Jason Zarri

Philosophical Logic Workshop

Bas van Fraassen

5/27/13

Ways Modality Could Be

~ On the Possibility and Prospects of Higher-Order Modal Logic ~

The modal logics of physical and metaphysical necessity are certainly at least as strong as Kρ: If A's truth is determined by the laws of physics/metaphysics, then A is true. But it is not clear that they are any stronger. For example, it is determined by the laws of physics that I do not accelerate through the speed of light. But why should this fact itself be determined by the laws of physics…? Similarly, I am not a frog, and so it is metaphysically possible that I am not a frog. But is that *fact true because of the essence of something…? The essence of possibility?*

— Graham Priest, An Introduction to Non-Classical Logic: From If to Is, 2nd edition, p. 47.

1. *Introduction*

In this paper I introduce the idea of a higher-order modal logic—not a modal logic for higher-order predicate logic, but rather a logic of *higher-order modalities*. "What is a higher-order modality?", you might be wondering. Well, if a first-order modality is a way that some entity could have been—whether it is a mereological atom, or a mereological complex, or the universe as a whole—a higher-order modality is a way that a first-order modality could have been. First-order modality is modeled in terms of a space of possible worlds—a set of worlds structured by an *accessibility relation*, i.e., a relation of relative possibility—each world representing a way that the entire universe could have been. A second-order modality would be modeled in terms of a space of spaces of (first-order) possible worlds, each space representing a way that (first-order) possible worlds could have been. And just as there is a unique *actual world* which represents the way that things actually are, there is a unique *actual space* which represents the way that first-order modality actually is.

One might wonder what the accessibility relation itself is like. Presumably, if it is logical or metaphysical modality that is being dealt with, it is reflexive; but is it also symmetric, or transitive? Especially in the case of metaphysical modality, the answer is not clear. And whichever of these properties it may or may not have, could *that itself* have been different? Could at least *some* rival modal logics represent different ways that first-order modality could have been?

To be clear, the idea behind my proposal is not *just* that some things which are possible or necessary might not have been so at the first order, as determined by the actual accessibility relation, but also that the actual accessibility relation, and hence *the nature or structure of actual modality, could have been different at some higher order of modality.* Even if the accessibility relation is *actually* both symmetric and transitive, perhaps it could (second-order) have been otherwise: There is a (second-order) possible space of worlds in which it is different, where it fails to be symmetric, or transitive. We must, therefore, introduce the notion of a higher-order accessibility relation, one that in this case relates *spaces* of first-order worlds. The question then arises as to whether *that* relation is symmetric, or transitive. We can then consider third-order modalities, spaces of spaces of spaces of possible worlds, where the second-order accessibility relation differs from how *it* actually is. I can see no reason why there should be a limit to this hierarchy of higher-order modalities, any more than I can see a reason why there should be a limit to the hierarchy of higher-order *properties*. There will thus be an infinity of orders, one for each positive integer, and each order will have an accessibility relation of its own. To keep things as clear as possible, a space of first-order points (i.e., of possible worlds) shall be called a *galaxy*, a space of second-order points, a *universe*, and a space of any higher order, a *cosmos*. However, to keep things as *simple* as possible, in what follows I will deal with but a single cosmos, and hence will *not* deal with modalities higher than the third order.

The accessibility relation is not the only thing that might be thought to vary between spaces of worlds: Perhaps the contents of the spaces can vary as well. While I presume that the contents of the worlds themselves remain constant—it makes doubtful sense to suppose that in one space some entity e exists in a world w and in another space e doesn't exist in that same world w—we may suppose that different spaces differ as to *which worlds they contain*, just as different worlds may differ as to which objects they contain. Thus we might have a higher-order analogue of a variable-domain modal logic. There seem, then, to be three ways in which spaces can differ: First, as to the properties of the accessibility relation; second, as to which worlds the relation relates; and third, as to which worlds or spaces are parts of their domains.

The paper will be structured as follows. In Section 2 I provide some reasons why one might want to pursue this kind of project in the first place. In Section 3 I outline the syntax and semantics of my proposed logic. Section 4 covers semantic tableaux for this system; and after giving the rules for their construction, I construct a few of them myself to establish some logical consequences of the system and give the reader a feel for how it works. In Sections 5, 6 and 7 I explore some of its potential philosophical implications for areas besides logic, namely the philosophy of language; metaphysics, including the metaphysics of modality and the philosophy of time, and finally the philosophy of religion, before concluding the paper in Section 8.

2. *Motivation*

Why should we adopt a framework such as the one I have just described? To motivate it, consider the fact that people have mutually conflicting intuitions about what the space of all (first-order) possible worlds is like. For example, does God exist in all, none, or only some worlds? Each of these positions, if true, is necessarily true, and if false, is necessarily false. God either exists in all, none, or only some worlds, and on the standard view the actual space of worlds could not have been different, because it is the only space of possible worlds that there is.

On the face of it, this is problematic for the view that conceivability implies possibility: Each of these positions has been believed, and by very able philosophers at that. What is believed is conceivable in some sense; otherwise, such "beliefs" would have no content. So each position is conceivable, but only one is possible. No matter which of them holds, conceivability doesn't imply possibility.

But maybe that's not *quite* true. Perhaps, though only one of these positions is actually true, and hence first-order possible, each is *second-order possible*. So maybe conceivability *does* imply possibility—at some order or other. Related considerations might apply to semantic content and possibility: If we can coherently mean something, it can be the case—at some order or other.

Or consider Kripke's doctrine of the essentiality of origins. According to him, for example, your parents are essential to your existence; any metaphysically possible world where you exist is a world in which you have the same parents that you do in *this* word, the actual world. Even if we assume that this is so as a matter of first-order metaphysical necessity, it seems to be higher-order contingent: Surely your essentially having the parents you do is not a matter of *logic*, but in that case it is logically possible for it to be otherwise. Furthermore, it doesn't seem to be a matter of logic that it is *(first-order) metaphysically necessary* that you have the parents you do, and hence it seems *logically possible* that it is *not* (first-order) metaphysically necessary that you have them as your parents. Perhaps, then, even if every metaphysically possible world in the actual space of worlds is one in which you have the parents that you actually do, the actual galaxy accesses a possible galaxy which contains metaphysically possible worlds where you have different parents.

Finally, and perhaps most importantly, consider the fact that there are several modal logics that are mutually inconsistent, in the sense that two or more such logics cannot correctly describe the structure of the same space of worlds. These logics all seem to be perfectly good, and perfectly *intelligible*, as systems of modal logic. Even if I suppose that S5, for instance, correctly represents the actual structure of metaphysical modality, I can wonder how things would have differed if the accessibility relation had lacked certain properties; and another system, say S4, can tell me exactly what characteristically *modal* arguments would have been valid if that had been so—or so it seems to me. S4 certainly constitutes a *representation* of the structure—or better, *a* structure—of metaphysically modality, even if it is *actually* a necessarily false one. *Qua* representation, it seems to me to have much the same status as our more customary representations of how certain things about our world would have turned out if certain other things about our world had been different. In much the same way that we take these representations to depict ways things could have been, why couldn't we take S4 and other modal logics to depict ways *modality* could have been?

I should make it clear that I do not expect the kind of framework I propose to *settle* the issue of how modality at any order actually is—no more than I expect ordinary first-order modal logic to *settle* (aside from first-order necessary truths) what is actually the case. What goes for the actual world goes for the actual space of worlds, and for all higher-order spaces of spaces. What I do hope for is that it will, if it proves to be coherent, help to clarify the terms of the debate about the way modality is—to help us to state the issues, and to see their interrelations, as clearly as we can.

3. *Setting up the System: Syntax and Semantics*

In this section I shall describe the syntax and semantics of my proposed system, which I shall call ‘HOML’, an acronym for Higher-Order Modal Logic.

3.1 *Syntax*

First, we have the *syntax*:

1) I shall use $\Box_n$ and $\Diamond_n$, respectively, for necessity and possibility operators of the order n, for positive integers 1, 2 and 3.

2) I shall use $\Box_n^m$ and $\Diamond_n^m$, respectively, for an iteration of m necessity or possibility or necessity operators of the order n.

3) For atomic sentences, we will use capital letters from the entire alphabet, with numerical subscripts appended to them if necessary.

4) As for connectives, we will use the symbols $\neg$, $\wedge$, $\vee$, $\supset$, and $\equiv$, respectively, for negation, conjunction, disjunction, material implication, and material equivalence.

5) For brackets one can use parentheses, (,) ; square brackets, [,] ; or curly brackets, {, }. It makes no difference which brackets are used where.

6) I will use 1 and 0 as truth values, 1 for *true* and 0 for *false*.

7) Finally, the usual recursive clauses for constructing *well-formed formulas*—*wffs*, pronounced “woofs,” for short—from atomic sentences will be adopted. All atomic sentences of HOML are wffs, and where p and q are arbitrary sentences of HOML:

1. If p is a wff, so are ¬ (p), $\Box_1$ (p), $\Box_2$ (p), $\Box_3$ (p), $\Diamond_1$ (p), $\Diamond_2$ (p), and $\Diamond_3$ (p).[1]
2. If p and q are wffs, so is (p ∧ q).
3. If p and q are wffs, so is (p ∨ q).
4. If p and q are wffs, so is (p ⊃ q).
5. If p and q are wffs, so is (p ≡ q).
6. Nothing else is a wff of HOML.

3.2 *Semantics*

1) Any set of worlds that is structured by an accessibility relation, or a higher-order counterpart, is a *space*, and its members are *points*. As above, a space of first-order points (i.e., of possible worlds) shall be called a *galaxy*, a space of second-order points, a *universe*, and a space of any higher order, a *cosmos*. Points of these orders shall be represented by expressions of the forms w^x, g^y, and u^z, as mnemonics, respectively, for 'world x', 'galaxy y', and 'universe z'.

2) Every space has within it an accessibility relation holding between its points, and as every non-base point is itself a space, it will have an accessibility relation holding between *its* points. Thus, when considering a given universe of galaxies, one must take into account the fact that there will be a different accessibility relation for each galaxy, and that the properties of these relations may differ. So in this universe, there will be an accessibility relation of the second order which holds between its galaxies, and many accessibility relations of the first order holding between the possible worlds within the galaxies. If the universe we are considering is u^1, I will call the accessibility relation that holds between its galaxies $R^1{}_g$: The superscripted '1' means that the relation holds within universe u^1, and the subscripted 'g' means that it holds between galaxies. If, within u^1, we are considering the galaxy g^3, I will similarly call the accessibility relation that holds

[1]However, if p is an atomic sentence, one doesn't have to put brackets around it.

between its worlds R^3_w. Here the '3' indicates that we are dealing with the relation that holds within galaxy g^3, and the 'w' indicates that it holds between possible worlds.

3) Model structures: A *cosmos* is a 6-tuple <**W**, **G**, **U**, **RW**, **RG**, **RU**> where:

W is a non-empty set, its members are possible worlds

G is a non-empty set of subsets of **W**, its members are galaxies,

U is a non-empty set of subsets of **G**, its members are universes,

RW = $\{R^y_w$: g^y in **G**$\}$ is a set of access relations, one for each galaxy, and defined on that galaxy, holding between possible worlds.

RG = $\{R^z_g$: u^z in **U**$\}$ is a set of access relations, one for each universe, and defined on that universe, holding between galaxies.

RU = $\{R_u\}$ (no superscript necessary in this case) is the unit set of the access relation defined on the cosmos, holding between universes.

An evaluation point is a triple $<w^x g^y u^z>$, with a world w^x in **W**, a galaxy g^y in **G**, and a universe u^z in **U** and such that w^x is in g^y and g^y is in u^z.

4) Truth-values are assigned to sentences relative to these points, like so:

For an atomic sentence, the truth-value depends only on w (it is the same for $<w^x, g^y, u^z>$ and $<w^x, g^i, u^k>$). Writing the evaluation function, which assigns the semantic values 1 and 0 to sentences as v (p) @ $<w^x g^y u^z>$, meaning the value of p at the triple $<w^x g^y u^z>$, I define:

1) $v(\neg p)$ @ $<w^x g^y u^z> = 1$ iff $v(p)$ @ $<w^x g^y u^z> = 0$

2) $v(p \wedge q)$ @ $<w^x g^y u^z> = 1$ iff $v(p)$ @ $<w^x g^y u^z> = 1$ and $v(q)$ @ $<w^x g^y u^z> = 1$

3) $v(p \vee q)$ @ $<w^x g^y u^z> = 1$ iff $v(p)$ @ $<w^x g^y u^z> = 1$ or $v(q)$ @ $<w^x g^y u^z> = 1$

4) $v(p \supset q)$ @ $<w^x g^y u^z> = v(\neg(p \wedge \neg q))$ @ $<w^x g^y u^z>$

5) $v(p \equiv q)$ @ $<w^x g^y u^z> = v((p \supset q) \wedge (q \supset p))$ @ $<w^x g^y u^z>$

6) $v(\Box_1 (p))$ @ $<w^x g^y u^z> = 1$ iff, for all worlds w' in g^y in u^z that w^x accesses, v(p) @ $<w' g^y u^z> = 1$.

7) $v(\Box_2(p)) @ <w^x g^y u^z> = 1$ iff, for all worlds w' in all galaxies g' in u^z that g^y accesses, $v(p) @ <w' g' u^z> = 1$.

8) $v(\Box_3(p)) @ <w^x g^y u^z> = 1$ iff, for all worlds w' in all galaxies g' in all universes u' that u^z accesses, $v(p) @ <w' g' u'> = 1$.

9) $v(\Diamond_1(p)) @ <w^x g^y u^z> = 1$ iff, for some world w' in g^y in u^z that w^x accesses, $v(p) @ <w' g^y u^z> = 1$.

10) $v(\Diamond_2(p)) @ <w^x g^y u^z> = 1$ iff, for some world w' in some galaxy g' in u^z that g^y accesses, $v(p) @ <w' g' u^z> = 1$.

11) $v(\Diamond_3(p)) @ <w^x g^y u^z> = 1$ iff, for some world w' in some galaxy g' in some universe u' that u^z accesses, $v(p) @ <w' g' u'> = 1$.

5) Definition of *satisfaction*: 1 is the sole designated value in HOML. A sentence p is *satisfied* with respect to a point of evaluation $<w^x g^y u^z>$ iff it is assigned a designated value at $<w^x g^y u^z>$. A set **X** of sentences is *satisfied* with respect to a point of evaluation iff every member of **X** is satisfied at that point.

6) *Logical consequence.* We say that a sentence p is a *logical consequence* of a set **X** of sentences, which we write as **X** $\models$ p, iff in every model, i.e., in every cosmos **C** where **X** is satisfied with respect to a point of evaluation, p is also satisfied at that point. If there is some cosmos **C** where **X** is satisfied with respect to a point of evaluation and p is not, we say that **C** is a *countermodel* to **X** $\models$ p. Furthermore, if **X** is the empty set and **X** $\models$ p, we say that p is a *logical truth* or *logically valid sentence*, and may imply write $\models$ p.

7) *Restricted logical consequence.* We say that a sentence p is a *restricted logical consequence* of a set **X** of sentences iff in every cosmos **C**, *such that C has certain desired properties that are not guaranteed by the semantics alone*, and where **X** is satisfied with respect to a point of evaluation, p is also satisfied at that point. The desired properties are properties of the accessibility relations involved in the cosmos, namely being reflexive (abbreviated as 'REF.'), symmetric (SYMM.), transitive (TRANS.), or serial (SER.). If p is a restricted logical consequence of **X** when we assume that the

accessibility relations have one or more of these properties, we write **X** $\models$ p (or $\models$ p if p is a *restricted theorem*, i.e., a consequence of the empty set if certain assumptions about accessibility relations are made) followed the names of the relation(s) and the abbreviation(s) of the assumed property(ies) enclosed in square brackets, followed by a comma, and then followed by any other relation(s) and the abbreviation(s) of the assumed property(ies) enclosed in square brackets, and so on, until they have all been mentioned. For example, if p is a logical consequence of X when we assume that R^y_w is symmetric, we write **X** $\models$ p [R^y_w SYMM]. And if p is a logical consequence of X when we assume that R^z_g is both reflexive and transitive, we write **X** $\models$ p [R^z_g REF. TRANS.]. As a final example, if p is a logical consequence of X when we assume that R^z_g is both reflexive, symmetric and transitive and that R_u is reflexive, we write **X** $\models$ p [R^z_g REF. SYMM. TRANS.], [R_u REF.].

4. *Semantic Tableaux.*

In this section I first describe the tableaux rules associated with HOML, and then construct some actual tableaux to establish some consequences of the system.

4.1. *Tableaux Rules*

I will use semantic tableaux to establish *derivability*. We say that a sentence p is a *derivable from* a set **X** of sentences, which we write as **X** $\vdash$ p, iff every tableau containing every member of **X** at the root node, followed by the negation of p at the root node is *closed*. A tableau is closed iff every one of its branches is *closed*, and a branch is *closed* iff for some sentence q that branch contains both q and the negation of q. Furthermore, every node of a tableau will be followed by a comma and the name of the evaluation point at which it holds. Tableaux for the truth-functional connectives will have the following forms:

$$\begin{array}{c} \neg\neg (p), \langle w^x g^y u^z \rangle \\ | \\ p, \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{c} p \wedge q, \langle w^x g^y u^z \rangle \\ | \\ p, \langle w^x g^y u^z \rangle \\ q, \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{\neg (p \wedge q), \langle w^x g^y u^z \rangle} \\ / & \backslash \\ \neg (p), \langle w^x g^y u^z \rangle & \neg (q), \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{p \vee q, \langle w^x g^y u^z \rangle} \\ / & \backslash \\ p, \langle w^x g^y u^z \rangle & q, \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{c} \neg (p \vee q), \langle w^x g^y u^z \rangle \\ | \\ \neg (p), \langle w^x g^y u^z \rangle \\ \neg (q), \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{p \supset q, \langle w^x g^y u^z \rangle} \\ / & \backslash \\ \neg (p), \langle w^x g^y u^z \rangle & q, \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{c} \neg (p \supset q) , \langle w^x g^y u^z \rangle \\ | \\ p, \langle w^x g^y u^z \rangle \\ \neg (q), \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{p \equiv q, \langle w^x g^y u^z \rangle} \\ / & \backslash \\ p, \langle w^x g^y u^z \rangle & \neg (p), \langle w^x g^y u^z \rangle \\ q, \langle w^x g^y u^z \rangle & \neg (q), \langle w^x g^y u^z \rangle \end{array}$$

$$\begin{array}{cc} \multicolumn{2}{c}{\neg (p \equiv q) , \langle w^x g^y u^z \rangle} \\ / & \backslash \\ p, \langle w^x g^y u^z \rangle & \neg (p), \langle w^x g^y u^z \rangle \\ \neg (q), \langle w^x g^y u^z \rangle & q, \langle w^x g^y u^z \rangle \end{array}$$

Tableaux for modal operators will have the following forms:

$$\begin{array}{c} \Diamond_1 (p), \langle w^x g^y u^z \rangle \\ | \\ w^x \ R^y{}_w \ w^i \\ | \\ p, \langle w^i g^y u^z \rangle \end{array}$$

(where w^i is new to the branch),

$$\begin{array}{c} \Box_1 (p), \langle w^x g^y u^z \rangle \\ | \\ w^x \ R^y{}_w \ w^i \\ | \\ p, \langle w^i g^y u^z \rangle \end{array}$$

(for all worlds that are members of g^y)

$$\begin{array}{c} \Diamond_2\,(p),\ \langle w^x g^y u^z \rangle \\ | \\ g^y\ R^z_g\ g^j \\ | \\ p,\ \langle w^i g^j u^z \rangle \end{array} \qquad\qquad \begin{array}{c} \Box_2\,(p),\ \langle w^x g^y u^z \rangle \\ | \\ g^y\ R^z_g\ g^j \\ | \\ p,\ \langle w^i g^j u^z \rangle \end{array}$$

(where g^j is new to the branch and w^i is a member of it), (for all worlds of all galaxies which g^y accesses that are members of u^z)

$$\begin{array}{c} \Diamond_3\,(p),\ \langle w^x g^y u^z \rangle \\ | \\ u^z\ R_u\ u^k \\ | \\ p,\ \langle w^i g^j u^k \rangle \end{array} \qquad\qquad \begin{array}{c} \Box_3\,(p),\ \langle w^x g^y u^z \rangle \\ | \\ u^z\ R_u\ u^k \\ | \\ p,\ \langle w^i g^j u^k \rangle \end{array}$$

(where u^k is new to the branch, g^j is a member of u^k, and w^i is a member of g^j), (for all worlds of all galaxies of all universes which u^z accesses)

In addition, where n can be either 1, 2 or 3, we have:

$$\begin{array}{c} \neg\,\Diamond_n\,(p),\ \langle w^x g^y u^z \rangle \\ | \\ \Box_n \neg\,(p),\ \langle w^x g^y u^z \rangle \end{array} \qquad\qquad \begin{array}{c} \neg\,\Box_n\,(p)\ \langle w^x g^y u^z \rangle \\ | \\ \Diamond_n \neg\,(p),\ \langle w^x g^y u^z \rangle \end{array}$$

4.2. *Some Semantic Tableaux*

I will now construct a few schematic semantic tableaux in order to establish some consequences of the system. First, I will establish that $\Box_2\,(p) \vdash \Box_1\,(p)$ [$R^1{}_g$ REF.] :

$$\Box_2 (p), \langle w^1 g^1 u^1 \rangle$$
$$\neg \Box_1 (p), \langle w^1 g^1 u^1 \rangle$$
$$|$$
$$\Diamond_1 \neg (p), \langle w^1 g^1 u^1 \rangle$$
$$|$$
$$w^1 \; R^1_w \; w^2$$
$$|$$
$$\neg (p), \langle w^2 g^1 u^1 \rangle$$
$$|$$
$$g^1 \; R^1_g \; g^1 \text{ [REF.]}$$
$$|$$
$$p, \langle w^2 g^1 u^1 \rangle$$
$$\mathbf{x}$$

Thus, if something is second-order necessary it is also first-order necessary. We can say that necessity is *hereditary downwards*.

Second, I will establish that $\Diamond_1 (p) \vdash \Diamond_2 (p)$ [R^1_g REF.] :

$$\Diamond_1 (p), \langle w^1 g^1 u^1 \rangle$$
$$\neg \Diamond_2 (p), \langle w^1 g^1 u^1 \rangle$$
$$|$$
$$\Box_2 \neg (p), \langle w^1 g^1 u^1 \rangle$$
$$|$$
$$w^1 \; R^1_w \; w^2$$
$$|$$
$$p, \langle w^2 g^1 u^1 \rangle$$
$$|$$
$$g^1 \; R^1_g \; g^1 \text{ [REF.]}$$
$$|$$
$$\neg (p), \langle w^2 g^1 u^1 \rangle$$
$$\mathbf{x}$$

Thus if something is first-order possible it is also second order possible. So possibility is *hereditary upwards*.

Third, I will establish that $\Box_2 (p) \vdash \Box_1 \Box_2 (p)$:

$$
\begin{array}{c}
\Box_2 (p), \langle w^1 g^1 u^1 \rangle \\
\neg \Box_1 \Box_2 (p), \langle w^1 g^1 u^1 \rangle \\
| \\
\Diamond_1 \neg \Box_2 (p), \langle w^1 g^1 u^1 \rangle \\
| \\
w^1 \ R^1{}_w \ w^2 \\
| \\
\neg \Box_2 (p), \langle w^2 g^1 u^1 \rangle \\
| \\
\Diamond_2 \neg (p), \langle w^2 g^1 u^1 \rangle \\
| \\
g^1 \ R^1{}_g \ g^2 \\
| \\
\neg (p), \langle w^3 g^2 u^1 \rangle \\
| \\
(p), \langle w^3 g^2 u^1 \rangle \text{ (from the first premise and “}g^1 \ R^1{}_g \ g^2\text{”)} \\
\mathbf{x}
\end{array}
$$

Thus, if something is second-order necessary, it is first order necessary that it is second order necessary.

I believe these three tableaux will suffice to give the reader an idea of how to construct tableaux for HOML, and an appreciation of some of its general features.

5. *Implications for the Philosophy of Language*

I believe that my account has consequences for the philosophy of language, specifically, for a variety of counterfactual conditionals known as *counterpossibles*. Counterpossibles are counterfactuals that have impossible antecedents. As an example, let us take once more Kripke's thesis of the essentiality of origins. Let's suppose that Kripke is right, and that one essentially has the parents one actually has. Let us also suppose that the framework I have proposed is correct, and additionally that it is second-order possible that one has different parents from one's actual ones. One can then make good sense of the counterpossible conditional, “If the thesis of the essentiality of origins were false, Quine could have had Carnap for a father.” On the standard semantics, this is true, but vacuously so. For granting the essentiality of origins the antecedent is impossible, and on the standard semantics all counterfactuals with impossible antecedents are true, including the conditionals “If the thesis of the essentiality of origins were false, Quine *could not* have had Carnap for a father,” and “If the thesis of the essentiality of origins were false, Quine would have been a fried egg.” On my approach, however, one could construct a semantics in which, if a sentence is not true at any world in the actual galaxy, one can look to possible worlds in other possible galaxies. The first conditional can then come out as non-vacuously true, and the second and third as non-vacuously false. Moreover, one need not invoke “intrinsically impossible” worlds—worlds which are impossible *full stop,* in and of themselves—but only worlds which are impossible *with respect to* worlds in the actual galaxy. (One could, of course, modify my framework to include such worlds, perhaps to accommodate counterpossibles whose antecedents contravene logical laws, but one would then have to make non-trivial changes to HOML.)

6. *Implications for Metaphysics*

I believe that my account is relevant to two branches of metaphysics; the metaphysics of modality and the philosophy of time.

6.1. *The Metaphysics of Modality*

In contrast to Section 5, I believe that my account has no significant consequences for the metaphysics of modality, with the exception of Lewis's modal realism. Those who are not Lewisian modal realists may take possible worlds be various abstract entities: properties, or sets of properties, or maximal sets of propositions, or Plantingan maximal states of affairs, or perhaps something different still. Whichever of these views one takes, I think adopting my framework simply gives one “more of the same”: either higher-order properties, or sets of sets of properties, or sets of maximal sets of propositions, or higher-order Plantingan maximal states of affairs (or sets of Plantingan maximal states of affairs). Something similar, I take it, goes for instrumentalism: If one can be an instrumentalist about ordinary possible worlds semantics, one can be an instrumentalist about HOML as well. If I'm right about all this, the higher-order versions of each of these views will have the same sorts of pros and cons as the first-order views of which they are extensions, and hence adopting my framework should leave the debate over the correct metaphysics of modality much as it was before.

I made an exception for modal realism because the notion of a higher-order accessibility relation doesn't make much sense on that view, but then first-order accessibility relations don't make much sense on that view either. On the Lewisian picture, possible worlds are maximal spatio-temporally connected concrete entities; and since his is a *reductive* account of modality, there's not much, if any, content to the 'possible' in 'possible world': Their “possibility” amounts to their *existence,* and perhaps also to the fact that it is analytically true that nothing existent can contain the Russell set, married bachelors, round squares, and the like. A primitive notion of relative possibility could then make sense only if a primitive notion of relative existence made sense, which (I think) it does not. And if the notion of relative possibility were not primitive, on the Lewisian picture, such relations would have to supervene on the non-modal properties and relations of possible worlds. It would then make no sense to suppose that such relations could be different from how they actually are. My framework, then, is incompatible with modal realism, if it is taken to be a reductive account of modality. There is, however, no incompatibility between my framework and the existence of multiple maximal spatio-temporally connected concrete entities, only with the idea that modality is correctly analyzed in terms of them.

6.2. *The Philosophy of Time*

There is no reason why a framework like mine must be limited to logical and metaphysical modality: One could extent HOML to get higher-order tense logic(s). "Possible worlds" would become moments of time, "possible galaxies" would become intervals of time, or even entire timelines, and could themselves be considered "second-order moments" of "second order time." The second-order accessibility relation would be replaced with *second-order earlier than* and *second-order later than* relations. The idea here would be that second-order moments represent different ways that first-order moments can be related by the first-order earlier than and later than relations, and that which of those first-order relations obtains could change over the course of second-order time. Second-order moments could also, of course, differ as to which first-order moments they contain, and hence which first-order moments are part of the first-order timeline could also change over the course of second-order time. One could perhaps use such a framework to model certain time travel scenarios in which something happens which seems to “change the past”. If the time-traveler was “originally” not part of a past interval, one can nevertheless suppose that there is a second-order-later second-order moment at which they are, and that their presence results in different first-order moments being part of the first-order timeline than those that second-order-were part of the first-order timeline of the first second-order moment. This sort of change in first-order time over higher-order time is importantly different from a dialetheic view: This kind of framework would not require that there be any point of evaluation at which a contradiction is true. None of this to say that I think this account holds of actual time: Personally, I'm sympathetic to tenseless views of time, but I think such higher-order-tense frameworks are interesting in their own right and are worth exploring.

7. *The Philosophy of Religion*

In this section we shall return to our main topic, which is metaphysical modality, only now as it relates to the philosophy of religion. If there are higher-order modalities, there are higher-order contingencies; and if there is also a necessarily existent, omnipotent God, He ought to have control over them. A restricted version of Descartes' view that necessary truths were subject to God's will would be true. God would not, on this view, have the power to make contradictions true, or to make two plus two equal to five; but He could decide, for example, to make the thesis of the essentiality of origins true or false by deciding which galaxy to actualize.

If God did have that kind of control over the modal landscape, it would have implications for the problem of evil; in particular, for the *soteriological problem of evil.* This problem involves the issue of how, on the Christian view and others like it, the existence of a just and loving God is compatible with some people being condemned to hell. This is especially problematic for *Molinist* views, according to which God has *middle knowledge*—that God can know, independently of which possible world is actual, what agents would freely choose to do in certain circumstances, even though those agents possess libertarian free will. God can thus choose to actualize those circumstances and in a sense exercise a non-causal control over what non-deterministic agents *freely choose* to do. The question, then, is why God didn't choose to actualize circumstances in which agents freely act in such a way that no one gets condemned to hell.

One way of responding to this is to say that human nature is such that that is possible, that some always freely choose to reject God. God simply can't actualize a world in which this isn't so. However, if my framework is the correct view of modality, God ought to have control over what "human nature" is like: Even if some always freely choose to reject God in every world in the actual galaxy, God could have chosen to actualize another galaxy in which this isn't the case. Why, then didn't God do so? One could respond that some freely choose to reject God not merely in every world in the actual galaxy, but in *every possible world there is*. But that would make the claim that some freely reject God co-extensive with a logical truth; and that, in my view, is an implausibly strong status for such a claim to have: If a claim doesn't hold as a matter of logic, there ought to be some logically possible world in which it is false. If my framework is correct, then, this Molinist defense against the soteriological problem of evil simply doesn't work.

8. *Conclusion*

In this paper I have examined the possibility and prospects of a novel logical framework, which I have called *higher order modal logic*. I have outlined its motivation, as well as its syntax and semantics. I have shown how to establish derivability claims about it via semantic tableaux, and have constructed a few of them myself. Finally, I have explored some of its implications for the philosophy of language, metaphysics, and the philosophy of religion. It is my hope that the reader will agree with me that it is a framework which is both intriguing in its own right and potentially fruitful, having interesting implications for our understanding of modality and a variety of other issues of philosophical significance.

Bibliography

Lewis, David. *On the Plurality of Worlds.* Blackwell Publishing 1986, Malden Massachusetts.

Plantinga, Alvin. *The Nature of Necessity*, Oxford University Press 1974, Oxford.

Priest, Graham. *An Introduction to Non-Classical Logic: From If to Is, 2nd edition*, Cambridge University Press 2008, New York, New York.