Mechanics and Mechanical Engineering
Vol. 21, No. 2 (2017) 185–191


# A Methodology of Orthopaedic Measurement Arm Workspace Determination

Agnieszka Kobierska
Leszek Podsędkowski
Piotr Rakowski
Szymon Rejmonczyk
Jakub Kaszowski
Kacper Olejniczak
Bartłomiej Golasiński
Kamil Kaczmarek

*Institute of Machine Tools and Production Engineering*
*Lodz University of Technology*
*Stefanowskiego 1/15, 90-924 Łódź, Poland*
*agnieszka.kobierska@p.lodz.pl*
*leszek.podsedkowski@p.lodz.pl*
*piotr.rakowski@edu.p.lodz.pl*
*szymon.rejmonczyk@edu.p.lodz.pl*
*jakub.kaszowski@edu.p.lodz.pl*
*kacper.olejniczak@edu.p.lodz.pl*
*bartlomiej.golasinski@edu.p.lodz.pl*
*kamil.kaczmarek@edu.p.lodz.pl*

Received (20 October 2016)
Revised (26 November 2016)
Accepted (18 December 2016)

This paper outlines general assumptions of work principle of displacement measuring arm that bases on accelerometers. Symbolic representation of arm's kinematic structure is shown. Also, an analysis of construction correctness has been verified in terms of the effectiveness of using accelerometers to measure individual links displacements. A method of acquiring information on workspace is presented. The workspace has been determined in two variants: without considering last link orientation and for given orientation of that link. Paper also describes a determination method of increments values of angular displacements for individual links of the arm in terms of obtained proper results and rational computation time of workspace scanning algorithm. Finally, there is a presentation of method for determining a cuboid space for arm accuracy and repeatability measurement in XYZ coordinate system.

*Keywords*: arm kinematic, workspace, accelerometers.

## 1. Introduction

The described medical device purpose is to conveniently support the surgeon with the information on the displacements of operated limb. The information is based on the data acquired form accelerometers sensors.

Due to fact that operation field is really limited in most cases, an appropriate size of measurement arm elements are crucial. Kinematic chain of such arm must provide correctness of work and needs to be highly ergonomic. One needs to remember that such device has to be submitted to medical purposes. The construction of the arm must be sterilizable and prepared to work in an aggressive environment.

MEMS accelerometers, magnetometers and gyroscopes are commonly used in various industrial and technological fields. The examples of using accelerometers and gyroscopes for dynamic systems position measurement can be found in [2][3][4]. Yet, the measurements of displacements using only accelerometers [1][5] are still a novel concept. The presented paper expands this problem. It bases on 3D-printed arm equipped with 7 14-bit acceleration sensors. Their purpose is to determine angular displacements of individual joints in relation to the gravity vector. A methodology of identifying workspace basing on point cloud taking into account limitations imposed by selected sensors is presented in this paper.

A number of tests were performed in order to determine arm's workspace and to select an optimal cuboid space inside it to conduct tests on accuracy and repeatability of the arm.

## 2. Kinematics of measurement arm

The measurement of angle deviation of each link in relation to gravity field vector was used in the kinematic chain of the measuring arm . The purpose of this measurement is to define position of the last link in a coordinate system of the base link of the arm.

Because of the work principle of the gravity sensors used in the measurement arm, the rotation of the joints around a vertical axis should be avoided, both on the stage of arm's kinematic definition as well as during its work. In case of joint's rotation around the vertical axis parallel to gravity vector, the displacement will not be noted. To avoid the occurrence of vertical rotation axis, an appropriate kinematic construction of the arm was developed (Fig. 1).

The unique construction makes it very difficult to define the workspace for the measurement arm. As a solution, basing on theoretical numerical model of the arm, an algorithm was created in MATLAB environment (Tab. 1). It performs all the necessary calculations and provides the coordinates of calculated points. Depending on used parameters, the number of points may oscillate between a couple and several millions. The selection of parameters depends on the application of generated points. As a result of calculations, a group of points was obtained. This point cloud was later used to define the surfaces enclosing the region with the biggest density of points. Those surfaces helped to determine the best reference point in the workspace to perform arm's functionality tests. It provides an optimal range of arm's end movement, with a conservation of its orientation for the purpose of data gathering.

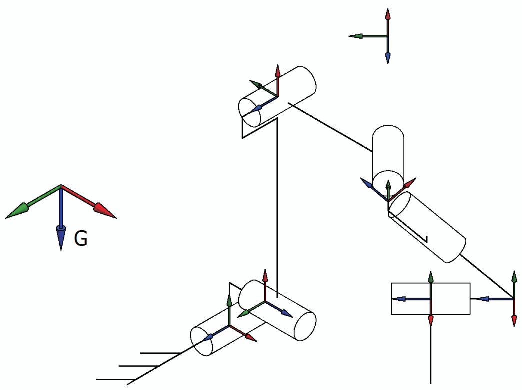


**Figure 1** Kinematics of measurement arm

**Table 1** Denavit–Hartenberg parameters of measurement arm

| Nr | Qi0 | $d_i$ | $a_i$ | $a_i$ |
|---|---|---|---|---|
| 1 | 90 | 0 | 10 | 90 |
| 2 | 0 | 13 | 50 | -90 |
| 3 | 0 | -30 | 0 | 45 |
| 4 | -45 | 53,926 | 0 | -90 |
| 5 | -90 | -34,127 | 0 | 45 |
| 6 | 0 | 16 | 0 | 0 |

## 3. The methodology of obtaining information on the workspace of an arm

One of the key elements in studies on measuring arm is the determination of its workspace. It will serve, inter alia, to identify measurement points for testing the accuracy and repeatability of the arm. Arm kinematic structure made it difficult to create a 3D model by conventional methods.

The first step to create a point cloud is to generate a set of points with coordinates X, Y, Z. This was achieved through the development of appropriate program in MATLAB. The first part of the program is to create a transformations matrices for individual members in accordance to Denawit-Hartenberg notation [6]. Next, nested loops are created corresponding to individual joints angular displacements, thus achieving all possible combinations of positions of the arm in space (Fig. 2). The inner loop is for calculating the position of the last link relative to the base. At this point, the program multiplies earlier created transformation matrices. Due to fact that these multiplications must be performed for each combination, it results in very long duration of the computations.

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & d_x \\ n_y & s_y & a_y & d_y \\ n_z & s_z & a_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The position of the last element, relative to the fixed base link of the kinematic chain is being obtained from the transformation matrix $H$, specifically from $H(4,1)$, $H(4,2)$ and $H(4,3)$ cells. The analysis of the obtained point cloud shows that the points resulting from the rotation of the last joints are placed in their immediate vicinity. Due to that fact they can be approximated to one point. Thanks to this operation it is possible to isolate the extreme points placed on the border of the workspace.

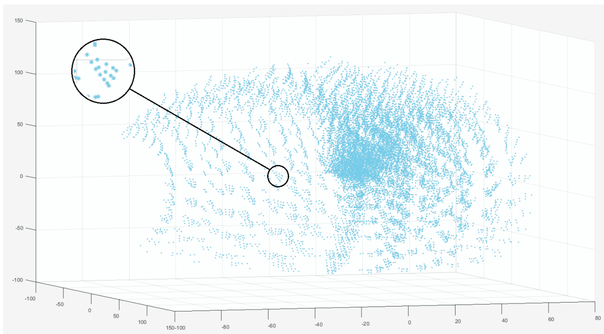

**Figure 2** The point cloud

The next step is a graphical representation of a set of obtained points transformed into closed area bounded by surfaces in 3D space (Figure 3). This step was made using Autodesk Inventor.
With Autodesk Inventor it was possible to eliminate points impossible to be reached because of the collisions of neighboring links.

To carry out the accuracy and repeatability measurement, it was necessary to create a workspace in which the last link will be parallel to the base one in all planes. This is due to the measuring station, on which the accuracy measurement will be performed. In order to achieve this, it was necessary to include a condition into the program that cells H (1, 1), H (2, 2) H (3, 3) of the matrix H (1) must fit the given range (near 1), dependent from the selected size of a single increment. Points that not meet this condition are not being saved in the generated file. This originates form the principle of homogeneous transformation, which says among other things that the mentioned cells describe the angular position of the vector connecting last link to the base [6].

The important issue is the proper selection of angular displacements increments of individual links in context of the obtained results and algorithm computation time. The problem is that making these increments too large result in workspace reproduction accuracy loss. At the same time, they cannot be too small as it increases the execution time of the program. The correct increment value was determined basing on the information on the time and accuracy of mapping of selected set of points for several small values. As a result, a generation time for small increment value has been estimated.

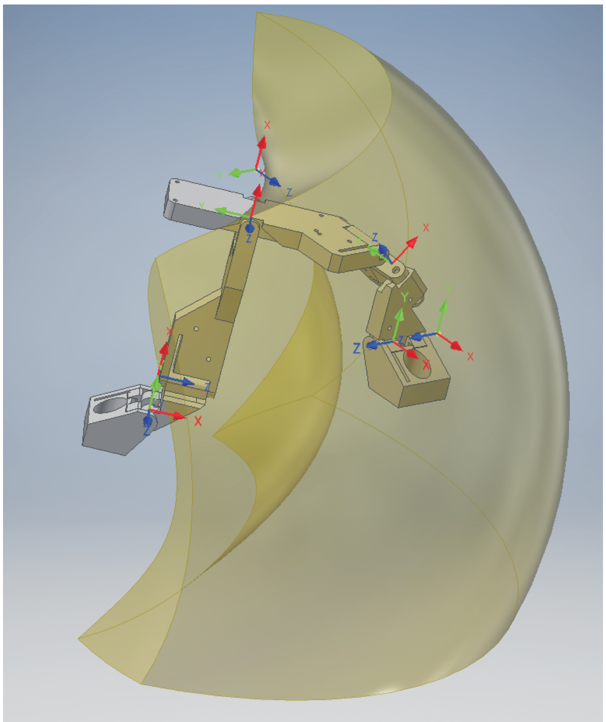

**Figure 3** Arm bearing surface model workspace

Basing on the workspace generated for kinematic chain with fixed orientation of last link, it was possible to determine the planes in which further accuracy tests will be performed. It was possible by placing the obtained points in 3D space. Three planes were needed (Fig. 3): XY, XZ and YZ where XYZ is the base coordination system. Planes cross-sections created lines which contained measurement points. The criteria of optimal plane selection was the local density intensification of obtained points and the arm motion range not causing orientation change of the last link. The planes were placed in the workspace parts where points density was the biggest. As a result, a cuboid workspace (Fig. 4) has been determined to be used in further analysis. The cuboid dimensions are 44x36x60 mm, with a center located in (70,18,15) point in the base coordinate system.

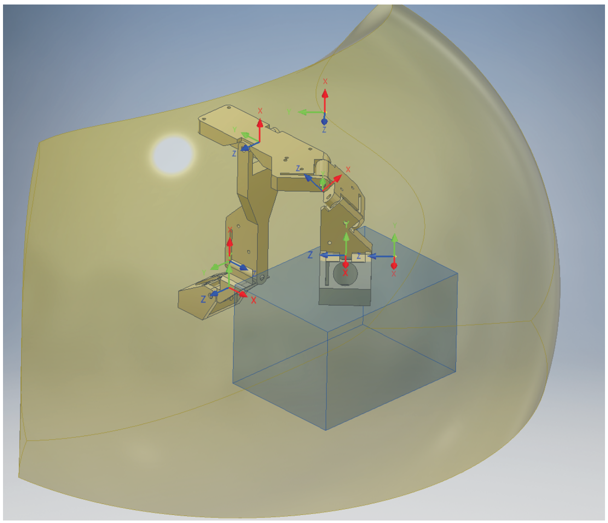

**Figure 4** Arm with determined cuboid workspace

## 4. Summary

A description of 6 DoF measurement arm kinematics has been created using analytical and graphical method. By using derived equations and theoretical dependencies an algorithm in MATLAB software has been created to determine arm's workspace.

The algorithm computation time has been optimized in terms of used increments values of each joint angular displacement while maintaining highest possible workspace mapping quality. The analysis provided a representative point cloud which was later reduced. It enabled simulation of last joint of the arm displacement. Collected data was used to determine available cuboid workspace in which accuracy and repeatability tests can be performed on the real arm [5]. During these tests none of the joints axis must be vertical. This paper presented a methodology of arm creation process and issues caused by kinematics limitations.

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