

Multi-track 2D Joint Signal Detection and Decoding for TDMR System Using Single Parity-check Coding

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Abstract— A combination of single parity check-code and Two-Dimensional (2D) Bahl Cocke Jelinek Raviv (BCJR) detector is used in this paper for joint detection and decoding of 2D interference channel. This is applied to a multi-track two dimensional magnetic recording (TDMR) system. The multi-level 2D BCJR detector handled the code constraint while performing joint detection and single parity decoding. Two coding approaches are presented in this paper: 1) parity bits are applied along-track direction only, separated with a dithered relative prime (DRP) interleaver, and 2) parity bits are applied in both along-track and across-tracks directions. The results show that the new constrained coded multi-track 2D BCJR joint detector provides improved performance with low detection complexity.

Keywords— Parity-check code, Bahl Cocke Jelinek Raviv (BCJR) algorithm, dithered relative prime (DRP) interleaver, 2D detection, 2D equalization.

I. INTRODUCTION

Two-dimensional magnetic recording (TDMR) is a promising technology aimed at delivering ultra-high densities for future magnetic recording systems. The TDMR system employs shingled write recording (SWR) for data writing and two-dimensional (2D) signal processing techniques for data recovery [1][13]. Among the 2D signal processing schemes is 2D detection used to detect 2D interferences, which is a key component in TDMR systems, wireless communications and optical storage systems [2].

In the TDMR system, joint track detection is carried out by applying a 2D detector that fully utilizes the interferences from both along-track and across-track directions. This makes the detection process more reliable and improves the system performance [3]. Although, full 2D joint-track detection can achieve the best possible performance, the complexity of the detector employed considering, the number of tracks w and bits k involved which is in the order of 2^{wk} , limits its deployment.

A handful variety of 2D detection schemes have been proposed by researchers. These correspond to various trade-off between computational complexity and detection performance. Some of these schemes include either: the use of linear equalizer to cancel interference in one direction and partial response maximum likelihood (PRML) detection in the other direction [4], or the use of full 2D soft-output Viterbi algorithm (SOVA) [5]. On the one hand, while applying a linear equalizer reduces the complexity of the detector, it suffers loss of performance especially during severe inter-symbol interference (ISI) and inter-track interference (ITI) conditions. On the other hand, full 2D SOVA gives very good

performance, but have high complexity compared to the former.

Apart from their computational complexity, most 2D detectors, especially maximum likelihood (ML) detectors, do not fully exploit the valuable information available from the other direction, since the detector is applied only along one of the directions [6]. This results in a poor performance when the interference from the other direction is high. An optimal 2D detector carries out multitrack joint full 2D detection across all tracks to fully exploit the 2D interferences present in both directions [5]. Although it suffers from high implementation cost, its implementation seems increasingly feasible in future recording systems with the current improvement in CMOS technology deployment [12]. In [7] and [5], a Multi-track 2D SOVA was designed to fully extract data jointly from multiple tracks in the presence of 2D interference. However, the trellis used in these detectors runs only in the down track direction thus reducing the detection performance when the ITI across-tracks is high. In [6], a proposed 2D SOVA concatenated with a Viterbi detector for detection of shingled magnetic recording (SMR) media was presented. Although the detection complexity was reduced, no coding or error control mechanism was considered.

Besides 2D detection, coding is also an important aspect of 2D signal processing for mitigating the effect of 2D interference. Iterative detection and decoding techniques are also used in 2D systems to combat 2D interference [14][15]. However, iterative decoding is suboptimal because its performance is highly dependent on the number of iterations performed and it becomes very difficult to implement in complex 2D situations.

In this paper, we present a constrained coded multi-track 2D Bahl Cocke Jelinek Raviv (BCJR) joint detector/decoder that performs joint signal detection and decoding of 2D interference signal within a single trellis structure. It extracts information from a two dimensional magnetic recorder using a shingled magnetic recording (SMR) media.

The rest of the paper is organized as follows. Section II provides an overview of the coding method, the SMR channel, and the detector used in this paper. In section III, the multi-level/multi-track 2D BCJR joint detector and decoder design is described. Section IV discusses the simulation model and gives an evaluation of the design. Section V presents simulation results, while section VI concludes the paper.

II. OVERVIEW OF CODING, CHANNEL AND DETECTOR

A. Data Encoding

Fig.1 depicts the block diagram of the system under study. Data to be stored were encoded using a single parity check code. The code appends a parity bit for every three consecutive bits of data based on odd parity-check constraint. The choice of the odd parity is to serve as a run-length limited (RLL) code for the media. A previous investigation shows that separating the first and the second parity bits with a dithered relative prime (DRP) interleaver improves performance [8]. In this paper, we present two encoding approaches: The first approach applies the parity-check constraint along the down track direction only, with the first and second parity bits separated by a DRP interleaver. Parity-check constraint is imposed in both along-track and across-track directions without interleaving for the latter.

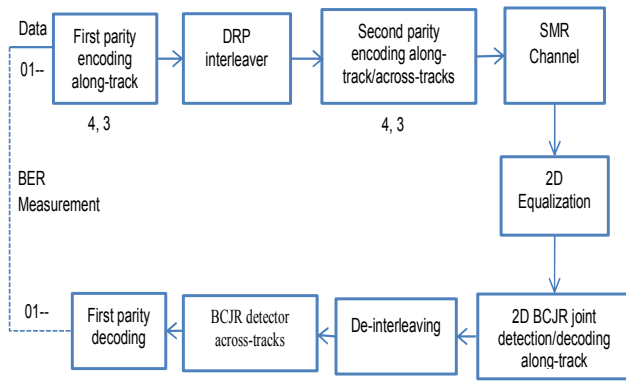


Fig.1. Proposed Multi-track 2D BCJR joint Detector/Decoder

The DRP interleaver was implemented in three stages using equations (1)-(3) [9],

$$I_a(i) = R \left\lfloor \frac{i}{R} \right\rfloor + (x \times i) \bmod R \quad (1)$$

$$I_b(i) = (s + i \times P) \bmod K \quad (2)$$

$$I_c(i) = W \left\lfloor \frac{i}{W} \right\rfloor + (y \times i) \bmod W \quad (3)$$

where $\lfloor \cdot \rfloor$ denotes the floor(x) function. x and y represent the read and write dither vectors of length R and W respectively, K is the interleaver length (total bits) and is a multiple of both R and W , s denotes the starting index, P is the periodic shift constant and is relative prime to K , and i ($i=0 \dots K-1$) represents the data bit address.

B. SMR Channel

SMR is one of the candidate recording techniques proposed for increasing the areal density of magnetic media beyond the conventional limit. It has the advantage of retaining the conventional recording head and media structure and is therefore very easy to implement [3]. The idea behind SMR is

overlapping of data tracks to squeeze data such that tracks are closely packed together in order to increase storage density. However, this leads to severe ITI across tracks, in addition to ISI along track, thereby necessitating the use of 2D signal processing to extract the data out of 2D interference [2][13].

To effectively extract the required information from the signal, in the presence of ITI and ISI, optimal full 2D detectors such as full 2D SOVA or full 2D BCJR are employed to carry out multi-track joint 2D detection across all tracks. The aim is to fully utilize the ITI available from neighbouring tracks in making final decision about the data.

In perpendicular magnetic recording (PMR) media, data bits are written by magnetizing the disk portion in opposite directions to represent binary bits 1 and 0. A magneto-resistive head is then used to read out the magnetic transitions from the media. Due to the randomness of the media grains, the transitions do not occur at the exact bit position. Rather, there is a random deviation from the bit position. The transition response representing the magnitude of the magnetic field read from a PMR media can be represented using the exponential function as shown in (4) [8][10].

$$h(b, t) = V_{\max} e^{\left(\frac{-1.34898^2 b^2 + t^2}{2T_{50}^2} \right)} \quad (4)$$

where V_{\max} is the signal peak voltage amplitude, T_{50} is the time taken for the signal to rise from $-V_{\max}/2$ to $+V_{\max}/2$, while b and t are locations of bits in the along-track and across-track direction respectively from the read head.

To account for the transition jitter due to deviation in the transition position t , jitter noise is modelled as a truncated white Gaussian variable with zero mean and standard deviation σ_j . Electronic noise, and other noises of significant importance are modelled as Additive white Gaussian noise (AWGN) $n_{b,t}$ with standard deviation σ_w . These are added to the final signal as represented in equation (5) [8]. Equations (4) and (5) are therefore, used in this paper for modelling the SMR channel.

$$h(b, t) = V_{\max} e^{\left(\frac{-1.34898^2 b^2 + (t + \delta t)^2}{2T_{50}^2} \right)} + n_{b,t} \quad (5)$$

C. 2D Equalization

In order to reduce the complexity of the detector, information signal from the channel is shaped by the equalizer to the desired target response. However, in a 2D multi-track system, two equalizers are applied in both directions for the required 2D target response to deal with interference from two directions across multiple tracks at the same time.

The equalizer coefficients required for shaping the signal to the desired target response along-track are computed by evaluating the matrix equation expressed in (6).

$$C = H^{-1}T \quad (6)$$

where C gives the required coefficients of the equalizer, H is the matrix approximation of the channel response, and T is a column matrix formed by padding the chosen target with zeros from both sides to make its size equal to the shaping equalizer.

D. Maximum A Posteriori (MAP)/BCJR Detection

The MAP detection is an optimal detection technique that determines the most probable bits of data in binary (1, 0) that are received. Unlike the ML-Sequence Detector (MLSD), which minimizes the probability of sequence error, the MAP detector tries to minimize the probability of bit error. It is a bit-wise detector that works bit-by-bit, thus providing soft information output, which is the information about the reliability of bit decision taken.

MAP detection can be implemented using the BCJR algorithm [11], a trellis based detection algorithm optimized for AWGN. The BCJR algorithm uses the trellis structure and the a priori probability (branch metric) to compute the a posteriori probability (APP) of the decoded bits 1 or 0 as defined by the joint probability equation expressed in equation (7) [11].

$$APP(x_i) = \sum_{s_{k-1}^{n'}, s_k^n} \alpha_{k-1}(s_{k-1}^{n'}) \cdot \beta_k(s_k^n) \cdot \gamma_k^{x_{k=i}}(s_{k-1}^{n'}, s_k^n) \quad (7)$$

The first and second terms of (7) represent the forward and backward state probabilities referred to as alpha and beta while the third term denotes the branch metrics or trellis path transition probabilities. Alpha and beta are computed recursively across the trellis section. The branch metrics (Gamma) are determined for transition from state s_{k-1} to s_k for branch n using (8).

$$\gamma_k^{x_{k=i}}(s_{k-1}^{n'}, s_k^n) = \exp \left[- \left(\frac{x_k - y_n}{2\sigma^2} \right)^2 \right] \quad (8)$$

where x_k is the received symbol or bit at time k , y_n is the ideal transmitted data for branch n at time k , and σ^2 is the channel noise variance. Equations (7) and (8) are used in the detection process.

III. MULTI-LEVEL/MULTI-TRACK BCJR JOINT DETECTOR DECODER DESIGN

A. Multi-level/Multi-track BCJR for ISI Cancellation and Single Parity Decoding

For a single track/two-level BCJR detector of a target length of 3, where one bit is received at a time, the number of branches leaving or entering a state is two and the number of states is four. However, for a multi-level system where more than one bit (k bits) are considered, the number of branch transitions per state is 2^k while the number of states S is equal to 2^{mk} , where m is the constraint length of the detector (Target Length-1). For instance, if two tracks are considered (ITI of two tracks), there are $n=4$ number branches per state and $S=16$ number of states (for target length 3). Similarly, when ITI from three tracks are considered, there are $n=8$ number of

branches per state and $S=64$ number of states (for target length 3). We consider a detector with target length 4 to take into account the parity check constraint imposed on the code. This enabled us to perform data detection and single parity decoding jointly using a single trellis.

In this paper, we consider a multi-track BCJR detector with ITI from two tracks ($k=2$) and a target length 4. This means the detector has $S = 2^{3 \times 2} = 64$ states and produces branch metrics for $2^2 = 4$ transitions per state. Therefore, a trellis with $S=64$ number of states and $n=4$ number of branches transiting from each state was used for detection.

The multilevel BCJR detector used the trellis structure and the branch metrics (eqn. (2)) to calculate the a posteriori probabilities (APPs) of having each transition, which represents the probability of symbols: [0, 0], [0, 1], [1, 0] and [1, 1]. That is, the probability of having symbols; "0" on main-track and "0" on succeeding track, "0" on main-track and "1" on the succeeding track, "1" on main-track and "0" on the succeeding track, and "1" on main-track and "1" on the succeeding track respectively. Since we are interested in performing detection and decoding jointly on the same trellis, the trellis structure is modified to accomplish this task. After each pairs of three data bits were received, the next branch transition reduces to one per state so as to satisfy the single parity check constraint of the code. This in addition to decoding the second parity bit in the data also reduced the complexity of the detection process.

Both alpha and beta require the branch metrics (Gamma) for their computation, Gamma is therefore computed first. Alpha values are determined from the beginning to the end of the trellis. The beginning of the trellis is first initialized with alpha for state ($n=0$) as 1 and the rest as zero. Then, the normalized values of alpha for all state nodes are computed by multiplying the gammas and the previous alphas of the state from which they originate and taking their sums until the end is reached. A similar procedure is applied for beta computation but, in the case of beta we start from the end of the trellis to the beginning. After all the alpha and beta computations are completed, the normalized APPs across each trellis section are determined for [0, 0], [0, 1], [1, 0] and [1, 1] using (7).

All the APPs are computed simultaneously with the second parity bits decoding. The second parity check bits are then removed from the detected data. For the encoding scheme where the first and second parity bits were separated with DRP interleaver, the data is de-interleaved to restore the data bits back to their original positions. De-interleaving is done in the reverse order starting with the inverse of equations (3), (2) and finally (1).

When all tracks are processed, the APPs are now stored and will be use as branch metrics for the BCJR running across tracks to cancel ITI.

B. BCJR Across-tracks for ITI Cancellation

Once all the tracks are processed, the saved APPs are used as branch metrics for the BCJR across tracks. The first bits of all the tracks are then considered in a successive fashion. That is, the first bit on track one is the first data followed by the first bit on track two as the second data and so on until all the data in the tracks are processed. The BCJR detector across tracks has a reduced trellis structure with two states and two branches per state. The APPs representing the four possible probabilities served as the total number of branch metrics. The alpha and beta recursions are computed using the initial conditions and the stored APPs as the new branch metrics. Once all the values of alpha and beta are computed from the beginning to the end of the trellis and vice versa, the APP of having either 1 or 0 is determined. This continues until all the data from all tracks are detected successively thereby detecting the data out of the ITI on the system.

C. First Single Parity Bit Decoding

Under the odd single parity-check constraint, the allowable data bits patterns are, 0001, 1000, 0100, 0010, 1101, 0111, 1110, and 1011. The first three bits are the information bits while the fourth bits are the parity bits. When all data in the tracks are processed, the APPs are then grouped accordingly to satisfy the code constraint depending on the direction along which the parity bit is applied. That is either along-track or across-tracks, as the case may be. MAP decoding is then implemented by finding the probability of each bit position in the allowable data bits patterns. The decoded parity bits are then discarded and the decision is taken based on the remaining APPs.

IV. SIMULATION MODEL AND DESIGN EVALUATION

A. Channel Simulation Model

In this paper, a two-track SMR channel with ITI of two tracks was considered. Initially, the data is coded to contain two single parity bits based on odd parity check constraints. Two coding approaches were implemented. In the first case, the parity check bits were applied along the track only, with the first and second parities separated with DRP interleaver. In the second implementation, we apply the parity check bits in both directions without interleaving. A sector containing 8 tracks and 4096 bits per track was assumed to hold the coded data with guard bands place between sectors. The guard band between the sectors contains -1s (zeros) written all through and the last track of a sector is assumed to be twice as wide as the preceding tracks. The latter ensures that the succeeding track is not overriding the extra width of the last track [6]. Also, a portion of the first track is set to contain -1s (0s) written all through. The total noise power was set to have 80% jitter noise power and 20% AWGN power. The SNR in dB of the overall signal is defined by (9).

$$SNR = 10 \log_{10} \left(\frac{V_p^2}{\sigma_w^2 + \sigma_j^2} \right) \quad (9)$$

where V_p is the peak voltage of the read back signal waveform. σ_j and σ_w are the standard deviations of jitter noise and AWGN respectively.

B. Design Evaluation

The coded multi-track 2D BCJR joint detector and decoder uses a linear equalizer of length 12 and target [0.4, 1.0, 1.0, 0.4] for data shaping along the track. The target length of 4 ensures that the detector handles the code constraint. The ITI from the two tracks serves as target across tracks.

Since ITI of two tracks ($k=2$) and a target length of 4 are considered, our multi-level BCJR along the track will have 64 (2^{mk}) states with 4 (2^k) incoming and outgoing branches per state. However, due the single parity check constraints of the code, the number of branches per state reduce to one after each three consecutive pair of data is received.

Additionally, the fact that a portion of the first track contains -1s written all through reduces the possible states to 8 with 2 branches per state and subsequently 1 branch after every pair of three bits are received. This further reduces the complexity of the detector and also improves performance. The same also applies for the last track.

After computing the APPs representing the probability of symbols [0, 0] up to [1, 1] across the trellis section, these values are saved for use in the ITI cancellation across tracks. The BCJR across tracks has two states with two branches entering or leaving per states. The branch metrics for the transitions are the saved APPs from the multi-level BCJR along tracks. The BCJR initially starts from one possible state with two branches, later diverges into a two state trellis with two branches per states, and finally terminates into one state. The APPs computed from the BCJR detector are then used for further processing (de-interleaving and single parity decoding as the case may be).

V. RESULTS

Fig. 2 shows the bit error rate (BER) performances of the multi-track 2D BCJR detector and the coded multi-track 2D BCJR joint detector and decoder for different T_{50} values. The T_{50} values represent the density levels along the track. The ITI level is set at 100% (read head covering equal portion of main track and side track). The plots "Uncoded" refers to the multi-track 2D BCJR detector without single parity coding while "Coded" refers to the coded multi-track 2D BCJR joint detector and detector employing single parity check coding with DRP interleaver separating the two parity bits along the track. As shown in Fig. 2, the coded scheme provides a significant gain of around 2.0 to 4.0 dB gain compared to the uncoded scheme depending on the value of T_{50} used.

The performance of the multi-track 2D detectors at $T_{50}=1.5$ for different ITI level (read head width) is shown in Fig. 3, which shows that the coded multi-track 2D detector outperformed the uncoded multi-track 2D detector at the different ITI levels. For both schemes, the detection performance improves as the width of the read head increases. This is more evident when the ITI from the side track reaches 100% [1.0, 1.0].

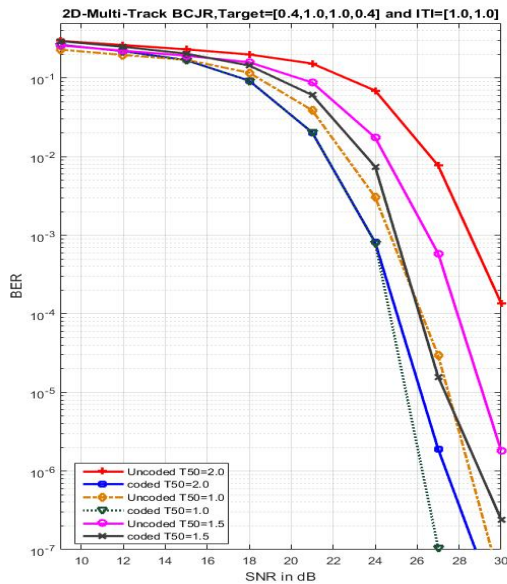


Fig. 2. BER performance comparison at different T_{50}

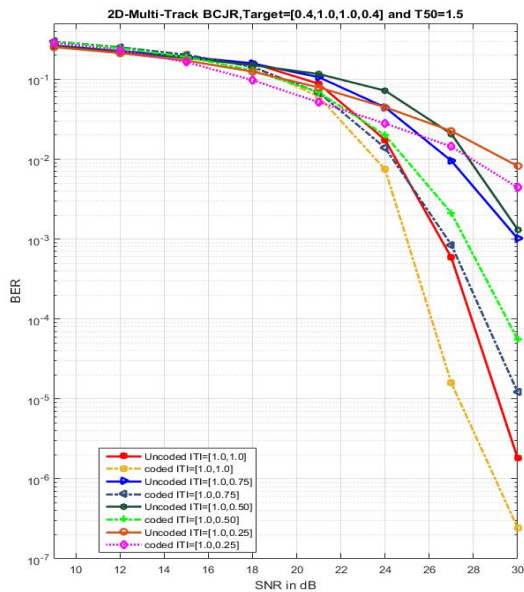


Fig. 3. BER performance comparison of different ITI levels

Fig. 4 shows the performance comparison of the multi-track 2D joint detector and decoder using the two coding approaches. At high density along-track and with 100% ITI from the side track, applying the parity bits along the track

direction only with DRP interleaving, provides better performance than applying the parities in both directions without interleaving. This is because the DRP interleaver reduces the noise correlation between the data bits and the parities by providing adequate spreading. This results in improved performance and also prevent error floor at high densities due to high SNR values, unlike the latter approach (applying the parity bits in both directions without the DRP interleaver) which is vulnerable to error floor at high density as the SNR level increases.

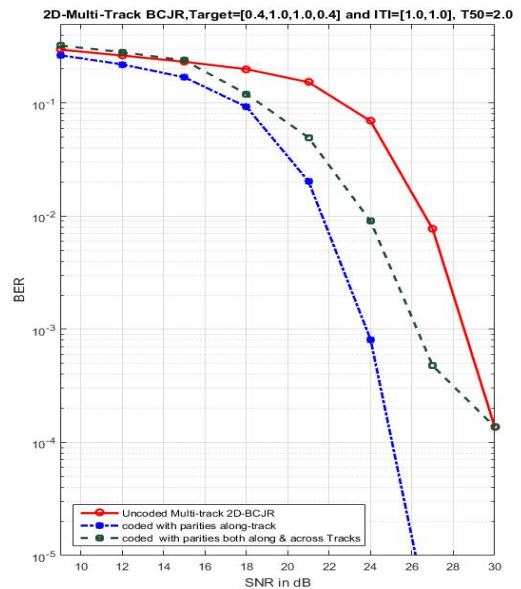


Fig. 4. Performance comparison of the two coding approaches

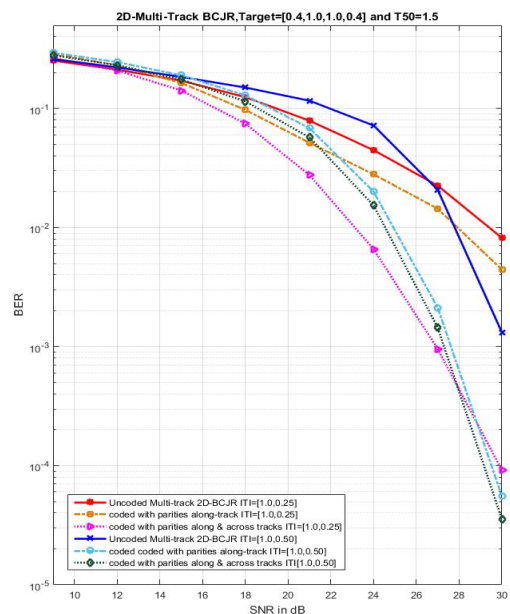


Fig. 5. Performance comparison of coding approaches at different ITI levels

Similarly, Fig. 5 compares the performance of the two coding approaches at low density along the track for different ITI levels. At low ITI level (25% ITI from side track), coding with parity bits in both directions provides better performance than the latter approach. However, as the ITI level increases (50% or higher), the coding with parities along the track separated by DRP interleaver outperformed the former approach. Also, at lower densities along the track direction the effect of error floor is not visible.

VI. CONCLUSION

This paper presented a constrained coded multi-track 2D BCJR detection strategy for performing joint detection and decoding of 2D ISI channel. Results presented showed that, performing multi-track joint detection and single parity decoding on the same trellis structure improves system performance. Compared with the multi-track 2D BCJR, the coded multi-track 2D BCJR detector achieved a gain of around 2.5 to 4.0 dB with significantly less complexity.

REFERENCES

- [1] R. Wood, M. Williams, A. Kavcic, and J. Miles, "The feasibility of magnetic recording at 10 terabits per square inch on conventional media," *IEEE Trans. Magn.*, vol. 45, no. 2, pp. 917-923, Feb. 2009.
- [2] C. K. Chan, et al., "Channel models and detectors for two-dimensional magnetic recording," *IEEE Trans. Magn.* vol. 45, no. 3, pp. 804-811, Mar. 2010.
- [3] Y. Shiroishi, et al., "Future options for HDD storage," *IEEE Trans. Magn.* vol. 45, no. 10, pp. 3816-3822, Oct. 2009
- [4] M. B. Abdulrazaq, M. Z. Ahmed, and P. Davey, "Two dimensional equalization of shingled write disk", presented in International Conference on Magnetism, ICM2015, July 2015 (Unpublished)
- [5] N. Zheng, K. S. Venkataraman, A. Kavcic and T. Zohang, "A study of multitrack joint 2-D signal detection performance and implementation cost for shingled magnetic recording", *IEEE Transaction on Magnetism*, vol. 50, no. 6, November 2014.
- [6] M. B. Abdulrazaq, M. Z. Ahmed, and P. Davey, "Concatenated 2D SOVA for two dimensional maximum likelihood detection", 23rd Telecommunications forum, TELFOR2015, November 2015.
- [7] N. Zheng, K. S. Venkataraman, A. Kavcic and T. Zohang, "Design of low-complexity 2-D SOVA detector for shingled magnetic recording", *IEEE Transaction on Magnetism*, vol. 51, no. 4, April 2015
- [8] M. D. Almustapha, M. B. Abdulrazaq, M. Z. Ahmed, M. A. Ambroze, and P. Davey, "Decoding and detection for magnetic recording channel using single parity coding", in digest of Asia-Pacific Magnetic Recording Conference, APMRC 2016, July 2016.
- [9] S. Crozier, and P. Guinand, "High-performance low-memory interleaver banks for turbo-codes. in Vehicular Technology Conference. IEEE VTS 54th Vol. 4, pp. 2394-2398, Fall 2001
- [10] E. Hwang, R. Negi, and B.V. K. Kumar, "Signal processing for near 10 Tb/in² density in two dimensional magnetic recording (TDMR)", *IEEE Transaction on Magnetism*, Vol. 46, no. 6, June 2010, pp. 1831-1861
- [11] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284-284, Mar. 1974.
- [12] K. Kim, "Future silicon technology," in Proceedings of European Solid-State Device Research Conference ESSDERC, 2012, pp. 1-6.
- [13] R. Wood, "Shingled magnetic recording and two-dimensional magnetic recording" (PDF). ewh.ieee.org. Hitachi GTS. October, 19, 2010. Retrieved August 4, 2014.
- [14] Y. Wu, J. O'sullivan, N. Singla and R. S. Indeck, "Iterative detection and decoding for separable two-dimensional intersymbol interference", *IEEE Trans. Magn.* vol. 39, no.4, pp. 2115-2120, July 2003
- [15] Y. Chen, and S. G. Srinivasa, "Joint self-iterating equalization and detection for two-dimensional intersymbol interference channels", *IEEE Trans. Magn.* vol. 39, no. 8, pp. 3219-3230, August 2013.