ON PERCEPTION

The numerical limit of perception

Abstract

In this paper I will try to determine the numerical limits of perception and observation in general. Unlike most philosophers who wrote on perception, I will treat perception from a quantitative point of view and not discuss its qualitative features. What I mean is that instead of discussing qualitative aspects of perception, like its accuracy, I will discuss the quantitative aspects of perception, namely its numerical limits. As it turns out, the number of objects one is able perceive is finite, while the number of objects our mind can imagine might be infinite. Thus there must be a level of infinity by which the 'number' of objects our mental world can host is bounded. I will use both philosophical assumptions and observations, and mathematical analysis in order to get an estimate of the 'number' of objects we could possibly perceive, which surprisingly turns out to be the first level of infinity or the number of natural numbers.

I will start by discussing the nature of concrete objects and the way we access them via perception. I will talk about mathematics as well and its relation with perception in order to justify myself for using mathematics as a tool in this paper. After some sections of discussions of various aspects of perception and imagination, I will finally be ready to make a counting and determine what I called "the numerical limits of perception".

1. Discussion on the notions of abstract and concrete

With the following arguments I will try to count, or at least create a conjecture or give an idea, of the perception field and the abstract and concrete objects in it. For the sake of this courageous objective one- in this case I- would have to dig deep into concepts and fields of study that still haven't found an answer to the deepest questions they pose. In order not to have an infinite paper, or an infinite book of arguments and counter arguments, I will have to go Platonic some times. Of course going Platonic, which means just stating the sentence "we all know that", is not something easily acceptable, but there is a reason for that which I would like to discus later. But anyway, besides the difficulty of building a valid argument, there is an initial threat to the achievement of my ultimate aim. There is no definition of abstract and concrete for starters. How can I even try to count sets of object if I don't know still what these sets are? Or for instance, there is not even a definition of what an object is even though there are many proposals. These threats in my way make me want to quit this job and declare myself unable to complete the task. But on the other hand, there is a conjecture in my mind that feels to be quite correct. So, while asking myself why "the result” “feels” to be right, I actually found myself having some good arguments and in a very advantaging position.

You see, the problem is quite simple. There is no definition on what is abstract and what is concrete, as mentioned earlier. But many philosophers, scientists and concerned people have discussed this topic for years, trying to give a definition, trying to give a general notion of what is abstract and concrete. I would have to think that everybody has his or her own understanding of what is abstract and concrete, and one thing that is concrete for somebody, is abstract to somebody else and vice versa. But that would not be the case as far as I observed. It seems actually, that the word abstract or concrete is used quite often in daily life, in small talks, so much so that it would be hard to imagine that there is not a well-accepted definition, or let’s say better, a way of expressing what is this concept that we all seem to know so well. And at this point dear old Plato would say that we are trying to discover a way to express what we all actually know. So how come we know what abstract and concrete is, how come we can make statements using those terribly delicate words so easily?

A way to see the problem would be to focus on what’s there in common for all people that creates such a certain convention of what is abstract and concrete. That would be simple, I would immediately jump to state loudly and firmly the word “perception”. It is reasonable after all. We all have a perception of the world that surrounds us. We just know what we percept as concrete and on the other hand what we perceive as abstract. If you ask a ten year old child if his desk at school is abstract or concrete, he would answer concrete with the simple argument that he can touch it. If we again ask this hypothetical child if beauty is abstract or concrete, he would answer abstract. Maybe the argument would be that he could never touch, see, smell or sense in any possible way this object named beauty. Well, this way of seeing the problem, through perception, gives us a good way of first understanding why everybody seems to have a clear thought of what is abstract and concrete, and second it gives me a good way to precede in a descriptive method the scientific aim of this paper. What if I just declared that what is concrete is so because I can perceive it “somehow”, and what is abstract is so because there doesn’t seem to be a way for me to perceive it. This is not something definite and precise, but at least it is a good starting point. It is good because I now can access the notions of abstract and concrete in a very constructive way. I wouldn’t be instead drowned in a seemingly crazy world of mine where my understanding of these notions would be so far away from what everybody feels of abstract and concrete, and doing so everybody who listens to me or reads what I wrote would try to translate my definitions and terminology in the way described above. And this is the advantage I got in hand that seems to relieve the heavy weight of not exactly knowing what I’m talking about. I would like to leave this matter Platonic as it is, so everybody understands what everybody says.

2. Relation between perception and mathematics: mathematical models

When people think about mathematics, usually they think wrong. There is this general idea that mathematics has nothing to do with real life, or “the real world”. Actually, as it will be argued in this discussion, mathematics is the closest thing to the real world that we know. Or in another way:

mathematics is the only language in which we can describe the real world, whatever that means. This statement is so strong and it seems like it is drawn out of just personal conventions of somebody who loves mathematics (in this case it would be me, and being somebody who does mathematics puts me in disadvantage while discussing). But there is more to it, there are aspects of mathematics that people seem to forget, sometimes for simple lack of knowledge, sometimes for the sake of not having to do with mathematics, or even worse, sometimes affected by personal delusions involving mathematics (people who don't understand mathematics usually underestimate it's value, try to deny relations between real world and mathematics or even claim that it is just a "tool" that sometimes is useful but generally it is not). On the other hand, people who are purely concerned about science and developing intellectual capacity, even if they don't understand quite fully functions, analysis or geometry, respect it and tend not to make emerging statements involving the term "mathematics".

So, in understanding what mathematics is about, one shouldn't think about the high school or college mathematics that maybe he couldn't get at all. It is surprisingly easy to make a discussion about the very nature of mathematics. Think for instance of how mathematics was developed. We know that the first times that mathematics was applied were because of the need to measure lands and make calculations in constructions in the ancient world. From this point there came abstract concepts that founded mathematics and what seems today something totally not related to the real state of affairs is actually the closest thing to the state of affairs. One of the reasons why people started to study mathematics, especially analysis and number theory, was to understand the structure of numbers. But think for instance, why there were numbers in the first place? Why did people include numbers in the language, and later use mathematics to discover their structure? Or think about geometry, the simple Euclidean geometry. Why did people use lines, planes and other geometrical structures in the first place? We can say that mathematics developed to understand these structures, but why there were these structures in the first place? These are simple, very basic questions, but surprisingly there seem not to be an immediate answer.

Let us forget for a moment about the structures like lines and numbers and why they were present in the first place. Think further of the relations that we put between them. For example, two distinct lines cannot intersect at more than one point. Now think about why this statement is true. We could say, there is no way that statement is not true, or the statement is "obvious", or there is no way to find out but just to accept it because it seems so, because we can see that two distinct lines cannot intercept in more than one point. This seemingly sloppy reasoning actually are not so sloppy. Can we paraphrase those sentences above, which have the same meaning? We could simply say that the statement "two distinct lines cannot intersect at more than one point" is true because I perceive it to be so and this gives the truth to the. This wouldn't be daring at all, this would just be recognizing mathematics as the way we express our perception of the outside world.

And now I can easily give a reasonable answer to the questions posed in the first paragraph: why there are numbers in our language, why there are structures like lines ore points that we use even unconsciously. The answer would be simple. Because these structures express the way we perceive the world surrounding us, the so called "real world". Mathematics feeds out of our perception and develops on it. There is a strong and deep connection between mathematical structures and the real state of affairs. This is why mathematics is the only language in which we can fully express what we perceive. Unintentionally we build mathematical models using mathematical structures. This happens to everyone. Think about simple statements involving numbers that each one of us state every day, like “there are two cows on that field”. The real state of affairs comes from perception, we perceive the field and cows standing on it, and we further perceive that the number of cows is two. But this sentence actually means the following: “the number of elements in the set formed by intersecting the set of objects standing on that field and the set of cows in the world is two”. So there is a, what we call “mathematical model” behind what seems a simple sentence.

Again I must repeat, we shouldn’t forget about the real state of affairs, or what we perceive. One should never confuse between perception and the language used to express what he or she perceives. And what I wanted to show, by the arguments and examples above, is simply the fact that mathematics is this language, this science, this way of modeling the world we see, that we use. This is why mathematical modeling is more than the applications in economy, physics or chemistry. Mathematics applies directly to perception, mathematics actually is perception and there is no way of denying it. I simply wanted to give an understanding of why there is such a relation between mathematics and what we perceive.

3. Concrete objects and their relation with time and technology

The notion of concrete I intend to use brings with it a very natural and reasonable discussion. A good example for discussion would be the “atom”. Again, if we ask somebody if atom is concrete or abstract, the answer would be concrete. And it is logic to give such an answer. But if what is concrete is what we perceive, one might have second thoughts. One may be forced, since we cannot see or feel in any way a particular atom, to accept the very unreasonable result that atom is abstract. But that wouldn’t be convenient because on the other hand, if we use some microscope we can perceive an atom. So what has the microscope to do with anything? Does the microscope make the atom concrete? But, if so, when the microscope wasn’t invented yet, was the atom abstract? So, I am forced to give an account of the relation between concrete and technology and time to be able to proceed with anything else in the first place.

I asked the very natural question: what has the microscope to do with anything we talked until now? Well, it is reasonable at this point to analyze the problem in terms of perception. I would say that the microscope is a technology that enhances our perception, and it seems to me quite a good point of start to understand what is going on. You see, there might as well be objects, like the atom was in the past, that we cannot yet perceive even with the help of present technology. But it is, in my opinion, very unnatural and unreasonable as well to think that those objects are abstract now but in a possible future they will be concrete. I don’t believe that for an object to be abstract or concrete is dependent on time.

Thinking this way, I find myself in necessity of revising a detail about perception. We all have perceptive tools, which are our sensors, but as we all know we perceive with the help of technology as well. Call it natural or unnatural perception, the fact is that when we talk about perceptive tools there is more than our sensors, there is also technology. And one may think that among these perception tools we have to include also future technologies, and that would be a very fair reasoning.

So another worry comes up after the above reasoning. We are pretty much incapable of making statements of what technology will be available for us in a possible future. So, how am I supposed or allowed to talk this way about perception? One may as well suppose that in a possible future, there would be no abstract objects left because the technology in hand would enhance our perceptive tools in such a way.

What I have to say about this "problem of technology" is very simple and logic to assume as well. In my view, technology is just an enhancement or improvement of our perceptive tools. As I will argue later in this paper, even with our perception enhanced at the maximum possible level via technology we still would not be able to perceive everything.

After the above arguments, I think it is safe to declare that an abject being abstract and concrete should not, by any means of reason, depend on time and technology. And even further, I gave a revised view on perception while giving the above arguments. Among perceptive tools, I included present and possible technology.

Recall carefully the "atom" example. The problem this example posed relied on the fact that it was very small and that we cannot perceive it without the help of the microscope. We may also think of the smallest object that can be perceived with actual technology. We can say that there is some smaller object, maybe constructed by cutting or dividing the previous one, and this smaller object would still be concrete (since earlier I argued that this fact shouldn't depend on time or possible technology to perceive). In the present time, think about this smaller object. Is it a part of our imagination? And still, I argued it is concrete. This makes me inclined towards thinking that what we imagine has a strong relation to what is concrete, or better, what could be concrete. And I wish to describe this relation using mathematical concepts and some little fair assumptions about the nature of perception itself.

4. Perception and imagination

In the previous section my arguments pointed somehow to a relation between perception and imagination. And I pointed out as well a main difference between perception and imagination. Things one perceives do not depend entirely on that individual. The position of the individual in time and space directly intervenes in what that individual perceives, at least in general. But this seems to vanish somehow when we think of objects an individual imagines. I have never seen a spaceship, so I have never perceived one. But I can imagine what a spaceship is. So, intuitively, in some sense the boundaries of perception, this inability to perceive some objects due to a lack of physical abilities or time, somehow vanish in imagination. I found interesting something that Albert Einstein said:

"Imagination is more important than knowledge. For knowledge is limited to all we now know and understand, while imagination embraces the entire world, and all there ever will be to know and understand."

I guess one can embrace a discussion on imagination in many angles. I would propose a particular one, one which will do to the cause in hand. Think about rabbits. We all heard of how fast they reproduce. And further, suppose I have a family of rabbits in my home. What I perceive is limited to the rabbits I have in my home, but I can imagine my entire home full of rabbits if I let them reproduce freely. Or think about a building. You can see maybe only half of it from where you are standing, but your imagination fills up for the remaining part you can't see even if you have never seen it. These examples orientate us to a very important intuitive statement, but before I state that I would like to give a mathematical example. Think about an infinite sequence; say the sequence of positive even numbers (2, 4, 6, 8 ...). There is no space where I can possibly write down the entire sequence and even if I have that kind of space in my disposal, time wouldn't be enough for me. So in a restricted time and space, I could only state or write finitely many terms of the sequence, there is a limit I reach. Suppose for example, in one hour time I could write down on a board or computer, the first 1000 terms of this sequence. But if you ask me what the 1001th term of the sequence is, I am able to answer that immediately. I can somehow imagine the sequence.

The above examples lead us somewhere, to some very important relation between perception and imagination. Time and space make our perception finite; we could never observe an infinite set of objects or data. I think this is a cruel fact of the world we live in. But somehow we stand to this cruelty nature poses on us: **where our finite perception stops our imagination starts**. And imagination is infinite, sends us everywhere we want to go and time space limits vanish amazingly. Somehow, and this is something I find extremely fascinating, we embrace infinity. So what I could say is that the infinite is inside us, not ultimately out of our reach. Actually I personally believe that this infinity carves a good portion of what our nature is. Besides the restriction in language, time, space and many other things we still have it and the way we see the world depends on it in a very strong way.

5. Imagination and perception

So, somehow imagination is related to perception. I embraced the view that where finite perception stops, infinite imagination starts and fills up what is left that we can't perceive. There is another angle from which I find crucial to see this relation in order to give a satisfactory description on how perception and imagination are related. In a previous example, I considered a building. And I said as well that even if we see half of it, we can imagine the other part even if we have never seen it before. But still, even if this gives us a hint on what the role of imagination is, it still leaves a gap. There should be a building in our perception field in the first place. In other words, there should be an inverse relation as well between imagination and perception. Somehow, what we imagine depends on what we perceive and my intention is to describe this relation using set theory in mathematics.

One of the questions I posed previously was "why there are numbers?" I also gave a bunch of arguments using mostly geometric examples of how we perceive and describe objects but I don't think I gave a clear answer to the question above. Well, at this point I think I am ready to give a good and reasonable answer. When we perceive the outside world, or the real world, somehow we can differentiate objects from one another. I don't believe that we see the outside world as a unique object entirely, but as a bunch of objects. Somehow, as we may recall from the example with the two cows standing on some field, we can tell that there are two objects, namely one cow and the other cow. Even if the cows are identical, we still see them as separate objects. I believe that this particular aspect of our perception is a fundamental one. And I think that this aspect of perception gives an answer to the posed question. There are numbers in mathematics to illustrate the fact that when we perceive the outside world we can differentiate objects from one another. And further, I strongly believe that this allows a set-theoretical approach to the problem of perception.

So, I can say that it is fair enough to say that whenever we perceive, we don't perceive just one object but a bunch of them. In other words, our perceptive field can be modeled as a set and the elements in this set are the objects that we perceive. This may seem quite trivial and logic but I find this conclusion to be essential and delicate. We are jumping from the real word to mathematics and as I stated before, the two of them should never be confused or mixed up. They may seem sometimes the same thing but this is not true. Sometimes the model is so clear, like in the above case, that one may think we actually are seeing the model.

So I have now a set in hand and I would like to give it a name, *the set of perception*. The elements would be objects we perceive and from arguments I have already given it is fair to say that this perception set is finite due to restrictions in space time. Naturally, thinking of the subject in hand from this angle, I approach to the necessity of another set. There are objects that we imagine as well, so it is fair, to consider another set which I would like to call *the set of imagination* or the imagination set. And again from previous arguments there are some things we can already say about this set. First, as stated above, the elements would be objects we imagine. And more importantly, this set, unlike the perception set, is infinite. This set theoretical approach to the problem, as you will better see later, puts me in a very advantageous position to proceed with further conclusions.

I have in hand now not one but two sets, the set of perception and the set of imagination, one finite and one infinite. I find it extremely important to describe how these sets are related and the first step was the construction of the sets themselves. I find the nature of imagination very intriguing. Think about a piece of wood with a hole inside it that has both ends in the surface of that piece of wood. We can see the two endings of the hole and maybe we cannot see how the hole is shaped. But in our minds we imagine the trail this hole takes, we give it a natural shape even if it does not correspond to the actual shape. So somehow, imagination is "dense", whatever that means. If you recall the building example, it also indicates that perception is incomplete. I suggest thinking about imagination as a completion of perception that leaves no empty spaces. In other words wherever we are standing, we are perceiving objects around us and at the same time we are imagining what there is beyond that.

Also I would like to give another example before I give some kind of conclusion. Suppose I see a girl. I see her as she is, and to my perception set one element is added naturally. But at the same time, I can imagine her with longer hair, various eye colors, wearing different clothes. I even can imagine her being taller or thinner, maybe I can imagine her even with infinitely long hair that spread over the entire universe. **One object in my perception set generates even infinitely elements in my imagination set. But to have some elements in the imagination set, I first need some element in the perception set.**

In other words, **imagination is generated by perception** and it is as well dense, it leaves no empty space. In set theory, this kind of relation is called an *extension*. So the imagination set is an infinite extension of the perception set and it is generated by this one. I believe as well, as I said before that it is a completion or it is “dense”. These aspects of perception and imagination, this set theoretic relation between them and this density that characterizes imagination, I believe, define the very nature of what we feel about abstract and concrete.

6. Imagination and the notion of “concrete object”

I have talked until now about imagination and discussed it because I find it extremely important in terms of the job I’m trying to do here. But I haven’t mentioned yet what imagination has to do with anything. What job has to be assigned to imagination when we want to describe concrete and abstract objects? Intuitively imagination has to do something with the notion of concrete since it has such a deep relation with perception and I am willing to give an account of it.

There are concrete objects in our world and we perceive them, sometimes with the help of technology as mentioned before in this work. And there is a perception set that I constructed in order to describe and group those objects into some set. **So every time we perceive something the perception set gains one element immediately.** The nature of this set is directly related to the actual world we live in. The objects in that set are observable somehow.

But obviously the objects in the set of imagination lack of such actuality, somehow they are not observable. I can imagine a man who is three meters tall; maybe I could call this man in my imagination a giant. Certainly it is fair to say that this man is not observable. So we may assume that we could never perceive such a giant in our life time. This way one may conclude that it is reasonable to consider this giant an abstract object. This giant is surely not actual but there is more to talk before calling him abstract. I would like to give another example to illustrate this doubt.

Instead of an unnatural giant we may think of some other unnatural object. Let’s talk about tables. In our actual world there are only finitely many tables, with different shapes and colors and many other things. But there might be some kind of table that is not actual, maybe with some sort of strange shape that we can imagine. I would like to call this unnatural table “the strange table”. So the strange table, as well as the giant, by parity or reasoning should be considered abstract because lack in actuality and thus cannot be perceived. Thinking about the strange table I may as well say that I can construct one. I can make this object in my imagination actual with the use of some technology. And maybe I can say that in some possible future some table artist will create it. And at that time the strange table will gain actuality and be considered concrete. But then, haven’t we concluded that an object like the strange table being abstract or concrete depends on technology and time? This I do not consider to be reasonable. And going back to the giant example I can assume that in a possible future there might be some technology to create this giant or that maybe nature will create this giant itself.

One may find tricky my above reasoning and talking about technology. It is logic to think that some kind of technology provides me the ability to construct the strange table, but a lab to produce a giant is a little bit awkward. Well, I would suggest forgetting about technology for a moment in order not to confuse ourselves. Instead I suggest thinking about it the following way: **if the giant were**

actual it would be concrete because we would be perfectly able to perceive it. And the same argument would hold for the strange table as well. If it were actual, it would be concrete. I would better put things this way: somehow the objects in our imagination are welcome in our actual world. The only thing that makes these objects not perfectly concrete is the lack of actuality. That is why I suggest thinking twice before calling objects in the imagination set abstract. Abstract is something else, something “not welcome” in our world.

I feel myself obligated to give account of another issue regarding this topic. These objects I was talking about might be referred to as “actualized possibles”. And Quine gives an argument, a brilliant one in my opinion, against considering these objects in ontology. The argument considers the problem of identity. According to Quine, unactualised possibles lack in identity. In his paper “On what there is” published in 1948, he gives the example of a possible fat man standing somewhere. He claims that we cannot consider this object ontologically because we wouldn’t be able to answer some basic questions about this fat man like “what color are his eyes” or “how much does he weight”. So this unactualized possible has a profound lack in identity. Seemingly the same argument holds against the objects in the imagination set that I created. How can the giant or the strange table be welcome in our world if they lack in identity?

There is a big difference between unactualized possibles and the objects in the imagination set. Previously I talked about the nature of imagination and concluded that it is somehow dense. I used the word dense because imagination leaves no empty gaps. Does the giant lack in identity? I can answer any question about this giant I imagine. He has an identity in my mind; I could even describe how he smiles. Maybe he wouldn’t be welcome in my home, but he is certainly welcome in our world! So the objects in the imagination set overcome this identity problem and in my opinion they do not deserve to be called unactualized possibles in the classic sense of the word. Sure they are unactualized, but not abstract at all. The only thing that makes them not concrete is not being actual and that’s all.

From a set theoretical point of view I stated before that **the set of imagination is an extension of the set of perception.** The identity discussion about the objects in the imagination set reveals something important about the nature of this extension. When we give account of concrete objects the first instinct is to consider only the set of perception and to consider everything else abstract. But this is unfair because objects in the imagination set have an identity and are welcome in our concrete world. So this is not just a set theoretical extension. This is a natural extension of the actual concrete world we live in. **And the extension is not abstract at all**. But it would be also daring to call it concrete since there is a lack in actuality. If the objects in the perception set define a concrete reality, the objects in the imagination set define a naturally extended concrete reality. And I believe we all feel the effect of this extension. I also believe that this extension is a blessing for us.

To put my arguments together I would say that **we shouldn’t call objects in the imagination set abstract.** They are just a concrete extension; they could one day be a part of the actual world. If they weren’t welcome, how could mankind reach this state of development and aim for more? There may be things we imagine that will never exist, but yet they are welcome in our world. And sometimes they affect us as much as actual objects. I will repeat it again, only the lack of actuality cannot make these objects abstract. Somehow these objects live on the same ground with us. This common ground is what I am willing to define somehow. There are objects which cannot coexist in the same ground with us, like beauty for example. A mere talk about abstract and concrete wouldn't even get close to explain the situation here.

7. The problem of infinity

I think we all agree about the fact that our perception has its limits. One can see those limits in different angles of course. As Locke pointed out, we can sense only some aspects of objects we perceive. For example we can see the object and smell it, but maybe there is some aspect of the object that we cannot recognize. One can talk about the limits of perception in this perspective. I believe that this perspective has to be embraced by other branches of science because there is nothing to describe about it. I would like to embrace the problem from a different angle. I think that perception is limited in some other way. When we perceive we never perceive only one object, and the existence of numbers in mathematics testifies it. I argued that that is the reason why we use numbers. I made it clear as well that this set of objects we perceive at any time is naturally extended by some other set of objects that we do not perceive but imagine. So we never leave ourselves incomplete. And I used the term 'welcome' because somehow those object we imagine could exist in our world with a definite identity. I didn't find it reasonable to call those objects abstract because the only thing that separates them from being concrete is the lack in actuality. It would be unreasonable to call them concrete as well. So I would like to call the objects in that natural concrete extension *almost concrete*. And I would like to work with this new notion I introduced and to discuss limits of perception in that sense.

So, almost concrete objects are objects that could very well exist in our concrete world. They are perceivable, that means if they existed we would perceive them. And, as I argued before, our imagination covers those objects. This is what I would like to define as the limit of perception. **The set of almost concrete objects is all there is to perceive for us and anything else out of the set would be unperceivable via our sense organs.** Perception is bounded by this set in this sense. We already know the nature of the objects in it but how many of them there are? How big can this set be? This is a question worth answering and this is worth wondering about. I believe it would be useless and impossible at the same time to count the concrete objects we perceive. But to count the number of perceivable objects is not useless at all. It would define our nature, the nature of perception and how we see the world. By answering this question we would be able to describe our limits in perception and put a definite relation between concrete, almost concrete and abstract.

It appears that the imagination covers all almost concrete objects. So, what I named before the imagination set would be exactly the set of all almost concrete objects, perceivable objects. **I argued before that this set of almost concrete objects is infinite.** The fact that this set is infinite doesn't mean it is not bounded or restricted somehow. From mathematics we know that there are different degrees of infinity, the smallest one and then bigger and bigger infinities. So the task would be to **determine how big the infinity of almost concrete objects is.** And that would determine how much our perception can take.

In other words, our ultimately enhanced perception which would leave no gaps - that would be equal to imagination - can take infinity. The problem I pose is the following: **to what order of infinity could our perception go?** Those almost concrete objects that we imagine, that part of the world that we give an identity to even though we cannot perceive it, is infinitely big. So I have to give an account of how big is that infinity, that concrete infinity we are talking about. I would like to make a small introduction of the orders of infinity in order for everybody to be familiar with those notions and the way I see the problem.

I would like to talk about infinity in a kind of non-mathematical way or at least involving less mathematical definitions than usual. One example of an infinite set is the set of natural numbers {1, 2, 3, 4, 5 ...}. Other examples would be the set of rational numbers or the set of real numbers. I would like to consider this last one, namely the real line. Further I would like to consider just a part of it, the segment [0, 1]. The segment [0, 1] is an infinite set because there are infinitely many numbers between 0 and 1 as we all know. A good thought experiment would be trying to imagine the set of natural numbers and the segment [0, 1]. When we imagine the set of natural numbers it is easy, the numbers just fit in our mind naturally; **they are put in a list** and it seems to be the case that we can imagine all of them. On the other hand when trying to imagine the set of numbers in the segment [0, 1] the same thing does not happen. We can imagine that set in a pictorial way as a part of the real line (as a whole) or we can try to categorize the set in different ways like radicals or rational numbers. One can easily feel that the two sets in hand are both infinite but not of the same kind of infinity. The infinity of the segment [0, 1] does not seem to fit in our imagination. In mathematics we make a distinction between these sets. The first type of infinity, or infinity of order 0, is a type of infinity that fits into a list. For example the set of even positive integers {2, 4, 6, 8 ...} fits into a list, meaning we can tell what the 1000th term is for example (it is 2000 in this case). **But that is not the case for the set of numbers in the segment [0, 1].** We cannot fit this set into an infinite list; we cannot tell what the 1000th term is, like in the previous case. This aspect is what makes this set impossible to imagine. And this is what makes us feel the difference between these kinds of infinities. **It seems pretty clear to me that we can only imagine the first order of infinity.**

So there are lots of **orders of infinity**. What I would like to do is to **determine the infinity associated with almost concrete objects.** Having an idea of how much we can imagine would help us to understand the nature of perception. The feeling is that **our imagination is restricted to what can be embedded in an infinite list** and I am convinced that that is the case. I would like to give an account of this hypothesis with a mathematical model and some aspects of numbers that reflect on the world we perceive.

8. A good question

For the sake of counting I would like to see almost concrete objects from a certain perspective. I would like to think about almost concrete objects as some kind of *deformation* of concrete objects. It is a mental process, a natural one I would say, the deformation that happens. An example would be a white ball which is concrete and we perceive it. We can imagine that ball being black and in this case the deformation would be a change in property that produces this almost concrete object, the black ball. Besides the change in color, we can make changes in size (if the diameter is 3 meter we can imagine it to be 5 meters). So we have these deformations of some of the properties the object has and they produce some almost concrete objects in imagination. The first thing one would like to discuss or clarify is what kind of properties that an object has can be deformed in such a way that the deformation will preserve the concrete nature of the object.

I would say that a definite answer cannot be given in this case. Of course we can deform properties like “being red” or “having 3 meters length” but we could not deform properties like “having a length”. This would lead us to a discussion about first and higher order properties. There are some qualities that the object has that are called immanent, meaning we cannot imagine the object lacking that property. And I find it difficult to talk about those properties. So I would like to leave the matter as it is because I believe we have a good idea of what properties can be deformed or not. I would rather talk about something else that concerns me. There are these properties that can be deformed and we know what they are somehow but how does this process of deformation occur ‘numerically’?

What I mean is that the each deformation produces an almost concrete object so the nature of the process would determine the number of almost concrete object welcome in the world we perceive. One may give the following very strong argument: *“If we accept the above deformations as the generators of almost concrete objects than there are uncountably many of them. Because the process of deformation occurs continuously and in this process we find ourselves producing uncountably many objects. Say you have a stick of length one meter and you deform it to a stick with length two meters. While deforming with your imagination you produce any stick with length between one and two meters. So you produce uncountably many of them and as a result our imagination of concrete is not embedded in a countable list of objects.”*

An intriguing matter pops up and it comes as follows: **is continuum present in perception or not?** I believe that the above reasoning has a flow in it and it comes from a confusion of perception and imagination with conception. The fact that we can conceive continuum does not mean that we perceive it, the same way that the fact that we conceive an object with no length does not mean me can perceive or imagine it. So I would put the problem differently: **is continuum conceived or perceived?**

9. The nature of a deformable property

In the end the whole problem comes to the continuum question, **whether a continuous process can be perceived as it is or not.** And recall that we can perceive with the help of various technologies, so the question becomes deeper when we think about it that way. Are we able to detect continuum in this world, with these perceptive means or continuum is a product of our conception of the world, a conception that completes the world we live?

When we talk about deformation in property there is an aspect I would like to remark in order to proceed. Various properties have different ways to deform in our imagination. There are metric properties such as length or width. These properties are *metric* in the sense that we can compare two lengths or two widths independently from the measurement we use. Even if the meter was not invented yet we could still be able compare the length of two objects. But properties like color apparently are not metric. We could not compare a blue object and a red object. On the other hand we can compare two red objects because there is dark red and lighter red and so on. Every deformation we make has some notion of metric involved. All deformable properties of an object somehow stand in some sort of continuous scale in our minds. We can consider it an aspect of our mind that allows us to compare objects we perceive. It is even a generator of imagination. How could we imagine objects we don't perceive if metric comparison between properties of objects was not possible?

Our intuition of the world pushes us in some direction: we need a sense of distance in property. In the world we conceive there is a distance in properties between any two abject and this is the reason why some objects resemble each other in our eyes and some do not, or resemble less. This is why I would like to get back **to the continuum problem from a metric perspecti**ve. Since I am considering deformable properties, I can use this aspect of these properties to approach the problem. All deformable properties are similar in some respect; there is a 'distance' we put between objects in virtue of these properties. So we can consider one of them to analyze continuum, color or length, and the analysis would hold for any deformable property.

10. An algorithm

For the sake of instantiation I would like to consider length (it is easier to talk about and to give examples of deformation in length). Suppose we want to deform the length of an object, in other words we want to imagine the same object with different length by deforming the length the object has. I want to discuss whether the deformation can be continuous or not in our mind and to do that I will give an algorithm.

Remember the microscope example: we can see an atom only by the help of the microscope but there may be smaller objects that current technology does not allow us to perceive and we can only imagine them. So we may think of how far we can go with our imagination. Should there be some kind of unit of deformation and how small would that be? In other words, how far can we go with imagination and the process of deformation of length? To start an analysis I will suppose without loss of generality that the length of the object is one unit and further I will model this length as the segment [0, 1] (we can do that since length is the only property we are interested in).

0 ———————————————— 1

Say I can imagine the object with a different length a_1 which is less than 1. Put the point a_1 in the segment.

0 —————————— a_1 ——————— 1

Since I can imagine the object with length a_1, we may argue that I can imagine the object with a length between a_1 and 1 or less than a_1. So we can go on with other steps. Each step will consist putting a point in the segment. At each step n we could say that we can imagine the object having length a_n. So this comes to filling the segment with points. More importantly, the problem of continuum comes to weather we can fill the segment continuously or not.

We have to consider when the process described by the above algorithm stops. The problem of necessitating an infinite amount of time is not a serious issue since imagination allows us to work without a time boundary; the same argument holds for space boundary. The first thing to determine here is whether the process stops after finitely many steps. Well, stopping after finitely many steps would mean that there is some finite length and that length would be the minimum length we can imagine. But that does not seem to be the case because no matter how small an object is we can imagine a smaller one, so the process stops after infinitely many steps. Since we are filling the segment with points, one may argue that we are done when we put all the points. But that is not the case: we have a sequence $\{a_n\}$ and the process ends when we cannot perceive any more, meaning when in our eyes the segment is covered with points and that means when the sequence becomes dense. When we imagine an infinitesimal small quantity, we do not divide it anymore; we cannot proceed because it is not the case that we can imagine smaller objects or that in a possible future there might be a technology that will allow us to perceive smaller objects.

We could never perceive continuum in nature. If something is dense, like the rational numbers in the real line, it fills our perception and we can no longer see empty spaces. We cannot perceive a distance between rational numbers; they fill all the line in our eyes. The same thing holds for the sequence of deformation a_n described in the algorithm: that sequence will be dense in perception but still will not cover the whole segment**. So a deformation that happens in imagination happens with a dense sequence of other deformations.** I would suggest thinking about it this way: the above process stops when we cannot imagine more, but **the fact that we cannot imagine more means that our imagination is full, it does not mean that there is nothing more.**

11. No continuum

There **is no continuum in imagination or perception**. Continuum is a product of conception. A deformation in our imagination happens discreetly, giving the illusion that it happens continuously. The smallest quantity we imagine is an infinitesimal small quantity and the basic principles of calculus are built on that quantity. In our imagination only a countable set can fit; only a list of objects is imaginable. If a set does not fit in a list we can only conceive the set but not imagine it. Any deformation contributes with countably many objects in our imagination, so **the number of objects we can imagine is countable, as big as the first order of infinity.** This is why we imagine the set of natural numbers so easily on one hand and on the other hand we cannot imagine the set of real numbers.

When God created humans, he prohibited us to see the whole thing. Maybe we would have a different opinion about the world if our perception was more powerful, if we could imagine or perceive an uncountable set. All conceptions we have about the world come from countably many perceptions, from a list of states we observe. We could never go beyond our conceptions because we could never go beyond our observations. **We could never imagine a world hosting uncountably many almost concrete objects because we wouldn't be able to perceive them. We perceive the world object by object and even when we cannot perceive any more we fill the remaining part object by object. Our mind does not accept anything but a list of objects, our ability to observe is restricted to countably many states of affairs. The concept of continuum is a way to make sense of it, it is an intuition conceived to complete the world.**

I hope to have made myself clear enough. To accept that what we perceive and imagine is countable is not easy, but when you think about it, it makes sense. I think I have given enough arguments, some of them intuitive and some of them mathematical. It is hard to accept that we cannot believe what we see and perceive, but this seems to be the case. All we have to do is trying to make sense of the world the way we see it, to accept our limits. I could go on with some applications of the result in conceptions about laws of nature or epistemology, but I would like to stop here and first reflect on the nature of the result I have drawn. I believe the conclusion of my work is itself powerful: even if not applied somewhere it reveals us an essential aspect of the way we see the world. The insight of the **numerical limits of perception** gives us enough to think about.