Root-Polynomial Color Homography Color Correction

Han Gong[a],* and Graham Finlayson[a]

[a]*University of East Anglia, Norwich, United Kingdom*

*Han Gong, E-mails: h.gong@uea.ac.uk

ABSTRACT

Homographies are at the heart of computer vision and they are used in geometric camera calibration, image registration, and stereo vision and other tasks. In geometric computer vision, two images of the same 3D plane captured in two different viewing locations are related by a planar (2D) homography. Recent work showed that the concept of a planar homography mapping can be applied to shading-invariant color correction. In this paper, we extend the color homography color correction idea by incorporating higher order root-polynomial terms into the color correction problem formulation. Our experiments show that our new shading-invariant color correction method can obtain yet more accurate and stable performance compared with the previous 2D color homography method.

KEYWORDS: color correction, root-polynomial, color homography

INTRODUCTION

Homographies are widely applied in solving computer vision problems such as geometric camera calibration [1], image registration [2], and stereo vision [1]. In geometry, a 2D Homography is used to mathematically model the geometrical transform between two images of the same 3D plane captured in two different viewing locations. Recent work [3, 4, 5, 6] also adopted the concept of a 2D homography mapping to model the mapping between chromaticities as oppose to image coordinates.

Provided with a calibrated color checker, the task of color correction is to find a mapping between the captured raw RGB of each color patch and its corresponding reference CIE XYZ [7]. In order to accurately estimate the mapping, a uniform shading field is usually required. In this paper, we extend the previous 2D color homography color correction idea for better shading-invariant color correction performance. Our idea is to incorporate higher-order color terms into the color correction problem formulation. Because it is important that any color correction function "scales with exposure", we adopt the recently developed Root-Polynomial formulation [8]. In our model, each RGB is mapped to a high-dimensional vector. For instance, when we use the 2[nd] order root-polynomial expansion, the input RGB to color correction becomes a 6-elment vector $[R \quad G \quad B \quad \sqrt{RG} \quad \sqrt{RB} \quad \sqrt{GB}]^\intercal$ and its output is still $[X \quad Y \quad Z]^\intercal$. The best shading independent mapping – a 3x6 color correction matrix – is then found by solving a higher order homography using the Alternating Least Squares (ALS) method [9]. Our evaluation shows that the root-polynomial approach delivers more accurate shading-invariant color correction result.

BACKGROUND

Color Homography

For the geometric planar homography problem, we write:

$$\begin{bmatrix} \alpha x \\ \alpha y \\ \alpha \end{bmatrix}^\intercal = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}^\intercal \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \triangleq \underline{x} = H(\underline{x}') \tag{1}$$

where (x, y) and (x', y') denote corresponding image points – the same physical feature – in two images. In homogeneous coordinates the vector $[a, b, c]^\intercal$ maps to the coordinates $[a/b, b/c]^\intercal$ and thus the scalar α cancels to form the image coordinate (x, y). For all pairs of corresponding points (x, y) and (x', y') that lie on the same plane in 3D space, Equation 1 models the relationship between their images [1].

In color, it is discussed in [3, 4, 6] that chromaticities across a change in capture condition (light color, shading and imaging device) are also a homography apart. Suppose that we have a RGB vector $\underline{\rho} = [R \quad G \quad B]^\intercal$ and we construct its homogenous coordinates $\underline{c} = [R/B \quad G/B \quad 1]^\intercal$ by diving $\underline{\rho}$ by its last element B. The first two elements of $\underline{c}$ become the chromaticities. We then have $\underline{c}'^\intercal = \alpha' \underline{c}^\intercal H$ where H is a 3×3 color correction matrix, $\underline{c}$ and $\underline{c}'$ are two corresponding pairs of chromaticities in its homogenous coordinates, and α' is an arbitrary non-zero unknown scale. The α' here encodes the unknown shading intensity of each RGB vector. This color correction model is exactly the same as the geometrical homography model in Equation 1 where the 2D image coordinates are replaced by chromaticities.

Root-Polynomial Color Correction

Higher-degree root-polynomials [8] were proposed for more accurate color correction. By its definition, the set of up to k^{th} order root-polynomial (RP) terms $\underline{P}_{n,k}$ in n variables is constructed as:

$$\underline{P}_{n,k} = \left\{ \rho^{-\frac{\underline{\beta}}{|\underline{\beta}|}} : \left|\underline{\beta}\right| \leq k \right\} \tag{2}$$

In our RGB intensity case (*i.e.* $\mathrm{n} = 3$), we adopt the root-polynomial expansions of the 2^{nd} order which is $[R \quad G \quad B \quad \sqrt{RG} \quad \sqrt{RB} \quad \sqrt{GB}]^\intercal$. For instance, an output XYZ is not only a weighted linear combination of linear terms $[R \quad G \quad B]^\intercal$ but the linear combination of both linear terms and the non-linear RP terms, which enable finer tuning for a mapping. Note that when a RGB vector is scaled by α', each RP term is also scaled by α'.

THEORY

Our hypothesis is that, by incorporating RGB polynomial expansion terms into the color homography framework, the accuracy of color mapping can be further improved. We first prove that the N-D color homography is still valid for root-polynomial color terms in Theorem 1.

Theorem 1. *Root-polynomial chromaticities across a change in capture condition (light color, shading and imaging device) are a homography apart.*

Proof Sketch. [3, 4, 6] proves that, up to an unknown shading scale α', the 3D RGBs (or CIE XYZs) under different capture conditions, are related by a 2D homography matrix (*i.e.* a full-rank matrix). When the shading intensities of different RGBs are the same, [8] proves that color root-polynomial terms under different capture conditions are also related by a full-rank linear transform matrix M. When exposure changes, the scaling term α' of each RGB vector is the same as its root-polynomial terms' scaling factors. Therefore, color homography holds for root-polynomial chromaticities. ■

Our RP color homography model can be written in a matrix form:

$$B = DAM \tag{3}$$

where A is a N×6 matrix denoting the input RP-RGB intensities, B is a N×3 matrix denoting the output XYZs, M is a full-rank 6×3 matrix, B is a N×N diagonal matrix whose elements are the shadings intensities of color patches. We adopt ALS [4, 9, 6], described in Algorithm 1, to solve for M where the superscript i indicates the i^{th} iteration. The effect of the individual M^i and D^i can be combined into a single matrix $D = \prod_i D^i$ and $M = \prod_i M^i$. In our case of 2^{nd} order RP chromaticities, the corresponding color homography matrix should be 6×6. However, in color correction, because we are interested in the output XYZs rather than all the output RP colors, we only need a 6×3 color correction matrix that maps 6D RP-RGB colors to 3D XYZs, up to unknown shading scales.

To stabilize the ALS optimization with RP terms, an additional shading smoothness regularization is necessary. In ALS, due to the presence of noise, larger number of modeling parameters can lead to incorrect shading estimation (*e.g.* Figure 2b). This is due to two factors: 1) ALS only ensures that each diagonal element of matrix D is fitted for an individual patch but does not guarantee that D is a smooth shading field (the scaling factors in D correspond to patches in the image. The patches viewed together should be "smooth"); 2) higher order polynomial leads to over-fitting. We adopt a DCT (Discrete Cosine Transform) shading regularization for shading smoothness [9]. The motivation is to approximate the 2D shading field image I_{D^i} (whose diagonal matrix representation is D^i) by using a linear combination of 2D DCT basis matrices. For instance, Figure 1 shows the first ten 2D DCT image bases. D^i is constructed as $I_{D^i} = \sum_{l=1}^{n} w_l G_l$ where G_l is the l^{th} DCT basis matrix (implemented as a diagonal matrix), w_l is its weight, n is the total number of DCT basis. Line 4 of Algorithm 1 is then replaced with $\min_{\underline{\mathrm{w}}} || \sum_{l=1}^{n} w_l G_l A^{i-1} - B \ ||_F$ where $\underline{w}$ is the vector of DCT weights. See the Appendix in [9] for the details on how to solve for $\underline{w}$. An example of the DCT-regularized shading estimation is shown in Figure 2c.

Algorithm 1: Homography from alternating least-square

1 $i = 0, A^0 = A$;
2 **repeat**
3 $\quad i = i + 1$;
4 $\quad \min_{D^i} \left\| D^i A^{i-1} - B \right\|_F$;
5 $\quad \min_{H^i} \left\| D^i A^{i-1} M^i - B \right\|_F$;
6 $\quad A^i = D^i A M^i$
7 **until** $\left\| A^i - A^{i-1} \right\|_F < \varepsilon$;

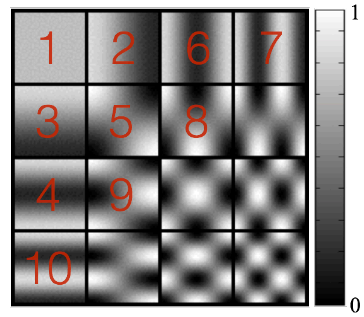
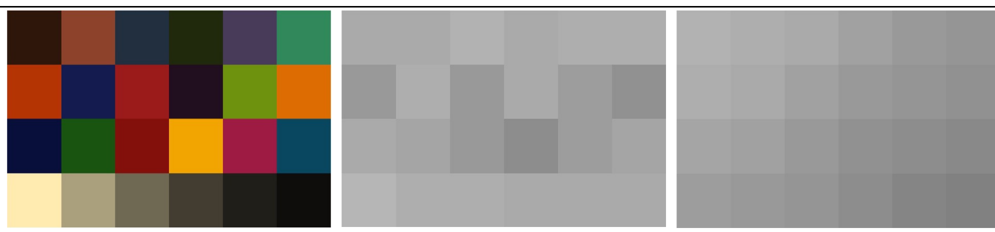


Figure 1: Examples of 2D DCT basis. The numbers in block indicates the index of basis which is in a zigzag order.



a) Captured RGBs b) Un-Regularized Shading c) DCT-Regularized Shading

Figure 2: The shadings of the capture RGBs (a) are estimated by naïve ALS (b) and ALS with DCT smooth shading regularization (c).

EVALUATION

In our experiment, we adopt a public dataset [3, 4, 6] which contains the captured RGBs with shading effects, the target calibrated XYZs, and its shading-removed RGB ground truth. We carried out the same experiment adopted in [3, 4, 6]: 1) A color correction model M is first trained from the RGBs with shading and its corresponding XYZs B_{ref}; 2) The model M is applied to convert the ground truth RGBs with a uniform shading field to a set of estimated XYZs B_{est}; 3) The CIE ΔE errors [7] between B_{ref} and B_{est} is computed as the measurement.

Table 1. Color Correction on CIE Lab ΔE Error

Method	Mean	Median	95%	Max.
Least-squares	3.70	3.30	7.73	8.39
2D Color Homography [4]	2.34	2.09	5.02	5.43
2D RANSAC color homography [3]	2.18	1.69	5.46	6.18
2^{nd} order RP color homography	**1.96**	**1.66**	**4.83**	**5.08**

Table 2. Color Correction on CIE Luv ΔE Error

Method	Mean	Median	95%	Max.
Least-squares	4.28	3.85	8.30	9.53
2D Color Homography [4]	2.73	2.41	5.82	6.54
2D RANSAC color homography [3]	2.58	2.11	6.62	6.94
2^{nd} order RP color homography	**2.12**	**1.93**	**5.01**	**5.76**

We compare our method with least-squares and two 2D color homography variants [3, 4, 6]. RANSAC color homography [3, 6] is an improvement over [4] that it applies the RANSAC scheme [10] to exclude outlier correspondence pairs for color correction. However, [3] is also known unstable due to the randomness introduced by RANSAC [10]. Table 1 and 2 show the color correction result measured by CIE Lab ΔE 1976 and CIE Luv ΔE 1976 errors. In both tables, our root-polynomial color homography color correction have improved the color correction accuracy in all measurements with a stable performance.

CONCLUSION

In this paper, we show that colors across a change in viewing condition (changing light color, shading and camera) can be related by a higher order root-polynomial color homography. The high-order homography color correction application delivers improved color fidelity compared with the previous 2D color homography color correction methods.

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