredients in *other* predicative elements playing different roles in the whole thought so decomposed).

This notion of a concept allows concepts a certain object-independence. What fell under a concept could be other than what does. No given object must fall under a concept for the concept to exist, or be the one it is. *Nothing* need fall under a concept for there to be one. If just one thing does so, the concept might still be object-independent in all the above ways. Not all concepts have all these features. There is the concept of being that very philosopher, Frege. But for Frege's existence there would have been no such concept. Since there is, nothing other than what falls under it could have done so no matter what. Similarly, being a whale is a way for a thing to be. But it might not have had there been no whales. What is built into this notion of a concept is just that such forms of object-independence are provided for.

A concept₁ may be called true *of* what falls under it, but not true outright. Nor does it represent what falls under it *as* that which it is a concept of. The concept₁ of being a smoker is what *requires* something for Sid to fall under it. It is indifferent to whether he meets that requirement. It is subject to no standard of correctness, or accuracy, which it may meet or fail to according to whether Sid smokes. It is in no business for which there might be such a standard of success—no business such as representing-as would be. Its indifference to whether Sid falls under it is part of its indifference to whether Sid exists.

This is to say that a concept₁ is not a predicative thought element. It does not *make* truth turn, even in part, on anything. It might be counted as an ingredient of such an element; that which the element puts to work as what relevant things need to be (or fall under). Or, again, one might rather count that which the concept is a concept of (in the unary case, a way for a thing to be) as the ingredient. Such is a matter of indifference just where one does not distinguish—as Frege sometimes does not—between a thought of something *being* thus and so and one of its falling under the concept of things so being. In any case, being an ingredient does not make something *Sinn*. Sid can be counted as an ingredient of a naming thought-element. *He* is put to work there as what truth turns on in the relevant way (what must be relevant ways). But Sid is certainly not a *Sinn*. One *might* equally see the ingredient in the naming element as a concept of a special kind: a concept of being that very thing, Sid. Such a concept, of course, requires Sid for its existence, and allows nothing but Sid to fall under it. Whether such a concept or Sid is an ingredient may cease to be a matter of indifference, notably, where one is concerned with the possibility of many concepts with the above two mentioned features—where one recognises many ways of making Sid the one on whom truth turns. If neither objects nor concepts₁ are *Sinne*, nor are they *Bedeutung*-elements, though objects, at least, may be ingredients thereof. Whether this would make an object a *Bedeutung* rests on a decision as to how to use that term. (Frege's *usage* suggests the answer yes; though whether it requires this depends on how that usage is to be explained.)

1.4. Concepts (Second Notion): Our second conception of a concept (a concept₂) also derives from Frege's perception of the needs of logic (or, this time, of a logical notation). To introduce the notion I will begin, as Frege does, with language. Frege suggests (e.g., (1892-1895: 130), (1897: 154)) that for logic's purposes the grammatical distinction of subject and predicate should be eschewed, in favour of a distinction parallel to that in mathematics between function and argument. There is a *grammatical* notion of subject and predicate; and there is a *logical* notion of naming and predication. The logical notion of predication is already in play here in the idea of a predicative thought-element: predication (in the unary case) is making truth turn on a way for a thing to be. The idea is: these two notions come apart.

In the simplest cases—words such as 'Sid smokes'—grammar and logic coincide. Here ' _ smokes' is the grammatical predicate: it forms a sentence with what is grammatically a name (or, more properly, a noun phrase); and in the expression of a thought it is responsible for identifying a predicative thought-element in the thought thus presented as decomposed—a thought element which makes truth turn on whether the right people *smoke*.

The logical notion, predicative element, though, generalises from this simple case to something which corresponds to *nothing* a grammar of a language need recognise. The linguistic correlate of this generalised notion is an open sentence. An open sentence is what is derived from a complete one by erasing one or more noun phrases (grammatically naming elements) from it, leaving blank spaces behind. In an expression of a thought, an open sentence with *n* blanks would identify an *n*-ary predicative thought-element (to be found in the thought expressed suitably decomposed). This would make truth turn on whether relevant *n*-tuples were a given way for an *n*-tuple to be. Where the whole thought-expressing sentence results from the open one by filling each blank with a (grammatical) name, the

relevant n-tuple for truth of the whole would be formed of the objects named in those fillings.

Our second notion of a concept enters the picture here as the answer to the question what an open sentence denotes. The idea will be: a concept₂ is what an open sentence, in the context of a whole one, in the context of its use to express a thought denotes. But this answer is meant to derive from a more general answer to the question what an open sentence denotes as such. The answer to this further question is: a function (where a function is what relates one domain of objects, its arguments, to another, its values, such that for each argument there is exactly one value). In 1904, Frege tells the following story about this (for double-open sentences). Consider the open sentence, '___ = sin ___'. We can understand this sentence as identifying, or denoting, a certain function (here, the sine function). So to understand it is to understand it as telling us, for any filling of the last blank (by a number) what is to go in the left blank: understanding this last filling as presenting an argument of the sine function, what value to put into the first blank. So, for example, 0 on the right would call for 1 on the left, $\pi/2$ would call for 0. The fillings, '0', '1', etc., denote numbers. Similarly, the open sentence should, here, denote a function. To find that function ask what remains constant as the fillings vary. What remains constant is what so relates all the fillings: just that mapping which maps 0 onto 1, $\pi/2$ onto 0 and so on. Such, then, is what the open sentence, so understood, denotes.

Suppose, now, that we consider the open sentence, '___ smokes'. If we fill the blank with 'Sid' we get a truth. If we fill it with 'Di' we get a falsehood, and so on. Frege's notion of *Bedeutung* begins from the idea that the *Bedeutung* of a whole thought is a truth-value. We can, then, understand '___ smokes' (at least in the context of the expression of a thought) as denoting a function. For a given filling of the blank as that function's argument, its value would be the *Bedeutung* of the whole thus formed: a truth-value. The *function* would be what remained constant as <filling, whole-*Bedeutung*> pairs remained constant. On the model of '___ = sin ___', this constant would be that mapping which mapped just those arguments into just those values.

There are the obvious differences between the two cases. Where '___ sin ___' is understood as denoting the sine function, the value of the function denoted is to be understood to be the filling of the first blank. If we view '___ smokes' as denoting a function, there is no place in the open sentence for a value. What would denote the relevant function on the understanding on which '___ sin ___' denotes the sine function, one would need an expression like '___ = ___ smokes' (in another style, 'y = smokes (x)'). Conversely, '___ = sin ___', blanks filled, may also be understood as the expression of a thought, as in the expression of the false thought, '17 = sin 2'. The function thus denoted would be one from pairs of numbers to truth values, with no place in the open sentence for the value this function takes. To denote *this* function in the style of '___ = sin ___', one would need something like 'z = sin(x,y)'. Perhaps the best approach here is to suppose that there are two relevant understandings of an open sentence, on each of which it denotes a function (though different ones on each). There is an understanding on which the value of the function is supposed to be mentioned in some blank in the open sentence, and an understanding where it is to be understood as the denotation (or *Bedeutung*) of a filled whole. Typically (but not necessarily) it is on the second understanding that an open sentence denotes a concept₂. For a concept₂ just is to be a mapping from (n-tuples of) objects to truth-values.

A concept₂ is a mapping. Though Frege tells us that we cannot really speak of being

identical with some given concept, if we keep that in mind, we can say this:

What two concept-words *bedeuten* is the same if, and only if, the concept-extensions belonging to them coincide. (1892-1895: 133)

So if we *were* to speak of concepts₂ as identical to, or distinct from, one another—if we were permitted to engage in such talk—we would know exactly how to go about it. A mapping is what a mapping does. For each mapping, there is what it maps into what. Whatever maps that into that is that mapping; whatever does not is not. concepts₂ are mappings. Thus, so it is with concepts₂.

One might argue as to whether the sine function ‘ought to be’ a mapping in above sense, or whether it is, rather, that ‘law’ which determines what *is to be* mapped into what. One might similarly argue as to whether a concept ‘ought to be’ a mapping in this sense, or, again, ought to be that which fixes which mapping is to bear some given title. The present position is that ‘concept’ lacks determinate enough content to give such questions an answer outright. They are not sensible questions until it has been decided what role a concept is to be assigned. As it happens, concept₂, conceived as mappings, are perfectly suited to serve as ingredients in predicative *Bedeutung*-elements. A mapping from objects to truth-values which takes on the value *true* precisely for the smokers registers *precisely* the world’s answer to the question whether Sid smokes—what in fact happens when truth so turns—insofar as what that answer is is determined by the fact that the thought in question is as to some given thing being *this* way: such as to smoke. It’s answer to that whole question is precisely the value into which *this* mapping maps the argument Sid.

Concepts₂ are intrinsically object-dependent in precisely the way in which concepts₁ are not. For a concept₂ (a mapping) to exist, and for it to be the one it is, is for it to map just *these* arguments into *these* values. Its existence depends on there being just *these* pairs of objects. Whereas a concept₁ may exist, be what it is, entirely independent of what objects (if any) fall under it. (Some concepts₁ do; some do not.) We *might* think of a concept₁ as denoting a concept₂: where a concept₁ is of some given way for a thing to be, it denotes that concept₂ which maps just those things into *true* which are that way. But such denotation would not be rigid. A bit less irony in the parent-child communication mix and Sid might have grown up non-smoker. The concept₁ *being a smoker* (the same for all that) would then have denoted a different concept₂. Object-independence—the very *point* of concepts₁—would defeat the whole point of concepts₂. These would no longer register the world’s answers to those questions thoughts pose.

In 1882 Frege speaks of a concept as what cannot have an independent existence, but “arises only through the decomposition of judgeable contents”. (1882: 118). There, and from thenceforth he also speaks of concepts as *unsaturated*:

A concept is unsaturated, in that it requires something of what falls under it; thus it cannot exist on its own. (1882: 118)

The first remark suggests that what the existence of a concept is dependent on is (in post-1891 terms) whole thoughts. The second suggests that its existence is somehow or other dependent on objects.

The first idea might be developed on these lines. For a concept to exist is for it to play a certain role (so for there to be precisely that role to be played) in whole thoughts; thus, for there to be a certain contribution to be made to making truth turn on how things are as given thoughts make truth thus turn. For there to be such a thing as a concept of being a smoker, there must be such a thing as making truth turn on who smokes—a thing which can only be done in the context of making truth *turn*, full stop, in some way or other on how things are. So there is no concept of being a smoker unless there are whole thoughts whose truth turns in one way or another on who smokes. One might also put this: for there to be a concept of being a smoker, there must be such a thing as its being true, or false, of things. But this presupposes there being such a thing as truth outright. Thus, whole *thoughts* first.

As for the second idea, what is intrinsic to a concept, for Frege, is precisely *not* that things must fall under it, but certainly that it requires things of what does. One might understand this as it requiring things which either fall under it or do not. For a concept₁ this idea might be developed on these lines. It is intrinsic to a way for a thing to be (so to a concept₁ thereof) to be general: being that way is something which, if possible at all, would be shared in common in indefinitely many cases of an object's being as it was. Equally for *not* being that way. But there cannot be something in common to ranges of cases unless there are cases for it to be common to—here, at the least, present in *or* absent from. In the case of a unary concept, such would be cases of an *object's* being as it is. So the existence of a concept₁ would be dependent on the existence of objects in this way: for there to be such a thing as the concept₁ of being a smoker, there must be objects which, in being as they are, either exemplify a thing's so being, or exemplify a thing's not. (Of course, in this sense of dependence, there is no reason to expect the existence of an object not to be equally dependent on the existence of concepts₁. The thing about an object is that it is *one* item which, inevitably, will fall under countless concepts₁. Take away the concepts, and there is nothing left of the objects either.)

In the event, later on (e.g., 1904) Frege inclines towards introducing the notion of unsaturation in the first instance as a notion applying to open sentences. Derivatively, it is also to be a feature of what open sentences denote. Which means, first, that it is a notion which applies to functions, and equally to *all* functions, not just ones which are concepts₂, but second, in what ever way it applies to functions, it *eo ipso* applies in just that way to concepts₂, which, among concepts, are the most immediate candidates for its application. An open sentence differs from the noun phrases eligible to fill its blanks, first, in that it need not belong to any grammatically significant category at all—it need not be any (grammatically) proper part of a sentence; second, in that, while, grammatically speaking, its blanks may be filled by noun phrases (or names), *it* is not eligible to fill the blanks in anything (construing filling blanks as restoring what is incomplete as an open sentence is to complete status); third, that, through its blank spaces, it determines just where completion would be needed to restore wholeness.

These three features identify a notion of *grammatical* unsaturation. An open sentence can *predicate* (express a predicative thought-element) only where occurring as completed: there *is* a predicative thought-element only where a whole thought has been expressed. Equally, and for just the same reason, a noun phrase can name (express a naming thought-element) only where a whole thought has been expressed. But a noun phrase (a potential name) contains nothing in it to fix just how it may be completed (grammatically). It might be completed by any open sentence, whether belonging to a grammatical category or not, and, if

this is n times open, then by it together with from 0 to n-1 other names, distributed in whatever pattern into its blank spaces. (This brackets issues of categorial restrictions on predication, e.g., predicating being blue of the score of the United-Sporting game.) As one might put this, an open sentence is, as a noun phrase is not, quite specific about its *Ergänzungsbedürftigkeit*.

Frege extends the notion of unsaturation to denotata of unsaturated expressions:

That peculiarity of the function-sign which we have called unsaturation naturally corresponds to something in functions themselves. This too we can call unsaturation, and thus distinguish them as fundamentally different from numbers. (1904: 665)

But he is at best frugal with his explanation of how the extension goes. If the denotata are just functions, one *might* see unsaturation in their very object-dependence. A function (mapping) is what remains constant (in denotation) across all variation in what fills the blank spaces (on that understanding of an open sentence, above, on which it identifies a function). It is that very same thing which, in each case, maps that argument into that value; the same thing for each pair of argument and value. Of course, there is no such same thing without the arguments and values which get mapped. In that sense what Frege said of concepts in 1882 is so of functions in general: they cannot exist independent of objects. Concepts₂ are unsaturated in this sense.

II

2.1. Predicative Nature: Part I developed a set of different things there may be an obstacle to speaking of. Each of these develops a different strand in Frege's thought. It was claimed of none that it is what Frege meant by the term *such-and-such*. Nor is it claimed that our two notions of *concept* are necessarily concepts of different things. For all it matters here, Frege may just not have seen these different things to choose between. In any event, insofar as Frege tries to explain why there should be an obstacle, he offers two suggestions. One is: that for which there is an obstacle has something called 'predicative nature'; predicating things of it as though it were an object is inconsistent with this nature. The other is: the obstacle arises for what it does because this is unsaturated. These diagnoses are clearly different. All functions are unsaturated. So, if this is the source of the obstacle, then it arises for functions in general, not just ones from objects to truth-values. There are various things 'predicative nature' might mean. But in any case it seems clear that this is something not all functions have. There is, it seems, nothing intrinsically predicative about the sine function. In fact, I will suggest, neither diagnosis is very promising. This section takes up the first.

Having predicative nature needs different understandings for our different candidates for the obstacle's object. A predicative thought-element has such a nature in that for it to be is for it to be making truth turn (somehow) on what is, what not, some given way for things to be (in some particular decomposition of some given thought); for its contribution to representing things (catholically read) as some given way to be representing the relevant things (plural of 'thing') as some given way—to *predicate* such-and-such of these things. A

predicative *Bedeutung*-element is predicative in that it registers the world's answer to a given thought's question of truth insofar as this question is fixed by the predicative thought-element to which it corresponds. A concept₁ has a predicative nature in that it is something fit for service as an ingredient in predicative thought-elements—it provides something for such elements to make truth to turn on. A concept₂ has predicative nature in that it registers the outcome of truth turning as some predicative thought-element makes it turn on how things are.

So, it seems, in the having of predicative nature there is something fundamental about predicative thought-elements. Let us start with them. To be a predicative element is, *per se*, to be doing a certain sort of work: to make the truth of some thought, on some decomposition of it, turn in part on which things are some given way for things to be. For any given such element, there is no such thing as being that element without doing that work, or being that element while doing other work for a thought-element to do. A predicative element could not, e.g., take the place of a naming element in any decomposition of any thought. There is just no such thing as that.

This is so on two understandings of being a predicative element. It is trivially so if a thought-element just is an element of a particular decomposition of a particular thought. Then any element in any other decomposition, or any decomposition of any thought, is not it. Nor is any other element in the decomposition it belongs to. But instead of understanding an element as the *doing* of such-and-such in a particular thought's making truth to turn as that thought makes it do, we can also think of an element as something there is to be done in *a* thought in its making truth turn as it does. To decompose a thought into, *inter alia*, a given predicative element is to bring it under a certain generality; to represent its way of making truth turn on things as a way of a certain kind; if there is such a kind, then there is, *per se*, a range of thoughts, each of which, in its own way, is of that kind. A predicative element, on this second understanding, is what is in common to all of these. On this understanding, too, to be a given element is to be the doing of given work (here understood just as something to be done). For no other thought element can the doing of this work be the doing of that other's. It thus remains so that no predicative element can replace ('vertreten') a naming element in any thought whatever.

Such corresponds to something Frege says about the fundamental difference he sees between 'concepts' and 'objects'—that difference which makes for the obstacle: that objects and concepts cannot take each other's place. If this were to mean that predicative and naming thought-elements cannot take each other's place, then indeed not. This, though, is irrelevant to the question whether there could be a thought *about* a predicative thought-element in which that element played the role of an object—one of being, or not, various ways there are for an object to be. For if there were such a thought about a predicative thought-element, that element need be *no* element in that thought. Not a predicative element, since the point is not to predicate of *that element* what it predicates of something. Nor a naming-element. A naming element in such a thought would make truth turn on how that predicative thought-element was. Of course that predicative element, in the nature of the case, would not make truth turn on *itself* as truth might turn on some object.

The question, though, is rather whether there could be a naming element in *some* thought which made truth turn on which ways that predicative element was—otherwise put, whether a predicative thought-element in one thought might be an *ingredient* in a naming thought-element in another. The core fact here is that the doing of one thing cannot be the

doing of another, so that no predicative thought-element is any naming thought-element, nor vice-versa. *Such* does not answer this question. No reason is yet in evidence why truth cannot be made to turn on what is so of, e.g., making truth turn on who smokes. That a predicative thought-element can never be exchanged for an element of another sort (in any decomposition of anything) is not, so far as it goes, a route to, or source of, the obstacle.

Such is not the end of the story. For Frege suggests another reason why no concept could ever be an ingredient of a naming element. It is this:

Objects and concepts are fundamentally different, and cannot take each other's place. ... Concepts cannot stand in the same relations as objects. To think them so to stand would be, not false, but impossible. (1892-1895: 130)

The suggestion is: one simply *cannot* predicate the same thing of a concept and an object. Objects form a category. There is what is essential for membership: that the question whether something is that very thing (is identical with it) makes sense. There is then what is predicable of what satisfies this condition. What is predicable of this is predicable of *just* this. Concepts form a category. There is, again, what is essential for membership: that the question what falls under it (a member) makes sense. There is what is predicable of what falls in this category. Again, what is predicable of this is predicable of *only* this.

Suppose, now, that what Frege suggests about concepts here were so of predicative thought-elements. Now suppose that A is an ingredient of some naming-element in a decomposition of some thought. It is thus treatable as an object. So what is predicable of it is what is predicable of objects. So the question what is identical with it makes sense. But, on the present suggestion about predicative thought-elements, the question, for any thought-element whether A is identical with it makes no sense. For predicative thought-elements, on present assumption, belong to a different category. So there is nothing it would be, for any predicative thought-element, and for any ingredient in a naming thought-element, for these to be one. In which case there would be an obstacle of the sort Frege sees to treating concepts as objects.

Similarly for our other obstacle candidates. Many objects are unfitted for service as an ingredient in a predicative thought-element. My friend Guy is. So is the apple in his hand. So are all his kin. So is the smoke point of peanut oil, or the divorce rate in Brazil. Guy is not a way for truth to turn on how an object is. Nor, equally, an apple. Nor that smoke point. *Being* Guy, or *being* an apple, is another story. These are (as Guy himself is not) ways for a thing to be. Truth may then turn on which things *are* those ways. So there is what is fit for predicative service and what is not. Where there is what *is* so fit, there is also the concept₁ of it. One may ask of these whether anything thus fit for predicative service, or any concept₁ thereof might also be fit for service as an ingredient in a naming thought-element. That ever so many things fit for such 'naming' service are unfit for predicative service simply fails to answer the question. What *might* force a negative answer on us? Frege's idea would if what he says about concepts applied to ways for things to be, or to concepts₁. It is hard to see what else might.

Similarly again for concepts₂. Can it really be that a mapping from objects to truth-values could not be an ingredient in a naming element in any thought whatever? What, e.g., of the thought expressed in, 'The mapping which maps Pia and Sid into *true*, all other objects

into *false*, is the denotation of any concept₁ under which precisely and only the present author's favourite couple fall? It is hard to see what might force a negative answer on us here unless, of course, what Frege says about concepts above holds of mappings from objects to truth values. In which case the above argument repeats itself for concepts.

All this gives us so far, though, is: if what Frege says about concepts and objects were true of any of our candidates, then we would face the obstacle he sees to speaking of that candidate. But *is* what Frege says right of any of our candidates, or anything else with 'predicative nature'? Nothing in what predicative nature is, or might be, seems to show this. Though our search for what would show it is not yet over, we might now note two counter-indications.

The first is this. If Frege is right, then concepts (whatever these are) *cannot* serve as ingredients in naming elements. Suppose we overlooked this and tried to speak so as to put them in such service. What would happen? How would our attempt founder? The crucial fact is: we would not be able to say of anything that it was identical with any concept. Such would, indeed, lead to grief. But it is not as if the trouble would be that, if we could make such statements, we would not know when to count what was said in them as true. Suppose, e.g., that concepts are concepts₂. We know exactly when to say (if only it could be said) when anything is a given concept₂. It is so when it maps just those things the concept does into just those truth-values it does. As for concepts₁, we are in no more difficulty about when to say something is a given concept₁ than we are as to when to say something is a given way for a thing to be; and there is no more difficulty as to that than there is as to ever identifying what it is that was said where something was. *Mutatis mutandis* for predicative thought-elements, and for predicative *Bedeutung*-elements.

If we follow later Wittgenstein, then what has just been said *demonstrates* that Frege was wrong about the obstacle. For, on his view, there is nothing it might be for a would-be identity statement to fail to be truth-evaluable, so to fail to make sense, except for us to find it "to stupid, or complicated, or something of the sort" so to regard it. (*Investigations* §136) What we are *able* to treat a truth-evaluable, the idea is, is so.

For Wittgenstein, the story ends there. But not for us. There is, though, a second point to keep in mind. If the distinction between objects and concepts is categorial in the way described, then, though one cannot predicate of a concept what one can predicate of an object, all the same there *is* what one can predicate of a concept. But there is no predicating anything of anything unless the subject of the predication participates in a relation which satisfies something of the form of Leibniz' law, applied to things of the category to which this subject belongs. The point about objects and identity is that there is no making sense of an object's having any property without supposing it to be re-encounterable: the very thing which is F also being that thing which is G, fails to be H, and so on; (as Frege insists) its being F (if so it is) being subject to joint investigation, and the object of stances by, different thinkers. Such a demand would apply equally, and for the same reasons, to concepts if, like objects, they have properties, whether or not of categorially different types. One *could* refuse to call the requisite 'Leibniz' law' relation between concepts 'identity'. Call it 'shmidentity' if you like, and insist that this is *not* identity. There is surely an air of whimsy in all this. Such, though, does not end the story.

2.2. Unsaturation: Unsaturation is a property of *all* functions, not just those special ones,

from objects to truth-values, which Frege (post-1891) calls concepts. The question is whether some unsaturated things might sometimes serve as objects. 'Serve as', here, means serve as ingredients in naming elements—ones which make truth turn, somehow, on how such-and-such object is. It is assumed here that nothing can *ever* serve as an object unless questions whether such-and-such is *identical with* (that very thing,) it make sense, further (often enough) have answers. As we have seen, if *this* were the only hurdle to be passed, then nothing would bar functions from such service. *If*, however, unsaturation bars something from such service, then *no* function can so serve. So, e.g., if a mathematician were to say, 'Let F be that function which maps each integer from 1-500 into its double, each integer from 501-1000 into its quadruple, all other integers into themselves', he would not actually have specified a *function*—a bit of an embarrassment.

It is perhaps for this reason that Frege *sometimes* modifies his stand on just *what* obstacle is posed to so speaking of a concept. In 1914, for example, when speaking of the role of logic in mathematics, he says,

We say, 'The function', and 'the concept', expressions which are hardly avoidable, and yet are unfit for purpose. The definite article gives these expressions the form of a proper name in the logical sense, as though they designated objects; precisely what they are not supposed to be doing here. The very essence of concepts, and of functions, unsaturation, is thereby concealed. Language presses us into an unsuitable expression. It is difficult to avoid this evil; but one can render it harmless by remaining conscious of this unsuitability. In which case one would also not confuse the value of a function with the function.(1914: 257)

Here, thus, it is not that the above mathematician cannot have *spoken* of a concept; it is just that one must keep in mind that such *is* what he has done.

In any case, being a function, hence being unsaturated, is something very different from having predicative nature. Correspondingly, for it to be unsaturation which generated Frege's obstacle (if such were so) would be a very different matter than for it to be predicative nature which did so. Functions do not, in general, have any predicative nature. The sine function, for example, has none. One can no more predicate the sine function of something than one can predicate Sid of it. Of course, as Frege notes, one can predicate of something being that very object, Sid. Similarly, so one would have thought—barring any unexpected obstacle—one can predicate being that very function, the Sine function, of something. Moreover, one can predicate being the *value* of the sine function for such-and-such argument of something. Starting with the sine function—a function from reals to reals—one can, in a way described above, construct a function from objects to truth-values. Such a function might, e.g., map pairs of numbers onto the true just in case the second was the sine of the first. Similarly, starting with Sid one can construct functions—one which maps an object into the true just in case that object is Sid, one which maps an object into the true just in case it is an ancestor of Sid's, and so on. That there are such constructs confers predicative nature neither on Sid nor on the sine function. So far, there is no more reason to think one cannot treat the sine function as an object—that it is unfit for duty as an ingredient in a naming

element—than there is to think one cannot treat Sid as an object.

Nor is it true that a concept cannot be put to work other than as ingredient in a predicative thought-element. As Frege also notes, a concept, while for all that a *concept*, can be, even necessarily, one under which exactly one object, such-and-such, falls—a concept of being that very object, such-and-such. Now, as noted already, an object cannot itself be a thought-element. A naming thought-element does a piece of work which an object such as Sid, or the boiling point of water, or Venus—or, one would have thought, the sine function—cannot do. The work is *making* truth (the truth of some representations) turn, in part, on how such-and-such object is. Such is work which a concept (of the special sort just mentioned) is eminently suited to aid in. A naming thought-element can be thought of as making truth turn, in part, on how that thing which (alone) falls under such-and-such singular concept is. (Truth would then turn on how what falls under it is; this last remaining the *ingredient* in present sense.) It is, in fact, pointful to think in this way about naming elements if, or insofar as, one thinks there can be different ways for a naming element to bind a thought, in the above way, to just one individual—different ways, e.g., for a *thought* to make truth turn on how Venus is. Of course, it is contentious whether this is so—a discussion lying without the present discussion. The point, though, is that there is nothing about a concept, in either of our two senses, which makes its involvement in a thought *intrinsically* involvement in predicative work.

It is hard to see what there is about the sine function which makes it intrinsically unfit for work as an ingredient in a naming element. On the other hand, there is something distinctive about those special sorts of functions, ones from objects to truth-values. When we decompose a whole thought-expression, not into its grammatical parts, but rather into an open sentence with fillings (of one sort or another) for its blanks, if we then think of that open sentence as in some sense designating, or speaking of, a function from objects (or n-tuples thereof) to truth-values, then there is no blank space in it whose function is to be filled with *values* of the function for fillings of the other blanks as arguments. Thus, the open sentence, ‘__ smokes’, as it occurs in ‘Sid smokes’, contrasts with the open sentence, ‘__ = sin __’, construed, as Frege sometimes does, as simply denoting the sine function. Perhaps the special way in which ‘__ smokes’, as above, refers to a function corresponds to its expressing what has a predicative nature: the (canonical) contribution of such an expression to the thought expressed is to make this a thought with a certain predicative thought-element (when decomposed in the way this expression of it suggests). Similarly, if we begin with a whole expression of a thought such as ‘17 = sin 3’, and abstract to the open sentence, ‘__ = sin __’ as there occurring, we can view this as denoting a function from pairs of numbers to truth-values—here according as the first number is, or is not, the sine of the second. The open sentence so construed would then express, in the above sense, a (binary) predicative thought-element, predicating being the sine of of a pair of numbers, as above.

What lesson is there here? If ‘__ = sin __’ is a way of designating the sine function—if such is an understanding it would sometimes bear—then, by parallel, ‘__ = __ smokes’, understood in the same way, is a way of designating that function which maps an object into the value true just in case that object smokes. Similarly, where ‘__ = sin __’ is abstracted from its role in the sentence ‘17 = sin 3’, thus understood as designating a function without providing any place for its values, but only places for its arguments, then that same function can also be designated, leaving a place for its values, by ‘__ = (__ = sin __)’. What this shows is that, even by Frege’s own lights, there is not just one (canonical) way of denoting a given

function. Frege sometimes seems to suppose that if an unsaturated expression of a certain sort denotes a function where predicative work is done, then it is only an expression of *that* sort which can ever denote that function. But, as we have seen, taking Frege at his word where he discusses what a function is, such is simply not so. A function denoted in one way in engaging in predicative work (that is, a function from objects to truth-values, denoted by a predicative part of some expression of a thought) may also be denoted in a different way where it is only a question of identifying *what* function it is. If, as appears so far, the sine function can be put to an object's work in some naming element—say, in an expression of the thought that the sine function is cyclical, with period π —then, so, too, for functions from truth-values, at least on some ways of denoting *them*. And such will be so unless it is unsaturation *per se*, and not just predicative nature *per se*—which generates Frege's obstacle.

Once again, then, we end at a now-familiar place. *If* functions and objects (that is, things fit for object-duty as ingredients in naming elements) are fundamentally different—so much so that they *cannot* stand in the same relations, *if* such is just intrinsic to being a function, then, of course, there is no such thing as a function occurring as an ingredient in a naming-element; simply nothing it would be for such a thing to be so. If not, though, then not. So the question is why we should believe Frege that there is such a categorial gulf between concepts and objects, now understanding 'concept' as just *function*. As per usual, we are still in search of any reason to believe this. Whatever such reason might be, unsaturation begins to appear as *not* the root of the evil. So, it now seems, neither unsaturation nor predicative nature is.

To sum up, thought-elements do not recommend themselves for hosting Frege's obstacle. Nor do *Bedeutung*-elements. What an element cannot ever do is be an element of another type. But if there *were* a thought in which an element was *named*, that named element would not be an element of *that* thought at all. It would be an *ingredient* of a naming element. Like Sid, it would be, not making truth turn on such-and-such, but rather just some such such-and-such. So it must be ingredients, specifically concepts, which are the host. But the features Frege mentions as peculiar to concepts do not generate the obstacle. For that we need an extra assumption which, while equivalent to the obstacle, seems, *so far*, equally gratuitous. We must look further.

2.3. Stratifying: As noted, neither Guy nor any kinsman is fit for predicative duty. Nor is Venus, the current housing shortage, the ph-level of Sid's pool, nor, as we have just seen, the sine function. One could go on. *Many* objects are unfit for predicative service: they are not ways for a thing, or for things, to be. What is wanted, though, is a converse point: nothing *ever* fit for service as predicative *ingredient* is *ever* fit for service as naming *ingredient*—and thus a stronger point, that *no* object can ever do predicative duty. We would thus have an ontological result: there are two absolutely distinct classes of things, 'objects' and 'concepts'; the distinction between these is categorial. Frege's route to this result appears to be through representation. In representing things as being some way or other, different sorts of work is done. There is naming; there is predicating. Doing the one is never doing the other. In each of these, something is put to a distinctive sort of work. Each has its distinctive way of representing that which is thus put to work. What is represented in the one way can never be represented in the other. There is no such thing as that. Does the ontology follow? A suspicion arises at this point that, as later Wittgenstein once put it, "We predicate of the thing what lies in the method of representing it." (*Investigations* §104). Perhaps mere suspicion.

Here, then, is how Frege makes the point:

We have here a word, 'Venus', which can never really be a predicate, although it can form a part of a predicate. The *Bedeutung* of this word can thus never occur as a concept, but only as an object. Kerry, too, would surely not wish to contest that there is something of this sort. But thus a distinction would be allowed between that which can only occur as an object and everything else, the recognition of which is most important. Nor would this distinction be erased if it were true, as Kerry thinks, that there are concepts which can also be objects. (1892: 194-195)

Two things we can agree on here. First, the word 'Venus' is not a predicate—not even if we could make sense of calling someone 'a Venus'. Second, Venus (the planet) is, like my friend Guy, *absolutely* unfit for duty as a predicative ingredient. So, third, as Frege says, there is, indeed, a distinction between that which can *only* occur as (in the role of) an object and everything else.

The mystery here lies in the word 'thus' in the second sentence. Frege admits that there would still be that just-mentioned important distinction even if there were also things which, while fit for predicative duty were also fit for duty as objects. Why, then, suggest that it is *because* 'Venus' can never occur as a predicate that what it denotes, the planet, can never 'occur as a concept'? Let us say that for 'Venus' to occur as a predicate would be for it to put what it denotes, or speaks of, to predicative work—as 'smokes' puts *being a smoker* to predicative work (where it does). What Venus denotes cannot be put to predicative work. This we know anyway. But why should we not suppose that, while 'Venus' cannot put what it denotes to predicative work, something else which denoted that same thing *could* do this? Of course, we should not suppose this because the denotation happens to be Venus. Cf. Guy. But since it is not yet ruled out that there should be *items* fit for both predicative and naming service, why should the possibility just scouted not arise for them, should there be such?

Frege also writes,

Not infrequently in logical investigation one needs to say something about a concept, and then to express this in the usual form for such predications, namely such that what is predicated is contained in the grammatical predicate. Accordingly one would expect the *Bedeutung* of the grammatical subject to be the concept; but due to its predicative nature this cannot without further ado appear as such, but must first be transformed into an object, or, more exactly, an object must stand in for it, which we designate by means of the prefix 'the concept', as in "The concept man is not empty." (1892: 197)

One claim here is that, due to the 'predicative nature' of a 'concept', anything which is a name (that is, which expresses a naming element in a thought) cannot be naming a concept. Again, this would make Frege's case for whatever concepts are thus to be understood to be. Once again we are on familiar ground. There is no one thing 'predicative nature' obviously must

mean, nor, equally, any one thing ‘concept’ must mean. But suppose that to predicate something of something is to make truth turn on what things are some given way there is for things to be. Then nothing in the nature of the predicating, or nothing we have yet seen, prevents either what makes truth so turn (a predicative thought-element), or on what it thus turns (a way for a thing to be) ineligible for itself serving as a subject of (of course different) predications—ways of making truth turn on which things are given ways where predicative thought-elements, and/or ways for a thing to be are among the things eligible for being (or not) those ways. If *bedeutet* is to be a technical term, one could, of course, stipulate that, for anything with ‘predicative nature’, in one of the above senses, it has not been *bedeutet* unless thereby put to predicative work. One cannot, though, rule out by stipulation the truth of some thought turning (on some decomposition of it) on which ways some way for a thing to be, or some given predicative thought-element, is.

Frege does not deny, however, that things may be predicated of concepts. Concepts may be represented as falling under higher-order concepts. *One* can bring a concept under another higher one. What one cannot do, he thinks, is to predicate of a concept what might also be predicated of an *object*, say, of Guy and his kin. So one thing one thus cannot do is to predicate something of a concept in putting it to work as an ingredient in a naming element—that is, by putting it to an object’s work. Perhaps there is a clue here to exactly *what* is blocked by the obstacle to saying what we *seem* to want to say; hence *why* it is.

Thomas Ricketts suggests something on these lines when, in a recent discussion, he remarks,

Kerry’s objection rests on a failure ... to appreciate how the difference ... between complete and incomplete parts of sentences and the thoughts they express evinces the striation of quantificational generality into levels. For Frege, the debate stops here. (2010: 198)

The crucial idea here—that on which Kerry’s objection founders—is to be: (In logical form) predication, and correspondingly quantification, is stratified—comes in hierarchies. We may begin, then, with the notion of an hierarchy. At its origin, this is an idea about the structure of representations (of something *as* something). If Frege is right, it has consequences for structures inherent in what does *not* represent, but, rather, there is to be represented as thus and so; ones which will mean, *inter alia*, that an ingredient in a naming element in any thought can be fit *only* for *that* representational duty.

But, to start at the origin, the hierarchy is one of thought-elements, or, more precisely, of abstractions therefrom, certain thought-element types. At ground level in this hierarchy are naming elements—things which make a thought’s truth turn on how something doing object’s duty is. At the next level are (first-order) (n-place) predicative elements. These combine with naming elements to form thoughts. So combined, they predicate being some way for an object to be of what the relevant naming element names. At the next level are second-order predicative elements. In our present vocabulary, these predicate things of ingredients in first-order predicative elements. They combine with first-order predicative elements to do so. And so on as far up the hierarchy as one cares to go. Where there is predication there is also opportunity for quantification. At first level, one may quantify with respect to a first-order predicative element—instead of *such-and-such is thus and so*,

something is, or *everything is*. Such is first-order predication. At second order, one quantifies with respect to a second-order predicative element. And so on on up.

So far, these are remarks about certain forms of representing: logical forms. The idea is: logical forms are formed of certain element types; it is intrinsic to a relevant type to be at a certain level in a hierarchy; a logical form is well-formed (that is, a logical form at all) only if the elements which form it are of suitable levels relative to one another. There is, of course, a trivial and familiar extension of this point to sets of forms. Suppose we have a stock of thoughts, each of a given logical form. Then if in one of these there is, e.g., such-and-such first-order predicative element, *that* element does not occur in a form assigned to any other thought as any other than a first-order predicative element. So it never, e.g., combines with another first-order predicative element in the way that a second-order predicative element would in predicating something of what it is the role of a second-order predicative element to predicate of. As said, this is a trivial point. Each type of element is essentially of *that* type.

A logical form, as here understood, is an abstraction from a decomposition of a thought. The central point of a decomposition is to identify features in a thought which it shares with others. To decompose a thought is to present it as instancing each of a given set of generalisations. Among the features in question are those relevant to the holding of *logical* relations (those to which such relations are sensitive). A central logical relation is (logical) entailment. But here I leave it to logic to say what relations concern it. Delete all other identifying features from an thought-element on a decomposition, and what is left is a logical form. Logic generates a stock of logical forms—the ones there are for a thought to take—out of some stock of (for it) significant element-types (marking the differences between the ways there are of contributing to participation in logical relations); as with any stock of items generated out of some fixed vocabulary, imposing on each form a certain structure.

So far we have a hierarchy of logical forms (and elements therein). But this hierarchy suggests two further hierarchies. Each is a hierarchy of things which relate in a certain way to elements. Now, though, to bring *things* into question, these elements need to be elements of (decompositions of) thoughts, rather than of logical forms. (Again, a decomposition can be thought of as a fleshed-out logical form.) There are *two* hierarchies here because in Frege's scheme of things objects serve two functions. First, they are ingredients in naming elements. That is, they are (at first-order) that of which things are predicated. Second, they are the *Bedeutung* of naming elements—those arguments of that function whose values for them record the world's answer to the relevant thought's question of truth.

In both hierarchies objects stand on the bottom step. They are zero-order. For the first of these, the crucial thing about objects is that they are that over which one quantifies in quantifying with respect to a given first-order predicate. In which case, what occupies the next step in the hierarchy should be that over which one quantifies in quantifying with respect to a first-order predicate. This, plausibly, is that which is *predicated* of an object in a first-order predicate, which is being a certain way there is for an object to be (or, speaking a bit more stiltedly, just that way for an object to be). So here one quantifies over ways for an object to be, or being them. At the next level, then, one would quantify over ways for a way for an object to be to be (or being them). And so on on up.

The second hierarchy would again have objects at zero-level. But the guiding idea here would be that an object is the *Bedeutung* of a naming element. At first-level, then, what is wanted is the *Bedeutung* of a first-order predicative element. In Frege's scheme of things this

would be a concept₂. At second-level what would be wanted is the *Bedeutung* of a second-order predicative element. This should be a second-order concept₂, that is, a function from functions from objects to truth-values to truth-values. And so on on up.

For purposes of the main point to come, you may choose either of these hierarchies you like. For convenience I will speak of the first. Either way, starting from a hierarchy of representational forms, we have move to a hierarchy of things. For the hierarchy of forms there was a trivial point: no form-element which ever occurs at any level in the hierarchy ever occurs at any other. Suppose that the steps in our hierarchies of things are similarly disjoint: what is found at one step in the hierarchy is found at no other. Then a parallel point holds for these. Anything found at level zero—the level of objects—is found nowhere else. Guy and his kin are found at the level; *hence* they are found nowhere else. *That* they are found nowhere else is nothing new. But the ‘hence’ is. For now we can say: if, say, some function from objects to truth-values were found at zero-level in the hierarchy of *Bedeutungen*, then it could never be the *Bedeutung* of any predicative element. Hence it is not found there. Similarly for ingredients in predicative elements. If one such were found at zero level in its hierarchy, it could not be found at level one—could not be a predicative element. We thus arrive at Frege’s result. *All* that is found at zero-level is what is *absolutely* unfit for predicative work. And there is *some* reason to think we arrive here with right. For, the idea would be, if something ever occurred as an ingredient in a first-order predicative element—if it were predicated of an object in *any* thought—then when it came to predicating things of it, this would *have* to be via a second-order predication. Which would not be so if the same thing ever occurred in the guise of an object in any thought, as it would if it occurred at zero level.

Have we, then, proven that there is an obstacle? If the hierarchy of forms imposes such hierarchies on things, then yes. But we do need to suppose that the hierarchy of forms imposes these hierarchies on things; notably, hierarchies with disjoint steps. At level zero go ingredients in naming elements. This prescription is meant to identify a definite set. But how? To speak slightly picturesquely, we would have to search through a certain set of decompositions of thoughts, and, wherever we came to a naming element, extract from it its ingredient. These would then accumulate to the content of level zero (in both hierarchies of things). What set would this be? It would be that set which, for each thought, contained all its decompositions as members. So to speak, the set would be formed through assigning to each thought there is, all at once, all the logical forms ever to be found in it.

Two assumptions have now emerged. That it is fixed *tout court* what decompositions each thought does admit of; that restrictions on co-occurrence in a corpus allow for assigning all decompositions to all thoughts at once. To see what these come to let us change perspective. The hierarchy of forms, as we are now trying to understand it, imposes constraints, not just on (or through) what logical forms there are, but also on what combinations of forms are permissible. Given Frege’s idea of the multiple decomposability of thoughts, it is not as though ‘to each thought its form.’ Rather, for any given thought, there may be many forms to carve out of it, as there are many decompositions to carve out of it. So the idea of permissible co-occurrence of forms can be seen as, in the first instance, one of constraints on co-occurrence of forms on an *assignment* of a form to each thought in some corpus of. In an assignment, the idea is, one thought cannot take on such-and-such form while another takes on such-and-such other. Or, more exactly, it is an idea of constraints on permissible assignments of decompositions, one each, to the thoughts of some corpus. (Since though, on present assumptions, a decomposition determines a unique form, we so far have

no reason to think that a form determines an ingredient, e.g., of a naming element.)

The idea would be: in making an assignment of decompositions to a corpus (one for each thought in it), one cannot simultaneously decompose some given thought into, *inter alia*, a naming element with such-and-such ingredient, and some given thought into, *inter alia*, a predicative element with that same thing as ingredient. (The point here does not require there to be no such legitimate notion *same thing*.) Such, the hierarchy tells us, would countenance illegitimate forms of predication. What this obviously does not rule out is that what is an ingredient of a naming element on one assignment of decompositions to *some* corpus cannot be an ingredient of a predicative element on another assignment to *some* corpus. The hierarchy of forms (a logical fact) cannot, from this perspective, so constrain the *extra*-logical work of carving decompositions out of whole thoughts. Such a possibility would spoil the idea that a hierarchy of forms does impose a hierarchy on things of any of the two sorts scouted. Nor, without such an absolute categorisation of things, would the existence of a hierarchy of forms deprive us, *per se*, of a notion *same ingredient* with which to state all the above.

What, though, of quantification? Suppose that, on some assignment to some corpus, there is a thought assigned the form of a universal quantification. Of just what must the predicate thus quantified be true for that *thought* to be true? Presumably not just of all of what *counts*, or is treated, as an object in some decomposition within the corpus. Well, determining what might count as an object (or be put to an object's work in a naming element) is, anyway, *extra*-logical work. *Is* there a rich enough stock of determinate answers to questions as to whether such-and-such is Sid? Such depends on just how agile, or mercurial, Sid would have to be were there such a thing. So, too, for determining what might so count for purposes of the assignment in question. *One* idea might be this. Consider a closure of the corpus under the operation universal instantiation, wherever instantiation is to a *permissible* decomposition (one which can co-occur with what was already there). Then count the quantification as true just in case all possible such instantiations are. (Again, such is not pure logical work.)

III

3.1. Without Sense: I turn now to the second remark in that pregnant paragraph of Conant's essay. It concerns the nature and status of logic—*chez* Young Wittgenstein (henceforth YW), and full stop. It is this:

The central source of confusion in Frege's thought about logic is located ... by the *Tractatus* in the one assumption that it shares with psychologism (that "widespread philosophical disease"): that logic is a science. ... It is only once one has broken with the idea that logic is a science that one is free of the disease. (1991: 141)

I do not disagree with Conant as to what YW *thought* was Frege's crucial mistake. Rather, what follows is a case that what YW *saw* as Frege's crucial misconception about logic and what (if Frege held it) *was* the crucial point are two very different things. There is a good idea

of Frege's (set out below) which YW, if not blind to, simply rejected. The absence of that good idea (in its entirety), inevitably blinds one to what is crucial here. The trajectory from YW to mature Wittgenstein (henceforth MW), seen one way, just is a slow coming to see the value of this crucial point. One might, not for this alone, see the path Wittgenstein followed from 1929 on as a return to Frege.

We must start, though, with what Frege's point was meant to be. A not implausible reading of him on this might begin with the idea that logic is even-handed. It is about any thought only insofar as it is about all. So the features of a thought to which it is sensitive—those by which a *logical* relation holds or fails to—are features one might find in a thought regardless of its subject matter. Hence, on the conception, *logic* has no special subject-matter. So decompose a *truth* of logic (which, the conception tells us, would be, like any truth, a *thought*) and you will find no predicative element in it which makes truth turn on what is, what not, some given way there is *for* things to be; or at least a way susceptibility to being which might identify, in the first-order case, objects as ones of some special kind (e.g., sublunary), in the second-order case, ways of some special kind for an object to be, and so on. *Such* thoughts, the idea is, would have nothing but their structure to make them true; something their structure would oblige with in the case of truths of logic.

Such structural truths, the idea continues, would reflect a structure intrinsic to a domain of things structured (or structurable) by those features which, combined as they do in a truth of logic, make for truth *per se*. The relevant features are ones on which the *truth* of thoughts depends—on which it *alone* depends in logical truths and falsehoods. So they are, too, ones on which the holding of relations such as logical entailment depends. So the structure these features impose on the domain in question, or that reflected in logical truths, is the most general intrinsic structure of truth-preserving paths. The nodes on such paths are, then, truth-bearers—those which bring truth into question—so *thoughts*, on Frege's notion of this. So the domain is a domain of thoughts.

So logic contains, *inter alia*, logical truths. But logical truths, on the conception, are thoughts stripped of all but their structure. Such thoughts would belong to a special region of the domain: that of the most general thoughts. Or so the conception has things. For, on it, starting from any thought without this region one can, through a series of *thought-preserving* transformations, derive a member of the region. (By 'thought-preserving' I here mean: transforming thoughts into *thoughts*.) Those operations are quantifications. Where there is a thought-element with some topic-specific content, replace it with a quantification over thought-elements of its type. Conversely, one can apply logical truths in inference by reversing the procedure: instantiating at enough points where there is a quantifier. Call all of this Frege-inspired conception F*.

One might well doubt at least one bit of this. The worry would be: if you strip all particular content out of a thought, what you are left with is not a thought. Making truth turn in part on who smokes *exemplifies* a particular *type* of contribution to a thought: that made by (or in) first-order predication. Making truth turn on what waddles, or what just fell on the rug, do so too. By contrast, if you strip away all particular content from an exemplar, what is left, it seems is no exemplar at all. Such *seems to* clash with the also-plausible idea that quantification is an operation preserving thought-hood. Nonetheless, there is something right in this idea.

And YW was suspicious of the idea that thoughts with all particular content stripped

away were still, for all that, (fully) thoughts. His suspicion, though, took the wrong form. It is expressed in the idea that laws, or propositions, of logic *say nothing*. YW does not deny that quantifications are thoughthood-preserving operations. They yield well-formed structurings of ways of legitimate contributions to a thought. Nor does he refrain from calling propositions of logic both propositions and true (nor contradictions *false*). Still, as Conant puts it, such things are degenerate cases of thoughts. They pose no *genuine* question of truth. To fail at this, one *might* think, is just to fail to be a thought. To bracket this issue I will henceforth call such things, in logic and beyond, as *dicta*.

For logic not to be a science might be any of several things, depending on just what explanatory, or predictive, ambitions one thinks intrinsic to a science. ('Predictive' in the sense in which a theory of English syntax predicts what is, what not, a sentence.) At the very least, though, one expects that there is a science only where there is some set of facts for the science to state: the science would consist (at least) in stating (or predicting) them. Logic, if its dicta say nothing, fails this minimum requirement. So, for the moment, it will do to take this as a reading of the idea that logic is not a science. YW may ask more. MW, as we will see, has a more subtle reading of the idea.

Here is one set of expressions, by YW, of the idea that logic's dicta say nothing:

The propositions of logic are tautologies. (6.1)

The propositions of logic therefore say nothing. ... (6.11)

Theories which make a proposition of logic appear substantial are always false. ... (6.111)

The propositions of logic demonstrate the logical properties of propositions by combining these into propositions which say nothing. (6.121)

It is the characteristic mark of logical propositions that one can recognise in the symbol alone that they are true ... (6.113)

Tautology and contradiction are not pictures of reality. They represent no possible state of affairs. For the one allows *every* state of affairs, the other none.

In the tautology the conditions of agreement with the world ... cancel one another, so that it stand in no presenting relation to reality. (4.462)

A tautology has no truth conditions, since it is unconditionally true; and the contradiction is true under no conditions.

Tautology and contradiction are without sense. (4.461)

And here is a rather different one:

Not only must a proposition of logic be incapable of being refuted by any possible experience, but it must also be incapable of being confirmed by any such. (6.1222)

There are different notions here which will soon prove to come apart. The first set and this last correspond naturally to different intuitive ideas.

Corresponding to the first set is an idea about representing-as (representing something as (being) something. There are two terms of a relation here: what is represented as (being) something; what it is represented as. It is with just this relation that truth can come into question. Now the idea is: where there is a genuine case of this relation, neither of these two parties is allowed to idle. Truth is the yield a genuine cooperative enterprise between them. Party of the second part poses a question of truth; makes truth turn on party of the second. Party of the second then, in being as it is, yields an answer to the question thus posed. In a logical dictum, as YW puts it, one can recognise ‘in the symbol itself’ that there is truth. Party of the second, so to speak, bars party of the first from any participation in the joint enterprise. It matters not at all how party of the second is. But, the thought would be, such is simply not representing-as; not the genuine holding of a relation between two parties. (*Anything* would do as well in that first position as *anything* else.) Such is one intuitive idea.

The last idea of saying nothing connects in a special way with an idea of logic not being a science. A thought might be: if experience *could* bear on whether some logical dicta was true, then if it is true, there is room for explaining why. But if there is such room, then there is room for a science whose task would be to explain this. Frege writes against the idea of such room in saying,

The question why, and with what right, we take a law of logic to be true, logic can answer only by reducing it to another law of logic. Where that is not possible, logic can give no answer. (1893: xvii)

He goes on to suggest: where logic has nothing to say as to *why* a law of logic is true, nor can anything else. This idea that nothing extra-logical can bear on whether logic’s dicta are true (read as with YW above) suggests that, necessarily, logic is immutable’ would be as it is no matter what—an idea Frege puts picturesquely in describing logic’s dicta as “like boundary stones set in an eternal foundation, which our thought can overflow, but never displace”. Thus, Frege tells us, their “authority for our thought”. (1893: xvi) Whether the intuition is rightly read either thus or as per YW is for later consideration.

What YW sees, or seems to, then, is that the rest of Frege’s conception of logic, as per above, clashes with the idea that laws of logic say something—are full-fledged thoughts. Logic’s dicta reflect the structure of a system. What is thus structured—what composes this system—are those things between which logical relations hold—thus, thoughts (though Frege and YW use that term in different ways). The clash is to be resolved by removing the idea that logic’s dicta *say* something; that they are themselves fully members of the system. The next step is to examine YW’s general notion of a system.

3.2. Systems: The *Tractatus* explains its use of ‘system’ through examples. What is in question in the examples is systems of ‘descriptions’: unquestionably things by which truth can come into question; ways of making truth turn substantially on how things are. These will be systems whose ambitions are much more limited than those of any Grand System. Such a special system does not aim to provide representations of things as *all* the ways there are for one to think things, but only for (some of) how things stand on some particular topic one might engage with. Each such system, the idea is, has a most general structure which is proprietary to it. This structure is reflected in some set of dicta the system generates. Many—

perhaps not all—of these are generated from the system's vocabulary, but put together in such a way as *not* to belong to the descriptions of things the system generates.

A limited system, like a language, generates some stock of descriptions from some given vocabulary and syntax. A structure is thereby imposed on each. A description's place within the structure is fixed both by elements it shares with others and by ways it contrasts with others, thus by the vocabulary (and/or structuring) others have and it lacks. To be a given member of the system just is to be something the system generates. It is thus, *per se*, to be structured as the system structures this. If I want to represent Sid, say, as I take him to be, it is to a considerable extent at my discretion just which way to represent him as being. The role of a special system is to exhaust such discretion. Once Sid is described as fitting some description chosen from a special system, all discretion over *how* to represent him as being is used up.

6.341 offers two illustrations. The first is a system for describing patterns of black dots (or lines) made on a white flat surface. The system's descriptions are defined in terms of a net, thought of as placed in a given orientation of the surface to be described. The net consists of cells at given addresses in the net, each of given size and shape. A description in the system consists of a sequence of ordered pairs. The first item in a pair is a cell's address. The second is either the description 'black' or the description 'white' according as the surface underneath the cell is predominantly black or predominantly white.

Examples of dicta of the system would be: (if its cells are hexagonal) that each ordered pair in a sequence describes a hexagonal portion of such-and-such size of the surface being described, or that a (well-formed) sequence of such-and-such length is a grammatically full description within the system. These, the point is, say *nothing* about the surface being described. Such illustrates the notion of saying nothing on which such is what dicta do. On the other hand, that the pattern on a given surface can be *completely* described by some description in the system (where 'complete' means that it is fully determined what the pattern is) does say something about the surface (given the system); and vice-versa.

Newtonian mechanics provides the second illustration. Here the system is to generate descriptions of mechanical states and interactions of rigid bodies—e.g., of that six-pack flying towards that windshield. The laws (and definitions) of Newtonian mechanics are (among) what fixes relations between such descriptions. So they are among the system's dicta. These are meant to be dicta, and hence to say nothing (about the world), in just the same way as, say the dictum that no area beneath a cell can count both as white and as black says nothing. (6.342) They merely say how those terms spelled 'velocity', 'mass', etc. are to be used. By contrast, 'The six-pack has a momentum of 76kgm/sec' does say something about the world.

So, it seems, there is a contrast here between two sorts of representation. The one, if in order, reflects the structure of the system, *hence* says nothing about the world. The other describes the structure of the world. So, presumably, *it* does not identify the structure of the system. For a dictum to be correct (in the only sense in which it might be) is simply for the system to have the structure it reflects; which is just the same as for it to *be* a dictum. A dictum, the idea is, *merely* reflects the system's workings. It thus in no way chances its hand. The structure its dicta reflect is *intrinsic* to a system. Such is intrinsic to being a dictum. For a dictum to be is for it to be so structured. Nor, we are to suppose, is the system's existence in any way hostage to how things are. A system in this sense is not something there might have

failed to be.

Special systems differ from a Grand System in several ways. Here are two. First, one can choose to describe the mechanical states of rigid bodies or not. Whereas, for a thinker, there is no choice as to whether to engage in representing-as. Second, there may be many systems of descriptions of the *same* thing—e.g. mechanical phenomena. A special system's dicta apply to all representing which exploits it, but not thereby to all representing, or even all representing on a given topic. Whereas there is no representing other than within the Grand System, thus none not subject to its dicta. Newtonian dicta reflect the *conferring* of content (the using up of the discretionary). The structure logic's dicta reflect (on our present conception of logic) is a structure intrinsic to *all* representing. There is no such thing as evading them. Which *seems* to mean: no such thing as representing being structured otherwise.

The dicta of a special system select between choices of how to represent things (as being). Those of the Grand System do not. Accordingly, while there might be a project of saying which choices a special system made, and there might be reasons for making these, there is no such project for the Grand System, so nothing parallel in it for which reasons might be given. Such adds content to the idea that logic is not a science. It is a way in which a special system, and its dicta, *contrast* with the Grand System. Are there interesting ways in which special dicta and logic's are the same?

3.3. Gedanken and Sätze: Concluding its illustrations of special systems, the *Tractatus* remarks:

And now we see how logic and mechanics relate to one another.
(6.342)

The discussion thus begun concludes:

Although the spots in our picture are geometrical figures, obviously geometry can say nothing at all as to their actual form and position. But the net is *purely* geometrical; all its properties can be given *a priori*.
Laws, like the principle of causation, etc., treat of the network, not of that which the network describes. (6.35)

One comparison: so, too, for logic's dicta. They, too, speak only of that most general network of truth-preserving paths which structure that system (a Grand One) to which any thought belongs. How, though, *do* special systems compare with Grand Ones?

As background to comparison I now introduce two central and closely linked ideas of Frege's. The first is that thoughts are multiply decomposable: there is no such thing as '*the* set of elements which compose a given thought', much less '*the* way a thought is structured.' Such is reflected in the fact that the same thought can be expressed in syntactically and semantically diverse sentences. Such is corollary to what he claims as most distinctive in his approach to logic (see 1919a): putting truth first; next whole questions of it. (One crucial application for Frege is in showing how the thought expressed in 'Sid smokes', where '—

smokes' *bedeutet* a certain concept, can also be expressed without *bedeutende* that concept at all—as in 'The concept *smokes* is satisfied by Sid'(cf. 1892: 199-200). A thought may be carved, as a goose may, into any of many diverse sets of parts. Like a goose, it is not generated out of some particular set of independently existing parts. (Such is integral to Frege's anti-psychologism: nothing but a role in a thought makes anything the right sort of thing to play it.) Here, too, the idea that a thought is an object of intellectual, and, necessarily, not sensory, awareness. A thought abstracts from representing-as a *pure* question of truth; distinguished from others *only* by its way of making truth turn on how things are. *Thus* it is that Mont Blanc, like Sid and the True, cannot be a thought-element.

Russell, missing the notion's rationale, had an aversion to Frege's notion *thought*, more generally, *Sinn*. YW seems to have inherited it. But, as with Russell, such is to miss the point—one which will now prove crucial for evaluating F*. A thought is a *pure* way of making truth turn on how things are. What it does can be broken up into subtasks. But, stripped of features, such as sensible ones, identifiable independent of it, no particular way of thus decomposing it can claim objective priority—can claim to be *that* way by which it is intrinsically identified as the one it is.

How, then, *is* a thought identifiable? The question brings us to Frege's second idea. It is that it is intrinsic to a thought to be that which 'can be grasped as the same by different thinkers' (cf. 1919b: 146). A thought needs no 'bearer'—no one to whom it relates in some special way. A thought need not be thought; *a fortiori*, not by anyone in particular. A thought marks (or just is) a point at which (indefinitely many) different thinkers may agree, dispute, jointly investigate, jointly find proof or refutation. It identifies something *one* can get right or wrong.

What, then, is intrinsic to a thought? One can say: when (in what cases) things would be as that thought represents them. Now, Frege also stresses, two thinkers *are* disputing or agreeing only where they agree well enough as to what it is to which they are agreeing, or about which they dispute. If we think of a thought as identified by when things would be as represented, then there, accordingly, what there *is* to recognise as to when it is that thought which is in question. Parties to a dispute over *its* truth would need to agree to enough of this. But suppose I now express a thought. Fictionalising slightly, I might do so in now saying that that six-pack will break the windshield. Maintaining the fiction (e.g., waving the question which windshield), it hardly strains credulity to suppose that there are those among us who understand me—that is, know what I said. Those who do so are, accordingly, equipped to recognise when things will have been as I said. They enjoy such capacity. We, then (they and us), thereby agree on that which identifies something there is to dispute or agree to. What we are equipped, or prepared, to agree to thus identifies a certain thought.

Now identify a thought however else you like. Is it the one we had in mind above? Such is a substantial question. Its answer depends, for one thing, on what is to count as *same thought*. But again it is no strain on credulity to suppose this a question we understand. If we do, then again we are capable of recognising when a thought would, when not, count as that one we had in mind. (The point stands no matter how occasion-sensitive the operation of such capacity may be.) We are thus equipped to recognise when, and how, that substantial question has been answered.

With which we return to the *Tractatus*' special systems. These reverse Frege's priorities. For Frege whole thoughts come *first*. In decomposing elements are carved out of these, *ad lib*,

so long as parts are in the same business as the whole, and jointly just *are* it. Whereas for YW it is some stock of elements, and a syntax, which come first. In a special system play the role of thoughts for him—significant propositions—are generated from this basis. So while Frege's thoughts are multiply decomposable, what play that role for YW—that by which truth can come into question—are precisely what is generated from the basis; to be which is to be structured in just that way thus imposed on them. To exhaust the discretionary in representing is thus to be so structured. And, as opposed to Frege, to represent in, or via, a member of some such system is to produce representing intrinsically so structured.

The representations of a special system are thus *not* multiply decomposable as it is intrinsic to a Fregean thought to be. They correspond, at best, to a decomposition of a thought. A thought in Frege's sense could not belong to a special system. At best it could be represented, or presented, in the system as decomposed in a particular way. Of course, a special system structures its elements in proprietary ways, reflected in its proprietary dicta. So *its* decompositions would be not just into given logical forms, but logical forms in which content to which logic is indifferent is also structured in particular ways.

Suppose, now, we choose a given thought—say, that the six-pack is flying towards the windshield with momentum 76kg/m-sec. Then we select a special system—say, the *Tractatus'* Newtonian-mechanical one. *Is* that thought representable in that system? If, but only if, it is structurable as some description of the system is. Seeing whether that is so is extra-systematic work. The system itself contains no resources for approaching the question. The point generalises to the question in what systems the thought just mentioned is representable. What might answer such a question? Here we may refer to Frege's second idea. Above I mentioned a thought: that the six-pack is flying towards the windshield. I spoke understandably. There *are* those who understand what thought was mentioned. They are thus able to recognise (well enough) when *that* thought has been mentioned. Now structure some representing as some given special system—e.g., the *Tractatus'* Newtonian system—does. Would those who thus grasp what thought is now in question be prepared to recognise it as so structurable—that member of the system as saying things to be as that thought represents them? Thus are such questions answered.

Unlike that Grand System which logic is meant to structure, a special system is made up, not of thoughts, but rather of structurings for a thought to admit of. To find a point in a special system at which some given thought is *represented* (decomposed in a particular way) is to engage in extra-systematic work. Relations between decompositions and thoughts are not a special system's concern. By nature, a thought is what *might* be represented (or expressed) in any of many different special systems. What had no place in a grand system would not *be* a thought. Not so what had no place (expression) within some special system. For all that, it might find expression in another. Similarly, a decomposition of a thought might, for all of its so being, find no place in some special system in which that thought was expressible. But it could not be a structure by which the thought was *not* related to others within a Grand System.

A Grand System, by F*, is composed of *thoughts*. If Frege is right in putting whole thoughts first, then it would be made up of precisely what a special system, for YW, is not. In any case, a Grand System differs from a special one in this: to represent, even about a given subject matter, one need not represent in any given special systems (there might be many, e.g., for representing the mechanics of rigid bodies); whereas if there is Grand System, there is no representing at all except within it.

3.4. Two Kinds of Saying Nothing: YW uses special systems to illustrate something. But one might now wonder whether what they do illustrate is quite what he meant them to. For a start, they exhibit *something* about the notion *saying nothing*. YW's own ideas of saying nothing, as cited above, divide in two (or three). There is, for a start, the simple idea of saying nothing as opposed to saying *something*, where this means: to say *such-and-such*, for some such-and-such. YW elaborates this idea on these lines: thought elements which may be combined with others so as to say something in present sense may also be combined with others, depending on the others, so as to 'cancel out', thus say nothing. 'All bachelors drink' says something. 'All bachelors are single' does not: being single is already contained in being a bachelor. Where elements cancel each other out in some such sense, one can recognise from 'the symbol itself'—from the thought, or its expression—that it is true (*casu quo* false).

This first simple idea is well displayed in the idea of a special system. The system combines elements into 'descriptions'—genuine representations of things as being thus and so. But these same elements can also be combined so as to cancel each other out. For example, in the system for describing patterns on walls, elements in a description of a cell as white, and those in a description of a cell as black can be combined into a proposition, 'If a cell is white, then it is not black'. But such combinations do not belong to the system of wall-descriptions generated. Rather, their failure at this marks them as in another role: they reflect the most general proprietary structure of the system. They are thus dicta. (It is implausible that all dicta of a special system can be so regarded. But such worries do not touch the present point.)

Such is an idea of a dictum of a special system saying nothing. It belongs to the system. But it does not belong to the stock of *descriptions*—representations of *things* as being thus and so—that the system generates. The system assigns it nothing as that which it says to be (or represents as being) so. Hence, in our first sense, it says nothing. *This* notion of saying nothing is a system-relative notion. So, in light of the last section, it has no, or no immediate, application to thoughts (on Frege's conception of what thoughts are). A thought has (or lacks) a *representation* in a system ('represent' here not *represent-as*, but rather *stand in for*, *vertreten*.) A thought which lacks a representative in the stock of descriptions some given special system generates might yet have one in another. Further, for all we know so far, a thought might have the status of a dictum in one special system, and that of a description in another. A simple example: Sid arrives at Vesuvio (for one of those upper-Douro weekends among the *familias-boas*), to be told by his hostess that his room is first floor, end left. He is thus *assigned* a room. There is no sense to the idea that the hostess has spoken *falsely* (though ineptly if there are no rooms on the first floor). Later that evening, after his last Porto and *bagaço do mesmo*, Sid wanders bewildered, looking for his room. His hostess tells him, 'It's first floor, last on the left'. Here a genuine description which she relies on memory to get right.

What special systems illustrate here is thus that saying nothing in our present sense is relative to something. At first pass it is relative to in what special system we locate a thought. But locating a thought in a special system—finding a description in the system which is a representative for the thought—is extra-systematic work, to be performed in conformity with Frege's two ideas. It is Hilary Putnam's point that where there is such extra-systematic work to be done, there is relativity to the world, or to the circumstances there obtaining. In what

special systems a thought is represented, and, indeed, what special systems there *are* in which to represent thoughts depends, or is liable to depend, on how things are. How a thought is decomposable, and, correlatively, where the notion *same thought* applies, is, by Putnam's point, occasion-sensitive, and, notably, *world-sensitive*. The key here is the point that where we succeed in expressing, or in mentioning a thought, or, relatedly, that way it represents things as being, there are then, potentially, those who understand our mention. At which point Frege's second idea, as elaborated above, applies. To which Putnam adds the idea that what is to be understood as to what was mentioned in such a case—how the mention was to be understood—is a complex made up of many, and potentially independent, strands.

There is a parallel to Putnam's idea in a simple point about understanding thought-expressions, that is, about recognising *what* thought was thus expressed (just *how*, if any way, things were said to be). Sid says to Pia, 'That blonde woman in the sequinned dress is singing 'My Way'. *Who*, if anyone, did he say to be singing 'My Way'? Suppose there is a conspicuous item (person, Martian, robot) who/which, in the circumstances, one would have taken Sid to be speaking of (and no other relevant alternative items). But that item is no woman. That garment is no dress (but a jumpsuit). Those are no sequins (but rhinestones)—and the song is not 'My Way', but 'As Time Goes By' (moreover, the singer is lip-synching). Sid purported to be speaking of something of all those things (in fact not so) were so. All this forms one strand, or packet of them, in how Sid's words were to be understood. The first-mentioned fact about the conspicuous item forms another. At most one of these strands in what was to be *supposed* can, in fact, *be* to be supposed as to what, if anything, Sid spoke of. If the second strand has this status,—if we insist on the sequins, the dress, and so on—then Sid spoke of no one, so expressed no thought. Then, what was *then* to be supposed—that Sid spoke of such-and-such conspicuous thing—is false. If this last is still to be supposed, then Sid said something false of, as it happens (say) a Martian in a wig. Applying Putnam at this point, the idea would be that things *could* be such that the second strand, and not the first, remains as what *is* to be supposed.

The parallel is, e.g., with the predicative element, *having momentum M*, in, say, that thought about the flying six-pack, and the question whether that thought has a representative in YW's Newtonian system (or whether there even is, or could be, such a system). It is a dictum of YW's system, as presented, that, in it, momentum is mass times velocity. Depending on time and place, that thought about the flying six-pack may have been to be supposed to be, *inter alia*, about a certain physical quantity, identified precisely, and essentially, by that dictum. As the world turned out to be, though, there is, and could be, no physical quantity so identifiable. Once again we can identify two distinct strands in what the thought in question would have been supposed to be. On the one hand, it would have been (to be) supposed to be a thought about a certain physical quantity which all solid bodies have (at a time) in some definite degree. On the other, it would have been (to be) supposed that the quantity in question was the one identified by conformity to the dictum. The world as it is, insist on this last and there is no such thought as that the flying six-pack had such-and-such momentum. So the first strand proves not so. Insist on the first, and there is such a thought, but it is not governed by the dictum. So the second strand proves not so. Putnam's brief here is that, first, the world's being as it is means, first, that the first strand *is to be* insisted on; and, second, that there is, in fact, no such system of descriptions as the one YW proposes.

Computing mechanical quantities in a Newtonian way remains, for most practical

purposes, a good way of representing most near enough, slow enough mechanical phenomena. Such representations might be generated by a system much like the *Tractatus*' Newtonian one. This system would use the term 'momentum' in a proprietary way much as that system for describing patterns of dots on walls uses 'black' in a proprietary way (on which it applies to cells which are *predominately* black). On this use of 'momentum', a six-pack would have momentum M just in case for all practical purposes it might as well have. The dicta of this system would not be those of the *Tractatus*' Newtonian system. They would not reflect a structure in which *momentum* was mass times velocity.

A second notion of saying nothing is contained in YW's remark (6.1222) that no possible experience could bear (so the world could not bear) on the truth or falsity of a *logical* dictum. For (the idea is) *such* dicta lack conditions on agreement with the world. If this *is* so, we now see, it would be an important point of contrast between special systems and a Grand one. A dictum of a special system is, *per se*, assigned nothing *by that system* as the such-and-such which is what it says. So, *so far*, it says nothing in our first sense. But this is compatible with the same thought which is represented by that dictum in that system being represented by a description in some other.

Putnam's idea strengthens this point. *If* Newtonian physics had been correct, it would have been just what it *was* for a physical quantity to be momentum that it equalled mass times velocity. So there would have been *no* special system which contained a representative for a thought which represented *momentum* as precisely this, where within that system that representative was a genuine description of the way things are (with, thus, a definite condition on agreement with the world). We would have had no idea what the such-and-such might be which one said to be so (other than just that momentum just is, by definition, that quantity) in expressing the thought that momentum is mass times velocity. So, as we see, whether *this* thought could be represented in a system by a genuine description of how things are (on present understandings of what this would be) depends on how the world is.

In the unstrengthened point above one might simply see a dissimilarity between special systems and a Grand one. What has one status in a given special system might have another status in another. But if a thought (and, so far, it is a thought, and not a representative) has a given status conferred on it by the Grand System (given such a thing), there simply *is* no other system in which it might enjoy any other. So, one would think, what is a dictum of the Grand System—that is, a logical dictum—could not be anything other than a dictum in any other system. There is simply no other system in which for it to enjoy such status. Which suggests that idea about logic's immutability contained in Frege's remark about boundary stones. Which suggests, in turn, that *here* our two notions of saying nothing converge.

But if a logical dictum could not but have been a dictum—if there is no such thing as anything in how things were having borne on whether it was a dictum or not—should this not also be true of more of what is necessarily so? Suppose Newtonian mechanics had been correct. Then momentum (what it spoke of as such) would simply have been mass times velocity, as there stipulated. So there would have been *no* system in which that thought was represented by what had a genuine truth-condition—by what engaged in a genuinely cooperative enterprise of representing-as. There would have been no way to provide a such-and-such to make good on the idea of that thought saying something on our first simple notion of that. But *this* would have been a world-dependent fact: the world would have provided no way of identifying such a such-and-such. For logic's dicta, it is in fact so that the world provides no such ways; so that there is in fact literally no saying, or even imagining,

just *how* their dictum-hood might be world-dependent. *Thus* far they line up with stipulations about momentum as things might have gone. Is there any stronger sense in which there is no such thing as logic's dicta having been other than they are?

If there *is* a Grand System, there is no alternative to it. So what is a dictum in *it* cannot fail to be a dictum elsewhere. So there *is* no way that a dictum's truth (or truth-inaptness) could turn on how things are. As one might expect if logic's dicta are conceived as per F^* . If a dictum is a thought stripped of all but its most general structure, it would be, one would expect, identified (exceptionally) only by some given structure; hence *not* be multiply decomposable. Whereas different representatives for a thought in special systems *ipso facto* decompose it differently. On the other hand, the point remains that decomposing a thought is extra-systematic work; in the case in hand, extra-logical work. Nothing said so far entails that how such extra-logical work was to be done, even where it comes to dicta, could not have been otherwise were things different enough from the way they are. It seems, then, that we are not yet positioned to say how our two notions of saying nothing connect, or fail to, in the special case of logic's dicta. All we can claim so far is to have distinguished these two notions.

3.5. Comparisons: His special systems introduced, YW comments, "And now we see the relative positions of logic and mechanics". Just what *do* we see? Most strikingly, a number of glaring *dissimilarities* between special systems and a Grand One. Below are three bearing on what logic is.

First Comparison. The dicta of a special system fix the way its elements exhaust the discretionary in how to represent: choose one and such choice is over. Logic's dicta cannot be understood as using up discretion: there is nothing discretionary about whether representing-as is governed by the structure they reflect. They do not merely lay down how some special form of representing is to proceed. They cannot be understood as in any way stipulative. They are nothing like the hostess' dictum, announcing on your arrival that your room is (*to be*) first floor, last on the left. In this respect they assimilate to your hostess' late that night reminder. There is a special system in which the content of what the hostess says is a dictum, and another in which that same content is a description. What is not at one's discretion is (to add pleonasm) *in fact* not. Which would seem to make for something for logic to be a science of.

Second comparison. A grand system, as presently conceived, is made up of thoughts, where a special system is not. A special system is composed of *decompositions*, or things intrinsically decomposing in just one way—decompositions, of course, structuring proprietary subject matter in proprietary ways. Its dicta reflect structure proprietary to it (and, given multiple decomposability of thoughts, not, *per se*, to its subject matter). Its intrinsic structure is imposed by relations sensitive to more than just logical form; non-logical relations. Its members are items, *to be* which is to be generated as *it* does, to be structured accordingly. Just so that a *thought*, as *multiply* decomposable, is not a member of any special system (though a decomposition of it might be). By contrast, the dicta of a Grand System would reflect the structure of the most general form of truth-preservation. Truth-preservation is a feature of transitions between *truths*. And it is *thoughts* which *are* truths (or falsehoods); thoughts which entail and are entailed, thus which might lie along truth-preserving paths. Hence, the idea is, thoughts compose The System.

Third comparison. What is a dictum of one special system may be a description within another. The hostess, working in one system, by *assigning* rooms to guests, sets up a new system for describing them, then uses this to tell Sid how to find his bed. If a Grand System contains its dicta, or if these are *thoughts*, then a logical dictum could not be contained in, *a fortiori* be a description in, any *alternative* system. There is none such. Which seems to make a law of logic *unlike* a principle of mechanics. Which, as we have seen, might inspire the idea that logic *could not have been* any other than it is. But care is needed here. No matter how things were, there would not have been any alternative to what was then the Grand System (if the idea of such a system makes sense). So logic's dicta would *not* have been represented in any alternative system. It is another matter whether what *are* logic's dicta *would* have been logic's dicta no matter how things might have been.

Logic's dicta (if those of a Grand System) do not say *something* (namely, that such-and-such) in the following sense. Just as a special system assigns its dicta nothing as that which they say, so, too, for a Grand System. The function of *its* dicta, like that of a special system, is not to engage in full-fledged (cooperative) representing-as. (Though, on the other hand, by contrast with dicta of special systems, nor are they in any way stipulative.) But this, even if admitted, need not mean that there could have been nothing which what, in fact, are logic's dicta would have said (in a different enough world). (There *might have been* no system for describing the world's mechanics which assigned the thought that momentum is mass times velocity something which is what it said (again, in genuine representing-as).

The second comparison is what raises the crucial problem for F^* . Logic is concerned with truth-preservation. It is thoughts which entail and are entailed; hence transitions between which preserve truth or not. Hence the idea was, it is thoughts which compose the Grand System. In this a Grand System would contrast with special ones, which are composed, not of *thoughts*, but of (what amounts to) thought-decompositions. The problem, in brief, is: given all else a Grand System is supposed to be, *can* it contrast with special systems in this way? Or might this, perhaps, be one respect in which a Grand System and special ones *are* parallel?

For a Grand System to be composed of thoughts is for that structure of it which its dicta reflect to be a structuring of thoughts. What its dicta reflect is, in first instance, a certain structuring of a domain with thoughts as elements: a structuring imposed by those relations between thoughts to which logic is sensitive—the logical relations, such as (logical) entailment. But what structures a domain also imposes a structure on each of its members. Here each member must be structured according to those features of it to which the holding of the just-mentioned relations (the logical ones) is sensitive. So, *if* the Grand System is composed of thoughts, its dicta would reflect, for each thought, a structure which is thereby imposed on it. It would be a structuring of just those features in each thought to which its participation with other thoughts in logical relations (e.g., its logically entailing, or being entailed) depends.

But if thoughts are multiply decomposable, and if, as Frege understands this, this means that, for a given thought there is no set of 'basic' elements which are *the* ones of which the thought is composed, then the idea of the last paragraph will not do. (Given attempts in the literature to bagatellise the idea of multiple decomposability, an example will help. A thought decomposable into, *inter alia*, an element which *bedeutet* a concept (in predicating being a smoker of Sid) is also decomposable, on Frege's view, in a way which includes no such element. (1892: 199-200)) For the whole point of decomposing thoughts is to identify

in them those features shared with other thoughts on which inferential relations depend—most notably, in the case of such things as predicative and naming elements, those on which the holding of logical relations depends. Conversely, a structure structured by logical relations (and the features to which they are sensitive simply would not be (given multiple decomposability) a structure of a domain of thoughts as such. A structure generated by those elements (or features) to which logical relations are sensitive simply would not structure such a domain. The relations between *thoughts* are structured in no such way. (In fact, what logical relations are sensitive to is actually a certain abstraction from a decomposition: what abstracts away from all in it (e.g., the particular content of a predicative element predicating *smoking* of relevant items) which is none of logic's concern.)

Suppose one forms a domain of thoughts (grand or not, but, say, closed under logical inference). Now, for each member of the domain, assign it some decomposition of which it admits. For each such assignment of decompositions to the whole domain there is *one* structure which logic's dicta reflect—a structure of relations between thoughts holding by virtue of those features assigned them on the relevant decompositions. No such structure is, as it stands, the structure of logical relations between thoughts *per se*. (No decomposition of a thought may, for Frege, claim objective priority.) Perhaps, then, the structure of logical relations between thoughts as such is a superimposition of all of the structurings corresponding to all possible such assignments.

But nor will this idea do. It presupposes, for a start, that for each thought there is a fixed determinate set which consists of all the decompositions the thought admits of. We already have reason to doubt this. One set of reasons comes from Putnam's point: what decompositions a thought admits of is liable to depend on how the world is. But the general point here is just that assigning decompositions to thoughts is extra-logical work; whereas when one conceives of logic as identifying the structure of a superimposition, as above, one turns what is extra-logical into logic's responsibility. In what ways *can* one decompose the thought that Sid smokes? Such depends on what else, besides that mention of it, would count as a mention, or expression, of that very thought. Would 'The concept *smokes* is satisfied by Sid' do the job? Frege thought so. Here I only pose the question. What about 'One of Sid's vices is inhaling tobacco'? Or 'Anyone identical with that very person, Sid, smokes'? And so on. Again I only pose the question. A route to an answer is in Frege's second idea above, applied in whatever occasion-sensitive way may be required for capturing what we are, in fact, prepared to recognise.

Assuming logic's dicta to reflect some structure of *some* system made up of some particular sort of member, that structure would have to be invariant across any assignment of decompositions to any set of thoughts. So it could not be a structure of a system of thoughts *per se*. The domain (or domains) it would structure would be made up of items, each structured, intrinsically, by a structuring of some set of features which are those to which logical relations are sensitive. Such items *might* be decompositions for thoughts to have. But then again, if it is none of logic's business which thoughts there are (e.g., nothing to logic whether there is, or is not, tobacco for there to be thoughts about), then, holding the extra-logical separate from the logical, the most suitable candidates for the role of relevant item here are just logical forms themselves. What Frege failed to see, if he really did hold F^* , is the just-mentioned incompatibility, hence its proper resolution. In this he would not have been true to himself. This lapse, however, if Frege suffered it, is not one YW was positioned to see, having confounded thoughts with a kind of *Satz*. All of which brings us to the next section.

3.6. Beyond F*: In 1917 Russell presented what is *almost* a version of F*:

Every logical proposition consists wholly and solely of variables ... You can consider stages of generalisations as, e.g.,

‘Socrates loves Plato’
‘x love Plato’
‘x loves y’
‘xRy’.

There you have been going through a process of successive generalisation. When you have got to xRy, you have got to a schema consisting only of variables, containing no constants at all, the pure schema of dual relations ... So that it is, as you might say, the pure form of all those propositions. (1918: 238)

Russell speaks here of *propositions* of logic, reached through ‘successive generalisation’, terminating when there is no more generalising to be had. So far, F*. Propositions reached in such a way would be among the most general. Where there was mention of an object, now there is only quantification over objects; where mention of a way for an object to be, now only quantification over *ways* for one to be. But Russell’s generalising operation does not end up where F*’s does. F*’s operation was meant to be thoughthood-preserving. Russell’s is not. A *thought* cannot consist merely of variables. Such are simply blanks to be filled. A question of truth so formed is just ill-formed, *no* question.

Perhaps just so that Russell also calls the ‘proposition’ thus arrived at ‘the pure schema of dual relations’. But if ‘xRy’ represents (stands for) a schema, then ‘x’, ‘R’, ‘y’, are not variables. There is no *instantiating* on them. (Thus quantifiers are not *missing* here.) Those letters, so understood, stand for, name, elements in the schema: a naming element, a (2-place) first-order predicative element, another naming element. The schema (what Russell also calls a ‘pure form’) is a form for a thought to take. That form is instanced (exemplified) in such things as *Sid freshened Pia’s glass*, *Sid owes Max a fiver*.

F* moves from more specific to more general by operations which *ought to* preserve thoughthood: quantifications. For YW the operation, carried *that far*, yielded, not *genuine* thoughts, but dicta. Russell’s operation of abstraction is plain and simply *not* thoughthood-preserving. It leads soon enough to what are neither thoughts nor dicta, to what *could* neither entail nor be entailed. *Quantifying* (thrice) over *Sid Freshened Pia’s glass*, one can arrive at any of a variety of thoughts, none a logical dictum. Abstracting à la Russell one gets to what is not a thought, but what may be of interest to logic. One arrives at a logically relevant form for a thought to take. A *proposition* of logic then might be that there *is* such a form for a thought to take; that *such* is a way of participating in logical relations. Is *Sid freshened Pia’s glass* merely a bad starting point for extracting laws of logic? Or is F* a bad conception of what a law of logic should be? What *should* Russell have been speaking of?

The upshot of the last section is that Russell should have been speaking of exactly what he did speak of: logical forms. A logical form is a form *for* a thought to take. It abstracts from

a decomposition for a thought to admit of, retaining from that decomposition only that information which is relevant to the holding of logical relations. Logical forms are, in the first instance, what logic is about. *They*, precisely, are what is purely logic's business, what involves no extra-logical work. On this conception, logic *does* have a specific subject-matter: logical forms. Logic's dicta are about these. One task for logic is then to tell us what logical forms there are; to generate these from some fixed 'vocabulary' (set of elements)—e.g., from such things as (being) (some given) naming element, or (being) (some given) first-order predicative element. Another task would be to generate truth-preserving sequences of (sets of) logical forms: for any element in such a sequence, and any later one, where any thoughts of that first set of forms (one thought for each form), and any thoughts of that second set of forms, if those first thoughts are true, then so are those second. A third task might be to say what a coherent assignment of forms to a specified set of thoughts might be. So, for example, it might be a proposition of logic that for any thought of Russell's form above (let us write it 'Fab'), if that thought is true, then so is the corresponding thought, $(\text{Ex})\text{Fxb}$. In that 'Fab', for example, that 'F' would stand for a two-place first-order predicative element (here occurring within a certain form). That it is of the type it is would be contained in its style—e.g., a capital letter. 'F' rather than 'G' would track recurrence, on an assignment of forms to a corpus of (decomposed) thoughts, of what it is thus assigned to.

The general idea is this. Take the thought that the six-pack is hurtling towards the windshield. It is none of logic's business how that very thought might be decomposable (except that any decomposition must be of *some* logical form). So nor does logic tell us *what* logical forms are to be carved out of that thought (except, again, only logical forms there are). But once that extra-logical work has been performed for some corpus, and an assignment of logical forms made to it accordingly, logic tells us what we must then live with.

F* began from the idea that logic is even-handed; equally about any thought as any other. As F* read this idea, those thoughts which were logic's dicta would need maximum generality. On the present conception even-handedness is achieved in another way. Logic is about something very specific. Logic's dicta do have specific, proprietary, content. But what logic is about is, of necessity, something to be found in any thought. It is intrinsic to a thought to entail and be entailed. So it is intrinsic to a thought to stand in logical relations. Logic just tells us all the possible forms of doing that.

A law of logic, like a law of mechanics, is a generalisation which holds across the relevant domain. If a conjunction is true, then so are all its conjuncts—a law of *conjunction*. A theory of mechanics aims to *state* what are in fact its laws. A *theory* of logic, or of inference, would aim to do the same. A *theory, though*, is but one of two familiar ways of expressing logic's content (neither way a *language*). The content of logic might also be expressed in a calculus. A (standard) calculus consists, first, of some vocabulary (set of signs), and some rules for generating from this some set of wholes (its 'formulae'); then, second, another set of rules for constructing sequences of such wholes. Its formulae represent (i.e., stand in for, incarnate) logical forms. Its sequences stand in for truth-preserving sequences of forms, or, if you like, forms of proof. An English sentence, say, 'Bachelors smoke', speaks of bachelors as smokers. A formula of a calculus, say, 'A&B', speaks of nothing. A *theory* of logic, by contrast, would be a set of *statements*, employing such predicates as, '___ is a predicative element', '___ is a naming element', '___ is a conjunction', and '___ follows from ___'. A theory which stated that all bachelors smoke would be committed to all bachelors smoking. If some do not, it is false. The sentence, 'All bachelors smoke' has no such

commitments, hence is not false. At best it merely speaks of something which is not the case.

The nice thing about a calculus' way of expressing logic is that it draws a clean distinction between logical and extra-logical work. A (successful) calculus expresses logic's content in its application to given corpuses of thoughts, each mentioned, or expressed, in some one particular way. Suppose, e.g., that a given corpus contains a thought which, recognisably, so far as it matters for our current purpose, would be true *just* where two other thoughts were true. (As may be, the thought is that Sid drinks and Pia eats.) Suppose that, in a given propositional calculus, 'A' and 'B' are atomic formula, and that, here, in our current assigning of forms to thoughts, they are not yet in use by us. Then the first thought above might be assigned that form (relative to all else in the corpus) which is named in the calculus by the formula, 'A&B'. If there is some other thought in the corpus which, recognisably, would be true just in case it is not so that that thought 'A' already stands for is true while some further thought (not yet assigned a form) is false, then, if the atomic formula 'C' in the calculus is not yet in use, we can, so far, in our assignment of forms to the whole corpus, assign this other thought a thought named (here) by the formula 'A→C'. If, now, both these assignments are to *true* thoughts, then 'C' has also been assigned to a true thought.

Our changed conception of logic brings with it a changed understanding of Frege's obstacle. Section II considered an idea which amounts to this: the hierarchy of logical forms (forms of ways to represent things things as being something) imposes a corresponding hierarchy on items in the world. There is a class of items which may be subjects of first-order predication, a class of items which may be subjects of second-order predication, and so on. Otherwise put, there is the class of items fit to be ingredients in naming elements, the class of items fit to be ingredients in first-order predicative elements, and so on. These classes are, as a matter of principle, disjoint. It is not just that the class of objects—the first class on the above lists—contains many things which could never function predicatively. Rather, nothing which could *ever* function as ingredient in a naming element could *ever* function predicatively. The hierarchy, in so categorising *things*, thus poses the obstacle. (I here illustrate with just one of the several hierarchies which *might* be involved.)

If logic were thought of as structuring some domain to which all thoughts belonged, there might be some plausibility in this idea. For such structuring presupposes some one assignment of decompositions, correlatively logical forms, simultaneously, to *all* thoughts (whatever such an 'all' might mean). Here are the ways *each* thought relates to *all* others, each a way it relates to all others full stop, independent of any extra-logical consideration—hence, each way co-existent, conjoinable, with each other. There is no room here for the idea of making assignments to corpuses, sometimes in this way, sometimes in another, where making one (sometimes) admissible assignment may exclude *then* making some other (sometimes) admissible one; no room for the idea that not all admissible assignments can be made at once. Then, perhaps, one or another of our hierarchies has metaphysical conclusions. For then the question whether what is *ever* put to the work of an object in a naming element can ever be put to the work of a concept in a predicative element is simply the question whether a certain combination of forms can occur in an assignment of forms to a corpus. There is no room for the idea that *different* assignments, perhaps to different corpuses, might treat the same thing differently. In rejecting F*, though, and conceiving of logic as about logical forms, *that* route from a fact about *representational* structure to a conclusion about the structure of the world collapses.

For any *assignment* of logical forms to any given corpus of thoughts, there will be those

items which function (somewhere) as ingredients in naming elements, so are treated (hence treatable) on that assignment as objects, those things which function as ingredients in first-order predicative elements, so are treated (and treatable) as concepts (concepts₁), those things which function as ingredients in second-order predications, and so on. Perhaps these respective classes of things must be disjoint if the *assignment* is to be coherent. Nothing follows from this as to whether what is an ingredient at one level on one assignment (say, some function) might be an ingredient at another level on another.

The *Tractatus* idea that logic's dicta say nothing is rooted in that conception of logic which Frege (by repute) and young Wittgenstein share. Conant suggests two reasons, within that shared conception, for so holding. One is that logic's dicta are too general. They speak of nothing in particular. They have nothing but their structure to make them true. And, to use the *Tractatus*' image, *pure* structure could do that only in its parts cancelling each other out. The other is: one could not decide any question of truth without presupposing all of logic; so one could not decide the question of truth a dictum raised (if it raised one) without presupposing that dictum itself. Hence there can be no such question to decide. On our new conception, the first idea is simply untrue. Something of the second may remain. As Putnam suggests (1994), perhaps one cannot so much as entertain the idea of the falsity of a logical dictum until confronted with sufficient details as to how such might be. Frege's laws of truth, for example, unfold a particular conception of truth, and of questions thereof. There are the *pure* questions there are to pose; each of these has an answer, 'Yes' or 'No'; any set of these has such answers simultaneously (whether some such question has an answer cannot depend on the answer to any other). Given all this the features of those connectives of thoughts which Frege recognises follow trivially. This notion *pure question of truth*, though, is an abstraction from our actual representing. That *this* sometimes misfires, so that a *statement* might be only contingently either true or false is beside the point. The abstraction is similarly resistant to many other *apparent* threats to these features of what underlies it. Still, such features are recognisable in it, and, recognisably, such as *might* prove *assumptions* with alternatives. A suggestion that truth does not have *quite* this structure need not prove unevaluable.

What *do* they say? And to what might this be hostage? They might, of course, say nothing in the sense that there is no saying *what* they say, though not in the sense that there is no such thing as the world bearing on whether what they say is so. At first glance such might seem so. A dictum might be, for example, that if a conjunction is true, then so are all its conjuncts. Informative, it might seem at first. But if we pursue the question what a conjunction is to be understood to be here, the answer is: by definition, a compounding of thoughts such that the compound is true just in case all its component thoughts are. At which point the law once again appears empty. (Though still not because of its great generality.)

But if logic has commitments, these plausibly lie elsewhere. Logic generates a set of logical forms. These are, according to it, the forms there *are* for thoughts to take. So wherever we identify a thought—wherever a question of truth arises, or is encountered—this thought is always correctly assignable *some* such form. Thoughts are so representable. No *thought* could fail to be assignable at least *some* form logic generates. For any form, there are thoughts which take it. Further, those transitions from thoughts to thoughts which logic licenses are, not just truth-preserving but thought-hood preserving, so that for any (correct) assignment of forms to a corpus, there is an extension of it which is closed under the inferences logic licenses. Conjunction, for example, distributes over disjunction. It is not as if all this is *just* a

theory; nor as if there is any specifiable way in which all, or some, of it *might* prove false. Still, on our present conception, laws of logic are substantial thoughts. Finding ways in which experience *might* bear on the truth of a thought is extra-logical work. Nothing in logic itself can rule out there being such a way. Logic, by definition, applies to *all* thoughts. Whether its specific dicta would *be* its dicta no matter what—thus unmovable boundary stones in an eternal foundation—is another question.

3.7. Concluding: In 1945 Wittgenstein wrote that in 1929 he was forced to recognise ‘grave errors’ in the *Tractatus* (1953: x). As he worked his way from these erroneous views to the changed view of *Philosophical Investigations*, a central locus of such errors proved to be the *Tractatus*’ conception of logic. Thus, in the *Investigations* the *Tractatus*’ idea that logic is not a science—that there could be nothing substantial about it, that its dicta were in principle immune to being borne on by experience—became the more nuanced idea that

Logic does not treat language—or thought—in the sense in which a natural science treats a natural phenomenon ... (§81)

Logic is unlike a science in that its applications are unlike those of a science. As Wittgenstein writes in §81, we “compare the use of words with games and calculi” Logic provides, as do those calculi which express it, forms in which to represent those representations whose relations to one another may concern us; forms by which (if assigned correctly) a particular sort of bearing of the truth of some thoughts on that of others is then made clear. But on the question what a thought’s form *really* is, or that of when a thought would be of such-and-such form, logic is silent. Assigning thoughts forms is extra-logical work, turning on extra-logical questions as to how *thought* is ‘really’ (or for some purpose) to be articulated into thoughts. Correspondingly, as to those questions it thus leaves to us, logic does not require them to have an answer (at least *tout court*). Logic does not, as the *Tractatus* supposes, and as a science might, describe ‘the most general scaffolding of the world.’ It provides ways of *structuring* representations of the world.

In our thoughts the ideal is immovable. You cannot step outside it. You always have to turn back. There is no outside at all; outside there is no air. —Where does this come from? It is like a pair of glasses on our nose, through which we see all that we do. We never think to take them off. (*Philosophical Investigations*, §103)

We predicate of the thing what lies in the method of representing it. We take that possibility of a comparison which impresses us for perceiving the most general state of affairs. (§104)

In philosophy we are too often blind to the place, and the burden, of extra-logical work. So it was with Frege in his difficulty about concepts and objects. We suppose thoughts, or propositions, to have ‘that formal unity’ (as he puts it in §108) that would be had, as it is by formulae of a calculus, by members of some determinate system, generated by given rules from a given vocabulary, or set of building blocks (the very thing Frege himself, in his best

moments, insisted that thoughts could not be). We fail to see the idea *a thought* as an abstraction from our historical representing; one which may be done in any of many ways for any of many purposes. A concern with such blindness, and its effects on our posing, and trying to answer, the particular questions we (philosophers) do permeates the *Investigations*, and precisely distinguishes it from the *Tractatus*—a work suffering from that very blindness itself.

Charles Travis

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