Modal Identification of Dynamic Properties
of the Cylindrical Grinder

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In the paper the method of model identification of the cylindrical grinder dynamic properties by means of experimental modal test was described. The method application, hardware solution as well as the procedure of carrying out the identification modal test of the cylindrical grinder was presented. The experiment was performed in order to acquire the frequency response function (FRF) of the cylindrical grinder. Having obtained the experimental FRF, the mathematical model of the response function was created. That mathematical model of the machine tool dynamic behavior can be applied in grinder and grinding holistic model. The conclusions regarding the application aspects of experimental modal analysis in order to identify dynamic properties of the machine tool were drawn.

Keywords: experimental modal analysis, grinding process, modal identification.

1. Introduction

Dynamic properties of machine tools include dynamics of the cutting or abrasive process as well as dynamic characteristics of the construction system of the machine tool including the machine elements, chuck, workpiece and cutting tool. The process of cutting performed during its working cycle is highly nonlinear and thus its dynamic properties investigation is complex and requires thorough analysis. That is the reason why new methods and research procedures have been developed i.e. experimental and operational modal analysis [1-3]. The operational modal analysis is time-consuming and requires performing analysis in machine working conditions what predestines that method rather for long-term diagnostics than dynamic model identification purposes [4].

The construction system of machine tool includes elements, the properties of which can be described by means of the mathematical equations of linear or quasi-linear characteristics. Nevertheless the construction complexity as well as numerous mutual influences of machine tool elements and units make mathematical description very laborious because the model is highly complex and detailed identification of the mathematical model parameters is required. The process of identification can be meticulously carried out by means of digital methods of signal acquiring, filtering, processing and curve fitting. Having identified every single parameter of the model one can obtain the mathematical form of the machine model which is suitable for further modelling process and simulation of the machine tool performance. The whole process of modelling carried out due to that procedure is very time–consuming and highly inefficient in industrial practice making this methodology suitable rather for purely scientific purposes.

Therefore another method of modelling seems to be more efficient and possible to apply without investigating every single unit characteristics. That method consists in masking the whole structure of the machine with a black box. The characteristics of that black box can be determined by means of identification modal test as far as the dynamics of the model is investigated. That approach benefits from direct experimental procedure which takes the whole machine structure into account and does not include any other simplifications required by detailed modelling of single units, mechanisms and their mutual interactions.

In order to create the holistic model of the grinder and grinding process it is crucial to identify the credible mathematical model of the dynamic behavior of the machine tool structure which is subsequently influenced by highly non-linear abrasive process of grinding. The reason of the research in this paper is to find out the effective way of the dynamic model creation. The identification modal test of the grinder structure was chosen as the method of determination its dynamic properties in the form suitable for further transformation to the mathematical formulae required in modelling.

2. Model of the cylindrical grinder

The object of research is the cylindrical grinder. The grinder is equipped with hydrostatic bearings of the grinding wheel spindle and hydrostatic slideways of the grinding wheel headstock and the workpiece table. The grinder is situated on the specially designed foundation which protects the machine from vibration sources from outside the machine tool. The dynamic model of the grinder was previously created by means of traditional method i.e. all parameters of the dynamic system (stiffness and damping) were derived experimentally or analytically one by one [5–7]. That approach is very time-consuming as far as the identification process in concerned. Additionally, all simplifications introduced into the model are of great influence on the simulation results since there is very poor share of experimentally derived parameters in the complicated structure of the whole model.

Therefore a new approach was applied which focused on the grinding process. All ambient processes and dynamic behavior of the machine tool structure affect the process of grinding directly or indirectly. The cylindrical plunge grinding process comprises several physical phenomena resulting in the shape modifications of ground

surface of the workpiece. Therefore modelling the depth of grinding which directly shapes the workpiece surface is of great significance [8]. During cylindrical plunge grinding the displacement of the grinding wheel headstock during in-feed should be fully transferred onto the workpiece shape in the form of reduction in the workpiece diameter. That ideal situation never happens due to the wheel wear process, contact phenomena and the machine tool structural deformation. All mentioned phenomena are time-dependent and meet the following Eq. (1):

$$u(t) = \delta_w(t) + \delta_s(t) + y_k(t) + y_m(t) \tag{1}$$

where:

  $u(t)$ – the feed displacement of the wheel headstock,
  $\delta_w(t)$ – the diameter variation of the workpiece due to cutting process,
  $\delta_s(t)$ – the diameter variation of the wheel due to its wear process,
  $y_k(t)$ – the deformation of the wheel and workpiece in the contact zone,
  $y_m(t)$ – the deformation of the machine tool structure.
For the modelling purposes the cylindrical plunge grinding process is graphically presented in Fig. 1.
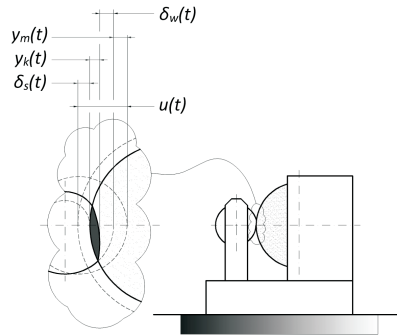

Figure 1 Illustration of the cylindrical plunge grinding process dependencies in the grinding zone

With this model it is possible to investigate the motion and dynamics of the most important elements in grinding i.e. the grinding wheel and the workpiece. The mutual position of the two elements directly determines shape of the workpiece surface. The dynamics of the grinding process comprises the self-induced vibrations which results in chatter observed on the surface of the workpiece and the wheel. That model includes components, which allow taking the regenerative self–induced vibration into the consideration. The diameter variation of the workpiece $\delta_w(t)$ and the diameter variation of the wheel due to its wear process $\delta_s(t)$ are the parameters which can be looped with the time delay in order to model the regeneration phenomena on the workpiece and the wheel.

  In that model one of the most important phenomenon is the machine tool structural deformation. It depends on the machine tool construction, its rigidity and

damping properties, characteristics of the joints between the fixed and movable units, applied materials properties etc. All those elements constitute the whole machine tool structural dynamic characteristics. That characteristics can be successfully derived by identification modal test of the whole machine tool structure. The experimental identification of the dynamic characteristics gives substantial results of the real machine tool. The mathematical form of that characteristics can be directly implemented into the simulation model of the analyzed grinder and grinding process.

3. Methodology

The modal identification test comprises deriving the dynamic properties of analyzed structure by means of experimentally obtained characteristics of input signal - excitation force and corresponding characteristics of vibration response signal of the system (acceleration, velocity or position). The modal identification test is subjected to the prerequisites of the experimental modal analysis. It returns information in the form which can be applied for further evaluation process in order to achieve the required information on the dynamic properties of the analyzed system. The information can be also applied in the model of the machine tool and the cutting process when the mathematical form of the obtained characteristics is properly defined. Additionally, the mathematical form of that definition must comply with the actual form of the whole model of analyzed system.

Modal analysis can be considered as transformation of a complex dynamic system into a set of single-degree-of-freedom systems [1, 2]. If the aim of research is to determine frequency of free vibrations of the system, modal analysis can be limited to simple identification modal test for deriving frequency response function (FRF) of the system. FRF is a transfer function and is expressed in frequency domain.
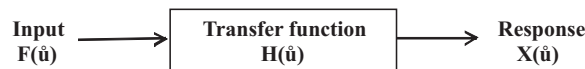

Figure 2 The linear mechanical system

For the linear system (Fig. 2) the following formula can be expressed – Eq. (2):

$$X\left(\omega\right) = H\left(\omega\right)F\left(\omega\right) \tag{2}$$

The transfer function $H(\omega)$ is the complex function which is formulated as follows – Eq. (3):

$$H\left(\omega\right) = \frac{X\left(\omega\right)}{F\left(\omega\right)} \tag{3}$$

The FRF reflects dynamic properties of the investigated system. It represents relation between the response of the system (e.g. position, velocity, acceleration) and input force for the certain pair of measurement points of response and excitation. The important feature of the frequency response function is the fact, that it describes dynamic properties of the investigated system regardless the type of

excitation signal (harmonic, impulse or random) during measurements. The FRF provides qualitative and quantitative evaluation of the dynamic system resonance of the machine tool [9].

The experimental modal analysis procedure can be performed by single input-single output (SISO) method, single input – multiple outputs (SIMO) or multiple inputs – multiple outputs (MIMO). These methods differ not only in the measurement equipment requirements but also in the required aim of the research and the accuracy of the analysis results.

SISO method is used mainly when only two–channel data acquisition measurement system is available and the resultant FRF can be limited to several point of measurement. It is usually applied when impact hammer test is performed (force input signal and acceleration output signal). The method can be applied as roving hammer (fixed accelerometer) or fixed hammer (roving accelerometer) procedure. In practice the roving hammer procedure is much more convenient because it does not require reinstalling the accelerometer from one point of measurement to another as it is in the fixed hammer procedure. The problems arise with normalizing input signal for every measurement point. Therefore averaging procedure must be introduced.

SIMO method enables application of the harmonic exciter as in input signal generator for simultaneous exciting many points of response measurement. It requires multi–channel data acquisition system and gives more accurate results than SISO method, because the same excitation signal is used for all analyzed response points while performing modal analysis.

MIMO is more hardware–requiring method but it returns much thorough information of the analyzed system than SISO or SIMO method. It is usually applied when the multi–channel data acquisition system is available. The selected input points can be excited with different force characteristics in order to enhance accuracy and efficiency of the modal analysis.

In the further analysis the SISO method was applied because the one general form of the FRF for the whole investigated structural system was requested.

4.   Identification modal test

The modal test was performed in order to obtain and evaluate the frequency response function (FRF) of the cylindrical grinder structure. The impulse excitation was exerted by means of a modal hammer. The response was measured with an accelerometer. The signals both from the hammer and the accelerometer were acquired and analyzed in the PULSE system. The PULSE LITE system [10] by Brüel&Kjær consists of the measurement module 3560L, the FFT analysis software, the accelerometer 4514 [11] and the modal hammer 8206-003 [12].

The points of input (excitation) and response (acceleration) measurement are indicated in Fig. 3. The impulse was exerted to the grinder wheel in the in–feed direction with the modal hammer *1*. The point was chosen on the wheel for it is the wheel surface where grinding force is acting during cutting. The accelerometer *2* was placed on the workpiece in the in-feed direction. The sensor was mounted on the workpiece with the bee wax, which is reported to as one of the best way of connecting the accelerometer to the measured surface [9].
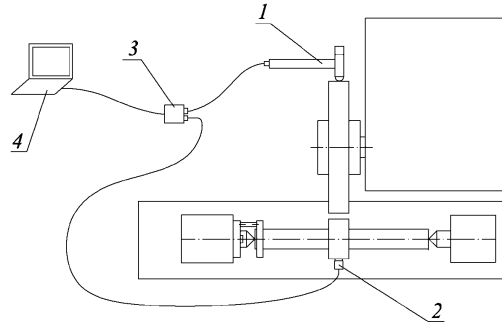
Figure 3 Diagram of the test stand of grinder's frequency response function: 1 – modal hammer, 2 – accelerometer, 3 – data acquisition system, 4 - computer

The accelerometer mounting point was chosen in the middle of the grinding width – the place of the intense acting of abrasive force. Measurement signals form the hammer and the accelerometer are connected to the data acquisition system *3* and the whole process is controlled by the external computer *4*. The modal test was performed without contact between the rotating grinding wheel and the workpiece since the frequency response function was meant to describe the dynamic interaction between the grinding wheel and the workpiece through the construction elements of the grinder. The process of grinding itself is modelled separately. It was not possible to perform the modal test of the machine during grinding process because of its high non-linearity. In the case of the modal test being carried out on the grinder the noise which can affect the measurement is mainly a noise in the measured output signal. The noise in the output signal may have its reason in external excitation caused by equipment working next to the grinder (hydraulic units) or externally induced vibrations from the foundations of the grinder.

In the case of noise in the output signal the H_1 estimator of FRF (Uhl 1997) is applied the value of which approaches the real FRF with the growing number of measurement repetitions. The H_1 estimator minimizes the output noise effect.

5.   Result of experiment

The experimentally derived H_1 estimator of FRF versus frequency ω is presented in Fig. 4.

In the diagram shown in Fig. 4 one dominant maximum is clearly visible the frequency of which is 272 rad/s. That indicates the dominant modal frequency of the examined system. The diagram of H_1 also shows local maximum the frequency of 155 rad/s. It is the frequency of rotating wheel (1480 rpm). It is a result of slight unbalance of the grinding wheel.

During measurement with 5 repetitions the coherence function was also derived. In the measured range of frequency the value of the coherence was not less than 0.93. It shows that the results were containing a slight noise of little significance.
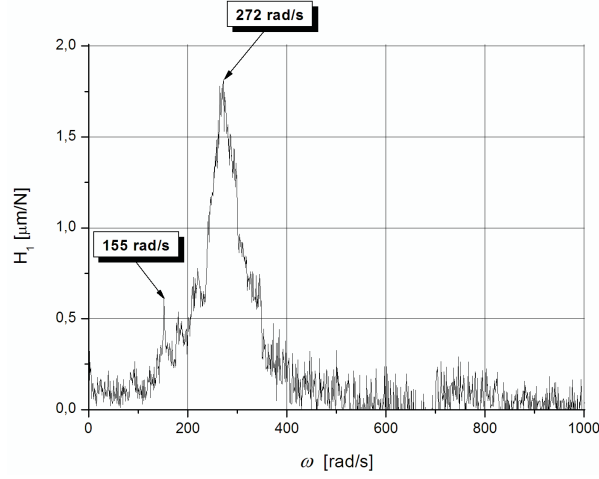
Figure 4 H_1 estimator of transfer function of the grinder

6. Deriving the mathematical form of FRF

In order to derive the mathematical form of the frequency transfer function the H_1–versus–frequency results were exported in the text form for further analysis. Subsequently, the data were imported into MATLAB system.

When the response function includes information on multiple modes of free vibration, its mathematical form can be described [9] as follows - Eq. (4):

$$FRF(j\omega) = \sum_{i=1}^{n} \frac{K_i}{T_{wi}^2 \ (j\omega)^2 + 2 \ T_{wi} \ \zeta_i \ (j\omega) + 1} \tag{4}$$

where:

n - number of modes,
K_i - gain factor for i^{th} mode,
T_{wi} - time constant for i^{th} mode,
ζ_i - dimensionless coefficient of damping for i^{th} mode.

Having analyzed the H_1–versus–frequency data (Fig. 2) the conclusion was drawn that the real system dynamic behavior could be described by means of mathematical formula for a system of one degree of freedom (DOF) with subcritical damping. Therefore the following function has been proposed - Eq. (5):

$$FRF\,(j\omega) = \frac{K}{T_w^2 \ (j\omega)^2 + 2 \ T_w\zeta\,(j\omega) + 1} \tag{5}$$

In the case of the machine tool as large as the cylindrical grinder, the approximation of the whole structure with the model of one DOF is a significant simplification. However considering the shape of the H_1 versus frequency graph, that approximation can be justified as a step in simplification the mathematical model. What is more, the model will be utilized to perform simulations of dynamic behavior of the

grinder structure in the most important place (contact of the grinding wheel and the workpiece) subjected to time-varying loads of the grinding force.

The parameters of the model of FRF were determined by means of MATLAB software. The Curve Fitting Toolbox procedures were applied in order to evaluate parameter values. The robust least square regression fitting method was applied.

The final form of the mathematical model is as follows - Eq. (6):

$$FRF\,(j\omega) = \frac{5,2 \cdot 10^{-8}}{1,3 \cdot 10^{-5}\,(j\omega)^2 + 0,007\,(j\omega) + 1} \tag{6}$$

In Fig. 5 the results obtained from the mathematical model of FRF are presented with a thick line whereas the real structure H_1 estimator is shown with a thin line. The juxtaposition enables comparison of the two graphs and evaluation of the similarity of the real system and its mathematical model. As it is visible there are some differences in the values as well as in the characteristics of the graphs, but high qualitative similarity of the real and modelled functions is undeniable. The correlation coefficient of the two graphs is 0,94.
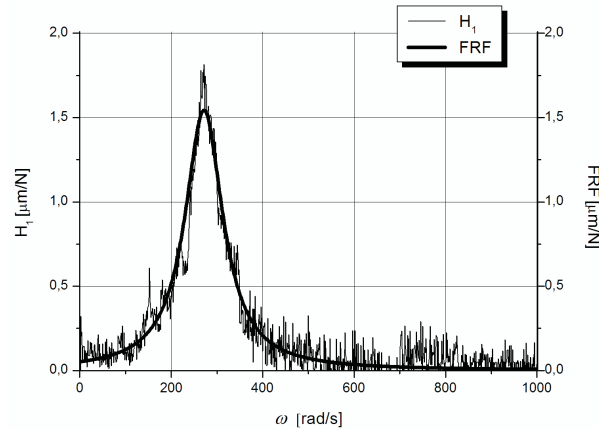

Figure 5 H_1 estimator of transfer function of the grinder and the mathematical model of the FRF

7. Conclusions

Considering high qualitative similarity of two graphs presented in Fig. 5 the decision was made to introduce the mathematical model of the FRF into the holistic model of the cylindrical grinding as a part which is responsible for the dynamic behavior of the grinder structure [8]. The model of the plunge grinding was created in order to perform simulations of vibration–assisted plunge grinding and the process parameters influence on the ground surface quality [13]. The same procedure of model parameters identification can be applied in the models of transverse grinding [14], centerless grinding, milling [15, 16] or machining with chatter [17–19].

In the case of performing time-consuming experimental research which requires long time of machine tool left idle it is crucial to find an effective and quick way of obtaining the dynamic characteristics of the machine. The procedure which is based on experimental modal test and deriving the frequency response function enables not only estimation the modal frequencies of free vibration of the analyzed system. It is also possible to obtain the mathematical form of the FRF after the approximation of the real system frequency response function (or its estimator) with the mathematical formula of properly fitted parameters. That simplifies the procedure of modelling and it is possible to apply it in industrial practice. The additional asset of the research procedure based on experimental modal test is analysis of the real object in its natural surroundings and working conditions.

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