

The Permissibility Solution to the Lottery Paradox – Reply to Littlejohn

Thomas Kroedel

ABSTRACT: According to the permissibility solution to the lottery paradox, the paradox can be solved if we conceive of epistemic justification as a species of permissibility. Clayton Littlejohn has objected that the permissibility solution draws on a sufficient condition for permissible belief that has implausible consequences and that the solution conflicts with our lack of knowledge that a given lottery ticket will lose. The paper defends the permissibility solution against Littlejohn's objections.

KEYWORDS: epistemic permission; justification; lottery paradox; probability

1. Introduction

A large and fair lottery is held; one of the tickets is going to win. It seems that for each ticket I'm justified in believing that it will lose. It seems to follow from this that I'm justified in believing that all the tickets will lose. But I'm certainly not justified in believing that all the tickets will lose. That, in a nutshell, is the lottery paradox.¹ I have suggested that the paradox can be solved if epistemic justification is conceived of as a species of permissibility.² (Call this solution the *permissibility solution*.) Clayton Littlejohn has objected that the permissibility solution draws on a sufficient condition for permissible belief that has implausible consequences and that the solution conflicts with our lack of knowledge that a given ticket will lose.³ This paper defends the solution against those objections.

¹ The lottery paradox is due to Henry Kyburg, *Probability and the Logic of Rational Belief* (Middletown, CT: Wesleyan University Press), 197.

² Thomas Kroedel, "The Lottery Paradox, Epistemic Justification and Permissibility," *Analysis*, 72 (2012): 57–60.

³ Clayton Littlejohn, "Lotteries, Probabilities, and Permissions," *Logos and Episteme*, 3 (2012): 509–514.

2. The Permissibility Solution

The lottery paradox arises from the following three claims, which seem individually plausible but jointly inconsistent:⁴

- (1-J) For each ticket, I'm justified in believing that it will lose.
- (2-J) If, for each ticket, I'm justified in believing that it will lose, then I'm justified in believing that all the tickets will lose.
- (3-J) I'm not justified in believing that all the tickets will lose.

Assume that epistemic justification is a species of permissibility. Assume, that is, that I'm justified in believing that p iff I'm epistemically permitted to believe that p .⁵ (For brevity, I'll simply use 'permitted' for 'epistemically permitted' in what follows.) If we rephrase (1-J)–(3-J) in terms of permissibility, we get:

- (1-Pe) For each ticket, I'm permitted to believe that it will lose.
- (2-Pe) If, for each ticket, I'm permitted to believe that it will lose, then I'm permitted to believe that all the tickets will lose.
- (3-Pe) I'm not permitted to believe that all the tickets will lose.

According to the permissibility solution, the clause 'for each ticket, I'm permitted to believe that it will lose' in (1-Pe) and (2-Pe) is ambiguous because different scopes can be assigned to 'permitted.' On a narrow-scope reading, the clause expresses separate permissions: that is, it expresses that I'm permitted to believe that the first ticket will lose, permitted to believe that the second ticket will lose, and so on. On a wide-scope reading, the clause expresses a single permission to have a number of beliefs, that is, the permission to believe that the first ticket will lose, believe that the second ticket will lose, etc. The

⁴ The nomenclature differs slightly from Littlejohn's, which in turn differs slightly from that of my "The Lottery Paradox, Epistemic Justification and Permissibility."

⁵ For a recent discussion of this claim, see Clayton Littlejohn, *Justification and the Truth-Connection* (Cambridge: Cambridge University Press, 2012), 42ff.

ambiguity can be brought out more fully by formalization. Assume that there are n tickets in the lottery. For $1 \leq i \leq n$, let t_i be the sentence ‘Ticket number i will lose.’ Let $B\phi$ be the sentence ‘I believe that ϕ ,’ and let $\mathbf{Pe}\psi$ be the sentence ‘It is permissible for me that ψ .’ We can then disambiguate (1-Pe) as follows:

(1-Narrow) $\mathbf{Pe}Bt_1 \ \& \ \mathbf{Pe}Bt_2 \ \& \ \dots \ \& \ \mathbf{Pe}Bt_n.$

(1-Wide) $\mathbf{Pe}[Bt_1 \ \& \ Bt_2 \ \& \ \dots \ \& \ Bt_n].$

Similarly, (2-Pe) can be disambiguated into the following two claims:

(2-Narrow) If $\mathbf{Pe}Bt_1 \ \& \ \mathbf{Pe}Bt_2 \ \& \ \dots \ \& \ \mathbf{Pe}Bt_n$, then $\mathbf{Pe}B[t_1 \ \& \ t_2 \ \& \ \dots \ \& \ t_n].$

(2-Wide) If $\mathbf{Pe}[Bt_1 \ \& \ Bt_2 \ \& \ \dots \ \& \ Bt_n]$, then $\mathbf{Pe}B[t_1 \ \& \ t_2 \ \& \ \dots \ \& \ t_n].$

Claim (3-Pe) is unambiguous and can be formalized as

(3-Unamb) $\sim\mathbf{Pe}B[t_1 \ \& \ t_2 \ \& \ \dots \ \& \ t_n].$

Let us consider the last two claims first and then continue from the top of the list.

Claim (3-Unamb) is clearly true. I know that it is false that all the tickets will lose; it would be grotesque if I were permitted to believe something that I know to be false.

Claim (2-Wide) seems to be true too. It is an instance of the following closure principle, which seems very plausible: if I’m permitted to have a certain set of beliefs, then I’m also permitted to have a single belief whose content is the conjunction of the contents of those beliefs.

Claim (1-Narrow) seems to be true. After all, it is highly probable that a given ticket will lose (provided that n is large enough), which seems to permit me to believe that it will lose. Since this holds for each ticket, we get (1-Narrow).

Now it may seem as though we are en route to paradox despite the disambiguation of (1-Pe) and (2-Pe). For (1-Narrow) may seem to entail (1-Wide). Given that we accept (2-Wide), this would yield the negation of (3-Unamb), contradicting our assessment of (3-Unamb) as true. There is, however, no need to accept that (1-Narrow) entails (1-Wide). It is a general feature of permissibility that it doesn’t agglomerate. That is, I might be permitted

to do this, permitted to do that, etc., without being permitted to do all of these things. For instance, I might be permitted to eat this piece of the cake, permitted to eat that piece of the cake, etc., without being permitted to eat the whole cake. Given the general failure of permissibility to agglomerate, it is reasonable to claim that epistemic permissibility doesn't agglomerate either. Thus, I might be permitted to believe this, permitted to believe that, etc., without being permitted to have all these beliefs at once. We can therefore accept (1-Narrow) while rejecting (1-Wide).

Claim (2-Narrow) is inconsistent with (1-Narrow) and (3-Unamb); we therefore have to reject (2-Narrow) given that we accept (1-Narrow) and (3-Unamb).

Here is a summary of our assessment of the claims (1-Narrow) through (3-Unamb):

(1-Narrow)	(1-Wide)	(2-Narrow)	(2-Wide)	(3-Unamb)
T	F	F	T	T

Our assessment bears on the solution of the lottery paradox as follows. If we disambiguate (1-Pe) and the antecedent of (2-Pe) uniformly, we take the paradox to be constituted either by the set {(1-Narrow), (2-Narrow), (3-Unamb)} or by the set {(1-Wide), (2-Wide), (3-Unamb)}. Either set is inconsistent, but we can straightforwardly deny one of the three claims in each case, viz. (2-Narrow) and (1-Wide) respectively. If we don't disambiguate uniformly, we get the sets {(1-Narrow), (2-Wide), (3-Unamb)} and {(1-Wide), (2-Narrow), (3-Unamb)}. The latter set, {(1-Wide), (2-Narrow), (3-Unamb)}, is the least interesting reading of the paradox. Whether the set is inconsistent depends on whether (1-Wide) entails the antecedent of (2-Narrow) (which, plausibly, it does), but at any rate all of its members except (3-Unamb) can be straightforwardly denied. The set {(1-Narrow), (2-Wide), (3-Unamb)} is more interesting. All of its members are true; *a fortiori* they are not jointly inconsistent. This dissolves the paradox if we take it to be constituted by {(1-Narrow), (2-Wide), (3-Unamb)}. It may also explain how we trapped in the first place. If we consider the first two claims of the original paradox individually, the most charitable reading of the first claim is the true claim (1-Narrow), and the most charitable reading of the second claim is the true claim (2-Wide). (The third claim is unambiguously true anyway.) This may make us inclined to accept the first

two claims of the original paradox. When we consider the all three claims collectively, however, uniformity takes precedence and we are led to one of the sets {(1-Narrow), (2-Narrow), (3-Unamb)} and {(1-Wide), (2-Wide), (3-Unamb)}, which are no longer consistent. Fortunately, the permissibility solution still allows for a principled rejection of one of the claims in each case.

3. The Sufficient Condition for Epistemic Permissibility

In the defence of (1-Narrow) presented in the previous section, I claimed that the high probability that a given ticket will lose permits me to believe that it will lose (provided the lottery is large enough). Generalizing from this case yields the following sufficient condition for epistemic permissibility:⁶

(High) If the probability that it is the case that p is sufficiently high on my evidence, then I'm permitted to believe that p .

By supporting (1-Narrow), principle (High) provides a rationale for an important aspect of the permissibility solution. Littlejohn objects, however, that a proponent of the solution can't afford an endorsement of (High).

His objection goes as follows.⁷ Suppose that I not only contemplate what I'm permitted or not permitted to believe but exercise the permissions to believe certain ticket-propositions. It seems to be all right for me to believe of just one ticket that it will lose. It is certainly *not* all right for me to believe of each ticket that it will lose. Suppose that I'm in the process of acquiring one belief after another to the effect that ticket number so-and-so will lose. Then at some point I will have reached a maximum number of such beliefs beyond which I'm no longer permitted to acquire additional ones. (Where this point lies is a vague matter, of course, but this need not worry us here.) The losing probability of each ticket

⁶ See Littlejohn, "Lotteries, Probabilities, and Permissions," 512.

⁷ See Littlejohn, "Lotteries, Probabilities, and Permissions," 512–513. I somewhat simplify the dialectic of Littlejohn's presentation.

remains the same, however; we therefore have a counterexample to the sufficiency principle (High). We also have a counterexample to (1-Narrow), since in the scenario some conjuncts of (1-Narrow) are false.⁸ Since the permissibility solution endorses both (High) and (1-Narrow), it has to be rejected.

A closer look at this objection reveals a fallacy. Here is a more rigorous reconstruction of the objection. Suppose that I come to believe the ticket-propositions in the order t_1, t_2 , etc. At some point I'll reach a proposition – call it t_{\max} – such that, intuitively, I would be epistemically blameworthy for acquiring ticket-beliefs beyond my belief that t_{\max} . According to (1-Narrow), I'm separately permitted to have each of these beliefs:

$$(4) \quad \mathbf{Pe}Bt_1 \ \& \ \mathbf{Pe}Bt_2 \ \& \ \dots \ \& \ \mathbf{Pe}Bt_{\max}.$$

Given that I don't acquire beliefs beyond my belief that t_{\max} , I don't seem to do anything wrong. We may therefore assume that – notwithstanding the general failure of permissibility to agglomerate – I'm also permitted to have all these beliefs at once:

$$(5) \quad \mathbf{Pe}[Bt_1 \ \& \ Bt_2 \ \& \ \dots \ \& \ Bt_{\max}].$$

Let Max be the sentence ' $Bt_1 \ \& \ Bt_2 \ \& \ \dots \ \& \ Bt_{\max}$ '; we can then express (5) more concisely as

$$(6) \quad \mathbf{Pe}\text{Max},$$

and the assumption that I believe t_1 through t_{\max} can simply be expressed as

$$(7) \quad \text{Max}.$$

The objection continues as follows. It would be impermissible for me to acquire the belief that $Bt_{\max+1}$ while continuing to hold the beliefs in Max. Thus, we have:

$$(8) \quad \sim\mathbf{Pe}[\text{Max} \ \& \ Bt_{\max+1}].$$

⁸ Littlejohn himself ("Lotteries, Probabilities, and Permissions," 512) merely claims that if one denies (High), the motivation for (1-Narrow) is unclear.

Let us introduce (epistemic) obligation as the dual of (epistemic) permissibility. In other words, if we let **Ob** ϕ be the sentence ‘It is (epistemically) obligatory for me that ϕ ,’ we have:

$$(9) \quad \mathbf{Ob}\phi \text{ iff } \sim\mathbf{Pe}\sim\phi; \text{ and } \mathbf{Pe}\phi \text{ iff } \sim\mathbf{Ob}\sim\phi.$$

Thus, (8) is equivalent to the claim that I’m obligated not to believe that the next ticket will lose while retaining the beliefs I already hold:

$$(10) \quad \mathbf{Ob}\sim[\mathbf{Max} \ \& \ Bt_{\max+1}].$$

It is uncontroversial that (epistemic) obligation is closed under logical equivalence.⁹ Since $\sim[\mathbf{Max} \ \& \ Bt_{\max+1}]$ is logically equivalent to the material conditional $[\mathbf{Max} \supset \sim Bt_{\max+1}]$, (10) thus entails

$$(11) \quad \mathbf{Ob}[\mathbf{Max} \supset \sim Bt_{\max+1}].$$

Earlier we assumed

$$(7) \quad \mathbf{Max}.$$

According to the objection, (7) and (11) entail

$$(12) \quad \mathbf{Ob}\sim Bt_{\max+1}.$$

That is, according to the objection, (7) and (11) entail that I’m obligated not to believe that the next ticket will lose. Given the duality of permissibility and obligation, (12) is equivalent to the claim that I’m not permitted to believe that the next ticket will lose, that is, to the claim that

$$(13) \quad \sim\mathbf{Pe}Bt_{\max+1}.$$

Claim (13), however, yields a counterexample to (High) and to (1-Narrow).

⁹ See Paul McNamara “Deontic Logic,” in *The Stanford Encyclopedia of Philosophy (Fall 2010 Edition)*, ed. Edward N. Zalta, <http://plato.stanford.edu/archives/fall2010/entries/logic-deontic/>, § 1.3.

The objection goes wrong at a crucial step. The inference from (7) and (11) to (12) is an instance of the principle of *factual detachment*, which says that

(FactDet) If $\mathbf{Ob}[\phi \supset \psi]$ and ϕ , then $\mathbf{Ob}\psi$.

Factual detachment is implausible, however. Bootstrapping cases like the following constitute counterexamples.¹⁰ It seems plausible that it's obligatory for me that if I believe that it's obligatory for me that p , then p ($\mathbf{Ob}[\mathbf{BOb}p \supset p]$). Given that I believe that it's obligatory for me that p ($\mathbf{BOb}p$), factual detachment would license the inference to the conclusion that it's in fact obligatory for me that p ($\mathbf{Ob}p$). It might be, however, that my belief that it's obligatory for me that p is completely irrational and that I do in fact have no such obligation. So we had better give up factual detachment. But then a gap in the objection emerges, and there is no obvious way of bridging it.

Where does this leave us? If we hold on to (High) and (1-Narrow) and thus reject (13), we have

(14) $\mathbf{PeB}t_{\max+1}$.

So I *am* permitted to believe that the next ticket will lose and I retain this permission even if I acquire this belief while holding on to my previously acquired lottery-beliefs. Given (8), however, I'm *not* permitted to believe that t_1 , believe that t_2 , ..., believe that t_{\max} , *and* believe that $t_{\max+1}$. Isn't this odd?

Compare the parallel situation in the cake example. Suppose that I eat the whole cake. I thereby do something that I wasn't permitted to do (i.e., eating the whole cake). But I also do a number of things that I was permitted to do (e.g., eating this or that particular piece of the cake). I do something wrong, and this isn't mitigated by the fact that some of the constituent actions of my wrongdoing weren't themselves wrong. If I want to avoid wrongdoing altogether, I should refrain from performing all the constituent actions. Nevertheless, if I do perform them, each of them is permitted. Likewise for the lottery: if I believe t_1 through $t_{\max+1}$,

¹⁰ See John Broome, "Normative Requirements," *Ratio*, 12 (1999): 398–419, 404–405.

I thereby do something I wasn't permitted to do (having all these beliefs together).¹¹ But I also do a number of things that I was permitted to do (e.g., believing that t_1). I do something wrong, and this isn't mitigated by the fact that some of the constituent cognitive actions of my wrongdoing weren't themselves wrong. If I want to avoid wrongdoing altogether, I should refrain from performing all the constituent cognitive actions. Nevertheless, if I do perform them, each of them is permitted. If you find the situation in the cake example acceptable, your assessment should carry over to the lottery case. If you don't, you might continue to find the situation somewhat odd. But this oddity, I take it, would be less worrying than the lottery paradox itself. (For better or worse, the oddity would also be more general, since it arises for epistemic and non-epistemic permission alike.)

4. Believing (and Asserting) What Isn't Known

Another objection of Littlejohn's draws on the claim that we can't have knowledge of any of the lottery-propositions.¹² Let p be the sentence 'Ticket number so-and-so will lose.' Given that I'm sufficiently reflective, I know that I don't know that p . Now consider the claim

(15) p , but I don't know that p .

It seems that the probability of (15) on my evidence is very high. Given (High), it follows that I'm permitted to believe (15). According to Littlejohn, believing (15) would be "deeply irrational," however.¹³ Say that knowledge is the norm of belief iff I'm permitted to believe only what I would know if I believed it. Then we can state the objection somewhat more generally as follows: if knowledge is the norm of belief, then the permissibility solution to the lottery paradox has to be rejected.

It doesn't seem promising to respond that my belief that a given ticket will lose would constitute knowledge after all. (At least in our setup; of course I may come to know *post*

¹¹ *A fortiori* this holds for the case in which I believe t_1 through t_n .

¹² See Littlejohn, "Lotteries, Probabilities, and Permissions," 512–513.

¹³ Littlejohn, "Lotteries, Probabilities, and Permissions," 512.

factum that a given ticket lost.) And certainly this couldn't be the case for *each* ticket, since knowledge entails truth and we stipulated that one ticket was going to win. Proponents of the permissibility solution should therefore tackle the objection head-on and deny that knowledge is the norm of belief. They should argue that this is simply the price we have to pay in order to solve the lottery paradox: in order to marry permissible belief to probability, we have to divorce it from knowledge first.

The denial of the knowledge norm for belief can be made more palatable by pointing out that it need not affect the knowledge norm of *assertion*. Saying '*p*, but I don't know that *p*' certainly sounds odd, and we may take this to show that I shouldn't assert what I don't know.¹⁴ But it would be consistent with this for beliefs of the form *p*, *but I don't know that p* to be acceptable.¹⁵ And even if *some* beliefs of the form *p*, *but I don't know that p* are objectionable, this might be purely because we're not justified in believing that *p* in the first place. A belief of the form *ticket number so-and-so will lose, but I don't know that ticket number so-and-so will lose* would then fall outside the objectionable class, since, given (1-Narrow) and the conception of justification in terms of permissibility, I have ample justification for believing that ticket number so-and-so will lose.¹⁶

¹⁴ See Timothy Williamson, *Knowledge and Its Limits* (Oxford: Oxford University Press, 2000), 238–269.

¹⁵ See Aidan McGlynn, "Believing Things Unknown," *Noûs* (forthcoming).

¹⁶ For helpful comments and suggestions, I would like to thank John Hawthorne, Benjamin Kiesewetter, Timothy Williamson, and the participants of a session of the seminar 'Recent Work in Theoretical Philosophy' that took place in Berlin in December 2012.