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Monetary Policy and Indeterminacy after the 2001 Slump*

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Abstract

This paper estimates a simple New Keynesian model of the U.S. economy, allowing for indeterminacy, over the period following the 2001 slump, an episode for which the adequacy of monetary policy is intensely debated. We find that only when measuring inflation with core PCE does monetary policy appear to have been sufficiently active to rule out indeterminacy. We then relax the assumption that inflation in the model is measured by a single indicator and re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the empirical inflation series. CPI and PCE provide better indicators of the latent concept while core PCE is less informative. Finally, we estimate an extended economy that distinguishes between core and headline inflation rates. This model comfortably rules out indeterminacy and confirms the view that the Federal Reserve put more weight on core PCE inflation when setting the policy rate during this period.

* *JEL codes* **E32**, **E52**, **E58**. *Keywords*: Indeterminacy, Taylor Rules, Great Deviation.

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1 Introduction

It has become prevalent to think of monetary policy in terms of nominal interest rate feedback rules. In certain situations, for example, loose monetary policy, these rules may introduce indeterminacy and sunspot equilibria into otherwise stable economic environments. Lubik and Schorfheide (2004) and many others suggest that, empirically, such sunspots-based instability was confined to the seventies and that the post-Volcker years can ostensibly be characterized by determinacy. The current paper extends this analysis to more recent data leading up to the Great Recession.

The issue of loose monetary policy during the 2000s is closely related to Taylor (2007, 2012), who asserts that the Federal Reserve kept the policy rate too low for too long following the recession of 2001. While Taylor does not touch the issue of indeterminacy, he nevertheless argues that this loose policy created an environment that ultimately brought the economy close to the brink. To bolster his thesis of an extra easy monetary policy, Taylor constructs an artificial path for the Federal Funds rate that follows his proposed rule. He characterizes this counterfactual rate's loose fitting to the actual rate as

> "[...] the biggest deviation, comparable to the turbulent 1970s." [Taylor, 2007, 2]

His view is disputed by many. Amongst them, Bernanke (2010) argues that Taylor's use of the headline consumer price index (CPI) to measure inflation in the Federal Reserve's reaction function is misleading. In fact, the Federal Reserve switched the inflation measures that inform its monetary policy deliberations several times over the last two decades. In particular, it moved away from the CPI to the personal consumption expenditure deflator (PCE) in early 2000. In turn, PCE was abandoned midway through 2004 in favor of the core PCE deflator (which excludes food and energy prices).[1] Bernanke (2015) revisits Taylor's exercise and constructs his own counterfactual Federal Funds rate using core PCE. Bernanke's verdict of the Federal Reserve's policy during the 2000s is inimical to Taylor's and he says that

> "[...] the predictions of my updated Taylor rule and actual Fed policy are generally quite close over the past two decades. In particular, it is no

[1] See Mehra and Sawhney (2010).

longer the case that the actual funds rate falls below the predictions of the rule in 2003-2005." [Bernanke, 2015]

Our paper sheds further light on this debate. It takes as a point of departure Taylor's claim of an analogy between the 1970s and the 2000s as well as one of the key recommendations for monetary policy that has emanated from New Keynesian modelling: interest rates should react strongly to inflation movements to not destabilize the economy. Phrased alternatively, if the central bank's response to inflation is tuned too passively in a Taylor rule sense, multiplicity and endogenous instability may arise. In fact, the U.S. economy of the 1970s can be well represented by an indeterminate version of the New Keynesian model as was shown by Lubik and Schorfheide (2004). Along these lines, the current paper turns Taylor's *too low for too long* story into questioning whether the Federal Reserve operated on the indeterminacy side of the rule after the 2001 slump. Knowledge about the economy's regime is important for policymakers because indeterminacy introduces sunspots and alters the propagation of fundamental shocks. Thus, for central banks to use models for policy analysis, a good understanding about the presence of (in-)determinacy is vital.

The empirical plausibility of a link between monetary policy and macroeconomic instability was first established by Clarida, Gali and Gertler (2000). They estimate variants of the Taylor rule and their research suggests that the Federal Reserve's policy may have steered the economy into an indeterminate equilibrium during the 1970s. Yet, they also find that the changes to policy which have taken place after 1980 – essentially a more aggressive response to inflation – brought about a stable and determinate environment. Lubik and Schorfheide (2004) reinforce this point but they refrain from using a single equation approach. They recognize that indeterminacy is a property of a rational expectations system and apply Bayesian estimation techniques to a general equilibrium model. Their results parallel the earlier findings that the U.S. economy veered from indeterminacy to determinacy around 1980 – largely as the result of a more aggressive response of monetary policy towards inflation.

Moreover, this monetary policy change had perhaps an even greater influence on the economy: the transformation from the Great Inflation of the 1970s to the Great Moderation is often conjoined to the conduct of monetary policy.[2] Yet, the

[2]See, for example, Benati and Surico (2009), Bernanke (2012), Coibion and Gorodnichenko

Great Moderation came to an end sometime during the 2000s, and it was followed by enormous economic volatility. Our aim is to examine the possible connection between this transformation and an alteration in the Federal Reserve's monetary policy. In particular, we concentrate on the effects of a possibly too easy monetary policy after the 2001 slump. We frame our analysis from the perspective of (in-)determinacy and conduct it under the umbrella of the Bernanke versus Taylor dispute by considering the measures of inflation that repeatedly occur in the discussion: CPI, PCE and core PCE.

Accordingly, we estimate a small-scale New Keynesian model allowing for indeterminacy over the period between the 2001 slump and the onset of the Great Recession, thus, the NBER-dated 2002:I-2007:III window to be precise. To test for indeterminacy, we employ the method of Lubik and Schorfheide (2004) to compute the posterior probabilities of determinacy and indeterminacy. We take as starting point the same basic New Keynesian model, priors and observables as Lubik and Schorfheide (2004). This strategy allows us to create a continuity between their and our results, which is important given the shortness of our period of interest.

We establish a number of new insights regarding recent U.S. monetary policy. For example, we can indeed expose a violation of the Taylor principle for most of the 2000s when using CPI to measure inflation. This finding supports the visual inspection checks based on single equations in Taylor (2012) who coined the phrase *Great Deviation* to refer to this period. Hence, the 2002:I to 2007:III period would appear to be best described by an indeterminate version of the New Keynesian model. Our upshot is different when basing the analysis on PCE data: we can neither rule in nor rule out indeterminacy. Finally, the evidence in favor of indeterminacy altogether vanishes when we use core PCE. Monetary policy then appears to have been quite appropriate. This conclusion parallels the insight from Bernanke's (2015) counterfactual Federal Funds rate. We thus establish that tests for indeterminacy are susceptible to the data used in the estimation.

We next consider whether our results are an artifact of the six year sample of data. To address this issue, we re-estimate the model on rolling windows of fixed length (23

(2011), Arias, Ascari, Branzoli and Castelnuovo (2014) and Hirose, Kurozumi and Van Zandweghe (2015).

quarters to match the length of the 2002:I-2007:III period) starting in the mid-1960s and focussing on the same inflation measure as Lubik and Schorfheide (2004) namely CPI inflation. The outcomes of the indeterminacy test performed on rolling windows are highly plausible. In particular, we identify only two broad periods (i.e. several consecutive windows) in which a passive policy has likely led to indeterminacy: the 1970s and the post-2001 period. The first period, which coincides with the span of the Burns and Miller chairmanships, exactly matches the indeterminacy duration, as well as the timing of the switch to determinacy in 1980, that Coibion and Gorodnichenko (2011) document. We take this analogy as a reassuring validation of our small sample approach, i.e. even though our period of interest is quite short, it is possible to infer meaningful information from it.[3]

We then attend the issue of how best to measure inflation in the New Keynesian model. We tackle the ambiguity between the theoretical concept and the empirical inflation proxies by employing the DSGE-factor methodology proposed by Boivin and Giannoni (2006). Accordingly, we combine various measures of inflation in the measurement equation and re-estimate our model. CPI and PCE emerge as better indicators of the concept of inflation than core PCE and indeterminacy cannot be ruled out.

However, the finding that indeterminacy cannot be ruled out may hinge on the fact that the baseline three-equations New Keynesian model features a single concept of inflation. To address this question, we finally turn toward an artificial economy that distinguishes explicitly between core and headline inflation. We find that the Federal Reserve was responding mainly to core PCE and was sufficiently active to comfortably rule out indeterminacy.

Perhaps most closely related to our work are Belongia and Ireland (2016) who, like us, evaluate monetary policy during the 2000s.[4] Belongia and Ireland (2016) estimate a time-varying VAR to track the evolution of the Federal Reserve's behavior throughout the 2000s. They find evidence of a change in the Federal Reserve's behavior away from stabilizing inflation towards stabilizing output and also of persistent

[3] Judd and Rudebusch (1998) is another example of an evaluation of monetary policy over similarly short sample periods.

[4] See Fackler and McMillin (2015), Fitwi, Hein and Mercer (2015), Groshenny (2013) and Jung and Katayama (2014) for related exercises.

deviations from the estimated policy rule. While similar in spirit to our results they do not address issues of indeterminacy.

Bianchi (2013) examines the Federal Reserve's policy post-WWII taking a Markov switching rational expectations approach with two monetary policy regimes (i.e. *Hawk* and *Dove*). Bianchi characterizes monetary policy in the early 2000s as Hawkish and identifies a switch to a Dove regime after 2005. His approach to deal with the issue of passive monetary policy is by requiring a linear representation of the Markov switching model to have a unique solution. Phrased alternatively, the regime transitions do not imply moving from determinacy to indeterminacy as both regimes are determinate. Hence, Bianchi's model cannot address questions involving sunspot equilibria as in our paper.

The remainder of the paper evolves as follows. The next section sketches the baseline model and its solution. Section 3 presents the econometric strategy and baseline results. Robustness checks are conducted in section 4. Section 5 relaxes the assumption that model inflation is properly measured by a single empirical indicator. In section 6 we consider an economy that features more than one inflation rate. Section 7 concludes.

2 Baseline model

The familiar three linearized equations summarize our basic New Keynesian model:

$$\begin{aligned} y_t &= E_t y_{t+1} - \tau(R_t - E_t \pi_{t+1}) + g_t \qquad \tau > 0 \qquad (1) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa(y_t - z_t) \qquad \kappa > 0,\ 0 < \beta < 1 \qquad (2) \\ R_t &= \rho_R R_{t-1} + (1-\rho_R)(\psi_\pi \pi_t + \psi_y [y_t - z_t]) + \epsilon_{R,t} \qquad 0 \leq \rho_R < 1. \qquad (3) \end{aligned}$$

Here y_t stands for output, R_t denotes the nominal interest rate and π_t symbolizes inflation. E_t represents the expectations operator. Equation (1) is the dynamic IS relation reflecting an Euler equation. Equation (2) describes the expectational Phillips curve. Finally, equation (3) represents monetary policy, i.e. a Taylor-type rule in which $\psi_\pi > 0$ and $\psi_y > 0$ are chosen by the central bank and echo its responsiveness to inflation and the output gap, $y_t - z_t$. The term $\epsilon_{R,t}$ denotes an exogenous monetary policy shock whose standard deviation is given by σ_R. The other fundamental disturbances involve exogenous shifts of the Euler equation which are captured by the process g_t and shifts of the marginal costs of production captured by z_t. Both variables follow AR(1) processes:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \qquad\qquad 0 < \rho_g < 1$$

and

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t} \qquad\qquad 0 < \rho_z < 1.$$

We denote by σ_g and σ_z the standard deviations of the innovations $\epsilon_{g,t}$ and $\epsilon_{z,t}$. Finally, the term $\rho_{g,z}$ denotes the correlation between the demand and supply innovations. Then, the vector of model parameters entails

$$\theta \equiv \left[\psi_\pi, \psi_y, \rho_R, \beta, \kappa, \tau, \rho_g, \rho_z, \rho_{g,z}, \sigma_R, \sigma_g, \sigma_z\right]'.$$

Indeterminacy implies that fluctuations in economic activity can be driven by arbitrary, self-fulfilling changes in people's expectations (i.e. sunspots). Concretely, in our simple New Keynesian model, indeterminacy occurs when the central bank passively responds to inflation changes, i.e. when $\psi_\pi < 1 - \psi_y (1 - \beta) / \kappa$.

To solve the model, we apply the method proposed by Lubik and Schorfheide (2003) in which case the full set of rational expectations solutions takes on the form

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t \tag{4}$$

where ϱ_t is a vector of model variables,

$$\varrho_t \equiv [y_t, R_t, \pi_t, E_t y_{t+1}, E_t \pi_{t+1}, g_t, z_t]',$$

ε_t denotes a vector of fundamental shocks and ζ_t is a non-fundamental sunspot shock.[5] The coefficient matrices $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{\mathbf{M}})$ and $\Phi_\zeta(\theta)$ are related to the structural parameters of the model. The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Indeterminacy can manifest itself in two ways: (i) through pure extrinsic non-fundamental shocks, ζ_t (a.k.a sunspots), disturbing the economy and (ii) through affecting the propagation mechanism of fundamental shocks via $\widetilde{\mathbf{M}}$.

[5] Under determinacy, the solution (4) boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

Table 1 - Priors and posteriors of DSGE parameters.

		Priors		Posterior Mean [5th pct, 95th pct]	
Name	Range	Density	Prior Mean (Std. Dev.)	CPI Indeterminacy	Core PCE Determinacy
ψ_π	$\mathbb{R}^+$	Gamma	1.10 (0.50)	0.84 [0.61,0.98]	3.01 [1.97,4.17]
ψ_y	$\mathbb{R}^+$	Gamma	0.25 (0.15)	0.19 [0.05,0.41]	0.28 [0.07,0.64]
ρ_R	[0,1)	Beta	0.50 (0.20)	0.83 [0.74,0.90]	0.76 [0.64,0.85]
π^*	$\mathbb{R}^+$	Gamma	4.00 (2.00)	3.28 [1.27,6.01]	1.99 [1.67,2.31]
r^*	$\mathbb{R}^+$	Gamma	2.00 (1.00)	1.15 [0.47,2.01]	1.40 [0.84,2.01]
κ	$\mathbb{R}^+$	Gamma	0.50 (0.20)	0.91 [0.51,1.41]	0.71 [0.31,1.19]
τ^{-1}	$\mathbb{R}^+$	Gamma	2.00 (0.50)	1.66 [1.00,2.49]	1.62 [0.95,2.48]
ρ_g	[0,1)	Beta	0.70 (0.10)	0.60 [0.45,0.73]	0.80 [0.72,0.87]
ρ_z	[0,1)	Beta	0.70 (0.10)	0.80 [0.68,0.89]	0.61 [0.49,0.74]
ρ_{gz}	[-1,1]	Normal	0.00 (0.40)	-0.28 [-0.72,0.17]	0.86 [0.57,0.97]
$M_{R\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-0.57 [-1.90,1.00]	
$M_{g\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	-1.99 [-2.92,-1.05]	
$M_{z\zeta}$	$\mathbb{R}$	Normal	0.00 (1.00)	0.41 [0.05,0.83]	
σ_R	$\mathbb{R}^+$	IG	0.31 (0.16)	0.16 [0.12,0.21]	0.16 [0.12,0.21]
σ_g	$\mathbb{R}^+$	IG	0.38 (0.20)	0.28 [0.18,0.40]	0.19 [0.14,0.25]
σ_z	$\mathbb{R}^+$	IG	1.00 (0.52)	0.74 [0.54,1.03]	0.62 [0.47,0.82]
σ_ζ	$\mathbb{R}^+$	IG	0.25 (0.13)	0.20 [0.12,0.30]	

Notes: The inverse gamma priors are of the form $p(\sigma|\upsilon,\varsigma) \propto \sigma^{-\upsilon-1}e^{-\frac{\upsilon\varsigma^2}{2\sigma^2}}$, where $\nu = 4$ and ς equals $0.25, 0.3, 0.6$ and 0.2, respectively. The prior predictive probability of determinacy is 0.527.

3 Estimation and baseline results

3.1 Data and priors

We employ Bayesian techniques for estimating the parameters of the model and test for indeterminacy using posterior model probabilities. The measurement equation relating the elements of ϱ_t to the three observables, x_t, is given by

$$x_t = \begin{bmatrix} 0 \\ r^* + \pi^* \\ \pi^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \varrho_t \tag{5}$$

where π^* and r^* are the annualized steady-state inflation and real interest rates respectively. Equation (4) and (5) provide a state-space representation of the linearized model that allows us to apply standard Bayesian estimation techniques. The technical appendix provides further details.

We use HP-filtered per capita real GDP and the Federal Funds Rate as our observable for output and the nominal interest rate. These choices follow Lubik and Schorfheide (2004) and make our baseline empirical analysis comparable to theirs in all dimensions but the sample period. To draw up our analysis in the Bernanke versus Taylor debate, we consider in turn three different measures of inflation: CPI, PCE deflator and core PCE (all expressed in annualized percentage changes from the previous quarter). The data covers the period between the 2001 slump and the onset of the Great Recession, i.e. 2002:I to 2007:III.

Table 1 reports our baseline priors which are identical to the ones in Lubik and Schorfheide (2004) and imply a prior predictive probability of determinacy equal to 0.527. Following Lubik and Schorfheide (2004) we replace $\widetilde{\mathbf{M}}$ in equation (4) with $\mathbf{M}^*(\theta) + \mathbf{M}$ where $\mathbf{M} \equiv [M_{R\zeta}, M_{g\zeta}, M_{z\zeta}]'$. We select $\mathbf{M}^*(\theta)$ such that the responses of the endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and the indeterminacy regions. We set the prior mean for $\mathbf{M}$ equal to zero.

3.2 Testing for indeterminacy

For each measure of inflation, we estimate the model over the two different regions of the parameter space, i.e. determinacy and indeterminacy. To assess the quality of the model's fit to the data we present marginal data densities and posterior model probabilities for both parametric zones. We approximate the data densities using Geweke's (1999) modified harmonic mean estimator. Table 2 reports our results.

Following Lubik and Schorfheide (2004) and Taylor (2007, 2012), we begin by using headline CPI to measure inflation. In this case, the data favors the indeterminate model: the posterior probability of indeterminacy is 0.90. This result suggests that Taylor's characterization of monetary policy in the aftermath of the 2001 slump as *too low for too long* is in fact consistent with indeterminacy and the view that the Federal Reserve has potentially veered the economy into instability.

Table 2: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Determinacy	Indeterminacy	Determinacy	Indeterminacy
CPI	-95.48	-93.28	0.10	0.90
PCE	-85.42	-85.75	0.58	0.42
Core PCE	-64.60	-71.58	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.527.

Yet, the upshot differs depending on which measure of inflation we employ in the estimation. Take Bernanke's (2015) suggestion that Taylor's counterfactual experiment should have been performed with core PCE. When making this choice, the posterior probability for our sample concentrates all of its mass in the determinacy region. This result flags that the Federal Reserve had not been responding passively to inflation during this period. However, the Humphrey-Hawkins reports to Congress document that the Federal Reserve based monetary policy deliberations on headline PCE from the beginning of 2000 until mid-2004. Since Taylor is particularly critical of the monetary policy from 2002 to 2004, we next measure inflation using headline PCE data. We repeat the estimation and the finding is now ambiguous: the probability of determinacy is 0.58. Phrased alternatively, we cannot dismiss the possibility of indeterminacy.

Table 1 reports the posterior estimates of the parameters for the model specification favored under CPI and core PCE respectively.[6] The estimated policy rule's

[6]The appendix reports results for parameter estimates when using headline PCE inflation data.

response to inflation, ψ_π, which essentially governs the indeterminacy, differs significantly depending on the way we measure inflation. In particular, when basing the estimation on CPI, the posterior mean equals 0.84 (with 90-percent interval $[0.61, 0.98]$). This result indicates that monetary policy violated the Taylor principle over the 2002-2007 period or in the words of Taylor:

> "[t]he responsiveness appears to be at least as low as in the late 1960s and 1970s." [Taylor, 2007, 8]

The opposite result ensues when using core PCE. In that case, the posterior mean of ψ_π is well above one at 3.01 (with 90-percent interval $[1.97, 4.17]$).

3.3 How important are sunspots and what drives the results?

Indeterminacy can manifest itself by affecting the propagation of fundamental shocks as well as introducing sunspot shocks. Given our above results, the question of how important sunspot fluctuations were during the 2000s comes up naturally. To answer this question, we study the propagation of shocks and the unconditional forecast error variance decomposition. A more detailed analysis can be found in the Appendix. Based on our estimation using CPI data, sunspots played only a marginal role with the most significant contribution being seven to eight percent in explaining the variances of the policy rate and inflation. However, indeterminacy qualitatively altered the propagation of demand shocks by changing the sign of the inflation response.

In sum, we find that indeterminacy outcomes are dependent on the inflation measure that is used. What is the intuition behind this result and which features of the data stand behind it? Headline inflation generally tends to be more volatile than core inflation that excludes the most volatile components, particularly in periods of persistent commodity price shocks. In fact, CPI and PCE are both more volatile than core PCE during our period of interest. This volatility feature of the data partly drives our findings through its influence on the estimates of the Taylor rule. With core PCE as the preferred measure of inflation, the monetary authority reacts to relatively small movements in inflation. In that case, any policy response to inflation has to be substantially larger for the estimation procedure to fit the Federal Funds rate data. In contrast, when measuring inflation with CPI, the estimated responsiveness to inflation turns out to be smaller due to the larger fluctuations of the inflation gap. As monetary policy fails to guarantee a unique rational expectations equilibrium whenever it is insufficiently active with respect to inflation, the posterior probability of indeterminacy is higher with headline than with core inflation.

Beyond the difference in the volatility of the inflation measures, another feature of the data that drives our (in)-determinacy results is a disconnect between core and headline inflation in face of persistent commodity price shocks. Our estimation based on CPI suggests that indeterminacy primarily affects the propagation of demand shocks. In particular, the parameter $M_{g\zeta}$ redirects the transmisson of this disturbance, making it look similar to a cost-push shock. This mix of disturbances helps the model fit the joint behavior of headline inflation (especially CPI), real activity and monetary policy during the 2002-2007 episode.

4 Sensitivity analysis

We now investigate the sensitivity of our results in various directions. The robustness checks involve testing for indeterminacy on rolling windows and alternative measures of output as well as using real-time data.[7]

We conduct further robustness checks that involve (i) estimating the policy parameters only, (ii) alternative priors for ψ_π, (iii) alternative measure of inflation, (iv) serially correlated monetary policy shocks, and (v) trend inflation. For all these tests, our results remain unchanged.

Rolling windows The size of our sample is undeniably short. So first and foremost, we want to assess the extent to which our results might be an artifact of the small sample. To do so, we re-estimate the model on rolling windows starting in the mid-1960's, and keeping the size of the windows fixed at 23 quarters to match the number of observations in our period of interest. Thus the first window is 1966:I-1971:III. We move the window forward one quarter at a time, and re-estimate all parameters each

[7] The Appendix conducts additional robustness checks that involve estimating the policy parameters only; alternative priors for ψ_π; an alternative measure of inflation; serially correlated monetary shocks; trend inflation. Our results are robust to all these extensions.

time.[8] Here we just consider CPI inflation as the Federal Reserve only began to base its monetary policy deliberations on PCE and core PCE in the 2000s. Moreover, doing so makes our results directly comparable to Lubik and Schorfheide (2004). Figure 1 presents the evolution of the posterior probability of determinacy for the U.S. economy from 1966:I to 2008:III. The end point is chosen to avoid obvious complications that emanate from hitting the zero lower bound. The graph suggests that the U.S. economy was likely in a state of indeterminacy during the 1970s. Thereafter, beginning with the Volcker disinflation policies, the economy shifted back to a determinate equilibrium. These findings are consistent with related studies such as Clarida, Gali and Gertler (2000), Lubik and Schorfheide (2004) and Coibion and Gorodnichenko (2011).[9] We take this correspondence as a justification for estimating our model on a short window.[10] Our paper documents a second shift after the 2001 slump now from determinacy to indeterminacy.

[8] This approach to estimate linear DSGE models was recently promoted by Canova (2009), Canova and Ferroni (2011a) and Castelnuovo (2012a,b). Rolling window estimation provides two benefits. It allows us to uncover time-varying patterns of the model's parameters, in particular, of the monetary policy coefficients. At the same time, the procedure permits us to remain within the realm of linear models and apply standard Bayesian methods.

[9] Figure 1 is comparable to Coibion and Gorodnichenko (2011, Figure 4). They report a moving average of the probability of determinacy which makes their series smoother than ours. Coibion and Gorodnichenko (2011) use a model with trend inflation. We explore such model in the Appendix.

[10] We furthermore experimented with the window length and the results appear to be robust.

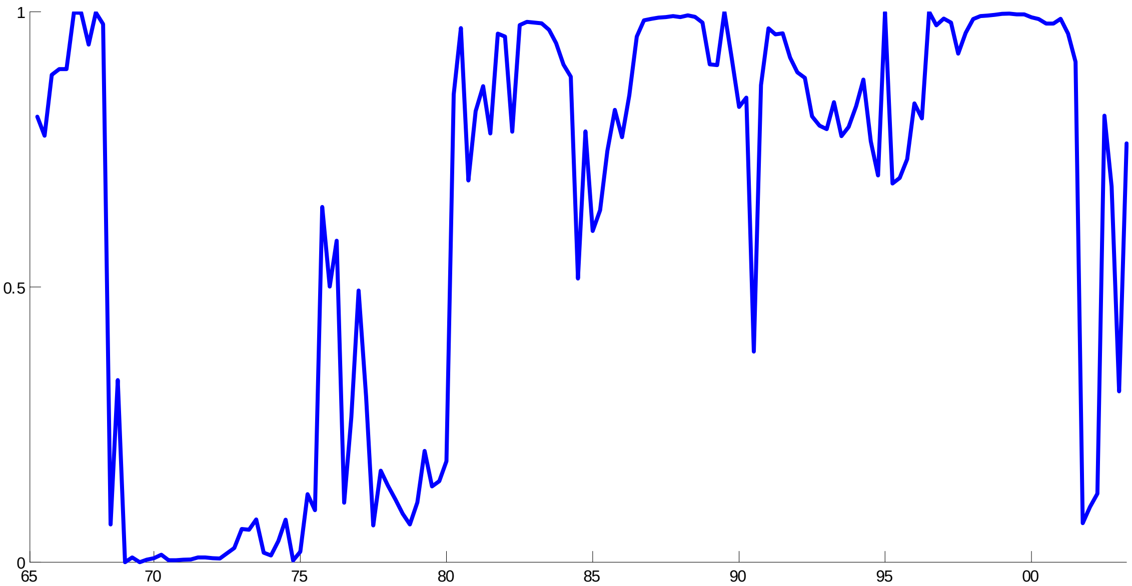

Figure 1: Probability of determinacy using rolling window estimation. The figure plots the probability at the first quarter of a window.

Alternative measures of output To make our baseline analysis comparable with Lubik and Schorfheide (2004), we used HP-filtered real GDP per capita to measure output fluctuations. However, as argued by Canova (1998), Gorodnichenko and Ng (2010), and Hamilton (2017) among others, HP-filtered data may induce spurious results. Accordingly, we now consider two alternative ways to gauge real economic activity. First, we replace the output trend extracted using the HP filter with the Congressional Budget Office's estimate of potential output as in Belongia and Ireland (2016) and others. Table 3 suggests that, again, our results remain robust. Second, we use output growth instead of an output gap measure. To this end, we assume that the artificial economy now features trend-stationary technology – it follows a deterministic trend as in Mattesini and Nisticò (2010) or Ascari, Castelnuovo and Rossi (2011).[11] Also, we no longer estimate the intertemporal rate of substitution, $1/\tau$, and instead set it equal to one to make the model consistent with balanced growth. Then, Table

[11] The measurement equation now writes $\gamma_{yt}^{obs} = \gamma^* + \Delta\widehat{y}_t$ where γ_{yt}^{obs} is the observed growth rate of output, γ^* stands for the steady state growth rate and $\Delta\widehat{y}_t$ is the first-differenced logarithm of detrended model output. The prior distribution of γ^* is $\mathsf{N}\left(0.5, 0.1\right)$.

3 shows that when using output growth, the case for indeterminacy becomes even stronger for CPI and PCE. Yet, it remains unchanged when measuring inflation via core PCE data.[12]

Table 3. Determinacy versus Indeterminacy (Robustness)

		Log-data density		Probability	
Inflation measure		Det.	Indet.	Det.	Indet.
CPI	CBO output gap	-97.89	-95.85	0.12	0.88
	Output growth	-93.29	-89.58	0.02	0.98
PCE	CBO output gap	-88.08	-88.18	0.53	0.47
	Output growth	-82.89	-81.80	0.25	0.75
	Real-time data	-83.32	-83.06	0.44	0.56
Core PCE	CBO output gap	-68.53	-73.63	0.99	0.01
	Output growth	-62.54	-67.58	1	0
	Real-time data	-65.85	-70.24	0.99	0.01

Real-time data One important distinction between CPI and PCE price indices is that the former are not revised (except for seasonal adjustments), whereas the latter go through repeated rounds of revision as more information becomes available. In particular, the PCE-based measure of inflation in Bernanke's (2010) speech is a real-time measure, which, as he argues, may exhibit considerable differences relative to the revised PCE data. Hence, like Orphanides (2004), we now take into account that monetary policymakers make decisions based on contemporaneously available information. Therefore, our estimation uses real-time data on output, PCE and core PCE from the Real-Time Data Set for Macroeconomists provided by the Federal Reserve Bank of Philadelphia. Table 3 confirms that our findings remain robust: we can confidently rule out indeterminacy when basing our estimation on core PCE, while there is a possibility that indeterminacy might have prevailed under PCE.

Further tests of robustness We conduct further robustness checks that involve (i) estimating the policy parameters only, (ii) alternative priors for ψ_π, (iii) alternative

[12] Given the indicated issues with HP-filtered data and the essentially unchanged results when employing output growth, the remainder of this paper concentrates on output growth.

measure of inflation, (iv) serially correlated monetary policy shocks, and (v) trend inflation.[13] For all these tests, our main result so far that the basic New Keynesian model provides mixed evidence about indeterminacy is robust.

5 Which measure of inflation to choose?

Our baseline estimations have delivered mixed evidence regarding the probability of indeterminacy for the 2002:I to 2007:III period. The results are consistently dependent on the specific inflation measure used in estimation – only with core PCE series can we comfortably rule out indeterminacy. However, each inflation proxy may only provide an imperfect indicator of the model concept. Put differently, all three measures of inflation may contain relevant information. In this line of thinking, we will now depart from the assumption that model inflation is measured by a single series and draw on Boivin and Giannoni's (2006) dynamic factor analysis of DSGE models.[14] In a nutshell, we want to exploit the information from all the inflation series in the estimation to deliver more robust results. We treat the model concept of inflation as the unobservable common factor for which data series are imperfect proxies. More concretely, the estimation involves the transition equation (4)

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t$$

or its determinacy equivalent

$$\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$$

and the measurement equation

$$\begin{bmatrix} \Delta GDP_t \\ FFR_t \\ \mathbf{X}_t \end{bmatrix} = \begin{bmatrix} \gamma^* \\ r^* + \pi^* \\ \underset{4\times 1}{\mathbf{0}} \end{bmatrix} + \begin{bmatrix} \mathbf{I}_2 & \underset{2\times 4}{\mathbf{0}} \\ \underset{4\times 2}{\mathbf{0}} & \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta y_t \\ 4R_t \\ \boldsymbol{\pi}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \mathbf{u}_t \end{bmatrix}. \qquad (6)$$

Here ΔGDP_t stands for the growth rate of per-capita real GDP, FFR_t denotes the Federal Funds rate, $\mathbf{X}_t \equiv [\Delta CPI_t, \Delta PCE_t, \Delta corePCE_t, \Delta DEF_t]'$ is the vector of

[13] In the model with trend inflation, it is no longer possible to analytically derive the indeterminacy conditions. Hence, we follow Hirose's (2014) numerical solution strategy for finding the boundary between determinacy and indeterminacy by perturbing the parameter ψ_π in the monetary policy rule (see also Justiniano and Primiceri, 2008).

[14] Canova and Ferroni (2011b) and Castelnuovo (2013) are recent applications.

empirical inflation proxies,[15] $\mathbf{\Lambda}$ =diag($\lambda_{CPI}, \lambda_{PCE}, \lambda_{corePCE}, \lambda_{DEF}$) is a 4×4 diagonal matrix of factor loadings relating the latent model concept of inflation to the four indicators, $\boldsymbol{\pi}_t \equiv 4[\pi_t, \pi_t, \pi_t, \pi_t]'$ and $\mathbf{u}_t = [u_t^{CPI}, u_t^{PCE}, u_t^{corePCE}, u_t^{DEF}]' \sim i.i.d.(\mathbf{0}, \mathbf{\Sigma})$ is a vector of serially and mutually uncorrelated indicator-specific measurement errors, with $\mathbf{\Sigma}$ =diag($\sigma^2_{CPI}, \sigma^2_{PCE}, \sigma^2_{corePCE}, \sigma^2_{DEF}$). We jointly estimate the parameters $(\mathbf{\Lambda}, \mathbf{\Sigma})$ of the measurement equation (6) along with the structural parameters θ. We calibrate π^* equal to 2.5 percent - a value roughly in line with the average of the sample means of the inflation series. We standardize the four indicators to have mean zero and unit variance. This standardization permits us to interpret the factor loadings, $\lambda_j s$, as correlations between the latent theoretical concept of inflation and the respective observables.[16] Our prior distribution for the loadings and measurement errors are $\lambda_j \sim Beta(0.50, 0.25)$ and $u_t^j \sim Inverse\ Gamma(0.10, 0.20)$ respectively. By employing a beta distribution, the support of the λ_j is restricted to the open interval $(0, 1)$ which is a necessary sign restriction.

Table 4 reports the resulting log-data densities which are -162.50 for determinacy and -161.83 for indeterminacy. Phrased differently, the posterior probabilities of determinacy and indeterminacy are 34% versus 66%, hence, we cannot rule out indeterminacy.[17]

Table 4: Determinacy versus Indeterminacy (DSGE-Factor)

Log-data density		Probability	
Determinacy	Indeterminacy	Determinacy	Indeterminacy
-162.50	-161.83	0.34	0.66

Notes: The prior predictive probability of determinacy is 0.527.

Table 5 reports the posterior estimates of the model parameters along with the factor loadings (i.e. the correlations between the latent factor and the proxies) as well as the standard deviations of the measurement errors. Conditional on both determinacy and indeterminacy the loadings on CPI and PCE are about three times

[15] DEF is the acronym for the GDP Deflator.

[16] See Geweke and Zhou (1996) and Forni, Hallin, Lippi and Reichlin (2000).

[17] We also replicated Lubik and Schorfheide (2004) with the DSGE factor model approach. The outcomes of the indeterminacy test for the pre-Volcker and post-1982 sample periods remain unaltered to this extension.

as large as the loading on core PCE. Furthermore, there is evidence of substantial indicator-specific component for core PCE as evident in the high standard deviation of its measurement error. These results imply that CPI and PCE provide better indicators of the latent concept of inflation, while core PCE, despite being promoted by Bernanke (2015), is less informative. In other words, while core PCE might better fit the Federal Reserve's behavior in isolation, the other inflation measures are more consistent with the New Keynesian model as a whole.

In sum, when taking the considered variants of the New Keynesian model, indeterminacy cannot be ruled out. What these model versions have in common though is that they all feature only one measure of inflation. In the next section we turn to an economy that explicitly differentiates between core and headline inflation rates.

Table 5 - Parameter Estimation Results (DSGE-Factor)

	Determinacy		Indeterminacy	
	Mean	[5th pct, 95thpct]	Mean	[5th pct, 95th pct]
ψ_π	2.13	[1.29,3.13]	0.80	[0.61,0.98]
ψ_y	0.30	[0.07,0.65]	0.21	[0.05,0.45]
ρ_R	0.81	[0.72,0.88]	0.81	[0.73,0.88]
r^*	1.00	[0.45,1.67]	1.23	[0.57,2.00]
κ	0.74	[0.41,1.15]	1.00	[0.57,1.49]
γ^*	0.53	[0.45,0.62]	0.51	[0.44,0.58]
ρ_g	0.79	[0.68,0.87]	0.60	[0.45,0.74]
ρ_z	0.68	[0.50,0.85]	0.70	[0.54,0.84]
ρ_{gz}	0.14	[-0.33,0.70]	-0.31	[-0.74,0.15]
$M_{R\zeta}$			-0.31	[-1.53,1.17]
$M_{g\zeta}$			-1.77	[-2.59,-0.95]
$M_{z\zeta}$			0.30	[0.01,0.62]
σ_R	0.18	[0.13,0.25]	0.16	[0.12,0.21]
σ_g	0.19	[0.14,0.27]	0.28	[0.18,0.42]
σ_z	0.69	[0.50,0.94]	0.73	[0.53,1.00]
σ_ζ			0.18	[0.12,0.27]
λ_{CPI}	0.76	[0.55,0.93]	0.57	[0.37,0.79]
λ_{PCE}	0.79	[0.59,0.95]	0.59	[0.40,0.82]
$\lambda_{CorePCE}$	0.28	[0.07,0.52]	0.21	[0.06,0.40]
λ_{DEF}	0.53	[0.31,0.77]	0.41	[0.23,0.64]
σ_{CPI}	0.31	[0.20,0.43]	0.32	[0.22,0.43]
σ_{PCE}	0.18	[0.10,0.31]	0.18	[0.10,0.29]
$\sigma_{CorePCE}$	0.91	[0.72,1.14]	0.91	[0.72,1.14]
σ_{DEF}	0.71	[0.56,0.90]	0.70	[0.56,0.88]

Notes: The table reports posterior means and 90 percent probability intervals of the DSGE-Factor model parameters.

6 An economy that distinguishes between core and headline inflation

Our baseline results on the issue of equilibrium determinacy were clearly dependent on the particular measure of inflation used in the estimation, thus leaving us with essentially the same dilemma that Taylor and Bernanke originally posed: should we measure inflation with CPI or Core PCE? In the previous section we have attempted to resolve this ambiguity by taking an econometric approach that draws on the DSGE-Factor analysis. Our estimation results there suggest that, for our period of interest, the concept of inflation in the basic New Keynesian model is more strongly correlated with broad indicators such as CPI and PCE than with narrower proxies such as core PCE. The immediate implication of this finding is that the indeterminate version of the model fits better than its determinate analogue.

However, the result that indeterminacy cannot be ruled out may hinge on the fact that the three-equation New Keynesian model features a single concept of inflation. Indeed, our DSGE-Factor approach forces the central bank to respond to the exact same measure of inflation (i.e. same combination of indicators) as the one that households consider in their consumption-spending decisions. But what (would be the consequences for equilibrium determinacy) if the Federal Reserve was actually focusing on core inflation in its conduct of monetary policy, as claimed by Bernanke (2015), while private-sector agents were looking at a different, broader, measure of inflation?

To address this question, we now turn toward a structural approach by employing an artificial economy that distinguishes explicitly between core and headline inflation, i.e. both inflation concepts simultaneously appear in the model.

6.1 Model

The artificial economy builds on Blanchard and Gali (2010) and Blanchard and Riggi (2013) who introduce imported oil into an otherwise standard New Keynesian model. We present the key aspects of the linearized model here and delegate the full description to the Appendix. Our exposition draws heavily on Blanchard and Gali (2010).

Oil is used by firms in production and by households in consumption. In particular, technology is given by a Cobb-Douglas production function that uses labor, n_t, and oil, m_t :

$$q_t = \alpha m_t + (1-\alpha)n_t \qquad 0 < \alpha < 1 \tag{7}$$

where q_t stands for gross output. Similarly, final consumption, c_t, is made up of domestically produced good, $c_{q,t}$, and imported oil, $c_{m,t}$:[18]

$$c_t = (1-\chi)c_{q,t} + \chi c_{m,t} \qquad 0 < \chi < 1. \tag{8}$$

Denoting the price of domestic output and the price of consumption by $p_{q,t}$ and $p_{c,t}$ respectively, and letting $p_{m,t}$ be the nominal price of oil, the following relationship arises between consumption-price inflation $\pi_{c,t}$ and domestic output-price inflation $\pi_{q,t}$:

$$\pi_{c,t} = \pi_{q,t} + \chi \Delta s_t \tag{9}$$

where s_t is the real price of oil, $s_t \equiv p_{m,t} - p_{q,t}$, which is exogenous. Following Aoki (2001) and Blanchard and Gali (2010), we interpret $\pi_{c,t}$ and $\pi_{q,t}$ as headline and core inflation respectively. Utility maximization by the household yields the standard intertemporal optimality condition

$$c_t = E_t c_{t+1} + E_t z_{t+1} - R_t + E_t \pi_{c,t+1} + d_t - E_t d_{t+1} \tag{10}$$

and the intratemporal leisure-consumption trade-off

$$w_t - p_{c,t} = \gamma(w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t]. \tag{11}$$

Here R_t denotes the nominal interest rate, d_t is a discount-factor shock, z_t is a shock to the growth rate of technology, w_t denotes the nominal wage and φ stands for the inverse Frisch elasticity. The parameter $\gamma \in [0,1]$ captures the extent of real wage rigidity where larger values indicate higher degrees of rigidity. Notice in the household's Euler equation (10) that the model-consistent real interest rate that drives consumption dynamics involves headline consumption price inflation. Domestic firms are monopolistic competitors facing nominal rigidities à la Calvo. Firms' profit-maximizing pricing decisions result in the familiar aggregate New Keynesian Phillips

[18] If the shares α and χ are set to zero, the economy boils down to a simple three-equation New Keynesian model, similar to the one we have used in the previous sections.

curve which governs the dynamics of domestic-good sticky-price inflation (i.e. core inflation):

$$\pi_{q,t} = \beta E_t \pi_{q,t+1} - \kappa \mu_t \tag{12}$$

where the slope coefficient $\kappa \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi}$, ξ denotes the probability of not being able to reset prices, β represents the household's discount factor and μ_t is the price markup over nominal marginal costs. Cost minimization by firms gives rise to the following demand for oil:

$$m_t = q_t - \mu_t - s_t. \tag{13}$$

The requirement that trade be balanced (as oil is imported) delivers the following relationship between final consumption and domestic output:

$$c_t = q_t - \chi s_t + \eta \mu_t \tag{14}$$

where $\eta \equiv \frac{\alpha}{\mathcal{M}^P - \alpha}$ and $\mathcal{M}^P$ denotes the steady-state gross markup. Value added (i.e. GDP), denoted by y_t, is given by:

$$y_t = q_t + \frac{\alpha}{1-\alpha} s_t + \eta \mu_t. \tag{15}$$

Monetary policy follows a Taylor rule which reacts to inflation, deviations of GDP from the balanced-growth path and the growth rate of GDP, $gy_t \equiv y_t - y_{t-1} + z_t$:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_\pi \{\omega \pi_{c,t} + (1-\omega)\pi_{q,t}\} + \psi_y y_t + \psi_{gy} gy_t] + \epsilon_{R,t}$$

where the monetary policy shock $\epsilon_{R,t}$ is i.i.d. $\mathsf{N}(0, \sigma_R^2)$. Notice that the central bank responds to a convex combination of headline and core inflation (with the parameter ω governing the relative weights; setting ω to zero implies that the central bank responds to core inflation only). As we have seen, the controversy between Taylor and Bernanke essentially boils down to the choice of the inflation measure in the monetary policy rule. By estimating ω, we will let the data speak as to whether the Federal Reserve was actually focusing on headline (Taylor, 2007) or core inflation (Bernanke, 2015). Lastly the structural disturbances s_t, z_t, and d_t are assumed to follow independent stationary $AR(1)$ processes:

$$s_t = \rho_s s_{t-1} + \varepsilon_{st} \qquad z_t = \rho_z z_{t-1} + \varepsilon_{zt} \qquad \text{and} \qquad d_t = \rho_d d_{t-1} + \varepsilon_{dt}.$$

We find that the *Taylor Principle* continues to hold in the Blanchard-Gali model.[19] In line with Carlstrom, Fuerst and Ghironi (2006), the indeterminacy condition is not dependent on any particular measure of inflation: as long as the central bank sets its response coefficient greater than unity to either headline or core inflation (or any convex combination of these two measures), such policy will ensure equilibrium determinacy.

6.2 Econometric strategy and results

To address typical identification issues, we calibrate a subset of the model parameters. We set the discount factor β to 0.99, the steady-state markup at ten percent, and the inverse of the labor-supply elasticity φ to one. Following the computations in Blanchard and Gali (2010) for their post-1984 sample period, we calibrate the shares of oil in production and consumption to $\alpha = 0.012$ and $\chi = 0.017$. Furthermore, we assume that shocks to the growth rate of technology are i.i.d., i.e. $\rho_z = 0$. We estimate the remaining parameters with Bayesian techniques. We use a loose Beta distribution centered at 0.5 to place an agnostic prior on both the wage-rigidity parameter, γ, and the weight on headline inflation in the monetary policy rule, ω. The other priors are similar to the ones we have used in the earlier sections and are reported in Table 6.

For our purpose, the main appeal of the Blanchard-Gali model is that it offers a micro-founded distinction between core and headline inflation which permits us to use both headline and core inflation data in the estimation. This approach will hopefully resolve some of the ambiguity that characterized our previous results.

At first, however, to maintain a continuity with our earlier findings, we estimate the new model using the exact same dataset with only three observables: the quarterly growth rate of real GDP per-capita, the Federal Funds rate and one of two alternative inflation rates, CPI or core PCE. Since we are initially using only one inflation series at a time, the weight ω in the Taylor rule is not well identified. Hence, when using CPI data, we calibrate this parameter to one, so that the central bank responds solely to headline inflation as in Taylor (2007). Similarly, when measuring inflation with core PCE, we set ω equal to zero, so that the monetary authority reacts to core inflation

[19] Figure A5 in the Appendix shows the determinacy region for combinations of ψ_π with the other policy parameters as well as with the degree of real wage rigidity γ.

as Bernanke (2015) suggests. Table 6 reports the posterior estimates while Table 7 gives the log-data densities. In line with all our previous results, the estimation favors the indeterminate version of the model whenever we use CPI data, while it unambiguously selects determinacy under core PCE. Since we are using our original dataset, we can compare the marginal data densities of the augmented economy with the ones of the baseline model shown in Table 3 (the row labelled 'Output Growth'). The fact that these densities are of similar magnitude indicates that the additional micro-foundations of the Blanchard-Gali model are not rejected by the data.

We can now move on to our next exercise: treating simultaneously both headline and core inflation as observables. Hence, our dataset will now include four variables. This step enables us to properly identify the commodity-price shock as well as the weight ω in the policy rule. First, we measure headline and core inflation using PCE and core PCE data respectively. Then, we consider CPI as the proxy for headline inflation, while still using core PCE data to measure core inflation. Using CPI and core PCE data simultaneously to estimate the model helps us tackle the controversy between Taylor and Bernanke in a more direct way.[20] Table 7 (cf. the two rows labelled "Four obs") shows that, no matter whether we measure headline inflation with PCE or CPI data, the whole posterior probability mass concentrates in the determinacy region. Looking at Table 6 (cf. the two columns labelled "Four obs"), the posterior mean of the weight on headline inflation in the policy rule, ω, is 0.25 with PCE data and 0.17 when we use CPI. Our estimation results therefore provide some empirical support for Bernanke's (2015) claim that the Federal Reserve was actively reacting to core inflation (as opposed to headline) during this period. Moreover, as anticipated, the parameters pertaining to the commodity-price shock are now better identified: the posterior mean estimates of ρ_s and σ_s are both significantly higher than the estimates we obtain when using only three observables.

A key parameter in the Blanchard and Gali (2010) model is the degree of real wage rigidity, γ. To sharpen the identification of this feature, we finally add real wage data, i.e. we ultimately employ five observables to estimate the model. We use

[20]However, this combination of headline CPI and core PCE data is not ideal to measure the theory's concepts of headline and core inflation: In the model, the core deflator is defined implicitly by excluding oil (the imported commodity) from the consumer's basket, without altering the weights of others goods. Yet, the CPI and PCE price index are assembled in different ways and attach different weights to different goods.

observations on hourly compensation for the non-farm business sector for all persons as a measure of nominal wages. To get real wages, we then divide this proxy by, alternatively, the PCE or CPI price deflator (depending on how we measure headline inflation). To circumvent the issue of stochastic singularity, we add a labor supply shock, ν_t.[21] As a result, the labor supply equation (11) becomes:

$$w_t - p_{c,t} = \gamma(w_{t-1} - p_{c,t-1}) + (1-\gamma)[\varphi n_t + c_t] + \nu_t. \tag{16}$$

Our main finding, that the data favors determinacy in this extended model, remains unchanged. The parameter estimate of γ becomes twice as large when we use real wage data, suggesting a substantial degree of real wage rigidity. This result contrasts with Blanchard and Riggi (2013) who find that real wages were highly flexible during the Great Moderation period. This divergence might be due to the different estimation strategy we employ. While Blanchard and Riggi (2013) adopt a limited-information approach that matches impulse responses to a commodity price shock in the DSGE model and in a structural VAR, we use a full-information Bayesian estimation with multiple shocks.

In summary, our estimation of the Blanchard-Gali model provides evidence that the Federal Reserve's monetary policy in the aftermath of the 2001 slump was responding mainly to core PCE and was sufficiently active to ensure equilibrium determinacy. These results line up with Bernanke's (2015) account.[22]

[21] As in Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010), we normalize the labor supply shock such that it enters the household's intratemporal optimality condition with a unit coefficient. This procedure improves the identification of the standard deviation of the labor supply disturbance and facilitates the convergence of the MCMC algorithm.

[22] Likewise we have estimated the model with CPI and core CPI data. Furthermore, we have also used real-time data on per-capita real GDP growth rate, PCE and core PCE inflation. Our results remain robust and are reported in the Appendix.

Table 6: Priors and posteriors for DSGE parameters.

Name	Priors	Posterior Mean [5th pct, 95th pct]					
		Three obs		Four obs		Five obs	
		CPI	CorePCE	PCE,CorePCE	CPI,CorePCE	PCE,CorePCE	CPI,CorePCE
ψ_π	G(1.1,0.5)	0.85 [0.63,0.98]	3.00 [2.01,4.14]	2.91 [1.94,4.03]	2.94 [1.99,4.03]	2.61 [1.57,3.86]	2.76 [1.69,4.03]
ψ_y	G(0.25,0.15)	0.22 [0.06,0.46]	0.28 [0.07,0.61]	0.30 [0.08,0.64]	0.30 [0.08,0.64]	0.11 [0.03,0.26]	0.07 [0.01,0.16]
ψ_{gy}	G(0.25,0.15)	0.47 [0.17,0.81]	0.28 [0.08,0.55]	0.29 [0.08,0.58]	0.30 [0.09,0.59]	0.62 [0.21,1.15]	0.69 [0.23,1.24]
ρ_R	B(0.7,0.1)	0.79 [0.70,0.86]	0.72 [0.61,0.81]	0.73 [0.62,0.82]	0.73 [0.63,0.82]	0.78 [0.66,0.88]	0.79 [0.70,0.87]
ω	B(0.5,0.2)	1	0	0.25 [0.08,0.47]	0.17 [0.06,0.32]	0.32 [0.10,0.59]	0.21 [0.06,0.41]
κ	G(0.5,0.1)	0.61 [0.45,0.80]	0.54 [0.39,0.72]	0.52 [0.38,0.70]	0.52 [0.37,0.69]	0.38 [0.25,0.53]	0.40 [0.25,0.57]
γ	B(0.5,0.2)	0.23 [0.07,0.46]	0.26 [0.07,0.50]	0.14 [0.04,0.28]	0.10 [0.03,0.19]	0.50 [0.30,0.68]	0.43 [0.24,0.60]
π^*	G(4,2)	2.92 [1.12,5.42]	1.96 [1.55,2.39]	1.99 [1.59,2.42]	2.02 [1.62,2.44]	1.95 [1.37,2.53]	1.99 [1.40,2.58]
r^*	G(2,1)	1.06 [0.43,1.85]	1.30 [0.72,1.99]	1.17 [0.64,1.75]	1.14 [0.60,1.75]	1.17 [0.59,1.84]	1.20 [0.59,1.87]
γ^*	N(0.5,0.1)	0.51 [0.39,0.64]	0.48 [0.38,0.60]	0.48 [0.37,0.59]	0.48 [0.38,0.59]	0.50 [0.37,0.64]	0.53 [0.39,0.66]
ρ_s	B(0.7,0.1)	0.70 [0.53,0.85]	0.70 [0.53,0.85]	0.88 [0.80,0.94]	0.90 [0.84,0.95]	0.88 [0.80,0.94]	0.91 [0.85,0.96]
ρ_d	B(0.7,0.1)	0.68 [0.52,0.81]	0.87 [0.79,0.93]	0.82 [0.72,0.91]	0.79 [0.68,0.89]	0.78 [0.66,0.88]	0.77 [0.64,0.87]
ρ_ν	B(0.7,0.1)	–	–	–	–	0.58 [0.39,0.81]	0.71 [0.50,0.90]
σ_z	IG(0.5,∞)	0.61 [0.46,0.80]	0.43 [0.34,0.55]	0.42 [0.33,0.54]	0.42 [0.33,0.54]	0.68 [0.52,0.89]	0.69 [0.53,0.89]
σ_R	IG(0.5,∞)	0.17 [0.12,0.24]	0.17 [0.12,0.23]	0.17 [0.12,0.24]	0.16 [0.12,0.23]	0.16 [0.11,0.24]	0.16 [0.11,0.22]
σ_s	IG(0.5,∞)	0.30 [0.15,0.59]	0.43 [0.16,1.00]	18.04 [14.1,22.9]	29.65 [23.2,37.9]	18.17 [14.08,23.36]	29.21 [22.77,37.40]
σ_d	IG(0.5,∞)	0.61 [0.26,1.08]	0.80 [0.53,1.21]	0.64 [0.42,0.99]	0.57 [0.39,0.84]	0.77 [0.48,1.17]	0.72 [0.45,1.07]
σ_ν	IG(0.5,∞)	–	–	–	–	0.62 [0.44,0.85]	0.80 [0.59,1.09]
σ_ζ	IG(0.5,∞)	0.20 [0.13,0.33]	–	–	–	–	–
$M_{z\zeta}$	N(0,1)	−0.36 [−0.63,−0.11]	–	–	–	–	–
$M_{R\zeta}$	N(0,1)	−0.17 [−1.12,0.90]	–	–	–	–	–
$M_{s\zeta}$	N(0,1)	0.01 [−0.71,0.76]	–	–	–	–	–
$M_{d\zeta}$	N(0,1)	−1.20 [−1.66,−0.87]	–	–	–	–	–

Notes: N stands for Normal, B Beta, G Gamma, and IG inverse gamma distribution. For each prior distribution, the parameters in parenthesis are the mean and standard deviation.

Table 7: Determinacy versus Indeterminacy

	Log-data density		Probability	
Inflation measure	Det.	Indet.	Det.	Indet.
Three obs (CPI)	-93.98	-88.06	0	1
Three obs (CorePCE)	-61.14	-67.33	1	0
Four obs (PCE, CorePCE)	-111.55	-123.16	1	0
Four obs (CPI, CorePCE)	-126.01	-138.31	1	0
Five obs (PCE, CorePCE)	-156.30	-161.86	1	0
Five obs (CPI, CorePCE)	-174.66	-181.61	1	0

Notes: The prior predictive probability of determinacy is 0.51.

7 Concluding remarks

Using the Taylor rule as a benchmark for evaluating the Federal Reserve's interest-rate setting decisions, some commentators have argued that monetary policy was too accommodative during the 2002-2005 period. Along these lines, this paper starts by estimating a basic New Keynesian model of the U.S. economy for the time following the 2001 slump. Our assessment of the Federal Reserve's performance varies with the measure of inflation that is put into the model estimation. When measuring inflation with CPI or PCE, we find some support for the view that monetary policy during these years was extra easy and led to equilibrium indeterminacy. Instead, if the estimation involves core PCE, monetary policy comes out as active and the evidence for indeterminacy dissipates. This divergence of results remains robust to several extensions. Our take is that each inflation series only provides an imperfect proxy for the model's concept of inflation. We re-formulate the artificial economy as a factor model where the theory's concept of inflation is the common factor to the alternative empirical inflation series. Again, extra easy monetary policy as well as indeterminacy cannot be ruled out. This finding, however, may hinge on the fact that the model features a single concept of inflation. Thus, we finally move to an economy that explicitly distinguishes between headline and core inflation. We find that the Federal Reserve was responding mainly to core PCE and was sufficiently active to comfortably rule out indeterminacy.

We chose to make these arguments while staying in relatively standard models.

This choice enables to establish a bridge from existing research to our study which we believe is important given the short sample period that we consider. We specifically did not add asset markets to the model or in the estimation. Thus, in terms of possible extensions, it would be worthwhile to introduce housing into the model and in the econometric analysis. It is our intention to pursue these lines of research in the near future.

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