Nicholas Webb [a], Alexandrina Buchanan [b]

[a] School of Architecture, University of Liverpool, Liverpool, United Kingdom,
[b] Department of History, University of Liverpool, Liverpool, United Kingdom

# Tracing the Past: Establishing Tolerances when Investigating Medieval Vault Rib Geometries using Digital Tools

## Introduction

This paper explores the digital documentation processes used to investigate the design and construction of medieval vaults in the British Isles. Here we can reopen investigations initiated in the past using techniques that are becoming widespread in the present, such as laser scanning and total station. We are also concerned with how our investigations are interpreted, particularly looking to the future once digital tools and techniques advance further still. We will focus our discussion on data collected at the east end of Wells cathedral in the south west of England and the choir at Ely cathedral in the east of England. Comparisons will be made between relative strengths and weaknesses of three different data collection methods as well as investigating the potential for human error by comparing the results when the same task is repeated several times.

Most Gothic vaults, as found in greater churches and cathedrals, work on the principle of first defining a series of ribs, with the in-between surfaces, or webbing, filled in afterwards either using, or at least subservient to, the geometry prescribed by the ribs already set out (Nussbaum and Lepsky 1999). This reverses the principle of stereotomy where the full geometry of a vault is designed prior to construction, which appears in neo-gothic vault examples in the nineteenth century. Our previous research suggests that to design and construct a medieval vault in the thirteenth and fourteenth centuries required the use of a tracing floor plan, where masons could experiment with 2D geometry in plan as well as projections from the plan to investigate the 3D design of each vault rib. This process resulted in a rich variety of vaults built throughout Europe and beyond which have long been studied to try to ascertain the different design and construction processes used. However, it is only now that digital surveying and analysis tools have become widespread that we can quickly and accurately document many vaults for analysis purposes; an issue which hindered pre-digital investigations (Willis 1842), as surveying vaults is logistically challenging because of their locations - often high up in church and cathedral interiors.

## Methodological questions

In surveying the vaults our primary documentation method has been laser scanning, resulting in detailed and accurate point cloud models, which can also be converted to mesh models. Recently we have also experimented with total station surveying, and previous investigations have used photogrammetry (Webb, Buchanan and Peterson 2016). Using the survey data, vault ribs are digitally traced along intrados lines (their outermost edge) enabling us to extract data such as the radius, arc centre point in relation to the impost line, as well as the boss or apex height (Fig.01). The high accuracies prescribed using digital tools, however, opens a series of questions in the interpretation of such data. For example, if a vault rib has a radius of 4480mm and another related rib has a radius of 4360mm, is this difference of 120mm significant? Three scenarios are proposed:

1. It represents an intentional change in design by the medieval masons.
2. It is a result of acceptable tolerance levels in the design and construction process, including settlement of the vaults post-construction.
3. It has occurred due to inaccuracies in our own documentation methods moving from the physical artefacts to the digital representations.

The first and second scenarios are beyond our control, yet, if found, could be important in aiding our understanding of medieval vault design and construction. On the other hand, the third scenario is within our control and therefore becomes the focus of this paper, and its potential impact on the first two scenarios.
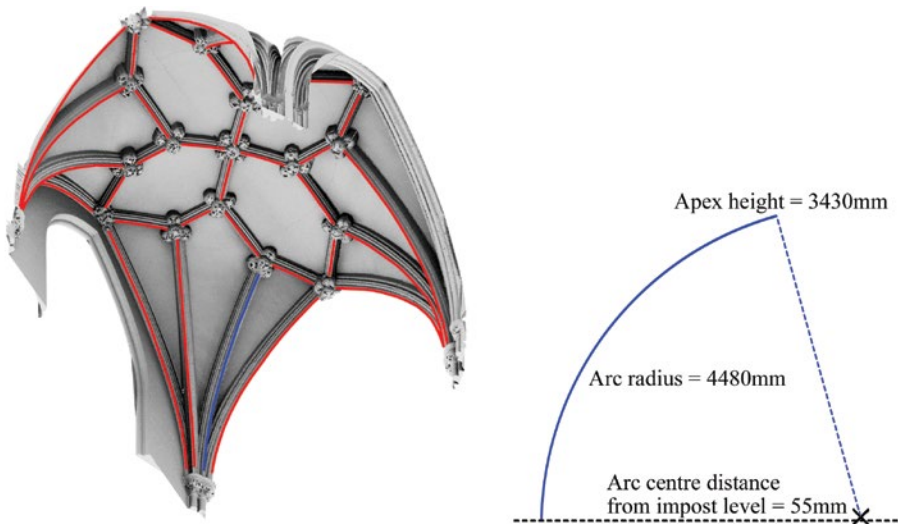


Fig.01. Tracings the mesh model of bay N5 at Wells cathedral, with rib Tr_Ti_2 extracted in blue to show its key geometry features.

### Process options

To investigate possible inconsistencies in the digital data collection, we must first understand the processes used, and the different options available. In tracing intrados lines our standard method uses mesh models. This is because, as a surface model, it is

easy to visually locate intrados lines, particularly compared with point clouds (Fig.02). The mesh is traced in Rhinoceros 5 using the 'three-point arc' tool requiring start and end points, followed by a further point along the arc to form its curvature. There have been cases where capturing the curvature of vault ribs are more problematic, such as in the Lady Chapel at Chester cathedral, where ribs snake their way upwards making single arcs harder to define. However, at our case study sites this is not a significant issue. The main assumed problem in tracing mesh models using the arc tool is that each arc is created of three points. Therefore, as hundreds of points are available to 'join the dots', it is highly likely that if traced multiple times, a slightly different curve will be created. Hence one aim of this study is to test the scale of differences when the same rib is traced several times.
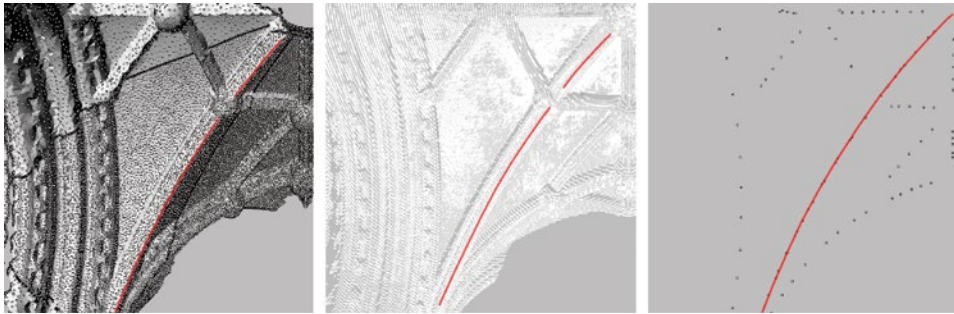


Fig.02. Three tracing methods used for the investigation. Mesh model (left), point cloud model (centre), and total station (right).

An alternative method to our standard mesh tracing procedure is to create arcs using the point cloud data itself (Fig.02). Here it takes time to locate the intrados lines amongst the other thousands of points, but once isolated, the 'circle: fit points' tool can be used to create a best fit curve through them. A further method is to use a Leica total station (Fig.02), where vault rib intrados lines are documented from the cathedral floor creating a series of digital x,y,z co-ordinate points. These are also joined together using the 'circle: fit points' tool in Rhinoceros 5. As the points along the intrados lines are the only objects in the digital model this is much simpler than using the dense point clouds. Potential issues with the total station method are intrados lines being inaccurately surveyed in-situ, such as not enough points being used to document them. We suggest seven points should be used across the breadth of each rib as a minimum.

### Lines of enquiry

Based on the above, we will first explore these three different digital tracing methods using vaults surveyed at Ely cathedral. Our aim for the project is to continue with the mesh model method, therefore by looking at the two additional methods we can test the reliability of our mesh model results. Also, we will investigate the possible role of human error when tracing vault ribs using the standard mesh model method by comparing the same arcs which are traced on three separate occasions, this time using the vaults surveyed at Wells cathedral.

For each of the two investigations, two comparatives can be made. Firstly, we will examine each rib against their duplicate ribs. In relation to the Ely data we will compare the same rib traced three times using different methods and in relation to the Wells data we will compare the potential variations when traced multiple times using the same mesh model. Next, we will examine individual ribs against given clusters of data, for example, at Wells cathedral we established a radius of 4599mm as significant, therefore individual rib data can be compared against all other rib data forming the same set.

Establishing clusters is important to the project, as it enables us to identify patterns in geometry, which may provide clues as to how the medieval masons designed the vaults. Significant data clusters were initially identified manually, then, as it became apparent that more scientific methods were preferable, SPSS Statistic was employed. Statistical clustering enabled us to identify different cluster patterns and centres. With hierarchical clustering we could test different numbers of clusters, and identify the centre of these using K-means clustering. The hierarchical clustering method associates each piece of data to its nearest cluster centre using a squared Euclidean distance, and we could then identify how many clusters were required for the arc radii, arc centre's distance from the impost level and the boss or apex heights. This was aided using means, standard deviations and Z-scores.

The statistical and technical nature of this paper is necessary, as it offers an opportunity to interrogate base data to ensure that findings are as accurate as possible. Scholarly investigations into medieval vault design and construction using digital surveying and analysis tools are increasingly prevalent across Europe and beyond, whereas scrutiny of the base data and methodology is less evident in published work, which could potentially lead to misleading results and hypotheses.

## Ely cathedral: comparing digital documentation methods

For this study, we utilised survey data from vaults of bay C8 in the choir, dated around 1328-1335, and the vaults of bay N8 in the north choir aisle, dated prior to the choir vaults (Pevsner and Metcalf 1985). Once all ribs had been traced using the three different documentation methods of mesh models, point cloud models and total station, we could record the required geometries and populate the results into a table, which were then investigated by comparing duplicate ribs, as well as rib clusters.

### *Comparing duplicate ribs*

By tracing the three different sets of survey data, we had three duplicates for each individual rib, therefore we could initially calculate the average rib data of the three, followed by the standard deviation figure. For example, in bay N8 the radius of rib D1 was 3423mm when traced using total station data, 3249mm traced using the mesh model, and 3341mm traced using the point cloud data. This gave a mean of 3338mm and a standard deviation of 87mm. Once we had calculated these for all ribs in bay N8 and C8, we worked out average standard deviations across all the data, which could then be used to establish satisfactory tolerances (Tab.01). In the data, we removed the measurements of the longitudinal ribs and one set of longitudinal tiercerons ribs in bay

C8, as these proved difficult to record using total station, therefore a fair comparison could not be made between all three sets of data. Ely cathedral was our first use of total station, and we have since rectified the problem which prevented successful documentation of such ribs.

Table 01. Standard deviation averages when tracing each rib three times at Ely cathedral.

| Averages | Arc radius | Arc centre distance from impost level | Boss or apex height of vault rib |
|---|---|---|---|
| Mean S.D. | 107mm | 84mm | 32mm |
| Median S.D. | 84mm | 77mm | 22mm |
| Maximum S.D. | 340mm | 250mm | 74mm |

Using table 01 to compare the full sample of 38 tracings (each traced using all three methods), we can be satisfied that most tracings will be reliable within the mean standard deviation figures, regardless of the method used to trace them. Table 01 also reveals that deviations are largest when analysing radii, and smallest when analysing the boss or apex heights. Finally, highlighting the maximum standard deviations in the table shows us that there are occasions where data is less reliable when comparing the three tracing methods. As such, we can use clustering to recognise possible poor data.

### *Comparing clusters of data*

As described previously, we used clustering to identify trends within the data. Once these were defined we could highlight anomalies. To do this we first created a mean of all data in the cluster, followed by its standard deviation. We could then look at the distance of each rib from that mean, as well as calculating its Z-Score, which is a method of quantifying data that is increasingly distant from the mean data. We can use the Z-Scores to predict whether a rib's data is within one standard deviation of the mean (that is a Z-Score between -1 and 1), and therefore whether we need to interrogate it further. Using this process, we could then check the tracings to look for errors in our process, and if found they could be re-traced. If the tracing process was deemed satisfactory, this then indicated a possible anomaly in the original construction of the vault.

Comparing Z-Scores outside of -1 and 1 in all three tracing methods, similar numbers of data were flagged in each case, suggesting all are equally reliable methods. In terms of deciding whether to continue with the standard mesh model tracing method, we can therefore look to the times taken to survey and trace the vaults in each case. This was 35-40 minutes for the mesh model, 65-90 minutes for the point cloud and 120-125 minutes for total station. As such, the mesh model method is easily fastest, so we will continue with this process for our project. It is likely that surveying and tracing vaults with point cloud and total station data will become more efficient with practice, however, we will use these as back-up datasets as we can now be confident our mesh model results are accurate within tolerances as demonstrated.

## Wells cathedral: comparing multiple tracings

The second study compares the results of ribs traced multiple times, this time all traced using the mesh model method. The aim here was to establish acceptable tolerance levels, given that we know results are likely to differ slightly with every tracing. For this study, we utilised survey data from Wells cathedral, specifically bay N2 and S5 of the choir aisle vaults, probably 1330s and the north east transept vault, assumed completed by 1329 (Draper 1981). Once each bay was traced on three separate occasions, we could again populate the data into a table for analysis.

### Comparing duplicate ribs

The first comparison was once more made by investigating each rib's three duplicate tracings, this time all using the mesh model method. A table recorded the rib radius, the distance of the arc's centre point from the impost level, as well as each rib's boss or apex height, which could then be used to calculate the average figures for each rib and to compare their standard deviations. In exactly the same way as the Ely cathedral data, we could then compile a summary table this time of the 54 ribs in the three sample bays, all traced three times (Tab.02).

Table 02. Standard deviation averages when tracing each rib three times at Wels cathedral.

| Averages | Arc radius | Arc centre distance from impost level | Boss or apex height of vault rib |
| --- | --- | --- | --- |
| Mean S.D. | 42mm | 26mm | 6mm |
| Median S.D. | 28mm | 15mm | 4mm |
| Maximum S.D. | 203mm | 170mm | 69mm |

We can be satisfied that data is acceptable within the average figures given in table 02, regardless of who or when the mesh model was traced. On the other hand, if data lies outside of these standard deviations, we should interrogate it further. For example, for rib Tr3 in bay N2, the first tracing gave a radius of 4625mm, the second 4846mm and the third 4653mm. The second figure seems high, so this tracing would need to be checked. In our standard methodology, however, we would only trace each rib once and re-trace it if there were issues with the figures as previously discussed. As such we would not usually be able to compare duplicate data for ribs, and therefore looking at clusters of data is more informative. Having said that, the confirmation of standard deviations in comparing duplicate ribs is reassuring that our methodology is satisfactory.

### Comparing clusters of data

Using SPSS Statistics we could again form clusters of data, this time for the vaults at Wells, and flag Z-Scores outside of one standard deviation from the mean of each cluster. For example we identified three clusters of radii, the largest being 4599mm. The first tracing of rib Tr1 in bay N2, which was found to be part of this largest cluster, had a radius of 4901mm and a Z-Score of 2.20. The second tracing

had a radius of 4743mm and Z-Score of 1.01 and the third had a radius of 4725mm and Z-Score of 0.95. This tells us that all three are above most figures in the cluster, however, we should be concerned about the first figure, particularly given its high Z-Score, and interrogate the tracing further. In reality, it appears that this rib does have a bigger radii, which again may give us clues on the design and construction of the vault. Similarly to the Ely cathedral data, we can use this process of creating clusters and Z-Scores to constantly critique and check our data.

## Reflections and ways forward

Using the findings in each case we can formulate a set of tolerances to work with when tracing individual ribs. Comparing Ely cathedral (different survey methods) and Wells cathedral (multiple tracings), we decided to base our tolerances on the former as the average standard deviations were higher (Tab.01). As such we have a high level of confidence that the tracings produced will be satisfactory within these tolerances, regardless of the method used to trace them. Additionally, because the tolerances were smaller for the multiple tracings at Wells cathedral (Tab.02), we are also satisfied with these. In the future, we will therefore use the figures in table 03 as a guide, working to the nearest 25mm.

Table 03. Suggested tolerances for future investigations, with working examples.

|  | Arc radius | Arc centre distance from impost level | Boss or apex height of vault rib |
|---|---|---|---|
| Tolerance | 100mm | 75mm | 25mm |
| Rib example 1 | 4480mm | 55mm | 3430mm |
| Reliable between | 4380 to 4580mm | -20 to 130mm | 3405 to 3455mm |
| Rib example 2 | 4360mm | 100mm | 3465mm |
| Reliable between | 4260 to 4560mm | 25 to 175mm | 3440 to 3490mm |

By testing our tracing methods through the different experiments, we have also established a refined methodology to follow for future case studies (Fig.03). The aim here is to identify a process, particularly when dealing with erroneous data. As such, if a rib is traced using the mesh model and, once clusters of data are identified, its data falls within one standard deviation of the cluster mean, the result will be deemed satisfactory (within the established tolerances as previously outlined). On the other hand, if the data falls outside of one standard deviation of the cluster mean, we will first check the 3D tracing for errors and if found, trace the rib again. If still problematic, this could be compared with ribs traced using additional survey methods, such as total station. If the result is still not within one standard deviation of the cluster mean, we could test additional clusters using SPSS Statistics. If additional clusters are found, the whole process is restarted. If no new clusters are found, the result can be confirmed as an anomaly.
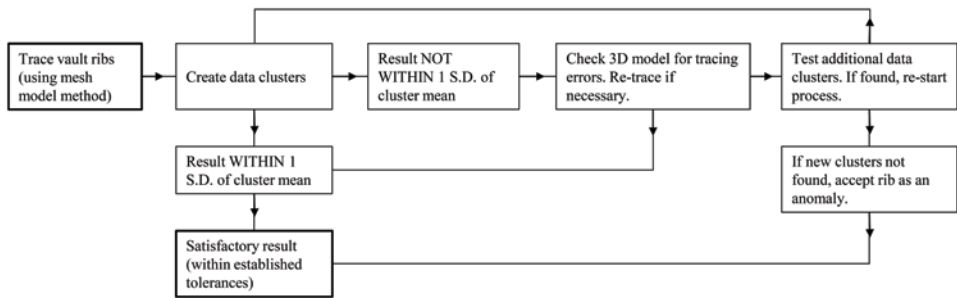
Fig.03. Proposed methodology to follow for future investigations.

It is highly likely that anomalies will be found in the data, due to the medieval construction processes as well as settlement. As such, the methodology cannot be used as an exact formula, and instead, along with the established tolerances, should be used for guidance only. In such research into vault design and construction however, we must re-state the importance of detailing the surveying and analysis methods used, which should fall under the same level of scrutiny as the results themselves.

## Acknowledgements

## Bibliography

Draper, Peter "The Sequence and Dating of the Decorated Work at Wells" in: *Art and Architecture at Wells at Glastonbury*. British Archaeological Association Conference Transactions IV for 1978. 1981, p. 18-29.

Nussbaum, Norbert; Lepsky, Sabine. *Das gotische Gewölbe. Eine Geschichte seiner Form und Konstruktion.* Wissenschaftliche Buchgesellschaft: Darmstadt. 1999.

Pevsner, Nikolaus; Metcalf, Priscilla. *The Cathedrals of England: Midland, Eastern and Northern England*. New York: Viking. 1985. p. 120.

Webb, Nicholas; Buchanan, Alexandrina; Peterson, J.R. "Modelling Medieval Vaults: Comparing Digital Surveying Techniques to Enhance our Understanding of Gothic Architecture" in: Herneoja, Aulikki; Österlund, Toni; Markkanen, Piia (eds.) *Complexity & Simplicity.* Oulu: ECAADE. 2016. p. 493-502.

Willis, Robert. "On the Construction of the Vaults of the Middle Ages". *Transactions of the Royal Institute of British Architects.* vol. 1 no. 2, 1842. p.1-69.