# Constraining spacetime nonmetricity with neutron spin rotation in liquid ${ }^{4} \mathrm{He}$ 

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#### Abstract

General spacetime nonmetricity coupled to neutrons is studied. In this context, it is shown that certain nonmetricity components can generate a rotation of the neutron's spin. Available data on this effect obtained from slow-neutron propagation in liquid helium are used to constrain isotropic nonmetricity components at the level of $10^{-22} \mathrm{GeV}$. These results represent the first limit on the nonmetricity $\zeta_{2}^{(6)} S_{000}$ parameter as well as the first measurement of nonmetricity inside matter.


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## 1. Introduction

The idea that spacetime geometry represents a dynamical physical entity has been remarkably successful in the description of classical gravitational phenomena. For example, General Relativity, which is based on Riemannian geometry, has recently passed a further experimental test: the theory predicts gravitational waves, and these have indeed been observed by the LIGO Scientific Collaboration and the Virgo Collaboration [1].

At the same time, a number of observational as well as theoretical issues motivate the construction and study of alternative gravity theories. Most of these efforts recognize the elegance and success of a geometric underpinning for gravitational phenomena and therefore retain this feature in model building. One popular approach in this context, known as metric-affine gravity [2], employs an underlying geometry more general than that of a Riemannian manifold. The basic idea behind this approach can be summarized as relaxing the metric-compatibility condition $D_{\alpha} g_{\beta \gamma}=0$ and the symmetry condition on the connection coefficients $\Gamma^{\alpha}{ }_{\beta \gamma}-\Gamma^{\alpha}{ }_{\gamma \beta}=0$. In general, this idea introduces two tensor fields
$N_{\alpha \beta \gamma} \equiv-D_{\alpha} g_{\beta \gamma}, \quad T_{\beta \gamma}^{\alpha} \equiv \Gamma_{\beta \gamma}^{\alpha}-\Gamma^{\alpha}{ }_{\gamma \beta}$,
relative to the Riemannian case known as nonmetricity and torsion, respectively.

[^0]The specialized situation in which the nonmetricity tensor vanishes $N_{\alpha \beta \gamma}=0$ and only torsion is nonzero represents the widely known Einstein-Cartan theory [3]. In that context, torsion has been the subject of various investigations during the last four decades [4]. Considering the question of the presence of torsion in nature as an experimental one has spawned numerous phenomenological studies of torsion [5-15] yielding bounds on various torsion couplings.

An analogous phenomenological investigation of nonmetricity has been instigated last year [16]. Paralleling the torsion case, that analysis treats the question regarding the presence of nonmetricity as an experimental one, and the nonmetricity field $N_{\alpha \beta \gamma}$ is taken as a large-scale background extending across the solar system. The particular physical situation considered in Ref. [16] lends itself to an effective-field-theory description in which $N_{\alpha \beta \gamma}$ represents a prescribed external field selecting preferred spacetime directions. Thus, such a set-up embodies in essence a Lorentz-violating scenario amenable to theoretical treatment via the Standard-Model Extension (SME) framework [17]. For example, sidereal and annual variations of physical observables resulting from the motion of an Earth-based laboratory through this solar-system nonmetricity background represent a class of characteristic experimental signals in that context [18].

The present work employs a similar idea to obtain further, complementary constraints on nonmetricity. The specific set-up we have in mind consists of liquid ${ }^{4} \mathrm{He}$ as the nonmetricity source. Polarized neutrons generated at the slow-neutron beamline at the National Institute of Standards and Technology (NIST) Center for

Neutron Research traverse the helium and serve as the nonmetricity probe. It is apparent that our set-up involves an Earth-based nonmetricity probe. Thus, the key difference between our study and that in Ref. [16] is that we examine the situation of nonmetricity sourced locally in a terrestrial laboratory by the ${ }^{4} \mathrm{He}$. This implies that the presumed nonmetricity in our case is comoving with the laboratory, and thus the neutron probe, which precludes certain experimental signatures, such as sidereal and annual variations. Instead, we utilize the prediction presented below that certain components of $N_{\alpha \beta \gamma}$ lead to neutron spin rotation in this system.

The outline of this paper is as follows. Section 2 reviews the basic ideas behind the effective-field-theory description of a background $N_{\alpha \beta \gamma}$ in flat Minkowski space and derives the resulting spin motion for nonrelativistic neutrons. This effect provides the basis for our limits on nonmetricity. The details of the measurement of neutron spin rotation in liquid ${ }^{4} \mathrm{He}$ including the experimental set-up are discussed in Sec. 3. A brief summary is contained in Sec. 4. Throughout, we adopt natural units $c=\hbar=1$. Our conventions for the metric signature and the Levi-Civita symbol are $\eta^{\mu \nu}=\operatorname{diag}(+,-,-,-)$ and $\epsilon^{0123}=+1$, respectively.

## 2. Theory

Our analysis is based on the approach to nonmetricity couplings taken in Ref. [16], so we begin with a brief review of that approach. The basic idea is to follow the usual reasoning that the construction of an effective Lagrangian should include all terms compatible with the symmetries of the model. In the present context, possible couplings between the background nonmetricity $N_{\alpha \beta \gamma}$ and the polarized-neutron probe need to be classified. Since we are interested in a low-energy experiment, we may disregard the neutron's internal structure and model it as a point Dirac fermion with free Lagrangian $\mathcal{L}_{0}=\frac{1}{2} \bar{\psi} \gamma^{\mu} i \overleftrightarrow{\partial_{\mu}} \psi-m \bar{\psi} \psi$, where $m$ denotes the neutron mass. Conventional gravitational effects are negligible, so that the flat-spacetime Minkowski limit $g^{\mu \nu} \rightarrow \eta^{\mu \nu}$ suffices for our present purposes.

The next step is to enumerate possible couplings of $\psi$ to the background nonmetricity $N_{\alpha \beta \gamma}$. This yields a hierarchy of possible Lagrangian terms $\mathcal{L}_{N}^{(n)}$ labeled by the mass dimension $n$ of the corresponding field operator:
$\mathcal{L}_{N}=\mathcal{L}_{0}+\mathcal{L}_{N}^{(4)}+\mathcal{L}_{N}^{(5)}+\mathcal{L}_{N}^{(6)}+\ldots$.
For the experimental set-up we have in mind, nonmetricity couplings affecting the propagation of neutrons are the most relevant ones. Moreover, $N_{\alpha \beta \gamma}$ must be small on observational grounds. We therefore focus on contributions to $\mathcal{L}_{N}^{(n)}$ that are quadratic in $\psi$ and linear in $N_{\alpha \beta \gamma}$. General arguments in effective field theory suggest that Lagrangian terms of lower mass dimension $n$ may be more dominant. Capturing the leading effects of all nonmetricity components then requires inclusion of Lagrangian terms up to mass dimension $n=6$ [16].

The construction of the explicit form of each individual contribution $\mathcal{L}_{N}^{(n)}$ is most easily achieved by decomposing $N_{\alpha \beta \gamma}$ into its Lorentz-irreducible pieces. These are given by two vectors $\left(N_{1}\right)_{\mu}$ and $\left(N_{2}\right)_{\mu}$, a totally symmetric rank-three tensor $S_{\mu \alpha \beta}$, and a rankthree tensor $M_{\mu \alpha \beta}$ with mixed symmetry [16]:

$$
\begin{aligned}
\left(N_{1}\right)_{\mu} \equiv & -\eta^{\alpha \beta} N_{\mu \alpha \beta} \\
\left(N_{2}\right)_{\mu} \equiv & -\eta^{\alpha \beta} N_{\alpha \mu \beta} \\
S_{\mu \alpha \beta} \equiv & \frac{1}{3}\left[N_{\mu \alpha \beta}+N_{\alpha \beta \mu}+N_{\beta \mu \alpha}\right] \\
& +\frac{1}{18}\left[\left(N_{1}\right)_{\mu} \eta_{\alpha \beta}+\left(N_{1}\right)_{\alpha} \eta_{\beta \mu}+\left(N_{1}\right)_{\beta} \eta_{\mu \alpha}\right]
\end{aligned}
$$

$$
\begin{align*}
& +\frac{1}{9}\left[\left(N_{2}\right)_{\mu} \eta_{\alpha \beta}+\left(N_{2}\right)_{\alpha} \eta_{\beta \mu}+\left(N_{2}\right)_{\beta} \eta_{\mu \alpha}\right] \\
M_{\mu \alpha \beta} \equiv & \frac{1}{3}\left[2 N_{\mu \alpha \beta}-N_{\alpha \beta \mu}-N_{\beta \mu \alpha}\right] \\
& +\frac{1}{9}\left[2\left(N_{1}\right)_{\mu} \eta_{\alpha \beta}-\left(N_{1}\right)_{\alpha} \eta_{\beta \mu}-\left(N_{1}\right)_{\beta} \eta_{\alpha \mu}\right] \\
& -\frac{1}{9}\left[2\left(N_{2}\right)_{\mu} \eta_{\alpha \beta}-\left(N_{2}\right)_{\alpha} \eta_{\beta \mu}-\left(N_{2}\right)_{\beta} \eta_{\alpha \mu}\right] . \tag{3}
\end{align*}
$$

With these pieces, the nonmetricity tensor can be reconstructed as follows [16]:

$$
\begin{align*}
N_{\mu \alpha \beta}= & \frac{1}{18}\left[-5\left(N_{1}\right)_{\mu} \eta_{\alpha \beta}+\left(N_{1}\right)_{\alpha} \eta_{\beta \mu}+\left(N_{1}\right)_{\beta} \eta_{\mu \alpha}\right. \\
& \left.+2\left(N_{2}\right)_{\mu} \eta_{\alpha \beta}-4\left(N_{2}\right)_{\alpha} \eta_{\beta \mu}-4\left(N_{2}\right)_{\beta} \eta_{\mu \alpha}\right] \\
& +S_{\mu \alpha \beta}+M_{\mu \alpha \beta} . \tag{4}
\end{align*}
$$

The sign changes in Eqs. (3), (4), and some subsequent equations relative to the corresponding equations in Ref. [16] arise due to differing conventions for the metric signature and for the sign of the Levi-Civita symbol. We also remark that although Eqs. (3) and (4) employ a notation similar to that for the irreducible components of torsion $T^{\alpha}{ }_{\beta \gamma}$ [14], the nonmetricity and torsion pieces are unrelated.

With this decomposition, the following Lagrangian contributions can be constructed [16]:

$$
\begin{align*}
& \mathcal{L}_{N}^{(4)}=\zeta_{1}^{(4)}\left(N_{1}\right)_{\mu} \bar{\psi} \gamma^{\mu} \psi+\zeta_{2}^{(4)}\left(N_{1}\right)_{\mu} \bar{\psi} \gamma_{5} \gamma^{\mu} \psi \\
& +\zeta_{3}^{(4)}\left(N_{2}\right)_{\mu} \bar{\psi} \gamma^{\mu} \psi+\zeta_{4}^{(4)}\left(N_{2}\right)_{\mu} \bar{\psi} \gamma_{5} \gamma^{\mu} \psi, \\
& \mathcal{L}_{N}^{(5)}=-\frac{1}{2} i \zeta_{1}^{(5)}\left(N_{1}\right)^{\mu} \bar{\psi} \stackrel{\leftrightarrow}{\partial_{\mu}} \psi-\frac{1}{2} \zeta_{2}^{(5)}\left(N_{1}\right)^{\mu} \bar{\psi} \gamma_{5} \overleftrightarrow{\partial_{\mu}} \psi \\
& -\frac{1}{2} i \zeta_{3}^{(5)}\left(N_{2}\right)^{\mu} \bar{\psi} \overleftrightarrow{\partial_{\mu}} \psi-\frac{1}{2} \zeta_{4}^{(5)}\left(N_{2}\right)^{\mu} \bar{\psi} \gamma_{5} \overleftrightarrow{\partial_{\mu}} \psi \\
& -\frac{1}{4} i \zeta_{5}^{(5)} M_{\mu \nu}{ }^{\rho} \bar{\psi} \sigma^{\mu \nu}{ }_{\partial_{\rho}} \psi \\
& +\frac{1}{8} i \zeta_{6}^{(5)} \epsilon_{\kappa \lambda \mu \nu} M^{\kappa \lambda \rho} \bar{\psi} \sigma^{\mu \nu} \overleftrightarrow{\partial}_{\rho} \psi \\
& +\frac{1}{2} i \zeta_{7}^{(5)}\left(N_{1}\right)_{\mu} \bar{\psi} \sigma^{\mu \nu} \stackrel{\leftrightarrow}{\partial_{\nu}} \psi+\frac{1}{2} i \zeta_{8}^{(5)}\left(N_{2}\right)_{\mu} \bar{\psi} \sigma^{\mu \nu} \overleftrightarrow{\partial}_{\nu} \psi \\
& -\frac{1}{4} i \zeta_{9}^{(5)} \epsilon^{\lambda \mu \nu \rho}\left(N_{1}\right)_{\lambda} \bar{\psi} \sigma_{\mu \nu} \overleftrightarrow{\partial_{\rho}} \psi \\
& -\frac{1}{4} i \zeta_{10}^{(5)} \epsilon^{\lambda \mu \nu \rho}\left(N_{2}\right)_{\lambda} \bar{\psi} \sigma_{\mu \nu} \overleftrightarrow{\partial_{\rho}} \psi, \\
& \mathcal{L}_{N}^{(6)} \supset-\frac{1}{4} \zeta_{1}^{(6)} S_{\lambda}{ }^{\mu \nu} \bar{\psi} \gamma^{\lambda} \partial_{\mu} \partial_{\nu} \psi+\text { h.c. } \\
& -\frac{1}{4} \zeta_{2}^{(6)} S_{\lambda}{ }^{\mu \nu} \bar{\psi} \gamma_{5} \gamma^{\lambda} \partial_{\mu} \partial_{\nu} \psi+\text { h.c. } \tag{5}
\end{align*}
$$

Here, the real-valued couplings $\zeta_{1}^{(n)}$ are taken as free parameters; they can in principle be fixed by specifying a definite underlying nonmetricity model. For the mass-dimension six term $\mathcal{L}_{N}^{(6)}$, we have only listed those contributions that contain the $S_{\mu \alpha \beta}$ irreducible piece; all other components of $N_{\alpha \beta \gamma}$ are already present in the terms $\mathcal{L}_{N}^{(4)}$ or $\mathcal{L}_{N}^{(5)}$ of lower mass dimension.

Equations (2), (3), and (5) determine the low-energy neutron effective Lagrangian in the presence of general background nonmetricity relevant for the experimental situation we have in mind. We note, however, that the terms (5) would generally be viewed as part of a more complete Lagrangian $\mathcal{L} \supset \mathcal{L}_{N}$ that also treats $N_{\alpha \beta \gamma}$ as a dynamical variable. The nonmetricity field equations then contain $\partial \mathcal{L} / \partial N_{\alpha \beta \gamma}$, and thus neutron source terms. This idea provides the justification for taking the ${ }^{4} \mathrm{He}$ nucleus as a nonmetricity source in the experimental set-up discussed below. The protons and electrons of the ${ }^{4} \mathrm{He}$ atom may produce additional nonmetricity contributions if these particles exhibit nonmetricity couplings analogous to those in Eq. (5). In what follows, we make no assumptions regarding the dynamics of $N_{\alpha \beta \gamma}$ or additional nonmetricity-matter couplings; we simply presume that the ${ }^{4} \mathrm{He}$ generates some nonzero nonmetricity.

A model refinement can be achieved by focusing on the leading contribution to $N_{\alpha \beta \gamma}$. Note that $N_{\alpha \beta \gamma}=N_{\alpha \beta \gamma}(x)$ must exhibit
a nontrivial spacetime dependence determined by the interatomic distance and the velocity of the ${ }^{4} \mathrm{He}$ atoms. However, the random nature of these two quantities suggests that the leading nonmetricity effects are actually governed by the spacetime average $\left\langle N_{\alpha \beta \gamma}(x)\right\rangle$. For this reason, we may take $N_{\alpha \beta \gamma}=$ const. in what follows. The nonmetricity contributions (5) then form a subset of the flat-space SME Lagrangian, a fact that permits us to employ the full repertoire of theoretical tools developed for the SME framework.

One such SME result relevant for the present situation concerns the observability of constant background fields [17,19]. For example, it is known that contributions associated with the couplings $\zeta_{1}^{(4)}, \zeta_{3}^{(4)}, \zeta_{1}^{(5)}, \zeta_{2}^{(5)}, \zeta_{3}^{(5)}, \zeta_{4}^{(5)}, \zeta_{7}^{(5)}$, and $\zeta_{8}^{(5)}$ can be removed from the Lagrangian-at least at linear order-via judiciously chosen field redefinitions. We may therefore disregard these terms in what follows. Their measurement would require situations involving nonconstant $N_{\alpha \beta \gamma}$, the presence of gravity, or the consideration of higher-order effects.

An additional simplification arises from the isotropy of the liquid helium. The ${ }^{4} \mathrm{He}$ ground state has spin zero, so anisotropies would have to be tied to excited states of ${ }^{4} \mathrm{He}$ or arrangements of the helium atoms involving preferred directions. However, the absence of polarization and the aforementioned random nature of both position and velocity of individual ${ }^{4} \mathrm{He}$ particles precludes sizeable, large-scale anisotropies. The leading background nonmetricity contributions generated by the liquid-helium bath can therefore also be taken as isotropic in the helium's center-of-mass frame. It follows that the present experimental set-up is only sensitive to the rotationally invariant pieces of $N_{\alpha \beta \gamma}$.

To uncover the isotropic content of $N_{\alpha \beta \gamma}$, we may proceed by inspecting its irreducible pieces (3). Clearly, components without spatial indices are rotation symmetric: $\left(N_{1}\right)_{0},\left(N_{2}\right)_{0}$, and $S_{000}$. Note that $M_{\alpha \beta \gamma}$ obeys the cyclic property
$M_{\alpha \beta \gamma}+M_{\beta \gamma \alpha}+M_{\gamma \alpha \beta}=0$,
which implies $M_{000}=0$. Further isotropic components in $S$ and $M$ with spatial indices must have spatial-index structure $\delta_{j k}$ or $\epsilon_{j k l}$, where Latin indices run from 1 to 3 . Since both $S$ and $M$ are symmetric in their last two indices, they cannot contain pieces of $\epsilon_{j k l}$. This only leaves contributions involving $\delta_{j k}$. But these do not yield independent isotropic contributions because both $S$ and $M$ are traceless. To see this, consider as an example a piece of the form $S_{0 j k}=s \delta_{j k}$, where $s$ is the isotropy parameter in question. But $S$ is traceless, so that we have $0=S_{0 \alpha \beta} \eta^{\alpha \beta}=S_{000}-S_{0 j k} \delta_{j k}=$ $S_{000}-s \delta_{j k} \delta_{j k}$. It follows that $3 s=S_{000}$ does not represent an additional independent isotropic contribution to $S$. An analogous reasoning applies to $M$, so that $\left(N_{1}\right)_{0},\left(N_{2}\right)_{0}$, and $S_{000}$ are indeed the only isotropic nonmetricity components.

The model determined by Eqs. (2) and (5) permits a fully relativistic description of all dominant nonmetricity effects on the propagation of both neutrons and antineutrons in the present context. Since our current goal is an analysis of the spin motion of slow neutrons, we may disregard all antineutron physics, and focus entirely on the $2 \times 2$ nonrelativistic neutron Hamiltonian $h=h_{0}+\delta h+\delta h_{s}$ resulting from our model Lagrangian (5). Here, $h_{0}$ is the ordinary nonrelativistic piece. The spin-independent nonmetricity contribution $\delta h$ is irrelevant for this work. The spindependent correction $\delta h_{s}$ resulting from Eq. (5) can be gleaned from previously established SME studies [20]. The result for both isotropic as well as anisotropic contribution reads

$$
\begin{aligned}
\delta h_{s}= & {\left[\left(\zeta_{2}^{(4)}-m \zeta_{9}^{(5)}\right)\left(N_{1}\right)_{j}+\left(\zeta_{4}^{(4)}-m \zeta_{10}^{(5)}\right)\left(N_{2}\right)_{j}\right] \sigma^{j} } \\
& +\left[\left(\zeta_{2}^{(4)}-m \zeta_{9}^{(5)}\right)\left(N_{1}\right)_{0}+\left(\zeta_{4}^{(4)}-m \zeta_{10}^{(5)}\right)\left(N_{2}\right)_{0}\right] \frac{\vec{p} \cdot \vec{\sigma}}{m}
\end{aligned}
$$

$$
\begin{align*}
& +\frac{1}{2}\left[\zeta_{5}^{(5)} \tilde{M}_{j \alpha \beta}+\frac{3}{2} \zeta_{6}^{(5)} M_{j \alpha \beta}+m \zeta_{2}^{(6)} S_{j \alpha \beta}\right] \frac{p^{\alpha} p^{\beta} \sigma^{j}}{m} \\
& +\frac{1}{2} \zeta_{2}^{(6)} S_{0 \alpha \beta} \frac{p^{\alpha} p^{\beta} \vec{p} \cdot \vec{\sigma}}{m} \tag{7}
\end{align*}
$$

This expression contains the leading contribution in the nonrelativistic order $|\vec{p}| / m$ for each nonmetricity component. In the above equation, we have set $\tilde{M}_{\alpha \beta \gamma} \equiv \epsilon_{\alpha \beta}{ }^{\mu \nu} M_{\mu \nu \gamma}$. Moreover, $p^{\mu}=$ $\left(p^{0}, \vec{p}\right)=\left(p^{0}, p^{j}\right)$ denotes the neutron's 4-momentum, and $\sigma^{j}$ are the usual Pauli matrices. Note that nonmetricity effects corresponding to $\zeta_{1}^{(6)}$ only produce spin-independent effects. They are therefore absent from $\delta h_{s}$ and cannot be determined by observations of neutron spin rotation.

## 3. Experimental analysis

To extract experimental signatures resulting from the nonmetricity correction (7), we analyze the aforementioned experimental situation, namely spin motion of a neutron as it passes through liquid ${ }^{4} \mathrm{He}$. As argued above, our Lagrangian (5) implies that neutrons, and hence ${ }^{4} \mathrm{He}$ nuclei, can generate nonmetricity. The injected neutron beam would then be affected by this nonmetricity background. Moreover, our "in-matter" approach permits us to search for short-ranged or non-propagating nonmetricity. In particular, this encompasses situations analogous to minimally coupled torsion, where the torsion tensor vanishes outside the spin-density source [4]. Such an approach rests on the premise that the probe penetrates the matter and that the effects of conventional Standard-Model (SM) physics are minimized. The ${ }^{4} \mathrm{He}-$ neutron system appears to be ideal in this respect for two reasons. First, the neutron mean free path inside liquid ${ }^{4} \mathrm{He}$ is relatively long allowing for the accumulation of the predicted spin-rotation effect. This is due to the small elastic and the essentially vanishing inelastic cross sections as well as rapidly decreasing neutronphonon scattering as $T \rightarrow 0$. Second, contamination of the nonmetricity spin rotation by ordinary SM physics can be excluded on the grounds that these conventional effects lie below the current detection sensitivity. This latter fact is explained in more detail below.

The rotation of the spin of a transversely polarized slowneutron beam is called neutron optical activity. It is quantified by the rotary power $d \phi_{P V} / d L$ defined as the rotation angle $\phi_{P V}$ of the neutron spin about the neutron momentum $\vec{p}$ per traversed distance $L$. The nonmetricity correction (7) leads to the following expression for the rotary power:

$$
\begin{align*}
\frac{d \phi_{P V}}{d L}= & 2\left(\zeta_{2}^{(4)}-m \zeta_{9}^{(5)}\right)\left(N_{1}\right)_{0}+2\left(\zeta_{4}^{(4)}-m \zeta_{10}^{(5)}\right)\left(N_{2}\right)_{0} \\
& +m^{2} \zeta_{2}^{(6)} S_{000} \tag{8}
\end{align*}
$$

where we have implemented the isotropic limit. The neutron rotary power is amenable to high-precision experimental studies and can therefore be employed to measure or constrain the combination of nonmetricity components appearing on the right-hand side of Eq. (8).

The experiment described in detail below measured the neutron rotary power to be
$\frac{d \phi_{P V}}{d L}=+1.7 \pm 9.1$ (stat.) $\pm 1.4($ sys $) \times 10^{-7} \mathrm{rad} / \mathrm{m}$
at the $1-\sigma$ level. Conversion to natural units together with Eq. (8) yields the following nonmetricity measurement:

$$
\begin{align*}
& 2\left(\zeta_{2}^{(4)}-m \zeta_{9}^{(5)}\right)\left(N_{1}\right)_{0}+2\left(\zeta_{4}^{(4)}-m \zeta_{10}^{(5)}\right)\left(N_{2}\right)_{0}+m^{2} \zeta_{2}^{(6)} S_{000} \\
& \quad=(3.4 \pm 18.2) \times 10^{-23} \mathrm{GeV} \tag{10}
\end{align*}
$$

We interpret this result as the $2-\sigma$ constraint

$$
\begin{align*}
& \left|2\left(\zeta_{2}^{(4)}-m \zeta_{9}^{(5)}\right)\left(N_{1}\right)_{0}+2\left(\zeta_{4}^{(4)}-m \zeta_{10}^{(5)}\right)\left(N_{2}\right)_{0}+m^{2} \zeta_{2}^{(6)} S_{000}\right| \\
& \quad<3.6 \times 10^{-22} \mathrm{GeV} \tag{11}
\end{align*}
$$

Disregarding the possibility of extremely fine-tuned cancellations between the various nonmetricity couplings in the constraint (11), we can estimate the following individual bounds:

$$
\begin{align*}
\left|\zeta_{2}^{(4)}\left(N_{1}\right)_{0}\right|<10^{-22} \mathrm{GeV}, & \left|\zeta_{4}^{(4)}\left(N_{2}\right)_{0}\right|<10^{-22} \mathrm{GeV}, \\
\left|\zeta_{9}^{(5)}\left(N_{1}\right)_{0}\right|<10^{-22}, & \left|\zeta_{10}^{(5)}\left(N_{2}\right)_{0}\right|<10^{-22}, \\
\left|\zeta_{2}^{(6)} S_{000}\right|<10^{-22} \mathrm{GeV}^{-1} & \tag{12}
\end{align*}
$$

The above limits represent the primary result of this work. To our knowledge, they provide the first measurement of $\zeta_{2}^{(6)} S_{000}$ as well as the first measurement of any nonmetricity component inside matter.

The measurement (9) performed at the NG-6 slow-neutron beamline at NIST's Center for Neutron Research has already appeared in the literature [21]. Neutrons with transverse spin polarization traversed 1 meter of liquid ${ }^{4} \mathrm{He}$ that was kept at a temperature of 4 K in a magnetically shielded cryogenic target. The neutron beam's energy distribution was well approximated by a Maxwellian with a maximum close to 3 meV . Paralleling the usual light-optics set-up of a crossed polarizer-analyzer pair, the experiment searched for a nonzero rotation in the neutrons' polarization. Further details of this measurement can be found in Refs. [22-27]. The result quoted in the above Eq. (9) represents the upper limit on the parity-odd neutron-spin rotation angle per unit length in liquid helium at 4 K extracted from the measured data.

The usual SM incorporates known parity-violating physics that can also lead to neutron spin rotation, for instance via interactions with electrons or nucleons. In fact, this phenomenon has been measured in heavy nuclei [28-30]. A convincing interpretation of the above nonmetricity constraint therefore requires a discussion of this SM background. From a theoretical perspective, parity violation in neutron-electron physics in the SM is well understood. In particular, it is suppressed relative to the parity-odd neutronnucleon interaction by the weak charge $\left(1-4 \sin ^{2} \theta_{W}\right) \approx 0.1$. The neutron-nucleon parity violation, on the other hand, is induced by quark-quark weak interactions. This system also involves the strong-coupling limit of QCD, which still evades solid theoretical tractability. Nevertheless, nucleon-nucleon weak-interaction amplitudes have been argued to be six to seven orders of magnitude below strong-interaction amplitudes at neutron energies relevant for our present purposes [31]. Although reliant on phenomenological input in the form of nuclear parity-violation data folded into a specific model, the value $d \phi_{P V} / d L=-6.5 \pm 2.2 \times 10^{-7} \mathrm{rad} / \mathrm{m}$ for the SM spin rotation in the ${ }^{4} \mathrm{He}$-neutron system is regarded as the most decent theoretical estimate [32]. Our experimental upper limit on nonmetricity (11) is larger than this SM-background estimate. For this reason, we disregard the remote possibility of a cancellation between SM and nonmetricity contributions to neutron spin rotation.

To determine additional limits on in-matter nonmetricity, one could also consider using data from other high-precision parityviolation experiments. One example in the context of neutrons are measurements of parity-breaking effects in atoms that are affected by the nuclear anapole moment and arise from parity-odd interactions between nucleons [33,34]. An idea for extracting nonmetricity constraints involving electrons could, for example, be based on the consistency between the theoretical SM result and the experimental value of the weak charge of the ${ }^{133} \mathrm{Cs}$ atom [35].

Additional nonmetricity components may become experimentally accessible with a set-up in which both the slow-neutron beam as well as the nuclear target are polarized: the aligned target spins would coherently generate large-scale anisotropic components of $N_{\alpha \beta \gamma}$, which were disregarded in our above analysis. High-sensitivity studies of this type have received considerable attention for quite some time [36]. The neutron-nucleus scattering amplitude exhibits a significant polarization dependence, an effect known as nuclear pseudomagnetic precession [37]: the neutron's spin precesses about the nuclear polarization vector as the neutron traverses the polarized medium. In the past, this method has been employed to determine the spin dependence of neutron-nucleus scattering cross sections for a number of nuclei [38]. However, the nuclear-pseudomagnetism spin-precession contributions from the strong neutron-nucleus interaction to such a measurement are substantial and currently evade theoretical treatment from first principles. It is therefore expected that the experimental reach regarding in-matter anisotropic $N_{\alpha \beta \gamma}$ components would be more modest than that in this study.

We finally mention that a high-precision transmission-asymmetry measurement utilizing transversely polarized 5.9 MeV neutrons was performed in a nuclear spin-aligned target of holmium [39]. This experiment explored the presence of P -invariant but T violating interactions of the neutron. The measurement yielded $A_{5}=\frac{\sigma_{P}}{\sigma_{0}}=+8.6 \pm 7.7$ (stat. + sys.) $\times 10^{-6}$. Here, $A_{5}$ denotes the transmission asymmetry for neutrons polarized parallel and antiparallel to the normal of the plane spanned by the neutron momentum and the spin polarization of the holmium target. An open question is whether or not polarized nuclear matter generates an effective $N_{\alpha \beta \gamma}$ that differs from that of unpolarized nuclear matter, and how such a difference would manifest itself in this experiment. That said, the neutron energy in this measurement remains nonrelativistic, so our above methodology should continue to be applicable.

## 4. Summary

In this work, we have considered the possibility of nontrivial nonmetricity in nature. We have argued that in this context an effective nonmetricity field could be generated inside a liquid ${ }^{4} \mathrm{He}$ target. We have shown that the spin of nonrelativistic neutrons traversing such a target would then precess. This prediction, together with existing data on neutron spin rotation in liquid ${ }^{4} \mathrm{He}$, implies the primary result of this work, namely the bound (11). To our knowledge, this is the first experimental limit on in-matter nonmetricity.

We have further concluded that it would be difficult to improve our bound via higher-precision spin-rotation data due to the conventional SM background arising from quark-quark weak interactions. However, other atomic and nuclear parity-violation tests may have the potential to yield complementary limits on nonmetricity interactions of neutrons and electrons. Moreover, polarized slowneutron transmission measurements through polarized nuclear targets could be studied with the approach presented in this work and may give bounds on additional in-matter $N_{\alpha \beta \gamma}$ components. We encourage other researchers to perform further nonmetricity searches using the general framework employed in this study with the aim to turn nonmetricity tests into a more quantitative experimental science.

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