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A Study of the Retention of Five Selected Mathematical Concepts of Seventh Graders in the Coulee Dam Public School System as Based on the Inductive and Deductive Methods of Teaching

James Prehm
Central Washington University

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A STUDY OF THE RETENTION OF FIVE SELECTED MATHEMATICAL CONCEPTS OF SEVENTH GRADERS IN THE COULEE DAM PUBLIC SCHOOL SYSTEM AS BASED ON THE INDUCTIVE AND DEDUCTIVE METHODS OF TEACHING

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James Prehm

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Dan A. Unruh, COMMITTEE CHAIRMAN

John E. Davis

John A. Schwenker

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CHAPTER I

THE PROBLEM AND DEFINITION OF TERMS

The launching of Russia's first space satellite in 1957, the increased technological needs of industry, and the increased use of automation and cybernetics by industry have all led to demands for changes in the mathematics curriculum of the public schools in the United States. Industrial leaders, mathematicians, scientists, teachers, and others have reexamined and studied the mathematics curriculum of our schools. The results of these studies have brought about both changes in the content of the mathematics taught in our schools, and changes in teaching methods.

The inductive method of teaching is the current approach to this teaching of mathematics. Modern mathematics materials have emphasized this method over the deductive method. It was the purpose of this study to compare these two methods of teaching in producing retention of mathematical concepts.

The Coulee Dam Schools have used modern mathematics materials for three years. It was decided that a study of the modern math teaching approach in contrast with the traditional approach might give some indication of the effectiveness of the two methods of teaching.

I. THE PROBLEM

The inductive and deductive methods of teaching for retention were compared in this study. It was also the purpose to determine whether there were significant sex differences in retention when the two methods were compared. Therefore, five hypotheses were tested.

Statement of the Hypotheses

Five null hypotheses were tested by this study. They were: (1) there would be no significant difference in retention between a group taught by the inductive method and a group taught by the deductive method; (2) there would be no significant difference in retention between girls taught by the inductive method and girls taught by the deductive method; (3) there would be no significant difference in retention between boys taught by the inductive method and boys taught by the deductive method; (4) there would be no significant difference in retention between girls taught by the inductive method and boys taught by the inductive method; (5) there would be no difference in retention between girls taught by the deductive method and boys taught by the deductive method.

Importance of the Study

There have been few studies of the effects of teaching methods on retention. The researcher believes that this area needs more exploration. It was suspected that an inductive approach to teaching would enable students to better retain concepts which they are taught. This study was an attempt to prove or disprove this idea.

It was also believed that part of the importance of this study lies in the tests which were used for the pretest and post-test. Most studies comparing modern math methods and traditional methods have made use of tests which were constructed to measure objectives of traditional math. This study made use of two forms of the *Stanford Modern Mathematics Concepts Test*, a comparatively new test which was designed to measure modern math objectives.

Limitations of the Study

Length of the study. Due to time limits, this study could not be conducted over the entire school year. It was decided to limit the study to the teaching of five concepts which took approximately five weeks scattered throughout the school year.

The number of students. Another limitation was due to the using of a test given at the end of the sixth grade for the pretest. As a result, students who had not taken this test were excluded from the study. Only twenty-eight students took part in this study: seventeen girls and eleven boys.

The size of the post-test. Since only five concepts were taught for this study, only part of the *Stanford Modern Mathematics Concepts Test* was used as the post-test. This measuring device consisted of fourteen test items, selected from the total test, which measured the concepts which were taught.

II. DEFINITION OF TERMS

Two terms, the deductive method of teaching and the inductive method of teaching, will be used throughout this thesis. Deductive reasoning and inductive reasoning are both used in problem solving. Max Black, as quoted by Burton, Kimball, and Wing, makes the following statement concerning these two methods of reasoning:

> In deduction we discover what is logically involved in given propositions: it supplies us with a valuable means of organizing and reorganizing our assumptions and beliefs. By means of induction we try to discover those generalizations that are true of the world in which we actually live (3:408).

With this in mind, the following definitions of the deductive method of teaching and the inductive method of teaching will be used in this thesis.

The Deductive Method of Teaching

The deductive method of teaching is that method which gives the students generalization. The students use these generalizations in solving various problems or performing certain tasks. No effort is made to search for relationships between concepts. This method would place more emphasis on drill and practice in mathematics (3:408).

The Inductive Method of Teaching

The inductive method of teaching is that method which leads students to discover generalizations and relationships between various concepts. In part this process may be teacher-guided. This teaching method is made use of in modern mathematics programs (3:408).

III. ORGANIZATION OF THE REMAINDER OF THE THESIS

Chapter II of this thesis is devoted to a review of the literature concerning comparisons of the inductive and deductive methods of teaching mathematics, studies of retention of mathematics concepts, and studies of sex differences in learning mathematics. Chapter III explains the procedure used in the study, and Chapter IV presents the findings, conclusions, and recommendations.

CHAPTER II

REVIEW OF LITERATURE

The mathematics taught in the early twentieth century focused on practical situations. Emphasis was placed on the practical uses of mathematics and little was done to show the inter-relationships of the various topics. Reaction to this type of teaching lead to the emphasis of meaning and understanding in mathematics (19:8).

In 1937 Pedro Orada made several suggestions to improve the teaching of mathematics. One of these suggestions was:

> The introduction of the mathematical concepts, combinations or processes in a form that will enable the pupils to generalize their learning experience (13:109).

He also believed that applying these generalizations and the learned skills would aid in the transfer of training to other fields of mathematics and to other subjects (13:109). So, dissatisfaction with the teaching methods of mathematics is not a new thing, and many studies have been made concerning this problem.

I. REPORT OF RESEARCH OF TEACHING METHODS

The Swenson Study

Esther Swenson ran a study of 332 second grade pupils in Saint Paul, Minnesota (4:397). Three methods of teaching arithmetic were used: a generalization method emphasizing meaning, a drill method, and a combination of the two. The group taught by the generalization method made the highest net gains on achievement tests, but the difference was not significant.

The Anderson Study

G. Lester Anderson obtained similar results in a study of 389 fourth graders in Minneapolis (4:398). The "meaning" method was not significantly better or worse.

The Miller Study

G. H. Miller ran a study of the meaning method versus the rule method of teaching mathematics to seventh graders in Los Angeles. He defines the rule method as that in which the instructor gives the students specific rules to be learned in order to solve problems (12:45). The meaning method makes use of definitions and principles of arithmetic (12:45). Students are encouraged to make useful generalizations. The emphasis is on meaning and understanding with this method.

This study was conducted in five schools during the second semester of the school year. The students were tested using the California Arithmetic Test to measure achievement, and a special test designed to measure the degree of understanding (12:47). Half of them were then taught using the meaning method, and the other half were taught by the rule method. At the end of the semester they were again tested using the same instruments.

It was concluded that the meaning method was more effective for teaching computation using fractions and that the rule method was superior for teaching measurements (12:48). The meaning method was also more effective in establishing comprehension of complex analysis (12:49). This would indicate a potential superiority in mastering difficult concepts. A third conclusion was that the meaning method was more effective in teaching children with high or average I. Q.'s (12:49). The rule method may be more effective with children with low I. Q.'s, although this may have been a result of some other factor.

The Keaney and Stockton Study

In 1958 Keaney and Stockton published a report of three methods for teaching percentage (7:294-303). The methods used were drill and rote memory, teaching to develop understanding, and a composite of the two.

Though the experiment lasted but a short time there appeared to be some evidence that the composite method was the more effective (7:302). They also stated that the understanding method might be best for developing problem solving ability. However, there were not statistically significant results to support these conclusions (7:302).

The Kushta Study

Kushta compared two methods of teaching algebra to ninth graders. One method was the topic approach, which emphasizes rules, and the other was the concept approach with its emphasis on unifying concepts underlying all mathematics (9:142).

This study involved five different schools and a total of 262 students. Each teacher taught two comparable classes; one by the topic method, and one by the concept method. At the end of the first semester the students were given an attitude scale and the Seattle Algebra Test, an examination to test understanding of the nature of mathematics (9:142).

Over-all there was no significant difference found in the degree of manipulative skills developed by the students (4:143). One school did report that the topic method students had performed significantly better.

The concept method developed a greater understanding of the nature of mathematics (9:143). The means at all the schools were higher for those classes taught by the concept method, and the over-all results were significantly in favor of this method.

At three schools the attitude scale showed that those students taught by the concept method changed toward a more favorable attitude of mathematics to a greater degree than those taught by the topic method (9:143). However, the other two schools found just as strong evidence favoring the topic method. As a result no conclusion could be made about these methods' effects on attitudes.

The Lankford and Pattishall Study

The *Review of Educational Research* reported that Lankford and Pattishall found a significant difference favoring the use of procedures to encourage pupils to think out independently the operations of adding and subtracting fractions (18:251).

The Tredway Study

The *Review of Educational Research* also reported that Tredway found that emphasis on the relationships between the elements of per cent was a more effective way of teaching than the usual textbook presentation (18:251).

Summary of Research of Teaching Methods

Of the seven preceding research studies of teaching methods only three reported significant differences in favor of an inductive approach. However, the other four reported no significant differences. On the basis of these studies, it would appear that the inductive method of teaching is at least as effective as the deductive method.

II. THE MODERN MATH PROGRAMS

The growing concern about the teaching methods of mathematics resulted in several studies which produced materials which emphasized the inductive method of teaching and which were termed "modern math." These studies began in the fifties and have continued into the sixties.

The University of Illinois Curriculum Study of Mathematics

The University of Illinois Curriculum Study of Mathematics (UICSM) started in 1951. During the 1952-53 school year, a freshman course of study was introduced at University High School on the campus. The results were encouraging and the study spread to other schools in Illinois and to other states (21:457). During this time the materials were constantly being revised.

The UICSM materials place great emphasis on a precise vocabulary, and understanding of basic principles through pupil discovery (14:19). Perhaps the UICSM philosophy can best be described by the following quote:

> A student who has been exposed to a diet rich in ideas is more resourceful than one who has been exposed principally to manipulative tasks (21:459).

The materials have been revised several times since the project started. These revisions have been the result of the experience of the teachers and of insights gained by the writing staff (21:462). The materials prepared so far have covered grades nine through twelve.

The School Mathematics Study Group

One of the largest efforts in improving the mathematics curriculum has been the work of the School Mathematics Study Group (SMSG). The materials developed by this group have combined the thinking of psychologists, teachers, mathematicians, and testmakers. The materials were first used during the school year of 1959-60. More than four hundred teachers and 42,000 students in grades seven through twelve were involved in this first trial (14:17).

The SMSG materials attempt to build a logical framework for the study of mathematics. The emphasis is put on basic principles and on concepts (14:18).

The revision of SMSG materials has been brought about by evaluations by teachers, mathematics advisors, and, in some cases, pupils (14:17-18). There have also been two or three major studies of the materials which have not yielded conclusive results (1:34).

These two are not the only experimental math programs in existance. However, they were two of the first and have had great influence.

The Studies of the Modern Math Program

The UICSM studies. Very few studies of UICSM materials have been published. One study compared the achievement of ninth graders taught by the traditional method with that of ninth graders taught by use of the UICSM materials (2:53). The upper third intelligence level of the class taught by use of the UICSM materials made a significantly greater gain in understanding math concepts.

The Williams and Shuff study. Williams and Shuff report of a study comparing junior high SMSG materials with traditional materials (23:495-504). No significant difference was found for the SMSG materials. However, greater gains in mathematical achievement were noted for the students of low ability who were taught using the SMSG materials.

The authors do point out that the tests used were those based on the traditional mathematics. Any content or concepts unique to the SMSG materials would not be measured.

The Minnesota National Laboratory study and the Weaver study. A study by the Minnesota National Laboratory comparing fourth grade classes using SMSG materials with those using traditional materials showed no significant difference as measured by the STEP achievement test (22:279). In a similar study Weaver reported that fourth and fifth graders using SMSG materials made gains in reasoning and computation which were at least equal to the gains made by those using traditional materials (22:279).

The Brown and Abell study. In the January, 1966 issue of *The Mathematics Teacher* Brown and Abell report the results of another investigation of SMSG materials (2:53). Ninety-two classes of students using these materials in grades seven through twelve did as well on a standard test as students nationwide had done.

The Ruddell study. Ruddell reports of a study comparing the achievement of seventh graders in a modern math program with that of seventh graders in a traditional program

(15:330-335). All the classes were homogeneously grouped according to scores on the California Test of Mental Maturity and the California Achievement Test. At the end of the experimental period they were given the Wide Range Achievement Test in Arithmetic, the STEP and SCAT tests, and a paper-pencil test for determining mathematics understanding.

The results of the STEP test showed a growth in achievement for the SMSG classes which was significantly greater than that of the traditional classes. All other results were not significant, but the SMSG classes scored higher on all tests.

The Lyda and Morse study. Lyda and Morse conducted a study with fourth graders using the Laidlaw modern math text (11:136-138). The pupils were taught for twenty-one periods of forty minutes each. The teaching stressed relationships in arithmetic and understanding. The pupils were tested before and after with the Dutten Arithmetic Attitude Scale, and the Stanford Arithmetic Test.

The results showed that negative attitudes had become positive and that positive attitudes had become even more positive. The achievement test also showed significant gains (11:138).

Summary of Results of Modern Math Studies

Most studies have shown little significant advantage for the modern math materials and methods of teaching. On the other hand, they have not been shown to be inferior to traditional materials and methods. Herbert F. Spitzer has the following to say:

> The observational reports of competent observers have been far more valuable than the findings of experimental studies in showing the importance of meaningful learning. There is a definite belief on the part of those who have observed pupil study that where meanings and understandings are emphasized, such learning is superior to that found where no such emphasis exists (14:8).

III. REPORT OF RESEARCH OF RETENTION

It has been shown that there is a significant loss of retention for all children over summer vacation (17:52-53). Kausmier and Feldhusen concluded that retention is the same for all levels of intelligence for both boys and girls (8:92). Several studies have been conducted comparing the effects of inductive versus deductive methods of teaching on retention.

The Miller Study

One of the findings of the aforementioned G. H. Miller study was that the inductive method was significantly more effective in producing retention of both arithmetic processes and understanding of the principles of arithmetic (12:48).

The Howard Study

In a 1947 study fifteen classes of fifth and sixth graders were taught using one of three different methods (5:25-29). The first method was deductive, the second was inductive, and the third was also inductive, but provided more practice with both written and verbal problems. The classes were tested, retested sixteen weeks later, and tested again at the end of the summer (5:28).

It was concluded that the inductive method provides significantly better retention, and that drill and practice is helpful for some concepts (5:29).

The Shuster and Pigge Study

Shuster and Pigge experimented with the time devoted to developmental activities versus the time spent on drill. Six classes of fifth graders were taught addition and subtraction of fractions with 75 per cent of the time devoted to meaningful activities and 25 per cent spent on drill. Another six classes were taught dividing the time evenly, and another six were taught spending 75 per cent of the time on drill (16:24-25).

It was concluded that there was no difference in the classes immediately after the teaching. However, the first two groups did significantly better on a delayed-recall test given six weeks later (16:30). It was concluded that skills are learned better when more time is spent on developmental activities and less on drill (16:31).

Summary of Research of Retention

The findings of the three studies of retention all indicate statistically significant results in favor of the inductive method of teaching. Apparently, the students' retention is better when taught through the inductive rather than the deductive method.

IV. REPORT OF RESEARCH OF SEX DIFFERENCES IN LEARNING MATH

It is popularly believed that mathematics is a subject in which boys excel. At least one study has shown that there is no correlation between arithmetic proficiency and masculinity (10:21). However, one method of teaching may be more effective with one sex than with the other.

The Stroud and Lindquist Study

Stroud and Lindquist gathered data from the Iowa testing program in the high schools from 1932 to 1939. They also received data from a 1940 testing program of grades three through eight (20:659). These data were analysed for sex differences in learning .

It was concluded from this study that boys are slightly superior to girls in mathematics. But, girls are superior in other subjects and are slightly better in algebra (20:667).

The Wozencraft Study

A comprehensive study of sex differences was made in 1955 (24:486-490). This was an attempt to discover whether there were sex differences in learning, and if they exist at all levels of ability.

Involved in this study were 564 third graders and 603 sixth graders in the Cleveland public schools (24:487). These pupils formed a random sample of the city school population. Low, average, and high ability groups were established according to I.Q. The sixth grade pupils were then tested for arithmetic reasoning and computation. The third graders were only tested for arithmetic reasoning. The tests used were the Stanford Achievement Tests.

It was found that the girls' mean was significantly higher in the low and average groups in the third grade. The boys' mean was greater for the high group, but the difference was not significant (24:488). The sixth grade girls in the average group scored significantly higher than the boys in arithmetic computation. There was no significant difference in the other two groups (24:489). No significant differences were found in arithmetic reasoning for any of the groups (24:489).

The author concluded that there was no significant sex differences between bright boys and bright girls, or between slow-learning boys and slow-learning girls. However, girls at the average level seem to do significantly better than boys (24:490).

Summary of Research of Sex Differences

The findings of the two studies of sex differences indicate that there is little difference between boys and girls in over-all mathematics ability. However, neither study examined teaching methods, which may be a factor.

V. SUMMARY OF CHAPTER II

While some studies have found a significant difference favoring the inductive method of teaching over the deductive, others have found no difference between the two. One fact should be noted. The standardized tests used in all of the studies reported tended to be constructed in favor of the deductive method. At the time of these studies no revisions of the instruments had been completed to deal with the modern math.

The two reported studies of retention seem to indicate that the inductive method of teaching produces better retention.

The studies of sex differences in learning mathematics seem to show that there is little difference between boys and girls.

CHAPTER III

THE STUDY

The research within this study is concerned with five major considerations. These were (1) the selection of the tests, (2) the selection of the subjects, (3) the selection of the concepts to be taught, (4) the teaching of the concepts, and (5) the testing.

I. THE SELECTION OF THE TESTS

The *Stanford Modern Mathematics Concepts Test* was chosen for use in this study. The intermediate level test was used as the pretest, and part of the advanced level test was used as the post-test.

The *Stanford Modern Mathematics Concepts Test* was selected because it was designed to measure the objectives of a modern math program (6:2). It was decided that this test would do a better job of testing the concepts taught than other tests.

II. SELECTION OF THE SUBJECTS

The subjects for this study were chosen from the seventh grade students in the Coulee Dam Public Schools. The students were divided into two groups, one to be taught inductively and the other to be taught deductively, according to their scores on the intermediate level of the *Stanford Modern Mathematics Concepts Test*. They had taken this test at the end of the sixth grade, in the spring of 1966. As a result, students new to the school system were not included in this study.

Mathematical Understanding of the Experimental and Control Groups

The students were match-paired as closely as possible by using their percentile scores on the pretest. Table I presents a comparison of the mean scores on the pretest for mathematical understanding of the experimental group and the control group.

TABLE I

PRETEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING: EXPERIMENTAL AND CONTROL GROUPS

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Experimental	14	44.1	24.6	0	1.78
Control	14	44.1	25.9		

As shown in Table I, both groups consisted of fourteen students. The means for the two groups were also equal.

Mathematical Understanding of Girls in the Two Groups

Table II illustrates a comparison of the mean scores on the pretest for mathematical understanding of the girls in the experimental group and the girls in the control group.

TABLE II

PRETEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING:
GIRLS IN THE EXPERIMENTAL AND CONTROL GROUPS

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Experimental	9	39.8	23.56	.086	2.95
Control	8	38.8	24.58		

Table II shows that there were nine girls in the experimental group and only eight in the control group. The difference between the means was not statistically significant at the .01 level of confidence.

Mathematical Understanding of Boys in the Two Groups

Table III, on page 23, presents a comparison of the mean scores on the pretest for mathematical understanding of the boys in the experimental group and the boys in the control group.

TABLE III

PRETEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING: BOYS IN THE EXPERIMENTAL AND CONTROL GROUPS

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Experimental	5	51.8	25.1	.034	3.25
Control	6	51.3	23.3		

Table III shows that there were only eleven boys in this study; five boys in the experimental group and six in the control group. No significant difference was found between the means at the .01 level of confidence.

Mathematical Understanding Within the Experimental Group

Table IV illustrates a comparison of the mean scores on the pretest for mathematical understanding of the girls and boys in the experimental group.

TABLE IV

PRETEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING: GIRLS AND BOYS IN THE EXPERIMENTAL GROUP

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Girls	9	39.8	23.6	.88	3.06
Boys	5	51.8	25.1		

Table IV on page 23 shows that though there was some difference between the means, the difference was not significant at the .01 level of confidence.

Mathematical Understanding Within the Control Group

Table V presents a comparison of the mean scores on the pretest for mathematical understanding of the girls and boys in the control group.

TABLE V

PRETEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING: GIRLS AND BOYS IN THE CONTROL GROUP

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Girls	8	38.8	24.6	.97	3.06
Boys	6	51.3	23.3		

Table V shows that the mean score of the boys was higher than the mean of the girls. However, the difference was not significant at the .01 level of confidence.

III. SELECTION OF CONCEPTS TO BE TAUGHT

The next step in the study was to select the five seventh grade math concepts to be taught. The concepts were selected with the post-test in mind. This was done in an attempt to choose concepts which were measured by at least two items on the test. The concepts chosen were rounding off, equations, common multiples, measuring, and area.

Rounding Off

Seventh grade students were expected to understand the concept of replacing one number by another number which is nearly equal to it. They were expected to round whole numbers and fractions off to any desired degree of accuracy.

Equations

Seventh graders were expected to understand the concept of equations as being mathematical sentences. They were expected to develop the skill of solving simple equations such as: $2n=12$, $n-7=21$, $3n+1=19$, etc. They were also expected to translate verbal problems into equations and to use the solutions of the equations in solving the problem.

Common Multiples

Students were expected to understand that common multiples of two or more numbers are numbers which are multiples of each of the original numbers; the smallest of these is called the least common multiple. They also learned how to find the least common multiple of two or more numbers.

Measuring

In earlier grades the students had been exposed to the idea of measuring being a comparison between the measuring instrument and the thing being measured. This was reviewed in the seventh grade and they were introduced to the concept of measuring always being approximate. In developing this idea they learned how to find greatest possible error and relative error.

Area

Students were expected to understand the concept of area as being that part of a plane enclosed by a simple closed figure. They learned the units which are used in measuring area, and how to find the area of rectangles, squares, triangles, parallelograms, and circles.

IV. THE TEACHING OF THE CONCEPTS

Time Spent in Teaching

Each class was taught during a period fifty minutes long. The total teaching time for the study was approximately five weeks scattered throughout the year. Both classes were divided into a slow-learning group, and a fast-learning group according to the ability they displayed in class and on chapter tests. Students could move from one group to the other. From ten to twenty minutes of each period was devoted to presenting the material to each group. The remaining time in the period was spent with individuals who needed help.

Materials Used

The basic text used for these two seventh grade classes was the modern math text published by Laidlaw Brothers. The Laidlaw math series is used in the first nine grades in the Coulee Dam Schools. This was the third year that this series had been used. Prior to its adoption a series employing a traditional approach had been in use.

An overhead projector was extensively used, and visual aids were used, particularly with the slow-learning students.

The rest of this section will deal with how the concepts were taught. The inductive method will be contrasted with the deductive method. The first example will be given in more detail than the others.

The Teaching of Rounding Off

Inductive method. The inductive method of teaching rounding off made use of the number line. A number line from zero to ten was first drawn on the board. The students were asked such questions as: "Is nine closer to ten or to zero?" This method helps them to picture what numbers a given number may be near.

The students were then told that this rounding off process is often used to make computations easier when only an approximate answer is needed. They made use of this to check problems which they did in later lessons.

At this point other examples were given for the class to round off. They worked with small numbers at first and then were gradually given larger numbers. For each example a number line was drawn so that the students could see the relationship of the number to other numbers near it.

Several of the students noticed that to round a number such as 15 off to the nearer ten a choice of either 10 or 20 could be made; for 15 is midway between 10 and 20 on the number line. They were given the rule that in such a case the digit to be rounded off to should name an even number. Fifteen is to be rounded off to the nearer ten. So, since the 1 in the tens place is an odd number it should be replaced by 2, which is even. Thus, 15 would be rounded off to 20. It was emphasized that this was just an arbitrary rule which had been agreed upon by mathematicians.

The number of examples worked in class depended on how well they seemed to understand the concept and on the amount of time available.

Deductive method. The deductive method of teaching this lesson emphasized rules for rounding off. As with the experimental group, the control group was told that rounding off is sometimes used to make computations easier when only an approximate answer is needed. Rounding off a number was defined as replacing it with another number, usually some power of ten, which is nearly equal to it. Rules for rounding off were then given to the students. The following illustrates how these rules were presented.

Suppose 83 is to be rounded off to the nearer ten. Look at the digit in the ones place. If it is greater than five, add one to the tens digit and put a zero in the ones place. If it is less than five don't change the digit in the tens place, but put a zero in the ones place. In this case 3 is less than 5, so 83 is rounded off to 80. The rule for rounding off when the critical digit is five was also given to them at this time.

Examples were gone over in class and they were given the same assignment as the experimental group.

The Teaching of Equations

Inductive method. The students in the experimental group were given a small number of simple equations for which they had to find the solution. They also had to be able to give some explanation of how they had solved the equations. The equations were discussed in class the next day and the four properties of equations were introduced to the students. They then discussed how these properties fitted with what they had done to solve the equations.

The same sort of discovery approach was used in teaching the translation of verbal problems into equations. It was hoped that the discovery approach would provide a better understanding.

Deductive method. The control group was simply shown the properties of equations and how they were used. This group spent most of its time on drill in solving equations and verbal problems.

The Teaching of Common Multiples

Inductive method. The experimental group was introduced to common multiples by listing several multiples of two different numbers. They could see that several numbers were common to both lists. The smallest of these, they were told, is called the least common multiple. They were then shown another method for finding the least common multiple, and an example of its use in finding a common denominator for adding fractions.

Deductive method. The control group was simply given the definition of the least common multiple and told how it could be found. No background information was given, nor were uses for it discussed.

The Teaching of Measurement

Inductive method. The experimental group discovered the concept of greatest possible error in measurement by the use of measuring devices. Relative error was also taught by having the students measure certain distances and compare their results.

Deductive method. The control group did a limited amount of measuring. They were shown how to find greatest possible error and relative error. No attempt was made to further their understanding by making actual measurements.

The Teaching of Area

Inductive method. The experimental group was shown the interrelationships between the geometric figures of which they learned. They were shown how the formula for finding the area of one figure was derived from the formula for the area of another figure.

Deductive method. The control group had to memorize the formulas for finding areas without benefit of learning the relationships between the geometric figures.

V. TESTING

Six weeks after the teaching of the last concept the students in both groups were tested to find which group had best retained the concepts. It was also planned to compare the girls in both groups with each other, the boys in both groups with each other, and the girls and boys within each group.

The testing instrument consisted of selected items from the advanced level form of the *Stanford Modern Mathematics Concepts Test*. Only those items from the test which tested the concepts taught were considered as part of the evaluation device for this study. However, the entire test was given and then an analysis of the selected items was made.

Fourteen of the sixty-four items on the test were selected as best measuring the five concepts.

VI. TREATMENT OF THE DATA

Mean scores for the two groups were obtained from the data and these were compared by use of the T-test. The means of the girls in both groups, the means of the boys in both groups, and the means of the girls and boys within each group were compared in the same way. A correlation of the pretest and post-test for both groups was also computed. These results and the conclusions will be presented in the next chapter.

VII. SUMMARY OF CHAPTER III

Students in the seventh grade were grouped for this study according to their scores on the intermediate level form of the *Stanford Modern Mathematics Concepts Test* which they had taken at the end of the sixth grade. Seventh graders who had not taken this test were not included in the study. One group was then taught five selected concepts by the inductive method and the other was taught the concepts by the deductive method. Six weeks after the last concept was taught they were tested using the advanced level form of the *Stanford*

Modern Mathematics Concepts Test. Fourteen selected items from this test were used as the measuring device.

A comparison of the test results for the two groups, of the girls' scores in the two groups, of the boys' scores in the two groups, of the girls' and boys' scores within each group, and a correlation of the pretest and post-test will be presented in the next chapter.

CHAPTER IV

FINDINGS, SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS OF THE STUDY

The results of the post-test for the two groups were compared using the T-test of significance. The same statistical approach was used to compare the girls in the two groups, the boys in the two groups, and the girls and boys within each group to find whether six differences existed with regard to the two different methods of teaching.

A correlation study of the pretest and post-test for the two groups was also undertaken. Since there was a small number of items on the post-test, it was concluded that a check of its correlation with the pretest was needed.

I. THE FINDINGS

<u>Comparison of the Classes</u>

The post-test results for the two groups were compared using the T-test of significance. Table VI on page 35 presents a comparison of the mean scores of the experimental and control groups on the post-test for mathematical understanding of the concepts. It also shows the correlation between the pretest and post-test for both groups. The correlation was calculated using the Pearson product-moment method.

Although Table VI shows a higher mean for the experimental group, the difference was not statistically significant at the .01 level of confidence. The coefficient of correlation between the two tests was high for both groups.

TABLE VI

POST-TEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING OF THE CONCEPTS AND CORRELATION OF THE PRETEST AND POST-TEST: EXPERIMENTAL AND CONTROL GROUPS

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Experimental	14	5.2	2.71	1.00	1.78
Control	14	6.2	2.62		

Correlation of Tests for Experimental Group 83

Correlation of Tests for Control Group 71

Comparison of the Girls

The post-test results for the girls in the two groups were compared using the T-test of significance. Table VII on page 36 illustrates a comparison of the mean scores of the girls in the experimental and control groups on the post-test for mathematical understanding of the concepts.

Table VII shows that the mean score of the girls in the control group was higher than the mean score of the girls in the experimental group. The difference was not statistically significant at the .01 level of confidence.

Comparison of the Boys

The post-test results for the boys in the two groups were compared using the T-test of significance. Table VIII presents a comparison of the mean scores of the boys in the experimental and control groups on the post-test for mathematical understanding of the concepts.

TABLE VII

POST-TEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING OF THE CONCEPTS: GIRLS IN THE EXPERIMENTAL AND CONTROL GROUPS

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Experimental	9	4.7	2.74	1.01	2.95
Control	8	6.0	2.60		

Table VIII shows that the mean score for the boys in the control group was slightly higher, but the difference was not statistically significant at the .01 level of confidence.

TABLE VIII

POST-TEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING OF THE CONCEPTS: BOYS IN THE EXPERIMENTAL AND CONTROL GROUPS

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Experimental	5	6.2	2.32	.20	3.25
Control	6	6.5	2.63		

Comparison of Girls and Boys in the Experimental Group

The post-test results for the girls and boys in the experimental group were compared using the T-test of significance. Table IX presents a comparison of the mean scores of the girls in the experimental group and the boys in the experimental group on the post-test for mathematical understanding of the concepts.

TABLE IX

POST-TEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING OF THE CONCEPTS: GIRLS IN THE EXPERIMENTAL GROUP AND BOYS IN THE EXPERIMENTAL GROUP

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Girls	9	4.7	2.74	1.09	3.06
Boys	5	6.2	2.32		

Table IX shows that the mean score for the boys in the experimental group was higher than the mean score for the girls. However, the difference was not significant at the .01 level of confidence.

Comparison of Girls and Boys in the Control Group

The post-test results for the girls and boys in the control group were compared using the T-test of significance. Table X on page 38 illustrates a comparison of the mean score of the girls in the control group and the mean score of the boys in the control group on the post-test for mathematical understanding of the concepts.

TABLE X

POST-TEST MEAN COMPARISON FOR MATHEMATICAL UNDERSTANDING OF THE CONCEPTS: GIRLS IN THE CONTROL GROUP AND BOYS IN THE CONTROL GROUP

Group	N	Mean	Standard Deviation	Calculated T-Score	Required T-Score
Girls	8	6.0	2.60	.36	3.06
Boys	6	6.5	2.63		

Table X shows that the mean score of the boys in the control group was slightly higher than the mean score of the girls. This difference was not significant at the .01 level of confidence.

II. SUMMARY OF THE STUDY

This study of retention tested five null hypotheses. The first was that there would be no difference in retention between a group taught inductively and a group taught deductively. The second was that there would be no difference in retention between girls taught inductively and girls taught deductively. The third was that there would be no difference in retention between boys taught inductively and boys taught deductively. The fourth was that there would be no difference in retention between girls and boys taught inductively. And the fifth was that there would be no difference in retention between girls and boys taught inductively.

Seventh graders in the Coulee Dam Schools were divided into two classes according to their scores on the intermediate level form of the Stanford Modern Mathematics Concepts Test. They had taken this at the end of the sixth grade; students new to the district were not included in the study. Five concepts were chosen to be taught to these classes. They were: (1) rounding off, (2) equations, (3) common multiples, (4) measuring, and (5) area. The experimental group was taught these concepts by the inductive method, and the control group was taught the concepts by the deductive method. Six weeks after the last concept was taught the two groups were tested using the advanced form of the Stanford Modern Mathematics Concepts Test. Fourteen items from this test were selected as the measuring device.

Analysis of the data indicated that the pretest and the post-test were closely correlated. The T-test of significance showed no significant difference between the means for all groups compared, and as a result all five null hypotheses were accepted.

III. CONCLUSIONS

The Comparison of the Two Classes

The data indicate that the group taught deductively, the control group, had a higher mean score than the experimental group. However, the difference between the two means was not statistically significant. Therefore, the null hypothesis that there would be no difference in retention between the group taught by the inductive method and the group taught by the deductive method must be accepted. This leads to the conclusion that in this research the deductive method was as effective as the inductive method for the retention of the five concepts: rounding off, equations, common multiples, measuring, and area.

The Comparison of the Girls

Though the data show that the mean score of the girls taught deductively was higher than the mean score of the girls taught inductively, the difference was not statistically significant. As a result, the null hypothesis which stated that there would be no difference in retention of the five selected concepts between girls taught inductively and girls taught deductively must be accepted. It appears that either method will produce as much retention of the concepts in girls as the other.

The Comparison of the Boys

The means of the boys' scores in the two groups were nearly equal. Though there was some difference, it was not statistically significant. The null hypothesis that there would be no difference in retention between boys taught inductively and boys taught deductively was accepted. Neither method of teaching appears to produce better retention of the concepts in boys than the other.

The Comparison Within the Experimental Group

The data show that there was no significant difference between the mean scores of the girls and boys in the experimental group. Therefore, the null hypothesis which stated that there would be no difference in retention between girls and boys taught inductively was accepted. It appears that the inductive method of teaching will produce as much retention of the concepts in girls as it will in boys.

The Comparison Within the Control Group

The difference between the mean scores of the girls and boys in the control group was not statistically significant. As a result, the null hypothesis that there would be no difference in retention between girls and boys taught deductively must be accepted. Girls and boys seem to retain about the same amount when taught by the deductive method.

Discussion of the Conclusions

Though the conclusions of this study seem to indicate that the inductive and deductive methods of teaching are equally effective in producing retention, this should not be taken as conclusive proof. There were several factors in this study which may have affected the results.

The group size. The size of the groups in this study was quite small: only fourteen in each. Such a sample may produce results which aren't representative of the population from which the sample was drawn.

The test size. The number of items on the post-test may not have been enough to thoroughly test the students' understanding. A small number of items increases the likelihood that the results of the test may be due to chance.

The students. Another factor which may have affected the results of this study was the questioning attitude of some of the better students in the control group. It appeared to the researcher that they tended to think inductively and to discover some of the relationships by themselves.

The test. A final factor which should be considered is the test from which the post-test items were selected. Though it is supposedly a test of concepts, it may in reality be more of a test of skills. It is true that some of these skills are unique to modern math programs, but knowing these skills does not assure an understanding of the underlying concepts.

The Correlation Study

The high values for the coefficient of correlation show that the pretest and post-test had a high degree of correlation. That is, a student who received the top score on one likely received a score at or near the top on the other. Likewise, a student who scored near the bottom on one likely scored near the bottom on the other.

The Researcher's Observations

Though the results of this comparative study seem to indicate no advantage of the inductive method over the deductive method, the researcher has made observations to the contrary.

Students taught inductively seemed to be better able to relate the concepts learned to other concepts. As a whole they seemed to have had a better grasp of the concepts when observed working in class throughout the year.

Interpretation of this Report

Because of the limitations of this study such as the human element, the type of district and services within the district, the number of concepts selected, and the fact that no test for retention was given prior to the test at the end of the sixth week, the reader is cautioned against making inferences from this study which may lie beyond the framework as previously defined. The validity of any conclusions would be based solely upon the situation in which the study was conducted.

Summary of the Conclusions

Analysis of the data has led to the conclusion that the inductive and deductive methods of teaching are equally as effective in producing retention of the five concepts. While there was a high correlation between the pretest and the post-test, it should be noted that the groups were small, the post-test contained few items, some of the students taught deductively appeared to think inductively, and the post-test may have measured skills rather than concepts. Based on observation, the researcher tended to favor the inductive method of teaching.

III. RECOMMENDATIONS

Further studies comparing the inductive and deductive methods of teaching should be attempted. There is still too much controversy about which method is the better, and too little research. This researcher would make some recommendations which might make such future studies more meaningful.

Large Samples

Samples should be as large as possible to be sure of getting a good cross-section of the student population. Small samples, as in this study, may adversely affect the results of the study.

Longer Study

It is also recommended that any study of retention be conducted over a longer period of time if possible. For instance, testing might be done six weeks after the teaching and then again after twelve weeks. This should better indicate how well concepts are retained.

Better Testing Instruments

One of the weaknesses of this study was the small number of items on the post-test. It is recommended that a test with more items be used, and a test may be constructed that would better measure understanding of mathematics concepts.

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