Maximum Ignorance Polynomial Colour Correction

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ABSTRACT

In colour correction, we map the RGBs captured by a camera to human visual system referenced colour coordinates including sRGB and CIE XYZ. Two of the simplest methods reported are linear and polynomial regression. However, to obtain optimal performance using regression – especially for a polynomial based method - requires a large corpus of training data and this is time consuming to obtain. If one has access to device spectral sensitivities, then an alternative approach is to generate RGBs synthetically (we numerically generate camera RGBs from measured surface reflectances and light spectra). Advantageously, there is no limit to the number of training samples we might use. In the limit – under the so-called maximum ignorance with positivity colour correction - all possible colour signals are assumed.

In this work, we revisit the maximum ignorance idea in the context of polynomial regression. The formulation of the problem is much trickier, but we show – albeit with some tedious derivation – how we can solve for the polynomial regression matrix in closed form. Empirically, however, this new polynomial maximum ignorance regression delivers significantly poorer colour correction performance compared with a physical target based method. So, this negative result teaches that the maximum ignorance technique is not directly applicable to non-linear methods. However, the derivation of this result leads to some interesting mathematical insights which point to how a maximum-ignorance type approach can be followed.

KEYWORDS: Colour Correction, Image reproduction

INTRODUCTION

The imaging sensors on cameras measure colour differently compared to human eyes. To faithfully reproduce colour, it is therefore necessary to convert the device specific RGB values to CIE XYZ tristimulus values. This process is called colour correction. Typically, the colour correction procedure involves measuring the device response for some physical targets, often viewed with respect to multiple lights. Mapping schemes can then be derived which *colour correct* RGBs to CIE XYZs. Colour correction methods include look-up tables, linear regression [1], polynomial regression [2], root polynomial regression and neural networks.

However, the more complex the model the more data is needed to get good results. Indeed, finding the best polynomial regression transform for multiple lights and surfaces can be tedious. Acquiring all the necessary images can take a few hours. Alternately, given a mathematical model of image formation and knowledge of the spectral sensitivities of a camera it is possible to generate images numerically. Importantly, colour correction transforms derived in this way can be based on very large data sets and the colour correction transform is delivered rapidly.

The *Maximum Ignorance* (MI) approach to colour correction take this approach one step further. Specifically, we assume that any spectrum, including the spectrum with negative values is equally likely to occur. Relative to this mathematical assumption it has been shown that the best colour correction matrix is the mapping which best takes the device specific spectral sensitivity functions onto the CIE XYZ colour matching functions [3]. This maximum ignorance transform is interesting as it also relates to the 'Luther' conditions which specify when perfect colour correction is possible. In particular, Horn [1] (and more recently Vora and Trussell [3]) has shown that perfect colour correction for any colour stimulus is possible if and only the Luther conditions are met, i.e. when the device sensitivities are a linear transform from the colour matching functions.

Arguably, however, for cameras that do not meet the Luther conditions, the MI assumption is practically not useful because it does not make physical sense. Indeed, spectra with negative power do not exist in nature, so making the assumption that they do exist has negative impact on the performance of colour correction.

Finlayson and Drew addressed this problem by introducing the concept of *maximum ignorance with positivity* (MIP) [4]. Under this assumption, all colour signals are assumed to be strictly positive and occur with equal likelihood. The colour signal is drawn uniformly and randomly from the interval $[0, P]$ (where P is an upper bound on the power at any wavelength). Finlayson and Drew presented an algorithm for MIP colour correction [4] which they showed depended only on the spectral sensitivities of the sensors (RGB and XYZ) and the autocorrelation of the spectra (which they computed in closed form for the MIP assumption). Experiments demonstrated that the MIP approach delivered much better colour correction than the MI method. Indeed, the results were found to be comparable to the physical target based approaches.

In this paper, we revisit and extend the concept of MIP. First, we reformulate the computation in terms of sensor response rather than spectral correlations. The advantage of doing this is that it allows us to consider non-linear correction schemes. In our second contribution, we show how we can derive the polynomial regression matrix given maximum ignorance with positivity assumptions (we call this MIPP). The practical importance of this work is also considered.

THEORY

Maximum Ignorance with Positivity

Suppose we represent a colour signal spectrum $C(\lambda)$ at 31 discrete sample wavelengths (10 nm, nanometre sampling) across the visible spectrum 400 to 700 nm. Let $\boldsymbol{C}$ denote the $31\times n$ matrix containing a set of n calibration colour signal spectra (one per column). Let $\boldsymbol{X}$ and $\boldsymbol{R}$ denote the 31×3 matrices containing the standard observer colour matching function and device spectral sensitivities. The camera and human observer response to the entire calibration set are captured by $3\times n$ matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ [5]:

$$\boldsymbol{P} = \boldsymbol{R}^{\intercal}\boldsymbol{C} \tag{1}$$

$$\boldsymbol{Q} = \boldsymbol{X}^{\intercal}\boldsymbol{C} \tag{2}$$

Mathematically, in colour correction we wish to map $\boldsymbol{P}$ to $\boldsymbol{Q}$. The least-squares solution to colour correction finds 3×3 matrix $\boldsymbol{M}$ which minimises:

$$\|\boldsymbol{MP} - \boldsymbol{Q}\|_F \tag{3}$$

$\|.\|_F$ above denotes the Frobenius norm (the square root of the sum of squared differences between $\boldsymbol{MP}$ and $\boldsymbol{Q}$). The Matrix $\boldsymbol{M}$ which minimises (3) is found in closed-form using the Moore–Penrose pseudoinverse:

$$\boldsymbol{M} = \boldsymbol{QP}^{\intercal}(\boldsymbol{PP}^{\intercal})^{-1} \tag{4}$$

By substituting (1) and (2) into (4), we obtain:

$$\boldsymbol{M} = \boldsymbol{X}^{\intercal}\boldsymbol{CC}^{\intercal}\boldsymbol{R}(\boldsymbol{R}^{\intercal}\boldsymbol{CC}^{\intercal}\boldsymbol{R})^{-1} \tag{5}$$

We can see from (5) that $\boldsymbol{M}$ depends only on the 31×3 sensitivities $\boldsymbol{R}$ and $\boldsymbol{X}$ and the 31×31 spectral autocorrelation matrix $\boldsymbol{CC}^{\intercal}$. The original MIP formulation uses a special spectral autocorrelation matrix. Suppose we wish to represent all possible colour signal spectra in the interval $[0, P]$, where P denotes the maximum spectral power per wavelength. Because the spectral autocorrelation matrix and its inverse both appear in (5), the magnitude of spectral autocorrelation matrix is not important. Hence without losing generality, we can assume spectra lie in the interval [0,1]. Under these conditions, according to [4] $\boldsymbol{CC}^{\intercal}$ is equal to:

$$[\boldsymbol{CC}^{\intercal}]_{ij} = \begin{cases} \frac{1}{3} & (i = j) \\ \frac{1}{4} & (i \neq j) \end{cases}, \tag{6}$$

Our new formulation is based on the expectation of the sensor response correlation. In order to tackle this problem, we first look at how to compute the expectation of the response of a sensor with a single colour channel. Let $\boldsymbol{r}$ denote the 31-vector containing sensor response curve from a single colour channel. Let $\boldsymbol{c}$ denote the 31-vector colour signal. The sensor response p is computed as a dot-product:

$$\begin{aligned} p &= \boldsymbol{r}^{\intercal}\boldsymbol{c} \\ &= \boldsymbol{r}\cdot\boldsymbol{c} \\ &= r_1c_1 + r_2c_2 + \cdots + r_{31}c_{31} \end{aligned} \tag{7}$$

Let us assume that each colour signal sample is an independent and identically distributed random variable with values between 0 and 1. The *expected* value of p is written as:

$$\begin{aligned} &E(p) \\ &= \int_0^1 \boldsymbol{r}\cdot\boldsymbol{c}\ \mathrm{d}\boldsymbol{c} \\ &= \int_0^1 \cdots \int_0^1 r_1c_1 + r_2c_2 + \cdots + r_{31}c_{31}\ \mathrm{d}c_1\,\mathrm{d}c_2 \cdots \mathrm{d}c_{31} \end{aligned} \tag{8}$$

We now need to apply the idea from equation (8) to equation (4). Now, let us explicitly write the least-squares matrix calculation in terms of the correlations of sensor responses:

$$\begin{aligned} \boldsymbol{M} &= \boldsymbol{QP}^{\intercal}(\boldsymbol{PP}^{\intercal})^{-1} \\ &= \begin{bmatrix} \boldsymbol{Q_1P_1^{\intercal}} & \boldsymbol{Q_1P_2^{\intercal}} & \boldsymbol{Q_1P_3^{\intercal}} \\ \boldsymbol{Q_2P_1^{\intercal}} & \boldsymbol{Q_2P_2^{\intercal}} & \boldsymbol{Q_2P_3^{\intercal}} \\ \boldsymbol{Q_3P_1^{\intercal}} & \boldsymbol{Q_3P_2^{\intercal}} & \boldsymbol{Q_{3*}P_3^{\intercal}} \end{bmatrix} \begin{bmatrix} \boldsymbol{P_1P_1^{\intercal}} & \boldsymbol{P_1P_1^{\intercal}} & \boldsymbol{P_1P_1^{\intercal}} \\ \boldsymbol{P_2P_1^{\intercal}} & \boldsymbol{P_2P_2^{\intercal}} & \boldsymbol{P_2P_2^{\intercal}} \\ \boldsymbol{P_3P_1^{\intercal}} & \boldsymbol{P_3P_3^{\intercal}} & \boldsymbol{P_3P_3^{\intercal}} \end{bmatrix}^{-1} \end{aligned} \tag{9}$$

In terms of equation (9) we would like to compute the expected values of $\boldsymbol{QP}^{\intercal}$ and $\boldsymbol{PP}^{\intercal}$. The terms in these two matrices can be computed if for arbitrary matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$ we can compute $(\boldsymbol{XY})_{ij}$. Denoting the i^{th} row of $\boldsymbol{X}$ as the vector $\boldsymbol{\alpha}$ and the j^{th} column of $\mathbf{Y}$ as $\boldsymbol{\beta}$, $E(\boldsymbol{XY})_{ij}$ can be computed by solving the following equation:

$$\begin{aligned} &E((\boldsymbol{XY})_{ij}) \\ &= E\big((\boldsymbol{\alpha}\cdot\boldsymbol{c})(\boldsymbol{\beta}\cdot\boldsymbol{c})\big) \\ &= \int_0^1 \cdots \int_0^1 \left(\sum_{i=1}^{31} c_i\alpha_i\right)\cdot\left(\sum_{i=1}^{31} c_i\beta_i\right)\ \mathrm{d}c_1 \ldots \mathrm{d}c_{31} \end{aligned} \tag{10}$$

For illustration let us suppose there are only 2 wavelengths then.

$$\begin{aligned} &E\big((\boldsymbol{\alpha}\cdot\boldsymbol{c})(\boldsymbol{\beta}\cdot\boldsymbol{c})\big) \\ &= \int_0^1\int_0^1 (c_1\alpha_1 + c_2\alpha_2)(c_1\beta_1 + c_2\beta_2)\ \ \mathrm{d}c_1\mathrm{d}c_2 \\ &= \int_0^1\int_0^1 c_1^2\alpha_1\beta_1 + (\alpha_1\beta_2 + \alpha_2\beta_1)c_1c_2 + \alpha_2\beta_2c_2^2\,\mathrm{d}c_1\mathrm{d}c_2 \\ &= \frac{\alpha_1\beta_1}{3} + \frac{\alpha_2\beta_1 + \alpha_1\beta_2}{4} + \frac{\alpha_2\beta_2}{3} \end{aligned} \tag{11}$$

Equation (11) can be extended to include 31 wavelengths, the derivations are not provided here.

Polynomial Maximum Ignorance with Positivity Colour Correction

In second order polynomial colour correction, we seek a more general form of the colour correction mapping. Specifically, we add squared and 'cross' terms to each RGB camera measurement: each input RGB is mapped to a 9-vector: $(r, g, b, r^2, g^2, b^2, rg, rb, gb)^{\intercal}$. Then, again, we can use the Moore-Penrose inverse to solve for the colour correction transform, although here $\boldsymbol{QP}^{\intercal}$ and $\boldsymbol{PP}^{\intercal}$ have the dimensions of 3×9 and 9×9 respectively.

The expectation for the cross-product terms – in the 3x9 matrix $\boldsymbol{QP}^{\intercal}$- between the linear terms and the polynomial terms are calculated using equation (12):

$$E\big((\boldsymbol{\alpha}\cdot\boldsymbol{c})(\boldsymbol{\beta}\cdot\boldsymbol{c})(\boldsymbol{\gamma}\cdot\boldsymbol{c})\big) = \int_0^1 \cdots \int_0^1 \left(\sum_{i=1}^{31} c_i\alpha_i\right)\cdot\left(\sum_{i=1}^{31} c_i\beta_i\right)\cdot\left(\sum_{i=1}^{31} c_i\gamma_i\right) \mathrm{d}c_1 \ldots \mathrm{d}c_{31} \tag{12}$$

As an example, to compute $E((\boldsymbol{r}\cdot\boldsymbol{c})(\boldsymbol{r}\cdot\boldsymbol{c})(\boldsymbol{x}\cdot\boldsymbol{c}))$ –the $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in equation (12) are redefined as the sensitivities of the red sensor and $\boldsymbol{\gamma}$ is redefined as the X colour matching function i.e. we compute$E(r^2x)$.

The expectation for the auto-product terms – the 9x9 matrix $\boldsymbol{PP}^{\intercal}$ – can be computed using equation (13):

$$E\big((\boldsymbol{c}\cdot\boldsymbol{\alpha})(\boldsymbol{c}\cdot\boldsymbol{\beta})(\boldsymbol{c}\cdot\boldsymbol{\gamma})(\boldsymbol{c}\cdot\boldsymbol{\zeta})\big) = \int_0^1 \cdots \int_0^1 \left(\sum_{i=1}^{31} c_i\alpha_i\right)\cdot\left(\sum_{i=1}^{31} c_i\beta_i\right)\cdot\left(\sum_{i=1}^{31} c_i\gamma_i\right)\cdot\left(\sum_{i=1}^{31} c_i\zeta_i\right) \mathrm{d}c_1 \ldots \mathrm{d}c_{31} \tag{13}$$

As an example, to compute $E((\boldsymbol{c}\cdot\boldsymbol{r})(\boldsymbol{c}\cdot\boldsymbol{r})(\boldsymbol{c}\cdot\boldsymbol{g})(\boldsymbol{c}\cdot\boldsymbol{b}))$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in (13) denote the spectral sensitivity of the red sensor and $\boldsymbol{\gamma}$ and $\boldsymbol{\zeta}$ are respectively redefined the spectral sensitivities of the green and blue sensors i.e. we compute $E(r^2gb)$

Although the process is quite tedious, we can derive a closed form answer to equation (12) and (13) that depends only on $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ and $\boldsymbol{\zeta}$, analogous to the derivation of equation (11). In this way, we can solve for the maximum ignorance with positivity polynomial regression. Space limitations do not allow us to present the derivation here.

EXPERIMENTS

We performed a colour correction experiment using a Nikon D5100 camera (see [6] for measurement details). Relative to this camera we computed the Maximum Ignorance, Maximum Ignorance with Positivity, and Polynomial Maximum Ignorance with Positivity colour correction transforms (two 3×3 matrices and one 3×9 matrix). We then apply these transforms to both synthetic and real camera data.

Simulation Experiment using Synthetic Colour Signal

We generated colour signals using all pairs of 102 illuminant spectra and 1995 reflectances [7] (over 200,000 spectra). We calculate RGBs and corresponding XYZs by numerical integration. We now apply each of the 3 maximum ignorance colour corrections to our RGBs to predict XYZs. To measure the fitting error, we used CIELAB ΔE. The results are shown in Table 1.

Table 1. CIELAB ΔE for Simulation Experiments

Method	**Mean**	**Median**	**95%**
Maximum Ignorance	5.25	3.54	13.02
Maximum Ignorance with Positivity	**3.16**	**2.14**	**8.64**
Polynomial Maximum Ignorance with Positivity	4.52	3.48	11.31

Experiment using real camera data

Under cloudy daylight, the XYZ values of a 24-patch X-Rite ColorChecker Classic were measured using a Photo Research PR-670. These colour signal spectra were integrated to form target XYZs. Great care was taken with the measurement geometry by measuring every patch the same way. The effect of shading across the target was normalised out. Then an image of the colour checker was taken. Per patch RGBs was averaged to return 24 camera measurements. As before, the RGBs were mapped to the target XYZs using our maximum ignorance transforms. The CIELAB ΔE was computed and the results are presented in Table 2.

Table 2. CIELAB ΔE for Experiment Involving Real World Data

Method	Mean	Median	95%
Maximum Ignorance	5.32	4.46	12.22
Maximum Ignorance with Positivity	**3.99**	**3.66**	**8.25**
Polynomial Maximum Ignorance with Positivity	6.25	5.02	12.5

CONCLUSION

In *linear* colour correction, Maximum Ignorance with Positivity (MIP) assumes that all possible positive spectra are equally likely. Relative to this assumption, colour correction depends only on the autocorrelation of the spectra and the device spectral sensitivities. However, to apply the MIP assumption to *non-linear* colour correction is much more complex. One of the contributions of this paper is to show how we can solve the problem of polynomial regression under the MIP assumption, by presenting the mathematics behind the modelling of the expected sensor response.

Experiments, however, demonstrated that the polynomial maximum ignorance assumption works less well than the antecedent methods. Speculatively, this underperformance can be explained by the higher order terms for 'unlikely' sharp spectra dominating the regression. But, the observation that we might examine performance for all spectra in terms of expected sensor responses is one that can be extended to other colour correction scenarios which we are investigating.

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