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Sensitivity Analysis of Monopiles' Fatigue Stresses to Site Conditions Using Monte Carlo Simulation

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ABSTRACT

The design of support structures of offshore wind turbines contains high number of design variables that influence load characteristics and structural responses. These variables are stochastic and cause many uncertainties. Some of them are examined in this study. It is investigated how scattering of site conditions and load parameters affect the structural response. It is exemplified in terms of stresses that contribute to the accumulated fatigue damage within a monopile substructure. Random sampling of combinations of site conditions and load parameters is performed in order to classify the effects of parameter scattering on the stress variability by means of Sobol' indices. Analysis shows that the highest influence on stress outputs have the variations in the load parameters. The reason is the sensitivity of the structural dynamical response to the wave height increase and decrease of distance between the wave peak frequency and the structural eigenfrequencies.

KEY WORDS: Offshore wind turbines, monopile structure, wave load, Monte Carlo simulation, sensitivity analysis, Sobol' indices.

INTRODUCTION

In the last decade, the offshore wind energy industry expanded greatly in order to meet requirements of the world's growing demand for energy and environmental-friendly solutions for energy resources. Despite of complicated and expensive transport and installation, it results with a higher electricity output compared to onshore wind turbines, due to the higher wind speeds and the lower turbulence level in offshore conditions. Wide available locations for potential sites are another reason that justifies the decision to go offshore (Wind Europe, 2016).

Steel monopiles are support structures mostly employed for offshore wind turbines, due to their simple design and relatively uncomplicated installation in shallow and medium waters. As offshore wind turbines (OWTs) are exposed to cyclic aerodynamic, hydrodynamic and mechanical loading, they are especially prone to fatigue damage. Beside the aerodynamic loads, the substructure is mostly influenced by wave-induced loads, which are the focus of this study. The design of

OWT support structures contains high number of stochastic variables in the modelling of structural properties and load characteristics. These variables cause many uncertainties which are nowadays in the design process mostly treated through semi-probabilistic approach by means of partial safety factors. However, this procedure does not give a clear insight into the importance of each input variance for the variability of the output. Therefore, in this study it is examined and compared how high are the influences of site condition input parameters' variances to the variability of extreme stresses in the structure. First, the numerical model of the structure is developed in finite element analysis tool Posedion. Then, 1000 numerical simulations of irregular sea state with random combinations of varying parameters are performed. For each of the simulations, extreme stresses in the cross section of interest are captured. Sets of obtained stresses are finally post-processed by means of sensitivity analysis. The share of each scattered input parameter in the final result variation is calculated by means of Sobol' indices.

METHODS

Finite Element Model of the Monopile Structure

Several different support structures can be employed for OWTs. A monopile support is considered here, as it currently represents the most common substructure application for OWTs, due to its simple design and easier installation compared to the alternatives. The monopile structure consists of a cylindrical steel tube with changing cross sections by mean of diameters and wall thicknesses. At the soil level, it has a diameter of 8 m. The structure is partially stuck into the seabed with an embedment length of 35 m. The soil is modelled as a set of nonlinear springs whose stiffness is obtained by means of p-y method (API, 2000). The monopile is designed for water depth of 40 m, where it has a cone part, which reduces the cross sectional diameter from 8 m to 6 m. It continues 18 m above the still water level, where it is connected with the wind turbine tower through transition piece.

The finite element model of the structure is built based on a realistic reference design (Dubois et al., 2014), using the finite element analysis tool Poseidon, developed at the Institute for Steel Construction of the

Leibniz Universität Hannover (Böker, 2010). The numerical 3D and beam models of the monopile structure with mudline and still water level are shown in Figs. 1~2.

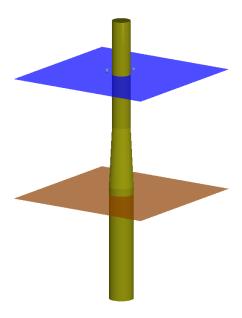


Fig. 1. Numerical 3D model of the monopile structure

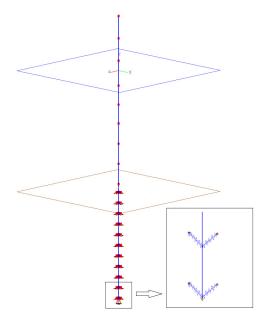


Fig. 2. Numerical beam model of monopile structure with detail of springs

Wave Load Modelling

Ocean waves are irregular and random in shape, height, length and speed of propagation. In the literature (IEC 61400-3, 2008; DNV-RP-C205, 2007; Germanischer Lloyd, 2012; Böker, 2010 and other authors), many methods are proposed to describe and model the wave load conditions for structural design purposes. For quasi-static response of structures, it is sufficient to use deterministic regular waves characterized by wave length and corresponding wave period and wave height, as no sophisticated dynamic analysis is conducted. However, slender structures with significant dynamic response, such as OWTs, require stochastic modelling of the sea surface and its kinematics by time series, by means of sea states. A sea state is specified by a waveenergy spectrum with a given significant wave height, a representative wave peak period, a mean propagation direction and a spreading function. In this work, wind-induced loads are not calculated and the uni-directional sea state is considered. In application, a sea state is usually assumed a random process that is stationary over a certain period of time. Depending on the conditions and purpose of the analysis, period of sea state stationarity can range from 30 minutes to 10 hours (IEC 61400-3, 2008).

The characteristics of a stationary sea state can be modelled by means of wave energy spectra. Wave spectra can be given in table form, as measured spectra, or by a parameterized analytic formula. The most appropriate wave spectrum depends on the geographical area with local bathymetry and the severity of the sea state. Models of wave spectra formulations that depend on characterizing parameters of a sea state, the significant wave height and the zero-up-crossing period, are used. These parameters are explained according to IEC 61400–3 (2008) and DNV-RP-C205 (2007):

Significant wave height, H_s

It is defined as the mean value of the 1/3 biggest wave heights recorded in the observed time series. This value is usually shown in scatter diagrams obtained from site measurements.

Mean zero-up-crossing period, Tz

It is a mean period of all successive up-crossings of the zero-water level of the water surface within the time series. A wave energy spectra usually depend on the wave peak period T_p , which is the period that contains the greatest amount of wave energy spectrum, and this value is most commonly given in scatter diagrams obtained from site measurements. The relation of T_z and T_p depends on the shape of the spectrum and can only be established in an approximate manner. IEC 61400-3 (2008) recommends the following relation:

$$T_p = T_z \sqrt{\frac{11 + \gamma}{5 + \gamma}} \tag{1}$$

where γ is spectrum shape parameter that depends on the chosen energy spectrum. The most frequently applied spectrums for describing the wind seas are Pierson-Moskowitz (PM-) spectrum and JONSWAP spectrum. The PM-spectrum was originally proposed for fully developed sea. The JONSWAP spectrum, which extends PM to include fetch limited seas, is used for modelling of sea state in this paper. It is based on the PM-spectrum, extended by the shape parameter γ :

$$S_{JS}(\omega) = nf \cdot S_{PM}(\omega) \cdot \gamma^{\exp\left(-\frac{1}{2}\left(\frac{\omega - \omega_P}{\sigma^* \omega_P}\right)^2\right)}$$
 (2)

Here nf stands for normalizing function given by:

$$nf = 1 - 0.287 \ln(\gamma)$$
 (3)

 $S_{PM}(\omega)$ is PM-spectrum defined by:

$$S_{PM}(\omega) = \frac{5}{16} H_s^2 \omega_P^4 \omega^{-5} \exp\left(-\frac{5}{4} \left(\frac{\omega}{\omega_P}\right)^{-4}\right) \tag{4}$$

 σ is bandwidth parameter, whose value is set in accordance to DNV-RP-205.

 γ is shape parameter. For γ = 1, the JONSWAP spectrum is equal to PM-spectrum. Here, the value of γ is set to 3.3, which is the value recommended for the location North Sea (DNV-RP-205).

The JONSWAP spectrum is expected to be a reasonable model for:

$$3.6 < \frac{T_p}{\sqrt{H_s}} < 5 \tag{5}$$

where T_p is in seconds and H_s in meters, and should be used with caution outside this interval.

Finally, the wave-induced loads are calculated by means of the Morison's equation (IEC 61400–3, 2008; DNV-RP-C205, 2001; Böker, 2010). The requirement for applicability of Morison's equation is the hydrodynamic transparency of the structure, i.e. the structure influences the wave flow only locally, without obstructing the free flow on a global scale. It is assumed that this is the case if the following condition is fulfilled:

$$D \leq \frac{\lambda}{5}$$

where D is the structure's diameter, and λ is wave length. Morison's equation, as following, is used:

$$f = 0.5 \cdot C_d \cdot \rho_{water} \cdot D \cdot u_{\perp} \cdot |u_{\perp}| + C_m \cdot \rho_{water} \cdot A \cdot u_{\perp}$$
 (6)

where

f is force per unit length of the member C_d is hydrodynamic drag coefficient

C_m is hydrodynamic inertia coefficient

 $ho_{\it water}$ is water density

D is diameter of the member in the respected section

A is cross section area of the member

 u_{\perp} is velocity of the flow normal to the member surface and

 u_{\perp} is acceleration of the flow normal to the member surface.

Hydrodynamic coefficients are adopted according to Germanischer Lloyd (2012).

Uncertainties Within the Modelling Process

Site conditions from data scatter diagrams are given as a result of many interpolations between site measurements and adopting average values of measurements over a certain time span, which introduces many uncertainties. Uncertainties in fatigue loads, according to Sørensen and Toft (2010), can be classified into aleatory and epistemic. Aleatory uncertainties are unavoidable due to the randomness in the nature of the

load processes. Epistemic uncertainties are knowledge based and can be reduced by gathering more information with more relevance. Sources of epistemic uncertainties can be:

- -Data uncertainty (due to measurement imperfections, for instance soil or mean sea level data)
- -Statistical uncertainty (due to estimations and averaged data, for instance wave peak periods and significant wave height from scatter diagrams)
- -Model uncertainty (due to the simplifications of real physical phenomena in the numerical model).

High number of stochastic variables also occur with the assumptions in design process of OWT support structures. This is another source of uncertainties, which affect the structural response. It is investigated which uncertain input parameters contribute the most to the uncertainties in extreme stresses that contribute to the accumulated fatigue damage to a high extent (Kelma and Schaumann, 2015). Therefore, the better insight into the importance of varying parameters, as well as possible need for reduction of input uncertainty is obtained. Influence of varying stochastic parameters to final stresses in the structure are investigated by means of global sensitivity analysis, namely through computing of Sobol' sensitivity indices (as explained in the section Sensitivity Analysis).

The investigated varying site conditions and load parameters are:

- -Significant wave height, H_s
- -Wave peak period, Tp
- -Inner friction angle of the soil, ϕ
- -Specific gravity of the soil, γ
- -Undrained shear strength of the soil, cu
- -Mean sea level (MSL)

 H_{s} and T_{p} are load parameters whose variation is given through scatter diagrams obtained by measurements on offshore sites. For the North Sea, parameter variations are taken from "EU UpWind" project (SES6 No 019945 UPWIND) named "UpWind Design Basis" by Fischer et al. (2010). For purposes of this study, wave parameters from scatter diagrams for K13 Deep Water Site are used. These diagrams show that the wind speed with the highest probability of occurrence is 9-10 m/s, so this wind speed is adopted for generating an irregular sea state.

Soil is a complex engineering material containing various geologic, environmental and physical-chemical processes. Therefore, all soil properties vary and cause many uncertainties. The sources of uncertainties can be categorized into inherent variability, measurement errors and transformation uncertainty. As a stochastic random field, soil variability can be described by mean value and coefficient of variation (Phoon and Kulhawy, 1999). In this case, soil is modelled according to API (2000), as a set of nonlinear springs whose stiffness is obtained by means of load/deflection method (p-y curves) in two normal directions, as shown in Fig. 2. Variability of relevant soil parameters has been investigated in the past by many researchers, but for purpose of this paper, coefficients of variations (COV) of soil parameters are used as recommended by Phoon and Kulhawy (1999) and shown in Table 1.

Table 1. Mean values and coefficients of variation of soil parameters

Soil parameter	Mean value	COV [%]
Inner friction angle, \$\phi\$	35 °	10
Specific gravity, γ	17.5 kN/m ³	10
Undrained shear strength, c _u	100 kN/m ²	35

In this work, the soil is considered to be a homogeneous material. The above-mentioned soil parameters are constant within a single simulation, but they are changing in every further simulation by sampling from predefined distribution functions.

Still water level is also scattering due to the sea tides. Compared to soil properties that contain high level of uncertainty due to the numerous difficulties in measurement, water depth and mean sea level are known with higher accuracy. Based on the wind farm data and previous research on sensitivity by Corbetta et al. (2014), it is taken that the standard deviation from the mean sea level of 40 m is 3.5 m.

Sampling of Varying Parameters

In a full probabilistic analysis, it is necessary to perform a large number of numerical simulations with varying parameters. For this purpose, the finite element code Poseidon is linked to an advanced software for stochastic analysis, namely OpenCOSSAN, which allows a more sophisticated uncertainty quantification and management (Patelli et al., 2014).

First, a qualitative analysis of the effect of the uncertainties is performed by using Monte Carlo Simulations to generate sample inputs in the predefined range (Archer et al., 1997). Then, the Sobol' indices are computed; these global sensitivity indices are based on the decomposition of the variance of the output to its individual input contribution. In this case, the correlation between the variation of significant wave height and wave peak period, shown by the available data history, is taken into account. However, given the complex dependency shown in the data, it was necessary to use a multivariate mixture distribution model (McLachlan and Peel, 2000), where a correlated bivariate Gaussian distributions are centred around each data bin mid-point of the bi-dimensional histogram, with a weight proportional to the height of the bin. Additionally, the mixture distribution is constrained for the upper and lower boundaries of the parameter values. Figs. 3~4. show the comparison between the discrete data scatter diagram and 1000 sets of H_s and T_p sampled in OpenCOSSAN. The Figures show a good agreement of the real data with the empirical distribution.

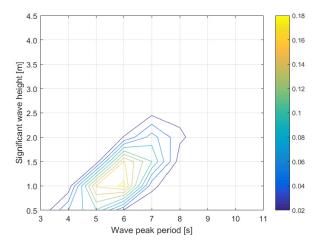


Fig. 3. Scatter plot of wave parameters $H_{\text{\tiny S}}$ and $T_{\text{\tiny P}}$ for wind speed of 9-11m/s (Fischer et al., 2010)

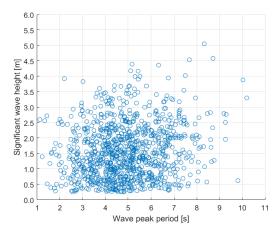


Fig. 4. Samples from Gaussian Mixture distribution model

Sensitvity Analysis

Global sensitivity analysis is one of the most relevant methods for risk analysis purposes. Variance-based sensitivity analysis is a form of global sensitivity analysis, that works within a probabilistic framework, often combined with numerical sampling methods i.e. Monte Carlo simulation or Latin Hypercube. The main principle, as described by Saltelli et al. (2008), is a decomposition of the output variance of the model or system into fractions. Next, each of this fractions of the output variance is attributed to the inputs or sets of inputs. These fractions are directly interpreted as measures of sensitivity. One of the variancebased sensitivity analysis method are Sobol' indices. It assignes uncertain input parameters to random variables, and examines the response of interest. Due to the uncertain input, there is uncertainty contained in the output as well. Sobol' indices give the insight of which fraction of the variance in output (response) can be attributed to each of the input random variables. Here, a direct variance-based measure of sensitivity, Si, is stated as in Eq. 6. It is called "first-order sensitivity index" or "main-effect index" and represents the contribution to the output variance of the main effect θ_i . Therefore, it measures the effect of θ_i alone, but averaged over variations of other input parameters. It is standardized by the total variance to provide a fractional contribution. For analytically tractable functions, Sobol' indices can be calculated analitically. However, in the majority of cases, they are estimated by means of some numerical method. It is also possible to form higherorder sensitivity indices as well as total-effect indices which include all variance caused by interactions between the input variables (Saltelli et al., 2008; Archer et al., 1997). In this paper, only the first-order sensitivity index (S_i) is used to describe fraction of the variance in response of interest (extreme stress), that can be attributed to each examined scattering input parameter alone.

$$S_{i} = \frac{V_{\theta i} \left[E_{\theta - i} \left[y \mid \theta_{i} \right] \right]}{V_{\theta} \left[y \right]} \tag{7}$$

where:

 θ_i are single uncertain inputs,

y is response of interest,

E[.] is expected value,

V[.] is variance.

In this work, 1000 random combinations of six varying parameters are sampled by means of Monte Carlo Simulation (MCS), and in each simulation extreme stresses are captured. This leads to large sets of outputs which are afterwards postprocessed using OpenCOSSAN, in the computing environment adjusted to the specific sampling needs of this research, explained in the previous section. Sobol' indices are calculated in order to obtain the level of importance for each examined varying parameter for the variance of extreme stresses.

RESULTS

Numerical simulations with random combinations of site conditions and wave load parameters are performed. Each of them simulates irregular sea states with a duration of 1800 seconds and a time step for stress capturing of 0.1 second, generated using the JONSWAP spectrum. For each time step, stresses for monopile's cross section at mudline level in the direction of wave attack are calculated. The reason is that the stress results have shown that it is the direction where the extreme stresses always occur. Example of a part of the stress recordings from the sensor at monopile's mudline in the direction of wave propagation for one simulation is shown in Fig. 5.

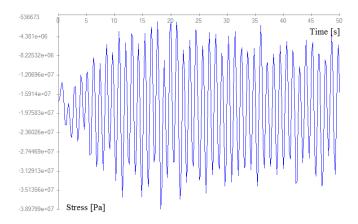


Fig. 5. Example of stress recording at mudline for the first 50 seconds of aleatory simulation with duration of 1800 s and the time step of 0.1 s

As the wave load is cyclic, also the stress values oscillate around its equilibrium position. Therefore, for each simulation there are two single outputs of interest: minimum and maximum stress in the direction of wave propagation. Measure of output variance which can be attributed to the input parameter's variance is reflected by means of Sobol' indices, which are computed after 1000 simulations. Figs. 6~7 show sensitivity measures of minimum and maximum stress results to the six varying parameters. The sensitivities are normalized with respect to relative changes of each parameter with respect to others, to offer the insight into the influence of single parameters to the global output sensitivity.

It can be seen that variation of the soil parameters has low affection to the output variation for both output cases – minimum and maximum stresses, although it changes the structural system characteristics. This is explained by the fact that a range of possible oscillations of these parameters is either very narrow (inner friction angle, specific gravity) as investigated by Phoon and Kulhawy (1999), or it does not affect the result variation heavily. In contrast, scattering of sea state parameters

and the mean sea level variation have a greater effect on the variation of stress results, as they influence the wave loads and the sea state model, and have a wider range of variance. Considering stresses, Fig. 5. shows that the cross section in the direction of interest is always in the area of negative stress. Therefore, the real extreme stress in this case is minimum stress. The highest variance in minimum stress output is attributed to the variability of the significant wave height (Fig. 6). Although the range of significant wave height variation is wide, lower probabilities for extreme values of significant wave height are taken into consideration within sampling. Therefore, the variance of extreme stresses is, by all means, in high fraction attributed to the variance of the significant wave height. Another indicator that minimum stresses are hardly affected by the water depth is the high impact of mean sea level to the result variation (Fig. 6).

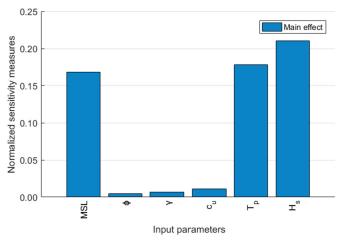


Fig. 6. Normalized minimum stress Sobol' sensitivity indices for six varying parameters

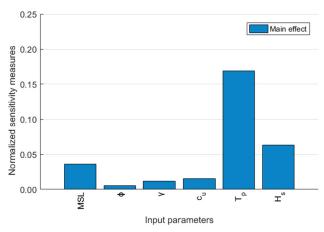


Fig. 7. Normalized maximum stress Sobol' sensitivity indices for six varying parameters

Considering the maximum stresses, they are result of measurements between two wave attacks, when the structure is heading to its equilibrium position after the wave attack. As the wave period dictates the oscillations, it is reasonable that the maximum stresses are highly affected by both mean zero crossing and wave peak period (which are correlated as given in Eq. 1), as shown in Fig. 7. If the distance between the wave peak frequency and the natural frequency of the system is decreasing, it leads to the higher dynamic amplifications, which raises stresses. This especially relates to decreasing the wave peak period, as it can lead to exciting the structure in one of its lower eigenmodes.

As the Sobol' sensitivity indices are estimated by means of the Monte Carlo Simulations numerical method, the accuracy of the estimators is dependent on the number of the samples N. The value of N can be chosen by sequentially adding point and calculationg the indices until the estimated values reach some acceptable convergence.

For the estimation of the Sobol' indices, 1000 random combinations of varying parameters are sampled, which leads to a high computational expense. As here only the first-order sensitivity indices are considered, it was possible to reduce the computational costs while keeping the satisfying accuracy by using the Random Balance Design method, for which a reference is made to Saltelli et al., 2008.

CONCLUSIONS

The traditional semi-probabilistic approach within the design of support structures of offshore wind turbines uses partial safety factors in order to cover unknown uncertainties within the design process. The gap in this approach is the absence of ability to determine the sources of the uncertainties and their contribution to the final result of interest. This research is conducted in order to better perceive importance of single parameters, whose values are scattering due to the stochastic nature of the processes, namely sea states and soil variability.

Numerical simulations, namely Monte Carlo Simulations, were used to sample random combinations of scattering parameters, and extreme stresses were calculated for each of them. Sobol' indices were used to decompose the variance of the extreme stresses to the single contributions of each varying parameter. Results have shown that the extreme stresses are to the highest extent influenced by wave load parameters, namely significant wave height and wave peak period. The influence of the variability of the water depth is significant as well. Sensitivity indices show that the scattering of examined soil parameters affects the stresses to a lower extent, compared to the others. The reason may be the narrow range of possible scattering, or simply the lower contribution of these parameters to the stresses. In contrast, the wave load parameters have a wider range of predefined variance. Furthermore, significant wave height affects the stress directly, while the wave peak period dictates the wave load oscillations, and can have a high contribution to stress in case that it becomes close to some of the lower eigenfrequencies of the system.

Sensitivity results are strongly influenced by the selection of the reference case (wind speed, offshore site location) as well as the selected parameter ranges. Therefore, in order to generalize the results, other cases, conditions and starting assumptions must be considered. Investigation of extreme stresses makes a good indicator for the further research with the complete fatigue damage of time series as an output. Because of lower computational costs, the variance of extreme stresses can be examined as a pre-design for the investigation of the variance of fatigue damage.

Next steps to be done are considering the wind-induced loads on the wind turbine and conducting the coupled analysis of wind and wave loads. Coupled load simulations with finite element software Poseidon and aeroelastic solver Flex5 are to be done in order to obtain the coupled eigenfrequencies and stiffness matrixes of the whole system

(substructure and wind turbine NREL 5 MW). Different wind speeds and wind-wave directionality (mis)alignment are to be included. Overall, including the wind loads on one hand, and calculating an actual fatigue damage on the other, will lead to the more realistic and accurate interpretation of results.

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