BBIOS: A Characterization of Evolutionary Algorithm Stability

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ABSTRACT

We report preliminary results of linking distinct parameter metrics and stability via a novel dynamical system stability characterization (BBIOS). We conduct EA trials to determine the extent of EA stability in parameter space neighborhoods defined by metrics. We capture EA performance loss due to perturbation.

CCS CONCEPTS

•**Mathematics of computing** → *Discrete mathematics;* •**Theory of computation** → *Design and analysis of algorithms;*

KEYWORDS

Evolutionary algorithm, dynamical system, perturbation, stability

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1 INTRODUCTION

It is known that changes in the control parameters of EAs form a dividing line between success and failure. This work introduces an EA stability characterization depending on parameter perturbation according to given input metrics. An EA is converted to a black box dynamical system and, via relaxation of control system stability over perturbation neighborhoods, a novel characterization of EA stability (BBIOS) is reached. The literature on EA stability with respect to control parameter perturbation is scarce, although individual articles on the stability of related algorithms do exist [1–3, 7]. These works ignore EA performance stability with respect to parameter perturbation. Work on this began relatively recently (the study [5] and a visual EA stability criterion [6]). The benefits of EA stability include lower EA performance variability and more consistent results, and it naturally follows that these may assist parameter tuning. This work expands on our works [5, 6], providing a preliminary theoretical basis to an experimental framework.

EA performance may be viewed as effectiveness of the fitness function or runtime. For simplicity, we choose *runtime* to be the number of generations required to solve a problem instance (equal to the number of fitness evaluations divided by population size). The EA control parameters determine how many individuals in each generation are produced by which operation. Let there be n such parameters, denoting this by the vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ with fixed sum. The totality of all such vectors is the parameter space $\mathcal{S}$. We make use of notation from [5, 6]. To each perturbed parameter vector $\mathbf{p}'$ associate the *perturbation* vector, $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ with $p'_i = p_i + \varepsilon_i$. The Manhattan and Chebyshev input metrics are

$$d_M(\varepsilon) = \sum_{i=1}^{n} |\varepsilon_i|, \quad d_C(\varepsilon) = \max_{1 \le i \le n} |\varepsilon_i|. \tag{1}$$

After sampling the parameter vector neighborhood

$$N'_{d_X,\beta}(\mathbf{p}) = \{\mathbf{r} \in \mathcal{S} \setminus \{\mathbf{p}\} : d_X(\mathbf{p}, \mathbf{r}) \le \beta\} \tag{2}$$

according to the metric d_X, the output (Kolmogorov) metric is defined on EA output space as

$$d_K(\mathbf{p}, \mathbf{p}') = \left(E_r(\mathbf{p}) - E_r(\mathbf{p}')\right)^2, \tag{3}$$

where $\mathbf{p}, \mathbf{p}' \in N'_{d_X,\beta}(\mathbf{p})$ and $E_r(\cdot)$ is the mean runtime from an EA run r times with a given vector. A penalty runtime is imposed if the EA fails to converge. The metric d_K indicates EA perturbation sensitivity (the change in output given a parameter perturbation). Previous work [5] reported that for $r = 1$ the variation in d_K over vector samples was high, suggesting more accurate results may emerge for larger r. EA parameters are tuned to try and achieve optimal performance. However results may vary due to small changes in parameter settings. We explore how EA stability may be characterized according to metrics d_M and d_C.

2 INPUT-OUTPUT STABILITY (IOS)

We take the notion of *IOS* from control systems [9]. To apply this to EAs we need the following notation. Let d_X be an input metric. Suppose that, at iteration $t \ge 0$, our EA has input the perturbation $\varepsilon(t)$ with size $\|w(t)\| = d_X(\varepsilon(t))$, output $y(t)$ and population $x(t)$. Let F, H be locally Lipschitz functions, F denoting an EA iteration on state $x(t)$ and H the output. We write the EA as a system

$$\begin{cases} \dot{x}(t) = F(x(t), \varepsilon(t)) \\ y(t) = H(x(t)) \end{cases} \qquad x(0) = \xi, \tag{4}$$

where the initial state of the system is denoted ξ and time is discrete.

Definition 2.1 ([9]). System (4) is IOS if there exist functions α, γ such that $y(t, \xi, w) \le \alpha(|\xi|, t) + \gamma(\|w\|)$ for all $t \ge 0$, with

i. α strictly increasing and continuous on $|\xi|$, and $\alpha \to 0$ as $t \to \infty$;
ii. γ strictly increasing, continuous and satisfying $\gamma(0) = 0$.

This definition states that the output size has upper bound depending upon the perturbation ε. However, this is time-dependent and so cannot be black box. Addressing this, we propose an IOS relaxation which gives a more fitting notion of EA stability.

3 BLACK BOX I-O STABILITY (BBIOS)

First, we set perturbation $\varepsilon(t) = \varepsilon$ for all t, implying a constant control. We then replace the term $y(t, \xi, w)$ with $v_{r,\ell}(\xi, \varepsilon)$, for r repeats and instance size ℓ (or some instance parameter). We

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remove dependence on the iteration number, t, meaning we focus only on initial state, input and end output. i.e., a black box. Let H be the function that after EA termination assigns the values

$$H(x(t)) = (\chi, g) \text{ if } t < S \text{ or } (\text{'FAIL'}, S) \text{ if } t = S. \tag{5}$$

This denotes output of a best solution, χ, when found, and the runtime $g = E_1(\cdot)$. If a solution is not found within S generations then 'FAIL' is output. Let $v_{r,\ell}$ be the list of outputs $H(x(t))$ produced by executing the EA r times on the same instance and parameter vector. The mean EA performance is the expectation $E[|v_{r,\ell}|] = E_r(\cdot)$. The conditions on functions F and H are weakened to be locally continuous and bounded. Finally, as a population metric is undefined, we replace $|\xi|$ with ξ. These relaxations give the following, in which the perturbation ε is applied to a vector $\mathbf{p}$.

Definition 3.1. The system (4) is *strongly* BBIOS with respect to input metric d_X, distance β and vector $\mathbf{p}$, if, for all perturbation vectors $\varepsilon \in N_{d_X,\beta}(\mathbf{p})$, there exist functions α, γ such that

$$E\left[\left|v_{r,\ell}(\xi,\varepsilon)\right|\right] \leq \alpha(\xi, r, \ell) + \gamma(r, d_X(\varepsilon)), \tag{6}$$

with α increasing in ℓ, continuous, and with decreasing variability for large r; and γ increasing, sub-exponential, continuous and satisfying $\gamma(r, 0) = 0$.

The term *strongly* refers to the BBIOS conditions being satisfied for all parameter vectors in the given neighborhood. The function $\alpha(\xi, r, \ell)$ represents the baseline EA runtime of a set of r EA runs, assuming a reference parameter vector $\mathbf{p}$. The term $\gamma(r, d_X(\varepsilon))$ relates to the (likely nonlinear) variability of r runs when the input distance $d_X(\varepsilon)$ is increased. This is the additional runtime due to perturbation. The condition $\gamma(r, 0) = 0$ means that a zero perturbation contributes zero to $d_K(\varepsilon)$. The functions α and γ are expressed in generations. The perturbative runtime γ depends on the input metric, implying that distinct metrics distinctly affect stability. We define the *stability radius* to be the maximal β^* such that (4) is strongly BBIOS with respect to the neighborhood N'_{d_X,β^*}.

BBIOS is the generalization of non-sensitivity of a vector $\mathbf{p}$ to a neighborhood $N'_{d_X,\beta}(\mathbf{p})$ under metric d_X. Increasing r effectively removes randomness from the baseline and perturbative runtimes, giving a lower bound for mean runtime in the limit $r \to \infty$. For small r, however, mean runtime is more variable and BBIOS relies upon mean behavior of the EA over r runs. So we may weaken the BBIOS definition, giving a more practical approximation.

Definition 3.2. The system (4) is ϵ*-weakly* BBIOS if ϵ is the maximal number such that for all perturbations $\varepsilon \in N'_{X,\beta}(\mathbf{p})$, we have $\mathbb{P}(\varepsilon \text{ does not satisfy (6)}) \leq \epsilon$.

This means that, given a stability radius $\beta > 0$ for the neighborhood $N'_{d_X,\beta}$, there exists some $\beta' > \beta$ and a small $\epsilon > 0$ such that the system is ϵ-weakly BBIOS on the neighborhood $N'_{d_X,\beta'}$.

4 EXPERIMENTS AND RESULTS

We compare BBIOS with experimental results obtained using the case study EA of [4]. For a range of β and each metric d_X we took a sample of $N - 1$ vectors in $N'_{d_X,\beta}(\mathbf{p}^*)$ at random. The optimal vector $\mathbf{p}^*$ (determined by experiment) was also included. The output distance (3) between all pairs $(\mathbf{p}_i, \mathbf{p}_j) \in Q^2$, followed by the mean and standard deviation was computed. In [5], we conducted similar experiments using $r = 1$ for d_M only. In this work we restrict stability to around optimal vectors. The left of Table 1 gives data for the Manhattan and the right for the Chebyshev metrics. Each row is a new sample. The mean value of input metric in a given row may be distinct from β as distances are measured over all pairs.

Table 1: Global statistics over S for $r = 3$ repeats and $N = 1000$ with bold text indicating stability radii.

β	$\overline{d_M}$	$\overline{d_K}$	$s(d_K)$	$\overline{d_C}$	$\overline{d_K}$	$s(d_K)$
4	-	-	-	5.11	19485	31852
6	-	-	-	7.03	21141	34936
8	10.36	19043	34350	8.85	24620	44975
10	12.42	20760	40445	**10.56**	**27245**	**44622**
20	22.53	21791	34814	18.73	48109	94250
30	31.83	26087	44721	27.51	71614	121566
40	**41.06**	**30508**	**51710**	31.86	120515	203171
50	50.69	38475	64946	35.68	124690	208462

The data indicate that BBIOS conditions (i) and (ii) are true, confirming the characterization is appropriate. Up to the stability radii in bold ($\beta = 10$ for the Chebyshev metric and $\beta = 40$ for the Manhattan) the output metric increases smoothly, and outside the radii, leaps in value (suggesting exponentiality).

5 CONCLUSIONS AND FUTURE WORK

BBIOS provides an effective characterization of EA stability. BBIOS also mitigates the variability of performance and outputs through the use of mean EA behavior. It provides a lower and upper bound in system performance characterized by instance and perturbation size. It depends on an EA possessing well-defined input metric and output metrics, and an instance parameter ℓ, making it readily applicable to general EAs. If $\mathbf{p}^*$ is an optimal EA parameter vector, then we conjecture (but will need to further investigate) that for all $\epsilon \in [0, 1)$ and as $r \to \infty$, ϵ−weakly BBIOS becomes strongly BBIOS. We will further explore our characterization of EA stability; for example, we will research and evaluate with parameter-finding methodologies such as `irace` [8]. Also, we wish to try out BBIOS on standard EAs and compare the stability of two (or more) different EAs in solving a given problem. Finally, we hope to test BBIOS on real-world problems with unknown optima.

REFERENCES

[1] S. Ben-David, U. von Luxburg, and D. Pál. 2006. A Sober Look at Clustering Stability. In *Learning Theory*. Springer, 5–19.

[2] O. Bousquet and A. Elisseeff. 2002. Stability and generalization. *J. Machine Learning Res.* 2 (2002), 499–526.

[3] N. Chakravarti and A. P. M. Wagelmans. 1998. Calculation of Stability Radii for Combinatorial Optimization Problems. *Oper. Res. Lett.* 23 (1998), 1–7.

[4] Matthew J Craven. 2008. An Evolutionary Algorithm for the Solution of Two-Variable Word Equations in Partially Commutative Groups. In *Stud. Comp. Intell. 153*. Springer, 3–19.

[5] Matthew J Craven and Henri C Jimbo. 2011. A Kolmogorov-Type Stability Measure for Evolutionary Algorithms. In *LNCS 6622*, P Merz and J-K Hao (Eds.). 26–37.

[6] Matthew J Craven and Henri C Jimbo. 2014. EA Stability Visualization: Perturbations, Metrics and Performance. In *GECCO 2014 Companion*, C Igel (Ed.). 1083–1089.

[7] M. V. Devyaterikova, A. A. Kolokolov, and N. A. Kosarev. 2011. The Analysis of the Stability of Some Integer Programming Algorithms with Respect to the Objective Function. *Russian Mathematics* 55, 4 (2011), 18–25.

[8] M. López-Ibáñez, J. Dubois-Lacoste, L. Pérez Cáceres, T. Stützle, and M. Birattari. 2016. The irace Package: Iterated Racing for Automatic Algorithm Configuration. *Oper. Res. Perspectives* (2016).

[9] E. D. Sontag and Y. Wang. 1999. Notions of Input to Output Stability. *Systems & Control Letters* 38, 4 (1999), 236–248.