"One of the beauties of Autograph is … that you don't really have to think": Integration of resources in mathematics teaching

Lina Kayali and Irene Biza

School of Education and Lifelong Learning, UEA, UK; L.Kayali@uea.ac.uk

This paper introduces part of a larger study on the use of technology, specifically mathematics-education software, by secondary mathematics teachers. It presents some of the data collected with the aim of investigating teachers' use of mathematics-education software: why are certain settings used, or underused, how are they used, and what are the reasons behind such use? The findings will be discussed by drawing on the documentational approach (Gueudet & Trouche, 2009) and teaching triad (Jaworski, 1994). The data collected is for one interview and one lesson observation with a secondary mathematics teacher. While the documentational approach provides an overview of the set of resources being integrated to achieve a specific goal, the teaching triad presents a lens to observe teachers' considerations when implementing a task in a mathematics lesson.

Keywords: tasks, documentational genesis, teaching triad.

Introduction

The complexity of the teaching profession imposes several factors that have impact on teachers' classroom actions that include not only their beliefs and knowledge but also their experiences and the educational context in which they act (Biza, Nardi, & Joel, 2015; Speer, 2005). Teachers, while planning and teaching, are considering the contexts they are working within: their students, school environment, curriculum, etc. In other words, any study of teachers' practices should take into account the different contextual conditions in which these practices develop including personalities, institutions, circumstances, epistemology, time issues and materials (Herbst & Chazan, 2003). The study we present in this paper is part of the PhD research of the first author that investigates mathematics teachers' ways of balancing the different elements in their working environment, especially when using technology (in this study mathematics-education software, i.e. software designed for mathematics teaching and learning purposes), by looking at their practices/intended practices within specific contexts. Furthermore, our work examines any gaps between intended technology use in mathematics classrooms and teachers' practices. To this aim we invite teachers' views on hypothetical classroom situations that involve teaching with technology in written responses and follow up interviews. Then, we observe teachers' use of technology in their classroom. In this paper we present preliminary analysis from one participant, Adam, by drawing on two theoretical perspectives: the *documentational approach* (Gueudet, Buteau, Mesa, & Misfeldt, 2014; Gueudet & Trouche, 2009) and *teaching triad* (Jaworski, 1994).

The documentational approach

The documentational approach looks at teachers' interactions with resources where a resource is defined as "anything that can possibly intervene in [a teacher's] activity", it can be an artefact (e.g. a pen or a mathematical technique), a teaching material, or even a social interaction (Gueudet et al., 2014, p. 142). Adler (2000, p. 207) adds that "resource" can be also "the verb re-source, to source again or differently". During their interaction with resources, teachers develop schemes. A scheme is a set of organised procedures carried out on an artefact (Gueudet & Trouche, 2009). It consists of "the *goal* of the activity; *rules of action*; *operational invariants*; and *inferences*" (Gueudet et al., 2014, p. 140, italics in original). Here, operational invariants are cognitive concepts established throughout the activity to be used in comparable situations (ibid). For teachers, these are "professional knowledge" (ibid, p. 142). The documentational approach talks about a two-way influence between a resource and a teacher: a resource affects the teacher's actions and knowledge; and the teacher's perceptions and experiences impact on the way the resource is used (ibid, p.140). A process of instrumental orchestration is required in order to organise the learning environment (e.g., space, time, dialogue) by the teacher, whose responsibility is to manage the process according to the requisites of the task (ibid). And when a teacher uses a set of resources according to a specific scheme for a specific goal, s/he creates a document. Such a development of a document is called *documentational genesis.* Thus, the documentational genesis is the process of a teacher developing schemes for adapting different sets of resources to achieve a specific target (ibid). The documentational approach studies the development of "structured documentation system[s]" that represent teachers' work and progress as a result of influencing and being influenced by different resources (ibid). In this study we have conjectured that the schemes developed by a teacher are dynamic and are being re-adapted from one situation to another and that the teaching triad (Jaworski, 1994) can help explore those schemes and adds to the documentational genesis as discussed in the next sections.

The teaching triad

Jaworski's (1994) teaching triad (TT) addresses classroom management as an act of harmony between three domains of activity: sensitivity to student (SS), mathematical challenge (MC) and management of learning (ML). These domains are evident when a teacher plans a lesson and start to think of how to consider teaching a specific mathematical idea (MC), his/her particular students' needs (SS), the best way to work on the task with the students (ML) (group work, individual work or classroom discussion). The same domains will be in play during lessons, but within a different context as this time the interactions with the students are happening and the teacher should respond on demand, in many cases by diverting from what was planned.

As Jaworski (1994) and Potari and Jaworski (2002) suggest from the "macro analysis" of classroom interactions, along with the TT domains, teachers' plans and practices are also influenced by social factors, such as: time pressure, having to go through a set syllabus, the requirement that students know specific things for exam purposes, expectations from the teacher, school ethos and the training provided for teachers. Such factors seem to be at the center of teachers' considerations and they include students' social culture, teaching resources and materials, syllabus, assessment schemes, time restrictions, room constraints, and cultural considerations of what constitutes good teaching practices (Goos, 2013, p.523). Hence, the TT domains, along with these factors, reflect the rich social interactions mathematics teachers have to balance. The TT can be "used as an analytical device (by researchers) and as a reflective agent for teaching development (by teachers)" (Potari & Jaworski, 2002, p. 351). Additionally, the teaching triad domains seem to be related to schemes' development in relation to resources. Sustaining the goals of the resource use, depends on how teaching is balanced by the teacher. Operational invariants can be derived from artefacts, mathematical concepts or social environments. All of the above can be used to satisfy a specific goal and produce a proper usage and inferences, but these are flexible techniques of balancing and rebalancing of the TT domains from one lesson to another. This work aims to investigate this conjecture in the preliminary analysis presented in this paper.

Technology in mathematics teaching - A view through the lenses of the documentational approach and the teaching triad

Technology software and hardware pieces are “artefacts” (Gueudet et al., 2014, p.141) and can be adapted to provide access to formal mathematical knowledge. They afford “opportunities for additional student actions, such as the manipulation of on-screen objects and the ability to make a range of mathematical inputs, which places an additional demand on teachers as they strive to make sense of a diversity of student activity in real-time” (Clark-Wilson & Noss, 2015, p. 95). Thus, interactions with technological resources influence teachers’ documentational geneses by developing the ways they organise their classroom activities and the way they manage learning situations with impact on the shape of the teaching process and on the way knowledge is communicated. When employing technology, resources become more complex, and so do the TT domains. Sensitivity to student becomes more evident (e.g. if students know more about technology than their teacher). Tasks can be more challenging for teachers to design, and the management of learning becomes more complicated with the higher chances of distraction. We also re-emphasise the importance of social factors when technology is used based on several premises. First, the technology use dependency on the teacher training provided (Gueudet et al., 2014; OECD, 2015). Second, the availability of hardware and internet connection (Bretscher, 2014; OECD, 2015). Third, the national curriculum obligations (OECD, 2015). Fourth, and most importantly, the education policies that aim to embed technology (ibid).

Methodology

This paper reports from the first phase of a project that looks at secondary mathematics teachers’ work with technology that involves participants’ written responses to *situation-specific tasks* with follow up interviews and classroom observations. Situation-specific task methodology has been suggested by (Biza, Nardi, & Zachariades, 2007, p. 301) where tasks are given classroom situations that “are hypothetical but grounded on learning and teaching issues that previous research and experience have highlighted as seminal; are likely to occur in actual practice; have purpose and utility; and, can be used both in (pre- and in-service) teacher education and research through generating access to teachers’ views and intended practices”. The study is conducted in England and participants are secondary school mathematics teachers with different levels of experience and training. The work is based on providing qualitative findings established on an interpretative research methodology (Stake, 2010).

In this paper we discuss the written response to the situation-specific task presented in Figure 1 (we call it the 3D Task), the follow-up semi-structured interview and the lesson observation (75 minutes) of one participant, Adam. Both interviews and observations were conducted by the first author. The situation described in the 3D Task regards an open investigative question to be given to the students. It did not suggest any specific use of technology, and left that to be decided by the teacher. It was also designed with a geometry question considering the affordances of geometry work with software such as Autograph (http://www.autograph-math.com/) or Geogebra (https://www.geogebra.org/), which were available at the school were the data were collected. We invited Adam to offer a written response to the 3D task and, then, we interviewed him, to clarify the answers and offer more elaboration where needed. The lesson was to a Year 12 class (17-18 year-old 4 female and 5 male students) and was audio-recorded. The foci of the observation was to look at Adam's use of resources, especially his use of Autograph or Geogebra which he said he frequently used and to the classroom management. At the time of the data collection, Adam was a mathematics teacher with four years' experience, during which he taught to students aged 12-18 years. He held degree in economics, a Postgraduate Certificate in Education for teaching mathematics at secondary level, and was about to finish master's degree in advanced educational practice. The school had interactive whiteboards, a computer lab available for booking, and Geogebra and Autograph software installed on all computers.

3D Task

A group of Year 11 students are asked the following question:

Design a milk container with capacity of 1L. What dimensions and which design uses less materials? Why?

- What are the mathematical ideas and activities addressed in this question?
- Would you use this question in class? Why or why not? What are the learning objectives for which you would use this question? Would you modify it?
- Would you use technology with this question? If yes, what type of technology? If no, why?
- If you were to use technology, how would you use it?
- What teaching approaches and resources would you suggest for this question?
- Do you anticipate any problems or challenges (either with students or resources)?

Figure 1: The 3D Task

A preliminary analysis of teachers' comments and interactions during interviews and lessons was performed for the transcripts (May, 2001). This was coded according to the Teaching Triad (Jaworski, 1994) into SS, ML, and MC. During each interaction, we explored the teacher's interactions with the resources, according to the the documentational approach (Gueudet et al., 2014). We then reviewed the results from the interview and observation, and offered a discussion according to TT and documentational approach together.

Adam's responses to the 3D Task and follow up interview

Adam identified mathematical ideas involved in the 3D Task, such as "volume", "surface area" and "calculus". In his response to the task, he frequently repeated the word "scaffold" to state that he would try to adapt the task according to the students' needs and "prior knowledge". He emphasised that he would not use the problem as it is because it needed a lot of scaffolding and it included "too many variables". During the interview he explained that the scaffolding would include giving hints and examples and even values to work on for weaker students. He said he will not use 1L in this problem, but would convert it to 1000 ml or use a bigger number:

> I think straight away students having to think of a length width and height that times to get 1 will be quite difficult for students… They might be able to go 1 1 1 and they might be able to go 2 ½ 1 or something like that. That will be it, they'll really struggle.

He wrote that he would use technology with the task for "gradient of curves on Autograph" and "to visualise the shapes". During the interview, when asked how and when he would use it during a lesson, Adam suggested "I think as a group activity. It wouldn't be the focus of the lesson though it would just be almost the point at the end" and he asked about the reasons behind his suggestion:

> I think it is because one of the beauties of Autograph is that it means that you don't really have to

think... I want the student to be thinking about problems and how to approach problem. I think almost Autograph gives you too much, too much help and then you don't have to think about the shape of the graph because you can just plot it in Autograph. And then other, obviously other reasons a lot of my students have never used Autograph even at key stage five[1]. So, to start understanding it, it will take quite a long time and a lot of effort just to get the students to understand it to start with

Adam said he will not use the task as it is because although it works well in "an ideal world", it does not go well with the way the syllabus is set. He anticipated problem with keeping track of calculations, prior knowledge and many involved variables (e.g. Adam suggested if a student chose to design a cylinder container, the case would be very confusing because s/he would have to think of adjusting the radius and height of the cylinder in order to find the minimum surface area).

Adam's teaching observation

The observation was on a revision lesson about solving simultaneous linear and modulus equations (i.e. equations that include absolute value). Adam started by moving a stick in the air in order to draw a specific graph, and asking the student to recognise the graph. One of these graphs was the *sine* graph, but the students seemed to be confused about what graphs were being drawn. Then, Adam asked his students to solve some problems that were displayed on the board. All of the problems apart from one (which was designed by Adam) were chosen from the textbook. During the lesson Adam used Autograph to check the answers given by the students, he entered the functions and the graphs came up on the screen which was projected on the board. Then a discussion/ demonstration of the algebraic solution was led by him on the white board. For example, when he was solving $y = |x + 2|$ and $y = 3$ simultaneously, he asked students to draw it and see the answers in the graph before solving algebraically: "You will get two points, you can see this graphically". Then, he started to write one of his student's algebraic answers on the board: "$3 = x + 2$ or $-3 = x + 2$". He then commented on the student's answer: "So, math says $x = 1$ or -1... What text book would say is $y = (x + 2)$ or $y = -(x + 2)$. Textbook would just say that, so I'll probably do it this way". Later, with the problem that followed, he commented that: "This is GCSE grade C[2] […] This is mark C[2] in C1[3]". He repeatedly encouraged the students to solve another problem he displayed on the board by saying that it is an "exam question". When the students asked Adam why they should learn modulus equations, he went to his computer and googled "when to use modulus equations in real life" and gave the answers accordingly "Distance, currency exchange…". Two of the students finished with the problems on the board earlier than the rest of the class, Adam gave them an extension problem which might have been suggested spontaneously in response to the need of extra work. The extension problem given was in two parts, the first asked for two different modulus functions that do not intersect, the second asked for two that intersect once. "Is that possible? Can

---

[1] Key stage five is post 16 school education i.e. for students aged 16-18.

[2] GCSE stands for the General Certificate of Secondary Education. It is the qualification taken by school students aged 14–16 in the UK (except Scotland). Its exams are graded on a scale of A* to U, with A* being the highest grade and U the unsatisfactory. A grade/mark C reflects an average progress (pass).

[3] C1 stands for Core1 and it refers to one of the mathematics textbooks, used at Adam's school, for students aged 16-18.

you give me two that intersect once?", Adam asked the class, and the dialog below followed:

Student A: $y = | x |$ and $y = 2 | x |$, shift across
Adam: Oh, ya it is.
Student A: Ya, you've translated it.
Student B: $y = | x - 4 |$ and $y = 2 | x |$.
Adam looked at the graphs on Autograph and nodded in what seemed like a hesitant agreement
Student C: Change the slope.
Adam amended the equations as student C suggested and wrote $y = 2 | x - 4 |$ *and* $y = 2 | x |$ *without commenting on student's B answer*

Adam did not follow up student's B response or student's C correction, but moved straight to a completely different activity by which he concluded the lesson. That incident seemed like a missing opportunity, as no immediate response was made by Adam.

Analysis

From his responses to the task and the observation we notice that Adam's resources were the textbook used at his school, help cards, a computer, Autograph, Excel, Google, interactive whiteboard, the stick he used at the beginning of the lesson observation, information about exam grades and questions, past experiences with students along with the mathematical concepts and methods. Adam's appreciation of Autograph ease of use as a tool for visual representation was evident, so he used the software to check students' work, and present graphical solutions before going for algebraic ones. So, he would ask his students to solve graphically, check that their graphical solutions are right according to the answers on Autograph, and then ask them to find the same answers algebraically. However, Autograph's use confused Adam when it came to student's B answer on which he seemed to hesitantly agree, because the intersection point was not visible within the displayed part of the graph. In this case, Adam missed the opportunity to use the full affordances of Autograph in order to improve student's B answer and to explain the correct answer to the rest of the class. There was no evidence that the rest of the class, apart from student C, realised where the problem was and how it was amended.

In terms of the TT, Adam indicated sensitivity to students "they don't know Autograph", "prior knowledge", "scaffold", "weaker students" (SS). In his teaching choices, he also showed consideration of the syllabus he had to follow, exam questions and the timeframe he had to adhere (MC and ML). The way he indented to use resources showed an attempt to balance mathematical challenges (MC) (e.g., change 1L, exercises from the textbook) with students' needs (SS) (e.g. students do not know how to use Autograph, providing extension question when needed), and management of teaching (ML) (e.g., use technology at the end of the lesson as a group activity, encouraging pair work when solving textbook exercises, graphing the equations to see the answers and then doing the algebraic solution because "Putting it in a graph might be easier") with attention to management of learning with technology (e.g. technology takes a lot of time).

Now, we will look at how Adam used the available resources to design and implement his teaching. Along with the textbooks that are being used at his schools, he mentioned he would also use help cards with hints or examples. These will help him "scaffold" and build on "prior knowledge", these terms seem to be adopted during Adam's teaching practice or teacher's education courses for reflection on students' needs (SS). Also, he drew on his teaching experience (as a resource) when he mentioned in the interview that students would struggle to "keep track of their calculations" (ML and SS). In terms of the Autograph as a resource, Adam would use it as a graphing software that helps visualise graphs and shapes and shows answers (ML and MC). The data showed the two-way influence between Adam and the resources. For example, Adam's belief that students do not think when using Autograph (SS) was influencing the way the resources were used, so he used Autograph to show or check answers (MC). Also, the resources available influenced the teacher's decision, so in this instance he used Autograph to show the graphical solution and then asked the students to do the algebraic solutions keeping the answers from Autograph in mind. Another example, Adam frequently used the textbook as a source for exercises, so the textbook influenced what mathematical challenge he gave the students (MC). At the same time, Adam used the textbook exercises along with Autograph, by doing so Adam's way of managing the teaching affected the way the textbook was used. The way Adam managed the teaching situation made us put a conjecture that his use of resources is connected with a potential scheme developed in order to properly use the resources available and achieve a specific goal, which in the lesson observation was revising the topic of linear and modulus simultaneous equations. We notice operational invariants like the use of Autograph has to be done as a class activity managed by the teacher on the board. However, we believe that more data and observations are needed to further investigate Adam's schemes.

Discussion and Summary

The preliminary findings we present in this paper are from a study on mathematics teachers' practices/intended practices in relation to the used resources and especially mathematics –education software. Here we discuss the responses to the 3D Task (Figure 1) and the follow-up interview with Adam, a mathematics teacher, as well as a lesson observation. His written responses to the task were more general that did not give particular details about his actions within the given context. However, the 3D Task facilitated the discussion with Adam in the follow up interview when he clarified his points.

Adam's attempt to balance the different domains of activity described by the teaching triad was evident in the interview and during the lesson observation; and his interactions with resources were influenced by considerations of these domains as well as considerations of exams' questions and grades, time management, and the syllabus. His use of technology resources was led by him on the board, because of his concern that he would be teaching mathematics and technology use if his students were to work independently or in pairs on computers. Although the teacher used Autograph frequently, his use was mainly for checking answers and displaying visual representations. This is due to his concern that Autograph offers excessive help and would stop students from thinking about the mathematical problems. The pilot observation proved that more clarification about the teacher's actions should be sought from future observations along with pre- and post- lesson interviews. This is because the pre- and post- lesson interviews will give more space for the teachers' interpretations of their classroom actions. More observations are also needed to explore the teacher's documentational work and investigate how the teaching triad helps clarify teachers' considerations when working with resources.

References

Adler, J. (2000). Conceptualising Resources as a Theme for Teacher Education. *Journal of Mathematics Teacher Education, 3*(3), 205-224.

Biza, I., Nardi, E., & Joel, G. (2015). Balancing Classroom Management with Mathematical Learning: Using Practice-Based Task Design in Mathematics Teacher Education. *Mathematics Teacher Education and Development, 17*(2), 182-198.

Biza, I., Nardi, E., & Zachariades, T. (2007). Using Tasks to Explore Teacher Knowledge in Situation-Specific Contexts. *Journal of Mathematics Teacher Education, 10*(4), 301-309.

Bretscher, N. (2014). Exploring the Quantitative and Qualitative Gap Between Expectation and Implementation: A Survey of English Mathematics Teachers' Uses of ICT. In N. Sinclair, N. Sinclair (Eds.), *The Mathematics Teacher in the Digital Era: an International Perspective on Technology-focused Professional Development. Mathematics in the Digital Era Vol. 2.* 43-70. Dordrecht: Springer.

Clark-Wilson, A., & Noss, R. (2015). Hiccups within technology mediated lessons: a catalyst for mathematics teachers' epistemological development. *Research in Mathematics Education, 17*(2), 92-109.

Goos, M. (2013). Sociocultural Perspectives in Research on and with Mathematics Teachers: a Zone Theory Approach. *ZDM: The International Journal on Mathematics Education*, *45*(4), 521-533.

Gueudet, G., Buteau, C., Mesa, V., & Misfeldt, M. (2014). Instrumental and Documentational Approaches: From Technology Use to Documentation Systems in University Mathematics Education. *Research in Mathematics Education, 16*(2), 139-155.

Gueudet, G., & Trouche, L. (2009). Towards New Documentation Systems for Mathematics Teachers? *Educational Studies in Mathematics,* 199-218.

Herbst, P., & Chazan, D. (2003). Exploring the Practical Rationality of Mathematics Teaching through Conversations about Videotaped Episodes: The Case of Engaging Students in Proving. *For the Learning of Mathematics, 23,* 2-14.

Jaworski, B. (1994). *Investigating Mathematics Teaching: A Constructivist Enquiry*. London Falmer Press.

May, T. (2001). *Social Research : Issues, Methods and Process* (3rd ed.). Buckingham Open University Press.

OECD. (2015). *Students, Computers and Learning: Making the Connection*. Paris: PISA, OECD Publishing.

Potari, D., & Jaworski, B. (2002). Tackling Complexity in Mathematics Teaching Development: Using the Teaching Triad as a Tool for Reflection and Analysis. *Journal of Mathematics Teacher Education, 5*, 351-380.

Speer, N. M. (2005). Issues of Methods and Theory in the Study of Mathematics Teachers' Professed and Attributed Beliefs. *Educational Studies in Mathematics, 58*(3), 361-391.

Stake, R. E. (2010). *Qualitative Research: Studying How Things Work.* New York: Guilford Publications, Inc.