The Math Teachers Know: Profound Understanding of Emergent Mathematics, by Brent Davis and Moshe Renert, Oxon, Routledge, 2014, 152 pp., £22.50 (paperback), ISBN: 0415858445

The Math Teachers Know develops a powerful set of understandings about the nature of mathematics for teaching. The authors conceptualise mathematics knowledge for teaching as an adaptive and emergent system, within the framework of complexity science, presenting a fuller picture of the study reported on in Davis and Renert (2013). They discuss the development and part formalisation of 'concept study' work (which I explain in more detail below) with mathematics teachers in which they have been engaged for some time (see e.g. Davis and Simmt, 2006; Davis and Renert, 2009). Arguing that mathematics knowledge for teaching is distributed and tacit, they make a persuasive case for the importance of collective action in the production of mathematics-for-teaching.

The book begins with an overview of key developments in research on mathematics for teaching, and locates this latest work in the field. Early researchers who sought a relationship between teachers' knowledge of advanced mathematics and their students' learning found a 'persistent lack of significant correlation' (p.7). The work of Shulman (1986), and subsequently Ball, Thames and Phelps (2008) and Ma (1999), significantly developed the discourse, enabling distinctions to be drawn between subject matter knowledge (SMK) and pedagogical content knowledge (PCK). A distinction appeared between 'mathematical knowledge that is structured to be used' and 'mathematical knowledge that is structured to be taught' (p.9). The terms 'specialised mathematics' (Ball) and 'profound understanding of fundamental mathematics' (Ma) began to have currency. In particular, researchers noted that teachers' mathematical knowledge is not static, but a dynamic system that is activated in the teaching moment. Davis and Renert find resonance in the assertion of Baumert et al. (2010) that, in the absence of pedagogical content knowledge, teachers' formal mathematics knowledge remains inert. Tracking the development of questions and answers about mathematics for teaching as the research gathered depth and complexity, they suggest a working definition of mathematics for teaching, as

> a way of being with mathematics knowledge that enables a teacher to structure learning situations, interpret student actions mindfully, and respond flexibly, in ways that enable learners to extend understandings and expand the range of their interpretive possibilities through access to powerful connections and appropriate practice (p.11).

They argue that teaching needs to be seen as a co-participation in the production of knowledge, with teachers as co-producers rather than as managers controlling the flow of established knowledge. T has implications for how teachers deal with what they do not yet know, since it removes from them the 'fount of all knowledge' label and replaces it with the designation of a co-enquirer, a co-learner.

In Chapter 2 the authors frame their work firmly in the domain of complexity science, arguing that teachers' mathematical knowledge is productively viewed as a complex phenomenon, i.e., a system that emergent and adaptive (see also Davis and Simmt, 2006). They suggest that a key metaphor for learning is coherence-maintaining. They discuss the role of the teacher as simultaneously expert and novice, and as an expert who is able to think like a novice. The concept of dynamic, emergent 'knowing' is seen as more productive than that of static, stable 'knowledge'. Much teacher knowledge is tacit, (see also Davis, 2011) and difficult to share with others or indeed to identify oneself. Knowledge production, or learning, is not logical but rather analogical and about establishing ever more complex webs of connection. These insights are central to the direction of this book, and to the authors' emerging response to the question 'How must teachers know mathematics for it to be activated in the moment and in the service of teaching?'.

Chapter 3 introduces concept study, a way of working with mathematics teachers which the authors have developed, in which profound mathematical concepts are studied in depth through participative group enquiry. Concept study may be seen as combining the power and focus of concept analysis with the collaborative structures of lesson study. It is described as offering 'opportunities to work together to re-form concepts in ways that render them more accessible to learners' (p.39). We are introduced to part of an 'entailments chart' focusing on multiplication, which is to feature strongly later in the book. The concept of multiplication can be represented or realised in various ways, which when elaborated give rise to different understandings of associated concepts, for example:

If multiplication is ______ then a factor is ________ then a product is ______

The reader is introduced to the concept of 'substructing', literally 'building beneath', mathematical concepts. This can be linked to the ideas of unpacking mathematics, or dismantling and rebuilding / reconstructing. The process of substructing concepts simultaneously enables teachers to find ways to make the mathematics accessible to learners, and to reconstruct their own understandings. I note that Ruthven (2011) argues that for teachers, reconstructing existing knowledge is as important as acquiring new knowledge. The authors also introduce mathematics for teaching as 'an open disposition', as against a specific collection of knowledge and skills. Thus, their 'way of being with mathematics' (first introduced in the definition above, p.11) develops into 'an open disposition towards mathematics'. Mathematics for teaching is again presented as an emergent phenomenon, i.e., a complex, living, learning system which is distributed across a network of people.

The following two chapters explore the processes and outcomes of concept study in some depth. A project that began some years ago, relatively informally with a group of teachers, has been developed into a Masters of Education programme. Thus the programme has gained greater continuity of participants and a stronger learning community than it had in its early days. We learn how the emergence of different realisations of concepts such as, for example, 'mutiplication', 'zero', within concept study shape teachers' understandings of the concept and where/how it is located in the mathematics landscape, and enable different entailments to be made. From here, the study group then move to re-form their ideas and develop conceptual blends. This represents 'a shift in emphasis from multiple meanings towards coherent and encompassing definitions' (p.71).

A key point made is that the insights generated collectively by the group were not available to any of them individually prior to engagement with the study. The emergence of new (to the group) mathematical knowledge required the combined effort of group activity. To me this is a central idea in the book and, although the authors' stance is from a complexity science perspective, I interpret here a very strong argument for a social constructivist view of learning and knowledge production.

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Davis and Renert spend some time discussing teachers' listening attitudes, identifying three main listening modes: evaluative, interpretative, and hermeneutic listening. They suggest that the lesson trajectory of an evaluative listener-teacher is mostly unaffected by student responses, and that of the interpretative listener-teacher is modified by student responses. However the lesson trajectory of the hermeneutic listener-teacher is *defined by* student responses. They argue that teachers' disciplinary knowledge of mathematics and their listening attitudes are inextricably intertwined. Again this underscores the idea of the co-construction of knowledge in the mathematics classroom. They offer an interesting comparison of the Davis and Simmt (2006) aspects of knowledge and knowing with the well-known Ball et al (2008) SMK and PCK model. The Davis and Simmt 'knowledge' dimensions (stable) map on to Ball's SMK categories, while the Davis and Simmt 'knowing' dimensions (dynamic) map on to Ball's PCK categories.

Chapter 6 brings concept study into the secondary mathematics classroom, through a case study of a particular teacher and his work on circles with 8th grade students. This illustration shows how a teacher's own knowledge is activated and articulated as he enacts an open disposition in working with and responding to students' mathematical questions in genuine problem-solving activities. Referring to the definition of mathematics for teaching, they explore why this teacher 'possesses and embodies' profound understanding of emergent mathematics (p.110). Davis and Renert contend that, despite the importance afforded it in mathematics education literature, there is little real problem solving in mathematics classrooms. Instead, students are often assessed on formal codified mathematics and replication of pre-established results. The work done by this teacher exemplifies an alternative way forward, creating powerful mathematical learning experiences.

In conclusion, the authors offer a response to the question 'how teachers must know mathematics for it to be activated in the moment and in the service of teaching?'. They acknowledge the work of Ma in establishing the construct of profound understanding of fundamental mathematics (PUFM). They offer the idea of 'profound understanding of emergent mathematics' (PUEM) – emergent because it is fluid and interconnected, is stimulated by interaction with others, and contains various realisations each of which transcends but includes previous ones. They argue that PUEM elaborates and transcends

Ma's PUFM, being coherent but evolving, and particularly being enactive and adaptive to students in the teaching moment.

The authors conceptualise knowledge as responsive, evolving, emergent and distributed among a body of educators. Therefore, not surprisingly, they underscore the importance of face-to-face engagement in learning, 'collective action in the production and interpretation of mathematics for teaching' (p.116). They offer examples of teachers' thoughts on the value of face-to-face engagement: one teacher commented that engaging in online communications was less effective than being together, since in online environments the teachers were 'unable to read one another' (p.112) and to establish the immediacy and flow afforded by direct in-person interaction.

Davis and Renert emphasise the necessity of an open disposition towards substructing mathematics. They argue that it is possible to nurture an open disposition in teachers, but that this takes time and requires collaboration. They recommend that mathematics teacher education programmes include concept study approaches. Finally, they comment on the impact that greater PUEM might have upon student attitudes and achievements. They acknowledge that more research is needed in this area and that operationalising such research may not be straightforward (how do we measure PUEM?). However, they argue that PUEM is a possible route to achieving the conceptual fluency needed in knowledge-based economies, and moreover that the alternatives to PUEM are in fact unengaging and limited forms of mathematics that are not suited to the needs of the world in the 21st century.

This is an important book, which adds a new dimension to the literature on mathematics knowledge for teaching, and brings together some key themes relevant to contemporary mathematics education. Messages that I will take away are:

1) that mathematical knowledge for teaching is complex, tacit and emergent, and cannot readily be codified in a ticklist,
2) that it is important for mathematics teachers to spend time unpacking, substructing, and reconstructing their own mathematical knowledge in order to render it accessible to learners,
3) that teachers' own development in mathematical knowledge develops and evolves in the process and context of preparing to teach, this provides the motivation. There is a resonance here with Ma's (1999) original recommendations: 'Address teacher knowledge and student learning at the same time' (p. 146); 'Enhance the interaction between teachers' study of school mathematics and how to teach it' (p. 147).

Davis and Renert offer us a detailed perspective on a way in which the interaction between study of the subject and how to teach it might take place – through concept study.

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