When mathematics touches physics:
Henri Poincaré on probability[1]

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ABSTRACT

Probability plays a crucial role regarding the understanding of the relationship which exists between mathematics and physics. It will be the point of departure of this brief reflection concerning this subject, as well as about the placement of Poincaré's thought in the scenario offered by some contemporary perspectives.

Keywords: probability; philosophy; mathematical physics, Poincaré.

1. Introduction

This paper is dedicated to the study of the role of mathematics in the conception of physics according to Henri Poincaré, where the calculus of probabilities is stressed as relevant to the elucidation of the internal relationship which exists among hypotheses, principles of rationality and the mathematical apparatus found in Poincaré's thought. It has two main scopes: first, showing that the set of works that constitute the basis of the conceptions of Poincaré concerning the calculus of probabilities can bring to light some relevant notions, including the maturation of his points of view; second, evaluating the relevance of his solutions concerning this subject facing the state of the art in the philosophy of science.

Thus, my speech will begin with an overview on the general sketch of Poincaré's epistemology, in order to introduce the main aspects of his understanding concerning mathematical physics and probability. A second moment is dedicated to brief considerations concerning Poincaré's solutions with respect to some of the contemporary themes on philosophy of science.

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2. Some preliminary remarks concerning the epistemological conception of Poincaré

In the pages of "*La science et l'hypothèse*", Henri Poincaré stretches a general scenario concerning the different ways of approaching of each expression related to the scientific knowledge, focusing their specific demands based on an epistemic delimitation done in function of the objects to which each particular branch of knowledge sleeps on. Arithmetic, geometry, mechanics and physics have their proper structural and methodological schemas drawn according to the peculiar vindication of the particular groups of objects under analysis in each discipline. Roughly speaking, the "construction of science", considering Poincaré's point of view, consists into an activity of human intellect that intends to organize, hierarchize, categorize, as well as interpret their objects. Some of these objects are the sensorial experiences (envisaged by physics and mechanics), abstract magnitudes (like the objects of arithmetic), or spatial - but not sensorial - magnitudes (the objects of geometry).

At a first sight, it looks like that Poincaré defends a position which fails concerning its internal consistency and that changes opportunistically according to the circumstances (the *savant* argues in favor of a pseudo-kantism in arithmetic, takes the direction of conventionalism when treating on geometry and mechanics, as well as elects a *sui generis* empiricism in physics); but a sharper look allow us to interpret the author's thinking by means of the Gerhard Heinzmann's notion of *pragmatic occasionalism*: no matter the different forms of approach in these disciplines, the conception of science in Poincaré can be seen as a whole whose parts are consistent, once focusing the way how the knowledge of objects envisaged by science is conceived. In resume, we can suggest that the intellect follows always similar traces, which are adapted according to the exigencies of their peculiar objects. Thus, the elaboration of the set of propositions that consolidate the scientific knowledge is always a consequence of the employ of intuition, which articulates the faculties of intellect to the subjects to be known in the process of scientific creation, fitting this assimilation under the imposition of its specific claims.[3] Thus, the process of scientific creation involves always a variable tension between reason and experience, between objectivity and subjectivity, because it can be seen as a type

[3] « Appeler la philosophie de Poincaré « conventionnalisme » est un mauvais choix conduisant aux dogmes de l'empirisme logique. Ainsi, je propose de l'appeler « occasionnalisme pragmatique ». Poincaré n'est pas une fois kantien, une fois conventionnaliste, une fois empiriste, une autre fois prédicativiste. Il défend toujours la même philosophie : dans un processus de va-etvient, l'intuition se rapporte à la fois à ce que l'on peut faire et, au niveau supérieur, à une connaissance propositionnelle. La construction et la description des objets n'est pas indépendante. De cette manière vont disparaître les apparentes incohérences dans sa philosophie qui empêchaient maints philosophes d'apprécier les vues philosophiques de Poincaré à leurs vraies valeurs » (HEINZMANN, 2006, p. 401-402).

of effort of reason to bring to light the objects of interest of science, objects that are dissimilar in what regards their origin and in the way by means of which we can elucidate them.

3. The construction of mathematical physics

Considering this scenario, the theoretical comprehension of physics finds its own difficulties, that come above all from the tension between reason and experience: at the same time that experience is “the only source of truth”,[4] it is quite clear that experimental facts aren’t able to guarantee alone the constitution of a theory, due to the fragmentary feature of individual experiences, even when expressed by a more or less precise language.[5] Thus, the claim for a perspective that can identify and aggregate what particular facts have in common is a *conditio sine qua non* of the reaching of a scientific theory properly so called.

In this way, the scientific laws and theories may be understood as efforts of induction and generalization, based on attempts of construction of hypotheses that, ideally, must be corroborated or discarded with the support of the principles of mathematical physics and of experience, whenever their experimental verification can be done.[6] These hypotheses can play several roles in this work of articulating the exigencies of reason with the way by which it interprets the relations revealed by facts. These implicit or explicit hypotheses are related to the unity of nature, to the possibility of translation of phenomena in focus into simplified theoretical models, to some kind of linearity between what we call “effects” and their “causes” (say, the former states of nature), to the plausibility of the “cuts” done concerning space and time, and so on. In this process, mathematics gains a central importance in physics.

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[4] « L'expérience est la source unique de la vérité : elle seule peut nous apprendre quelque chose de nouveau ; elle seule peut nous donner la certitude. Voilà deux points que nul ne peut contester » (POINCARÉ, 1902, p. 157).

[5] « Ne pouvons-nous nous contenter de l'expérience toute nue ?
Non, cela est impossible ; ce serait méconnaître complètement le véritable caractère de la science. Le savant doit ordonner ; on fait la science avec des faits comme une maison avec des pierres; mais une accumulation de faits n'est pas plus une science qu'un tas de pierres n'est une maison » (POINCARÉ, 1902, p. 157).

[6] « À quelle condition l'usage de l'hypothèse est-il sans danger ?
Le ferme propos de se soumettre à l'expérience ne suffit pas ; il y a encore des hypothèses dangereuses ; ce sont d'abord, ce sont surtout celles qui sont tacites et inconscientes. Puisque nous les faisons sans le savoir, nous sommes impuissants à les abandonner. C'est donc là encore un service que peut nous rendre la physique mathématique. Par la précision qui lui est propre, elle nous oblige à formuler toutes les hypothèses que nous ferions sans elle, mais sans nous en douter. » (POINCARÉ, 1902, p. 165-166).

The relationship between mathematical physics and mathematics is more than a happy neighborhood or a well-traced partnership.[7] The examples introduced by Poincaré in the chapter 5 of "*La valeur de la science*", named "*L'analyse et la physique*", have the scope of elucidating this relation, and the terms employed by the author are important to the understanding of the *locus* of each discipline: he intends to show « ce que la physique reçoit de la mathématique et ce que la mathématique, en retour, emprunte à la physique » (POINCARÉ, 1905, p. 104). The verbs "recevoir" and "emprunter" suggest, by themselves, an asymmetric relationship between mathematics and physics: mathematics gives, while physics lends. This relationship is very important to mathematics, but essential to physics, and it is not the same thing. I think that this idea is sound with the main proposals found in Poincaré's epistemology. I'll expose their main aspects disconnectedly.

First, mathematics must be seen as a resource which physics simply cannot dispense, once it is attached to its own framework. Two of the most important aspects of this deep integration can be quoted as follows: the relevance of mathematical language and the role it plays in the construction of physical theories.

Mathematical language is a privileged tool to express with rigor and precision the generalized notions of physics. Poincaré attributes to the aesthetics preoccupations of the mathematicians (once they have in mind the primacy of the beauty of their demonstrations and proofs) their aptitude on the construction of symbolic systems that satisfy the exigencies of the human intellect. Mathematics, which in "*Science et méthode*" Poincaré defines as "the art of giving the same name to different things"[8] propitiates, say, the way by which the discourse of physics gains its better (and necessary) form. Thus, words well chosen correspond to the language that makes the "mathematical spirit" flows, allowing an attachment to forms only, including the forms that we can apprehend from the facts (POINCARÉ, 1905, p. 106).

Thereby, the generalization which corresponds to a physical law is linked with the process of abstraction; that's how the *savant* transcends the concrete facts in the name of a choice, regarding the general comprehension of the relations which compound experience,

[7] « La physique mathématique et l'analyse pure ne sont pas seulement des puissances limitrophes, entretenant des rapports de bon voisinage ; elles se pénètrent mutuellement et leur esprit est le même. C'est ce que l'on comprendra mieux quand j'aurai montré ce que la physique reçoit de la mathématique et ce que la mathématique, en retour, emprunte à la physique » (POINCARÉ, 1905, p. 104).

[8] « Je ne sais si je n'ai pas déjà dit quelque part que la mathématique est l'art de donner le même nom à des choses différentes. Il convient que ces choses, différentes par la matière, soient semblables par la forme, qu'elles puissent pour ainsi dire se couler dans le même moule. Quand le langage a été bien choisi, on est tout étonné de voir que toutes les démonstrations, faites pour un objet connu, s'appliquent immédiatement à beaucoup d'objets nouveaux ; on n'a rien a y changer, pas même les mots, puisque les noms sont devenus les mêmes » (POINCARÉ, 1908, p. 32).

and allow him to know "the hidden harmony of the world".[9] The example offered by him concerning the contributions of Maxwell's electrodynamics is far from being humble: Poincaré argues that Maxwell proposes his contributions above all due to his capability of "thinking on vectors", and his "high degree of intimate sense of mathematical analogies", what allowed him to develop theories that would have an empirical proof only after twenty years. In this case, the sentiment of mathematical symmetry was decisive and determinant, facing the lack of an empirical proof in order to corroborate it. And even the achievement of this empirical proof is a consequence of the previous claims of mathematical physics (POINCARÉ, 1905, p. 106-109).

To sum up, theories on mathematical physics could be interpreted as generalizations framed by means of mathematical models. These models serve us right in the effort of elucidating the valid *rapports* among, say, the elements of facts (POINCARÉ, 1902, p. 174), through an intended objective form, changed into a general rule, once observed the hypotheses formerly assumed.

Second, regarding what mathematics borrows from physics, Poincaré stresses in "*La science et l'hypothèse*" that the creative faculty of human intellect is able to find new ways of overmatching its difficulties, a faculty that the "spirit doesn't use unless the experience give him a reason" (POINCARÉ, 1902, p. 55). This idea is exemplified in the mentioned text of "*La valeur de la science*", where Poincaré is less laconic. Once mathematics can be considered as a typical abstract activity of creating and treating on numbers and symbols, the claims for the solution of physical problems can serve as guides to what kind of combinations (among the infinite possible ways and forms) mathematicians must pay their attention, something that exceeds even the capability of human imagination.[10]

The opportunity of solving problems is a major aspect of the importance of physics to mathematicians, but that's not all: experience gives us also a kind of guideline to the reasoning

[9] « En résumé le but de la physique mathématique n'est pas seulement de faciliter au physicien le calcul numérique de certaines constantes ou l'intégration de certaines équations différentielles.
Il est encore, il est surtout de lui faire connaître l'harmonie cachée des choses en les lui faisant voir d'un nouveau biais.
De toutes les parties de l'analyse, ce sont les plus élevées, ce sont les plus pures, pour ainsi dire, qui seront les plus fécondes entre les mains de ceux qui savent s'en servir » (POINCARÉ, 1905, p. 108).

[10] « L'histoire le prouve, la physique ne nous a pas seulement forcés de choisir entre les problèmes qui se présentaient en foule ; elle nous en a imposé auxquels nous n'aurions jamais songé sans elle.
Quelque variée que soit l'imagination de l'homme, la nature est mille fois plus riche encore. Pour la suivre, nous devons prendre des chemins que nous avions négligés et ces chemins nous conduisent souvent à des sommets d'où nous découvrons des paysages nouveaux. Quoi de plus utile ! » (POINCARÉ, 1905, p. 109).

on mathematics, as well as suggestions of reasoning. Poincaré exemplifies his thesis arguing that the mathematical hypotheses sometimes are inspired on the physical world, and some of the mathematical creations lack of rigorous proofs just because they were established in advance to these rigorous proofs. Poincaré appeals again to the history of science in order to give an example to his argument, mentioning how Klein's use of analogy between the properties of electric current and the Riemann surfaces allowed him to achieve a solution concerning the latter. A physical property serving as model to reasoning on geometry! (POINCARÉ, 1905, p. 112).

But let us stress that the proximity between mathematics and physics cannot allow us to neglect the essential difference between these instances of science: mathematics flows from *a priori* synthetic truths, while physics claims for the experimental facts as the only source of truth, according to the author. There exists a qualitative difference between them, and if Poincaré stresses the importance and the relevance of the inspiration offered by the physical world in the invention of mathematics, it cannot be taken as kind of mathematical empiricism. The correct understanding of "receiving" and "lending", as stressed above, imposes a necessary distinction of these instances. Even sharing a mutual importance and the same spirit, they diverge essentially. Thus, in the conception of physics, mathematics plays the role of a resource, while the intellect turns experience compatible with those relations that mathematics can express, share and elucidate by its own ways. And the calculus of probabilities is a very important element for the understanding of this process.

4. The calculus of probabilities

The notion of probability on Poincaré is crucial because this is the instance where the mentioned tension between reason and experience show itself more clearly. Here, the rigor and perfection of mathematical constructions keep in touch with the fragmentary character that is typical of sets of phenomena (evidencing the lack of data in order to offer a higher degree of certainty), and with the limitations of our knowledge (when we fail in the effort to establish sufficient wide and precise scientific laws) and, even so, we still take for granted the possibility of predictions, something absolutely necessary to science. In this paper, my remarks will be restricted exclusively to the central aspects of this subject.

The difficulties related to probability and hazard are introduced by Poincaré mainly in the eleventh chapter of "*La science et l'hypothèse"* and in the fourth chapter of "*Science et méthode"*. In both texts, as usually, Poincaré introduces the questions of probability and of hazard by means of its apparent contradictions and difficulties: how could we understand the laws of probability, if probability opposes to certainty? How could we understand the laws of hazard, if hazard is the antithesis of law?[11] The apparent contradiction inherent to the laws of probability and hazard is derived from a perspective strongly oriented by his deterministic beliefs[12]: if causality rules all the relationships that govern the facts to which physics is dedicated to understand, we must look for the probability, as well as its factual counterpart, the hazard, exactly when (or where) something in this process fails.

But what fails? From a deterministic point of view, roughly speaking, the inferences traced in order to understand the world are centered on three aspects: the former state of affairs, the latter state of affairs, and a rule which governs the transition from one to another. Probability gains importance when our knowledge of one or more of these elements is not sufficiently well stated in order to guarantee a satisfactory degree of certitude. This is basically the schema traced by Poincaré with the scope of explicating the mechanism of probability in physics, the field where probability imposes the hardest challenges, once it is in physics that the hazard plays a genuine role.

The definition of hazard is also articulated in the function of Poincaré's deterministic position, and there are three explications introduced by the author in this context: first, the existence of minimal causes that can produce great effects (as in the case of instable equilibrium); second, the complexity of the causes that, due to their intricate and composite factors, escape from our control in what refers to their interactions (as the function that rules the distribution of falling drops of rain, which depends on the ionic distribution on space, the shifting air flows, their mutual dependency, and so on); and a third way, less important, where the convergence of causes apparently disconnected contribute to give rise to an unexpected consequence (like the highly improbable situations which we all know by heart from the cartoons).

But no matter probability and hazard being strictly related notions, at first sight it looks like weird that the fourth chapter of *"Science et méthode"* is employed as an introduction to the second edition of the book *Calcul des probabilités*. But I really don't think that it happened due to a kind of opportunism, as it may seem. My opinion is based on an interpretation

[11] « Le nom seul de calcul des probabilités est un paradoxe : la probabilité opposée à la certitude, c'est ce qu'on ne sait pas, et comment peut-on calculer ce que l'on ne connaît pas ? Cependant, beaucoup de savants éminents se sont occupés de ce calcul, et l'on ne saurait nier que la science n'en ait tiré quelque profit. Comment expliquer cette apparente contradiction ? » (POINCARÉ, 1902, p. 192).

[12] « Nous sommes devenus des déterministes absolus [...] Tout phénomène, si minime qu'il soit, a une cause, et un esprit infiniment puissant, infiniment bien informé des lois de la nature, aurait pu prévoir dès le commencement des siècles » (POINCARÉ, 1912, p. 2).

concerning some tenuous differences between these versions of 1896 and 1912, with respect to the form of presentation of the text.

The first edition is clearly a textbook. It brings as subtitle “leçons professées pendant le deuxième semestre 1893-1894” and the indication that it was written by Albert Quiquet. In turn, the second edition, if compared to the first, brings the change of the “lessons” into “chapters”, modifications concerning the division of contents (“mathematical expectation”, for example, that is a section in the first edition, gives the name of the third chapter in the second edition), as well as the inception of an introduction concerning the hazard. These are details that make me wonder that the difference between the first and the second editions of the *Calcul des probabilités* are not only introduced in the spite of achieve a better form, aesthetically or linguistically so to speak.

I’m convinced that these differences indicate the change of a didactic book into a wider reflection concerning this subject, or, in a better formulation, that the second edition treats the calculus of probabilities into a more general frame, related to a philosophical reflection concerning the hazard and how it can be assimilated by science. Let us remark also that this interpretation is soundness with the maturation of his ideas and the transition of his subjects of interests during his career, something that Laurent Rollet gladly calls “le parcours du Poincaré-scientifique au Poincaré-philosophe” (ROLLET, 1999, p. 134), no matter the details and nuances involved in this question being too subtle to be treated so superficially in this paper.

I quote here a Poincaré’s interesting observation at the first chapter of the *Calcul des probabilités*. The so-called *problème du jeu de bolles* is a very simple situation: two players with the same degree of ability are playing; Pierre will throw two balls, Paul will throw just one; the winner is the player who gets the greater approach to the target. The question is: what is the probability of Paul’s victory? The response of Poincaré is articulated in the following way: after showing that the reasoning which precedes the arrangement of the data of the problem can be established by two different strategies, the author argues that:

> La definition complète de la probabilité est donc une pétition de principe : comment reconnaître que tous les cas sont également probables ? Une définition mathématique ici n’est pas possible ; nous devrons, dans chaque application, faire des *conventions*, dire que nous considérons tel et tel cas comme également probables. Ces conventions ne sont pas tout à fait arbitraires, mais échappent à l’esprit du mathématicien qui n’aura pas à les examiner, une fois qu’elles seront admises. (POINCARÉ, 1912, p. 28).

And here the central point to my argument:

> Ainsi tout problème de probabilité offre deux périodes d’étude : la première, métaphysique pour ainsi dire, qui légitime telle ou telle

> convention ; la seconde, mathématique, qui applique à ces conventions les règles du calcul. (POINCARÉ, 1912, p. 29).

The existence of a kind of instance that precedes the mere application of mathematics (and that maybe Poincaré name as “metaphysics” in the lack of a better name) and that is directly related to, say, “the way things are in the world”, can be taken as a good reason to understand why Poincaré introduces the study of the calculus of probabilities with some considerations about the hazard, its causes and definitions. The challenge involved in the comprehension of nature consists largely on a tentative of bringing to light the way by which the relations showed by experience are true, something that involves variable degrees of uncertainty and how this uncertainty can be expressed mathematically.

In general, the examples given by Poincaré concerning the calculus of probabilities always indicate the application of supervening principles on the formulation of the strategies of solution,[13] like the principle of sufficient reason. These principles always determine *how* we will apply mathematics in physics. Therefore, the ideal feature of the theoretical constructions in physics may be seen as the systematic use of pre-established hypotheses and conventions introduced in order to give rise to an understanding of the causality of the facts, but always immerged into a variable degree of uncertainty, especially when involving prediction.

On the other hand, the calculus of probabilities when focused on mathematics is essentially different, no matter their cases can be treated in a similar way. Mathematical problems don’t treat on hazard properly so-called, instead of what physics do. If some mathematical problems can be treated by means of the probability (for instance, “how many algorisms “5” are found until the n^{th} decimal of π ?”) that isn’t because we must presuppose a metaphysic which supports some kind of mathematical realism, but just because the genesis of π is complicated, what make us think about it in the same way we are accustomed to do when treating on a set of complicated causes in the branches of physics. Thus, no matter the exigency of a delimitation and a strategy of action (which are introduced by convention), probability in mathematics can be solved within the limits of mathematical theory, and do not suffers the menace of hazard, but only the uncertainty generated by the terms of the formulated problems and the impossibility of given a complete solution to them.

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[13] « Pour entreprendre un calcul quelconque de probabilité, et même pour que ce calcul ait un sens, il faut admettre, comme point de départ, une hypothèse ou une convention qui comporte toujours un certain degré d'arbitraire. Dans le choix de cette convention, nous ne pouvons être guidés que par le principe de raison suffisante. Malheureusement, ce principe est bien vague et bien élastique et, dans l'examen rapide que nous venons de faire, nous l'avons vu prendre bien des formes différentes. La forme sous laquelle nous l'avons rencontré le plus souvent, c'est la croyance à la continuité, croyance qu'il serait difficile de justifier par un raisonnement apodictique. Mais sans laquelle toute science serait impossible » (POINCARÉ, 1902, p. 213).

In resume, mathematics plays a fundamental role in physics, but is always subordinated to higher principles, by means of what the analogy between the world and the mathematical framework is drawn, the inferences are done, and, above all, the way by which this assimilation between the state of affairs and the mathematical order is conceived. Anne-Françoise Schmid synthesizes well this relationship, when she affirms that "experimental sciences have the function of applying mathematics into the reality" (SCHMID, 1978, p. 65). But the role of intuition, that according to Heinzmann (2002, p. 2) can be conceived as an instrument of scientific invention which carries a heuristic function, cannot be disregarded as the guide and last reference to this process.

5. A contemporary overview concerning the relation between physics and mathematics

Hartry Field's "*Science without numbers"* is a good example of how one can conceive the relationship between mathematics and physics on a very different way. The book is centered into a defense of nominalism as a legitimate way of interpreting science, and physics in particular. The idea that rules his monograph is sketched with the scope of undermining the Quine-Putnam argument of indispensability. Once taken from granted that the indispensability argument is the only realist argument that must be considered seriously,[14] the author proposes that theoretical entities and mathematical entities are substantially different: theoretical entities are indispensable in order to erect scientific theories; mathematical ones do not. Based on this idea, Field proposes an alternative way of constructing Newtonian theory of gravitation.

The author intends to show that the supposed inextricability between mathematics and physics is no more than a false impression: historically, traditionally, mathematics and physics have been walking together, but Field offers a model which allows treating on the notions of physics by other ways (exclusively employing the quantification of theoretical entities). If we agree with him (and I could indicate a couple of reasons to not doing so), treating physics mathematically turns no more into a necessity, but a possibility. The best possibility, no doubt. But still *a* way, and not anymore *the* way of doing science.

Under a completely divergent perspective, Michel Paty argues in favor of recognizing a deep relation between mathematics and physics. He intends to show that more than a

[14] "I will argue that mathematical entities are not theoretically indispensable: although they do play a role in the powerful theories of modern physics, we can give attractive formulations of such theories in which mathematical entities play no role. If it is right, then we can safely adhere to a factionalist view of mathematics, for adhering to such a view will not involve depriving ourselves of a theory that explains physical phenomena and which we can regard as literally true" (FIELD, 1980, p. 8).

relationship between numbers and symbolic manipulation, mathematics involves necessarily a great set of qualitative solutions. Then, the link between these two disciplines cannot be thought as a mere opportunity of applying mathematic conceptions to the “translation” of phenomena into a commode language. In the same way, mathematics doesn’t reduces only to a set of numbers and their relations, but transcends these features in the name of the scientific practice, when the theoretical physics can be expressed as a rational construction determined by the mathematization of concepts.[15]

Based on the ideas sketched above, the thought of Poincaré can be understood as an intermediary solution considering those proposed by Michel Paty and Hartry Field, when he vindicates an intrinsic correlation between mathematics and physics - Poincaré says that the latter simply couldn’t exist without the former[16] - but keeping it still as a *resource* available to physics. However, the arguments proposed by Michel Paty aren’t too far from Poincaré’s conception. In fact, the great distinction between these two formulations is centered into the level of mutual dependence that can be attributed to one and other branch of knowledge in their mutual relationship: Paty looks like flirting with an empiricist approach of mathematics, while, on the other hand, his conception of physics as “qualitative concepts in relation” (and not a kind of translation of facts into a mathematical language) apparently aligns him with a nominalist position, that couldn’t be corroborated by Poincaré’s point of view.

6. Conclusion - a quick look towards contemporary perspectives

There are several other important questions to be discussed if we intend to consider how defensible is the solution of Poincaré concerning this subject facing the contemporary perspectives on philosophy of science. I’ll try to summarize one of them as well as suggest a direction of research in this conclusion.

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[15] « La physique, [...] entretient, parmi les « sciences de la nature », un rapport privilégié avec les mathématiques. Ce rapport n’est pas tant un rapport d’« application », comme s’il ne s’agissait que de plaquer des relations mathématiques sur des résultats de mesures physiques, qu’un rapport de *constitution* et de *nécessité d’existence*, en ce sens que les mathématiques sont incorporées à la physique qui seulement avec leur aide peut penser ses objets et leurs relations. Un théorie physique est une élaboration *sui generis* de concepts quantitatifs en relation, et non pas une « traduction » en termes mathématiques de concepts qui pourraient être conçus de manière seulement « qualitative », entendue au sens de qualités sensibles. Le sens physique des concepts physiques est indissociable de leur forme mathématique, et c’est ce qui fait le sens profond du renouvellement de la physique moderne à partir des XVII^e^ et XVIII^e^ siècles » (PATY, 2005, p. 110).

[16] « Les théories mathématiques n'ont pas pour objet de nous révéler la véritable nature des choses ; ce serait là une prétention déraisonnable. Leur but unique est de coordonner les lois physiques que l'expérience nous fait connaître, mais que sans le secours des mathématiques nous ne pourrions même énoncer » (POINCARÉ, 1902, p. 215).

First, the deterministic perspective was launched into a great turbulence since the advent of quantum mechanics, once it introduces two main problems regarding the philosophical background of deterministic laws: first, the impossibility of predicting future events in a deterministic way; second, considering a situation of experimental measurement, quantum mechanics doesn't allow to indicate the preceding necessary causes of an event.

This problem was exhaustively debated above all by positivists and neokantians, and we do not need to get in the details of the discussion. It serves us right merely to recognize that the sense of "being determinist" changed after quantum mechanics and, considering that this is a major aspect that surrounds all the Poincaré's thesis concerning probability, it cannot be neglected. But I believe that the general sketch given by Poincaré fits well to a "post-quantum determinism", especially concerning the central idea that rules hazard and probability according to him, say, the minimal causes that can produce great effects and the complex ones who escape from our control.

Second, the efforts in order to understand the links between mathematics and physics can be treated in terms of clarification of the *status* to be attributed to theoretical entities of mathematics and physics, as suggested by Field in his quoted book, but not necessarily falling into his solution, or focusing on the difference between mathematical and physical entities as being that "what exists" or "what doesn't exist". Instead, it can be a different point of departure in order to understand how these elements are mutually related and employed in scientific practice, in the spirit of Poincaré's informal/intuitive approach, what can bring to light new options to a pragmatic perspective of considering science on the meta-theoretical level.

Quand les mathématiques touchent la physique :

Henri Poincaré et la probabilité

La probabilité joue un rôle crucial en ce qui concerne l'éclaircissement du rapport entre mathématiques et physique. Ce sera le point de départ d'une brève réflexion à ce sujet, ainsi que sur le placement de la pensée de Poincaré dans le scénario offrant par quelques perspectives contemporains.

Mots-clés : probabilité; philosophie; physique mathématique ; Poincaré.

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