On Econometric Analysis of Structural Systems with Permanent and Transitory Shocks and Exogenous Variables*

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January 14, 2007

Abstract

This paper considers the implications of the permanent/transitory decomposition of shocks for identification of structural models in the general case where the model might contain more than one permanent structural shock. It provides a simple and intuitive generalization of the influential work of Blanchard and Quah (1989), and shows that structural equations for which there are known permanent shocks must have no error correction terms present in them, thereby freeing up the latter to be used as instruments in estimating their parameters. The proposed approach is illustrated by a re-examination of the identification scheme used in a monetary model by Wickens and Motta (2001), and in a well known paper by Gali (1992) which deals with the construction of an IS-LM model with supply-side effects. We show that the latter imposes more short-run restrictions than are needed because of a failure to fully utilize the cointegration information.

JEL Classifications: C30, C32, E10

Key Words: Permanent shocks, structural identification, error correction models, IS-LM models.

*The research of the first author was supported by ARC Grant DP0449659.

1 Introduction

The fact that macroeconomic variables are often integrated rather than co-variance stationary has been increasingly accepted and has affected the design of models describing them. Moreover, variables often seem to be co-integrated and this has also led to the specification of models so as to reflect such a phenomenon e.g. DSGE models which feature a single technology shock need to make it an integrated variable to ensure that variables such as output and consumption are co-integrated. An implication of a co-integrated system then is that the shocks to it will be both permanent and transitory and methods to reconstruct such shocks using a VAR are now well known.

Shocks are now regarded as the driving forces of macro-economic systems. Often we wish to attach "names" to the shocks in order to deliver some economic content to our explanations of the evolution of variables either on average or over particular historical episodes. Thus we increasingly see reference to "technology shocks", "preference shocks", "risk premium shocks", "mark-up shocks". Such shocks, often referred to as structural are fundamentally *unobservable* and cannot be identified without reference to an economic model. The effects of structural shocks on the evolution of the macroeconomy can be either permanent or transitory, and one of the purposes of this paper is to consider a general approach to the identification problem if it is known *a priori* which of the structural shocks are permanent.

There has been work on this before. For example, the body of research initiated by Blanchard and Quah (1989) stipulated that there were demand and supply shocks, with the latter being permanent and the former transitory. Their approach was tantamount to making one of the structural equations of their two variable system have an unobservable permanent shock as its error term while the second equation of the system had a transitory shock that had no long-run effect on output. Generalizations of this approach involved having more structural equations whose errors are transitory shocks e.g. Gali (1992) but retaining the assumption of a single permanent shock.[1] In the next section of the paper we will generalize this methodology using a much simpler approach than in the original paper and its descendants. This is due to the fact that in our approach we focus on the permanent structural shocks rather than the transitory ones as in previous research. Moreover, our

[1] Gonzalo and Ng (2001) consider models with more than one permanent shock, but adopt the triangular approach to the identification of shocks which is not generally applicable.

methodology easily extends to there being a number of permanent shocks, which is appealing as there has been a growing interest in DSGE models with more than one permanent shock e.g. Edge et al. (2005).

Analysis of small open economies often features foreign variables that are treated as being weakly exogenous. These variables enter into the structural equations as drivers of the system, but are determined outside the economic model being estimated. Sometimes these exogenous variables are modelled with a statistical model such as a VAR or a VECM, and the shocks into such a system can then be regarded as the observable shocks. When such exogenous variables are present we would model the macroeconomic variables with a Structural VAR with exogenous variables (SVARX) or SVECMX system, rather than an SVAR or SVECM. These systems resemble those specified in the traditional simultaneous equations literature, being differentiated solely by the unrestricted dynamics.[2] Consequently, as one might expect, the exogeneity assumption yields potential instruments for estimating the structural equations. In the literature there is a recognition that, apart from exclusion restrictions and prior coefficient values that are associated with these exogenous variables, one might also be able to find some extra restrictions that stem from the assumptions that some of these exogenous variables represent permanent observed shocks. One paper which makes this case is Wickens and Motta (2001).

In the next section of this paper we set out the system to be analyzed and show how knowledge about the coefficients attached to cointegrating errors can be useful for estimating structural parameters and for identifying structural shocks. The main result of the paper is derived in section 3 where it is shown that structural equations for which there are known permanent shocks must have no error correction terms present in them, thereby freeing up the latter to be used as instruments in estimating their parameters. Section 4 shows that when the number of exogenous $I(1)$ variables equals the number of endogenous $I(1)$ variables, the coefficients of the cointegrating errors are not zero and have known values - a result established in Wickens and Motta (2001). Section 5 looks at two examples of our approach- one re-visits the model used in Wickens and Motta (2001), and the other examines Gali's (1992) well known paper which deals with the construction of an IS-LM model with supply-side effects. We show that the latter imposes more short-run restrictions than are needed because of a failure to fully utilize the

[2] See Garratt, Lee, Pesaran and Shin (2006, Ch. 6)) for further details.

cointegration information. Section 6 ends the paper with some concluding remarks.

2 Preliminary Analysis

Suppose we have a Structural VAR(1) system in n $I(1)$ variables of the form

$$\mathbf{A}_0\mathbf{z}_t = \mathbf{A}_1\mathbf{z}_{t-1} + \mathbf{e}_t$$

which can be transformed to

$$\mathbf{A}_0\Delta\mathbf{z}_t = -\mathbf{A}(1)\mathbf{z}_{t-1} + \mathbf{e}_t,$$

where $\mathbf{A}(1) = \mathbf{A}_0 - \mathbf{A}_1$. As will become apparent the assumption of a VAR(1) is not restrictive for the conclusion we will reach. The reduced form of the above SVAR(1) is given by

$$\begin{aligned} \Delta\mathbf{z}_t &= -\mathbf{A}_0^{-1}\mathbf{A}(1)\mathbf{z}_{t-1} + \mathbf{A}_0^{-1}\mathbf{e}_t && (1) \\ &= \mathbf{\Pi}\mathbf{z}_{t-1} + \boldsymbol{\xi}_t && (2) \end{aligned}$$

Now suppose that there are $r < n$ cointegrating relations in this system, namely $\mathbf{\Pi}$ is rank deficient so that $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times r$ full column rank matrices. Then

$$\Delta\mathbf{z}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \boldsymbol{\xi}_t \tag{3}$$

and

$$\mathbf{A}_0\Delta\mathbf{z}_t = \mathbf{A}_0\boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{e}_t = \boldsymbol{\alpha}^*\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{e}_t \tag{4}$$

is the SVECM.

The central task in SVAR systems is to estimate the n^2 coefficients of $\mathbf{A}_0$, n of which can be fixed by suitable normalization restrictions. The remaining $n(n-1)$ coefficients need to be identified by means of *a priori* restrictions inspired by economic reasoning. A number of different identification schemes are possible depending on the nature of the available *a priori* information. Each identification scheme produces a set of instruments for $\Delta\mathbf{z}_t$ and so enables the estimation of the unknown parameters in $\mathbf{A}_0$. It is clear from (4) that, if $\boldsymbol{\alpha}^*$ is known and $\boldsymbol{\beta}$ is able to be consistently estimated, then $\boldsymbol{\beta}'\mathbf{z}_{t-1}$ can be used as instruments. It is this feature that is exploited in the paper.

In this paper we distinguish two cases in which $\boldsymbol{\alpha}^*$ is known. In the first we show that the knowledge of a particular structural equation (or equations) having a permanent shock (shocks) implies that the value of $\boldsymbol{\alpha}^*$ in those structural equations will be zero. In the second, when the number of co-integrating vectors equals the number of stochastic structural equations then

$$\boldsymbol{\alpha}^* = \begin{pmatrix} -\mathbf{I}_r \\ \mathbf{0}_{(n-r)\times r} \end{pmatrix}.$$

The latter case effectively requires that some of the elements of $\mathbf{z}_t$ are exogenous variables. This result was established by Wickens and Motta (2001). Thus knowledge regarding the nature of the structural shocks and the presence of exogenous variables in a system can be a useful source of information for identifying shocks.

3 Some Structural Shocks are Permanent

Now let us decompose $\mathbf{z}_t$ into p endogenous (y_t) and q exogenous (x_t) variables, with $n = p + q$. The part of the SVECM equation in (4) relating to the endogenous variables will be given by

$$\mathbf{B}_0\Delta\mathbf{y}_t = \boldsymbol{\alpha}_y^*\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{C}_0\Delta\mathbf{x}_t + \boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t$ are the structural shocks of interest. Suppose that the first r shocks in $\boldsymbol{\varepsilon}_t$, denoted by $\boldsymbol{\varepsilon}_{1t}$, are known to be transitory and the remaining $p-r$ shocks, $\boldsymbol{\varepsilon}_{2t}$, are permanent. For simplicity the exogenous variables will be taken to evolve as $\Delta\mathbf{x}_t = \mathbf{v}_t$, where $\mathbf{v}_t$ is i.i.d.$(\mathbf{0}, \mathbf{V})$, and so these will also be permanent shocks. The structural equations that have as their errors the $p-r$ permanent shocks $\boldsymbol{\varepsilon}_{2t}$ can be written as

$$\mathbf{B}_{21}^0\Delta\mathbf{y}_{1t} + \mathbf{B}_{22}^0\Delta\mathbf{y}_{2t} = \boldsymbol{\alpha}_{y,2}^*\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{C}_2^0\Delta\mathbf{x}_t + \boldsymbol{\varepsilon}_{2t}.$$

To explore the implications of the above permanent/transitory decomposition of the structural shocks for their identification, consider first the common trends representation of (3) (see, for example, Johansen (1995, Ch. 3))

$$\mathbf{z}_t = \mathbf{z}_0 + \mathbf{F}\sum_{j=1}^{t}\boldsymbol{\xi}_j + \sum_{i=0}^{\infty}\mathbf{F}_i^*\boldsymbol{\xi}_{t-i},$$

where $\mathbf{F} = \boldsymbol{\beta}_\perp (\boldsymbol{\alpha}'_\perp \boldsymbol{\beta}_\perp)^{-1} \boldsymbol{\alpha}'_\perp$, with $\boldsymbol{\alpha}'_\perp \boldsymbol{\alpha} = \mathbf{0}$ and $\boldsymbol{\beta}' \boldsymbol{\beta}_\perp = \mathbf{0}$, so that $\mathbf{F}\boldsymbol{\alpha} = \mathbf{0}$. Hence

$$\begin{aligned}
\mathbf{z}_t &= \mathbf{z}_0 + \mathbf{F} \sum_{j=1}^{t} \begin{pmatrix} \mathbf{B}_0^{-1} \boldsymbol{\varepsilon}_j \\ \mathbf{v}_j \end{pmatrix} + \sum_{i=0}^{\infty} \mathbf{F}_i^* \boldsymbol{\xi}_{t-i} \\
&= \mathbf{z}_0 + \mathbf{F} \begin{pmatrix} \mathbf{B}_0^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_q \end{pmatrix} \begin{pmatrix} \sum_{j=1}^{t} \boldsymbol{\varepsilon}_{1j} \\ \sum_{j=1}^{t} \boldsymbol{\varepsilon}_{2j} \\ \sum_{j=1}^{t} \mathbf{v}_j \end{pmatrix} + \sum_{j=0}^{\infty} \mathbf{F}_j^* \boldsymbol{\xi}_{t-j}.
\end{aligned}$$

In order for $\boldsymbol{\varepsilon}_{1j}$ to have only transitory effects we must have[3]

$$\mathbf{F} \begin{pmatrix} \mathbf{B}_0^{-1} & \mathbf{0}_{p \times q} \\ \mathbf{0}_{q \times p} & \mathbf{I}_q \end{pmatrix} \begin{pmatrix} \mathbf{I}_r \\ \mathbf{0}_{p-r} \\ \mathbf{0}_q \end{pmatrix} = \mathbf{0},$$

namely

$$\mathbf{F} \begin{pmatrix} \mathbf{B}_0^{-1} \begin{pmatrix} \mathbf{I}_r \\ \mathbf{0}_{p-r} \end{pmatrix} \\ \mathbf{0}_q \end{pmatrix} = \mathbf{0}.$$

However, since $\mathbf{F}\boldsymbol{\alpha} = \mathbf{0}_{n \times r}$ and therefore it follows that

$$\begin{pmatrix} \mathbf{B}_0^{-1} \begin{pmatrix} \mathbf{I}_r \\ \mathbf{0}_{p-r} \end{pmatrix} \\ \mathbf{0}_q \end{pmatrix} = \boldsymbol{\alpha}\mathbf{Q},$$

where $\mathbf{Q}$ is an arbitrary $r \times r$ non-singular matrix. But, since $\mathbf{x}_t$ is weakly exogenous, there are no ECM terms in the equations for $\Delta \mathbf{x}_t$ and it must be that

$$\begin{bmatrix} \mathbf{B}_0^{-1} \begin{pmatrix} \mathbf{I}_r \\ \mathbf{0}_{p-r} \end{pmatrix} \\ \mathbf{0}_q \end{bmatrix} = \begin{pmatrix} \boldsymbol{\alpha}_y \\ \mathbf{0}_{q \times r} \end{pmatrix} \mathbf{Q}$$

or equivalently

$$\mathbf{B}_0 \boldsymbol{\alpha}_y \mathbf{Q} = \boldsymbol{\alpha}_y^* \mathbf{Q} = \begin{pmatrix} \boldsymbol{\alpha}_{y,1}^* \\ \boldsymbol{\alpha}_{y,2}^* \end{pmatrix} \mathbf{Q} = \begin{pmatrix} \mathbf{I}_r \\ \mathbf{0}_{p-r} \end{pmatrix}.$$

[3] These restrictions are necessary and sufficient and apply irrespective of whether the transitory shocks are correlated or not.

This in turn implies that $\boldsymbol{\alpha}_{y,2}^{*} = \mathbf{0}_{(p-r)\times r}$, namely *the structural equations for which there are known permanent shocks must have no error correction terms present in them*, thereby freeing up the latter to be used as instruments in estimating their parameters.

A simple demonstration of this result follows by considering the structural equations that have permanent shocks. These will have the form

$$\mathbf{B}_{21}^{0}\mathbf{y}_{1t} + \mathbf{B}_{22}^{0}y_{2t} + \mathbf{C}_{2}^{0}x_{t} = \mathbf{G}_{2}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{2t}.$$

Now, if it is known that $\boldsymbol{\varepsilon}_{2t}$ is a vector of permanent shocks, it will be an $I(1)$ process of the form $\Delta\boldsymbol{\varepsilon}_{2t} = \boldsymbol{\eta}_{2t}$, where $\boldsymbol{\eta}_{2t}$ is $I(0)$, and we can eliminate the unit root in the error term $\boldsymbol{\varepsilon}_{2t}$ by differencing these equations to obtain

$$\mathbf{B}_{21}^{0}\Delta\mathbf{y}_{1t} + \mathbf{B}_{22}^{0}\Delta\mathbf{y}_{2t} + \mathbf{C}_{2}^{0}\Delta\mathbf{x}_{t} = \mathbf{G}_{2}\Delta\mathbf{z}_{t-1} + \boldsymbol{\eta}_{2t}.$$

Hence the lagged ECM terms are excluded from these equations and can therefore be used as instruments to estimate some of the unknown coefficients in $\mathbf{B}_{21}^{0}$ and $\mathbf{B}_{22}^{0}$ i.e. the contemporaneous coefficients in the equations that have the permanent shocks. In each of the equations it is possible to consistently estimate r of these coefficients, making $(p-r)r$ in total. Identification of the remaining $(p-r)(p-r-1)$ coefficients in $\mathbf{B}_{21}^{0}$ and $\mathbf{B}_{22}^{0}$, apart from the $p-r$ normalization restrictions, requires further *a priori* information, for example, by assuming that $\mathbf{B}_{22}^{0}$ is a lower triangular matrix as proposed by Gonzalo and Ng (2001). Only if $p=2$ and $r=1$ will no further restrictions be needed once the permanent shock is identified.

This simple demonstration shows why the assumptions we made about the evolution of $\mathbf{x}_t$, the fact that the system is a VAR(1) etc. are not important, as the only operation that needs to be performed is a differencing of variables in the equations that have permanent structural shocks. Moreover, it enables us to give a simple interpretation of the widely-used Blanchard-Quah procedure and also to show how it extends to different contexts from the one they were working with. In that procedure there are two equations and these are associated with one $I(1)$ variable and one $I(0)$ variable. Call these y_{1t} and y_{2t} (in the original paper these are the log of GNP and the level of the unemployment rate). The equation corresponding to y_{1t} is assumed to have a unit root error term so all the variables entering in it need to be differenced i.e. after differencing the equation to be estimated has Δy_{1t} as dependent variable and $\Delta y_{2t}, \Delta y_{1t-1}, \Delta y_{2t-1}$ as independent variables (there are extra regressors if the VAR was higher than first order). Because there is no cointegration there are no instruments available from that source. However, since y_{2t} is $I(0)$ it is possible to use y_{2t-1} as an instrument for Δy_{2t} - see Pagan and Robertson (1998) *inter alia* for the analysis done in this way. A restriction needs to be enforced upon the parameters of the second equation to ensure that y_{1t} enters into the system as Δy_{1t} i.e. if the second equation in levels was

$$y_{2t} = \gamma_{12} y_{1t} + \beta_{11} y_{1t-1} + \beta_{22} y_{2t-1} + \varepsilon_{2t}$$

then we need $\gamma_{12} = -\beta_{11}$. This was not apparent in the original treatment as it was just assumed that the VAR to be estimated was in the two variables Δy_{1t} and y_{2t}.

Let us use the approach of this paper to consider other scenarios. In particular, suppose that the variables were both $I(1)$. If there is no co-integration then it is clear that y_{2t-1} is a poor instrument for Δy_{2t}, so that instruments would need to be found from some other source, perhaps from any exogenous variables that are excluded from the equation. It should be observed that there might be an issue about which structural equation should be taken as having the unit root error term. One way of investigating this stems from the fact that it will be the equation that doesn't have an ECM term in it. Of course we have to be able to estimate the parameters of that equation in order to check the presence or absence of an ECM term and this would require some extra instruments. A situation where this strategy is possible is where some recursive structure is placed on the system so that the lagged ECM term is not necessary as an instrument.

Note that our simple derivation of the Blanchard-Quah methodology comes because we focus upon the implications of the permanent shock for the specification of the structural equation and not the effects of the transitory ones. In their analysis it is the fact that transitory shocks have no long-run impact which is used to show that the equation for Δy_{1t} will involve differences in Δy_{2t}, but the argument to establish this is reasonably complex. Moreover, using the approach of this paper it is very easy to analyze systems that have more than two variables and which include a combination of $I(1)$ and $I(0)$ variables - in particular it is clear that the structural equations involving the permanent shocks must have all variables appearing in that equation in differenced form, even if they are $I(0)$ variables.

It is also interesting to note that, in many DSGE models, one of the structural equations is a Cobb-Douglas production function of the form

$$y_t - l_t = \alpha(k_t - l_t) + \varepsilon_t,$$

where y_t is the log of output, k_t the log of the capital stock, l_t the log of labour input and the technology shock ε_t is generally assumed to be an $I(1)$ variable i.e. $\Delta\varepsilon_t = \eta_t$. Hence the structural equation to be estimated can be transformed to

$$\Delta(y_t - l_t) = \alpha(\Delta k_t - \Delta l_t) + \eta_t$$

and instruments are needed for $\Delta k_t - \Delta l_t$. In most DSGE models l_t is an $I(0)$ variable, but there is co-integration between y_t and k_t, so that the capital-output ratio $y_t - k_t$ is an $I(0)$ variable. Therefore, using the result established above, $y_{t-1} - k_{t-1}$ could be used as an instrument for $(\Delta k_t - \Delta l_t)$. Of course there are also lags of Δk_t and Δl_t excluded from this equation and these might provide other instruments. To assess how useful the lagged ECM term might be as an instrument we use the RBC model set out in Ireland (2004), but change the technology process to one having a unit root. Then one finds that the correlation between $y_{t-1} - k_{t-1}$ and $(\Delta k_t - \Delta l_t)$ would be .729, making it an excellent instrument, and therefore α should be quite accurately estimated.

It is also useful to observe that, once the structural equations have been estimated, the quantities $\sum_{j=1}^{t} \mathbf{v}_j$ and $\sum_{j=1}^{t} \hat{\boldsymbol{\eta}}_{2t}$ represent the common permanent components ("common trends") of the system.[4] Defining $\boldsymbol{\zeta}_t = (\hat{\boldsymbol{\eta}}_{2t}', \mathbf{v}_t')'$ the permanent component of the variables will be

$$\begin{aligned} \mathbf{z}_t^p &= \mathbf{z}_0 + \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}_\perp'\boldsymbol{\beta}_\perp)^{-1}\sum_{j=1}^{t}\boldsymbol{\zeta}_j \\ &= \mathbf{z}_0 + \mathbf{K}\left(\sum_{j=1}^{t}\boldsymbol{\zeta}_j\right) \end{aligned}$$

Notice that, since $\boldsymbol{\zeta}_j$ is observable, $\mathbf{K}$ might be consistently estimated by regressing $\mathbf{z}_t$ against $\sum_{j=1}^{t}\boldsymbol{\zeta}_j$, as the error term in the regression is $I(0)$ by definition. This provides a structural model of the permanent component of a series i.e. it is one implied by the theoretical structure.

[4] If $\Delta\mathbf{x}_t$ has serial correlation the permanent components of $\mathbf{x}_t$ would need to be found using the Beveridge-Nelson decomposition. These would then replace $\sum_{j=1}^{t} \mathbf{v}_j$ as the permanent components of the exogenous variables. See Garratt, Lee, Pesaran and Shin (2006, Ch.10) for a detailed account of the derivation of permanent components in cointegrating systems with weakly exogenous variables.

4 Cointegration with Weakly Exogenous Regressors

Let the structural system to be estimated have the form

$$\begin{aligned} \mathbf{B}_0\mathbf{y}_t + \mathbf{C}_0\mathbf{x}_t &= \mathbf{B}_1\mathbf{y}_{t-1} + \mathbf{C}_1\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t && (5)\\ \Delta\mathbf{x}_t &= \mathbf{v}_t && (6) \end{aligned}$$

Then, defining $\mathbf{C}(1) = \mathbf{C}_0 - \mathbf{C}_1, \mathbf{B}(1) = \mathbf{B}_0 - \mathbf{B}_1$, and recognizing that $\mathbf{y}_{t-1} = \mathbf{y}_t - \Delta\mathbf{y}_t$, $\mathbf{x}_{t-1} = \mathbf{x}_t - \Delta\mathbf{x}_t$, we would have

$$\mathbf{B}(1)\mathbf{y}_t + \mathbf{C}(1)\mathbf{x}_t = -\mathbf{B}_1\Delta\mathbf{y}_t - \mathbf{C}_1\Delta\mathbf{x}_t + \boldsymbol{\varepsilon}_t.$$

Because the RHS of this equation is $I(0)$, when the number of co-integrating vectors is equal to p it must be the case that $\begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \end{pmatrix}$ is a set of co-integrating vectors. If we can be certain that the co-integrating vectors are unique then it has to be the case that[5]

$$\boldsymbol{\beta}' = \begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \end{pmatrix}. \tag{7}$$

To work out the implications of this, let $\mathbf{z}_t = (\mathbf{y}_t', \mathbf{x}_t')'$, write (1) as $\mathbf{A}(L)\mathbf{z}_t = \mathbf{e}_t$, and partition $\mathbf{A}_0$ and $\mathbf{B}_0$ accordingly as

$$\mathbf{A}_0 = \begin{pmatrix} \mathbf{B}_0 & \mathbf{C}_0 \\ \mathbf{0}_{q\times p} & \mathbf{I}_q \end{pmatrix}, \text{ and } \mathbf{A}(1) = \begin{pmatrix} \mathbf{B}(1) & \mathbf{A}(1) \\ \mathbf{0}_{q\times p} & \mathbf{0}_{q\times q} \end{pmatrix}.$$

Consequently, since $-\mathbf{A}_0^{-1}\mathbf{A}(1) = \boldsymbol{\alpha}\boldsymbol{\beta}'$, it must be that

$$-\mathbf{A}_0^{-1}\begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \\ \mathbf{0} & \mathbf{0} \end{pmatrix} = \boldsymbol{\alpha}\boldsymbol{\beta}',$$

or (using (7))

$$\begin{aligned} -\begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \\ \mathbf{0} & \mathbf{0} \end{pmatrix} &= (\mathbf{A}_0\boldsymbol{\alpha})\boldsymbol{\beta}' = \boldsymbol{\alpha}^*\boldsymbol{\beta}' \\ &= \begin{pmatrix} \boldsymbol{\alpha}_1^* \\ \boldsymbol{\alpha}_2^* \end{pmatrix}\begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \end{pmatrix}, \end{aligned}$$

[5] Conditions for exact identification of the cointegrating vectors, $\boldsymbol{\beta}$, are discussed, for example, in Pesaran and Shin (2002). For a model with r cointegrating vectors exactly r restrictions must be imposed on each of the r cointegrating vectors to achieve exact identification of $\boldsymbol{\beta}$.

implying that $\boldsymbol{\alpha}_1^* = -\mathbf{I}_p$, $\boldsymbol{\alpha}_2^* = \mathbf{0}$, and so $\boldsymbol{\alpha}^*$ has known values. Notice that the proof above doesn't require the system to be a VAR(1) since the co-integrating vectors will always be $\boldsymbol{\beta}' = \begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \end{pmatrix}$, and that one can consider a more general form for $\Delta\mathbf{x}_t$, although it must exclude any co-integration terms involving $\mathbf{y}_t$, namely it is sufficient for $\mathbf{x}_t$ to be weakly exogenous or long run forcing for $\mathbf{y}_t$.

It's clear that the key element in the proof is that the number of co-integrating vectors is the same as the number of the endogenous variables, p, as that enables us to identify $\boldsymbol{\beta}'$ with the elements making up long-run responses - $\begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \end{pmatrix}$. This fact enables us to produce a simple demonstration of the origin of the result. First, write (5) as

$$\begin{aligned} \mathbf{B}_0\Delta\mathbf{y}_t + \mathbf{C}_0\Delta\mathbf{x}_t &= -\mathbf{B}(1)\mathbf{y}_{t-1} - \mathbf{C}(1)\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t \\ &= -\begin{pmatrix} \mathbf{B}(1) & \mathbf{C}(1) \end{pmatrix}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t \\ &= -\boldsymbol{\beta}'\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t \end{aligned}$$

Hence it is immediately apparent that $\boldsymbol{\alpha}_1^* = -\mathbf{I}_p$ i.e. the coefficient on the ECM term is known. The requirement that there must be the same number of co-integrating vectors as $\dim(\mathbf{y}_t)$ is clearly very limiting but might be of use in analyzing open economies where there are often exogenous $I(1)$ variables. Thus an example would be the model in Justiniano and Preston (2006) if foreign output was an $I(1)$ process.[6] In that instance there would be two endogenous $I(1)$ variables- consumption and output - and one exogenous $I(1)$ variable - foreign output- so that the conditions are satisfied for the application of the result.

5 Further Applications

5.1 Wickens and Motta's Four Equation Monetary Model

Wickens and Motta (2001) give an example which has four equations.

$$\begin{aligned} i_t &= \rho + \Delta p_t + \varepsilon_{it} && (8) \\ m_t - p_t &= \mu + y_t - \lambda i_t + \varepsilon_{pt} && (9) \\ \Delta y_t &= \gamma + \alpha\Delta y_{t-1} + \varepsilon_{yt} && (10) \\ \Delta m_t &= \mu + \theta\Delta m_{t-1} + \varepsilon_{mt}, && (11) \end{aligned}$$

[6] In their model it is $I(0)$ but there is no reason to make that assumption.

where i_t is an interest rate, p_t is the log of the price level, m_t is the log of the money supply and y_t is the log of real output. All the four structural shocks, ε_{it}, $\varepsilon_{pt}, \varepsilon_{yt}$, and ε_{mt}, are assumed to be uncorrelated. These are their equations (22)-(25). It is clear that y_t and m_t are $I(1)$ and strongly exogenous. This leaves two potentially $I(1)$ variables. If both variables were $I(1)$ there would be two cointegrating vectors. In their paper they state "there is only one long-run structural relation among the $I(1)$ variables, namely $m_t - y_t - p_t$, and hence only one co-integrating vector" (p. 380), which clearly identifies i_t as $I(0)$.[7] Clearly, the number of co-integrating vectors is equal to the number of $I(1)$ endogenous variables so that the conditions for the result discussed in section 4 to hold apply.

The task is to estimate the parameters of this system, specifically λ. To apply the approach of this paper we simply re-write the second equation as

$$\Delta p_t = -\mu + \Delta(m_t - y_t) - (p_{t-1} - m_{t-1} + y_{t-1}) + \lambda i_t - \varepsilon_{pt},$$

which shows that the ECM term has a -1 as its coefficient. Hence we can re-express the money demand equation as

$$\Delta(m_t - p_t - y_t) = -\mu - (m_{t-1} - p_{t-1} - y_{t-1}) + \lambda i_t - \varepsilon_{pt}$$

The problem is that i_t is an $I(0)$ variable and will be correlated with the error term ε_{pt}, but it is clear that the lagged ECM term $(m_{t-1} - p_{t-1} - y_{t-1})$ provides an instrument for i_t, and so λ can be estimated by IV.[8] When we do this we estimate λ to be 2.2, much lower than found by Wickens and Motta (they found a value of 8.9 using a very different estimator). But the diagnostic statistics indicate that there is serial correlation in the equation. This raises the issue of whether there should be other lagged variables in this system. Normally in SVAR work the number of lags in an individual equation are determined by the lag order needed to get an adequate representation of the data by a VAR. It seems that Wickens and Motta utilized a VAR(1), but both AIC and BIC point strongly towards it being a VAR(2). Indeed, in the m_t and p_t VAR(1) equations, the Durbin-Watson tests applied to the residuals are .85 and .56, again pointing to the need to have a higher order VAR.

If the VAR is second order then one would need to augment the equation above with $\Delta m_{t-1}, \Delta y_{t-1}, \Delta p_{t-1}, i_{t-1}$, and i_{t-2}. In the case of this specifi-

[7] The fact that i_t is to be taken as $I(0)$ is repeated in their footonte 14.

[8] A first order deterministic trend is also present in the equation

cation the instrumental variables estimate indicates that the interest rate component in the equation can be written as $73\Delta i_t + 9.9 i_{t-2}$. It should be said that none of these coefficients is significantly different from zero.

Finally, one of the principal conclusions of the paper is that "it is found that impulse response functions do not display the liquidity effect". (p. 385). This conclusion seems at odds with the evidence. The liquidity effect describes the initial impact of a monetary shock upon interest rates. To compute this we can ignore any lagged values, output and non-monetary shocks in (8)-(11), so that the condensed system will be

$$\begin{aligned} i_t &= p_t \\ p_t &= m_t + \lambda i_t \\ m_t &= \varepsilon_{mt} \end{aligned}$$

Consequently the impact of the monetary shock upon the interest rate will be $\frac{\partial i_t}{\partial m_t} = (1-\lambda)^{-1}$. For any value of λ above unity there will be a liquidity effect. So the estimate they report of λ (and ours) is completely consistent with a liquidity effect.

5.2 Gali's IS/LM Model

A second example involves the IS/LM system developed in Gali (1992). Gali's presentation is of the case where there are four $I(1)$ variables, the log of GNP at 1982 prices (y_t), the yield on three-month Treasury Bills (i_t), the inflation rate in the CPI (Δp_t), and the growth in M1 (Δm_t). He indicates that there are two co-integrating vectors among these four variables and identifies them as $\zeta_{1t} = i_t - \Delta p_t$ and $\zeta_{2t} = \Delta m_t - \Delta p_t$. Gali then works with an SVAR in terms of the variables $\Delta y_t, \Delta i_t, \zeta_{1t}$ and ζ_{2t}.

To analyze Gali's method within our framework we need to elicit the implications for the system to be estimated of the fact that there are four $I(1)$ variables y_t, i_t, Δp_t and Δm_t and two co-integrating vectors among them. First, collect the four $I(1)$ variables in the vector $\mathbf{w}_t = (y_t, i_t, \Delta p_t, \Delta m_t)'$ and assume that $\mathbf{w}_t$ evolves as an SVAR(1) (this is simply for the purposes of exposition). Second, since there are two co-integrating vectors among $\mathbf{w}_t$, there must be two $I(1)$ common trends, which we will identify with two of the shocks in the SVAR - these shocks are both $I(1)$ and the innovations into them are the permanent shocks. Finally, given there are two co-integrating vectors we know that the SVAR can also be written as an SVECM of the form

$$\mathbf{B}_0\Delta\mathbf{w}_t = \boldsymbol{\alpha}^*\boldsymbol{\zeta}_{t-1} + \mathbf{B}_1\Delta\mathbf{w}_{t-1} + \boldsymbol{\varepsilon}_t \tag{12}$$

where $\boldsymbol{\alpha}^* = \mathbf{B}_0\boldsymbol{\alpha}$, $\boldsymbol{\zeta}_{t-1} = \left(\zeta_{1t-1}, \zeta_{2t-1}\right)'$ as per our earlier discussion. Gali works with a higher order SVAR but, as we will see, the analysis performed does not depend upon the exact order for the underlying VAR. The VECM corresponding to the SVECM in (12) is

$$\Delta\mathbf{w}_t = \boldsymbol{\alpha}\boldsymbol{\zeta}_{t-1} + \mathbf{B}_0^{-1}\mathbf{B}_1\Delta\mathbf{w}_{t-1} + \mathbf{B}_0^{-1}\boldsymbol{\varepsilon}_t \tag{13}$$

We will want to work with $\Delta\mathbf{z}_t$, where $\mathbf{z}_t = (y_t, i_t, \zeta_{1t}, \zeta_{2t}y_t)'$, and to convert the SVECM in $\mathbf{w}_t$ into one involving $\Delta\mathbf{z}_t$. This is done by using $\Delta\mathbf{z}_t = \mathbf{H}\Delta\mathbf{w}_t$, where

$$\mathbf{H} = \begin{pmatrix} \mathbf{S} \\ \boldsymbol{\beta}' \end{pmatrix},$$

and $\mathbf{S} = (\mathbf{I}_2, \mathbf{0}_2)$ is a selection matrix that selects Δy_t and Δi_t from $\Delta\mathbf{w}_t$. The following sequence of transformations then does the conversion.

$$\begin{aligned} \mathbf{B}_0\mathbf{H}^{-1}\mathbf{H}\Delta\mathbf{w}_t &= \boldsymbol{\alpha}^*\boldsymbol{\zeta}_{t-1} + \mathbf{B}_1\mathbf{H}^{-1}\mathbf{H}\Delta\mathbf{w}_{t-1} + \boldsymbol{\varepsilon}_t, \\ \mathbf{B}_0\mathbf{H}^{-1}\Delta\mathbf{z}_t &= \boldsymbol{\alpha}^*\boldsymbol{\zeta}_{t-1} + \mathbf{B}_1\mathbf{H}^{-1}\Delta\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t, \\ \mathbf{B}_0^*\Delta\mathbf{z}_t &= \boldsymbol{\alpha}^*\boldsymbol{\zeta}_{t-1} + \mathbf{B}_1^*\Delta\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t. \end{aligned} \tag{14}$$

We now assume that the permanent shocks are the first two structural shocks, and so there are no ECM terms in those equations. This means that (14) can be written explicitly as

$$\Delta y_t + b_{12}^{0*}\Delta i_t + b_{13}^{0*}\Delta\zeta_{1t} + b_{14}^{0*}\Delta\zeta_{2t} = b_{11}^{1*}\Delta y_{t-1} + b_{12}^{1*}\Delta i_{t-1} + \overline{\mathbf{b}}_1^{1*}\Delta\boldsymbol{\zeta}_{t-1} + \varepsilon_{1t}, \tag{15}$$

$$\Delta i_t + b_{21}^{0*}\Delta y_t + b_{23}^{0*}\Delta\zeta_{1t} + b_{24}^{0*}\Delta\zeta_{2t} = b_{21}^{1*}\Delta y_{t-1} + b_{22}^{1*}\Delta i_{t-1} + \overline{\mathbf{b}}_2^{1*}\Delta\boldsymbol{\zeta}_{t-1} + \varepsilon_{2t}, \tag{16}$$

$$\Delta\zeta_{1t} + b_{31}^{0*}\Delta y_t + b_{32}^{0*}\Delta i_t + b_{34}^{0*}\Delta\zeta_{2t} = \alpha_{31}^*\zeta_{1t-1} + \alpha_{32}^*\zeta_{2t-1} + \mathbf{B}_{31}^{1*}\Delta\mathbf{z}_{t-1} + \varepsilon_{3t}, \tag{17}$$

$$\Delta\zeta_{2t} + b_{41}^{0*}\Delta y_t + b_{42}^{0*}\Delta i_t + b_{43}^{0*}\Delta\zeta_{1t} = \alpha_{41}^*\zeta_{1t-1} + \alpha_{42}^*\zeta_{2t-1} + \mathbf{B}_{41}^{1*}\Delta\mathbf{z}_{t-1} + \varepsilon_{4t}. \tag{18}$$

Thus (15)-(18) incorporate the information coming from a knowledge that $\mathbf{w}_t$ are $I(1)$, that there are two co-integrating vectors, and that the permanent shocks are associated with the first two structural equations.[9]

[9] Note that the only equation in which permanent shocks can appear are the first and second equations, as the remaining equations determine $I(0)$ variables. It has to be the case that there are two $I(1)$ structural shocks in the original SVAR in w_t if there are two $I(1)$ shocks in the VAR.

To estimate (15) we need three instruments. Since $\boldsymbol{\zeta}_{t-1}$ is not present in this equation these exclusion restrictions provide two instruments. To get the third we need some short-run restriction. Selecting $b_{12}^{0*} = -\ b_{12}^{1*}$ makes the term $b_{12}^{0*}\Delta i_t + b_{12}^{1*}\Delta i_{t-1}$ become $b_{12}^{0*}\Delta^2 i_t$, and then Δi_{t-1} is available as an instrument. As detailed in Pagan and Robertson (1998), this is exactly the equation estimated by Gali as the first equation of his SVAR. Consequently, there is no difference in regard to our strategy for estimating this equation to what Gali does, except we see that there is a short-run restriction that is implicitly imposed in his SVAR - it is not a consequence of long-run information.

Consistent estimation of the second equation, (16), also needs three instruments. Again two of the instruments are provided by the elements of $\boldsymbol{\zeta}_{t-1}$. The residuals, $\hat{\varepsilon}_{1t}$, represent the third instrument, as the structural shocks are assumed to be uncorrelated with each other. However, this equation is different to the one estimated by Gali. Since he works with an SVAR in $\Delta y_t, \Delta i_t, \zeta_{1t}, \zeta_{2t}$, the equation he estimates is of the form (see Pagan and Robertson p. 213).

$$\Delta i_t + b_{21}^{0*}\Delta y_t + \gamma_{23}^{0*}\zeta_{1t} + \gamma_{24}^{0*}\zeta_{2t} = b_{21}^{1*}\Delta y_{t-1} + b_{22}^{1*}\Delta i_{t-1} + \overline{\boldsymbol{\gamma}}_2^{1*}\boldsymbol{\zeta}_{t-1} + \varepsilon_{2t}. \quad (19)$$

This differs from (16) because it involves the *level* of the co-integrating errors and *not the changes*. Because of this difference Gali does not have $\boldsymbol{\zeta}_{t-1}$ available as an instrument, and is therefore forced to impose two short-run restrictions (as well as using the residuals of the first equation as an instrument, as our method does). Consequently, it should be clear that it is not necessary to use short-run restrictions if one follows through the implications of Gali's $I(1)$ and co-integrating assumptions.

To understand the reason behind these differences, note that only values of $\Delta\boldsymbol{\zeta}_t$ must appear in equation (16) - the shock in the Δi_t equation is a permanent shock and the equation must be constrained to have a zero long-run effect on the $I(0)$ variables, $\boldsymbol{\zeta}_t$. The differenced dependent variable Δi_t arises in eliminating the original $I(1)$ shock that was in the levels SVAR. Implicitly Gali treats the shock of this equation as transitory when in fact it is not. This may stem from a confusion of the stochastic nature of the shock and its effects. The shock ε_{2t} is an $I(0)$ process but it has a permanent effect upon i_t. In the original SVAR there will be an $I(1)$ shock η_t but in the SVECM it becomes $\varepsilon_{2t} = \Delta\eta_t$ The long-run effect of ε_{2t} is non-zero even though it is an $I(0)$ process. In summary, the co-integration restriction that

Gali assumes must mean that the Δi_t equation can be estimated using $\boldsymbol{\zeta}_{t-1}$ as instruments for the $\Delta\boldsymbol{\zeta}_t$, and he has therefore not used all the information that is available.

Moving on to the third and fourth equations, (17) and (18), we see that these are identical to the third and fourth equations in Gali's SVAR since any equation with both levels and differences in $\boldsymbol{\zeta}_t$ can be re-written as one in the levels of $\boldsymbol{\zeta}_t$. Consequently, one estimates them in exactly the same way as Gali does. Cointegration provides no instruments for these equations.

It should also be clear from the derivation above that the structural coefficients in $\mathbf{B}_0^*$ are not the same as the original structural coefficients $\mathbf{B}_0$. Because

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix},$$

and when

$$\mathbf{B}_0 = \begin{pmatrix} 1 & b_{12}^0 & b_{13}^0 & b_{14}^0 \\ b_{21}^0 & 1 & b_{23}^0 & b_{24}^0 \\ b_{31}^0 & b_{32}^0 & 1 & b_{34}^0 \\ b_{41}^0 & b_{42}^0 & b_{43}^0 & 1 \end{pmatrix},$$

then

$$\mathbf{B}_0^* = \mathbf{B}_0\mathbf{H}^{-1} = \begin{pmatrix} 1 & b_{12}^0 + b_{13}^0 + b_{14}^0 & -b_{13}^0 - b_{14}^0 & b_{14}^0 \\ b_{21}^0 & 1 + b_{23}^0 + b_{24}^0 & -b_{23}^0 - b_{24}^0 & b_{24}^0 \\ b_{31}^0 & b_{32}^0 + 1 + b_{34}^0 & -1 - b_{34}^0 & b_{34}^0 \\ b_{41}^0 & b_{42}^0 + b_{43}^0 + 1 & -b_{43}^0 - 1 & 1 \end{pmatrix}.$$

Therefore, the coefficients in the SVAR that Gali (and we) use are combinations of the coefficients in the original SVAR system. Whether this matters depends on whether short-run restrictions on the structural equations make sense in the modified system.

There is another issue that the analysis above clarifies. There must be two permanent shocks in this system - in the equations for Δy_t and Δi_t. The presence of $\Delta^2 i_t$ in the Δy_t equation ensures that the shock in the Δi_t equation has no long run effect upon output, and so it will be a permanent shock affecting nominal magnitudes, while the shock in the first equation will be the supply side shock. But there is nothing in either ours or Gali's estimation strategy that imposes the restriction that the supply side shock will not have a permanent effect upon the nominal magnitudes $i_t, \Delta m_t$ and Δp_t. To avoid that one must have the coefficient of Δy_t and Δy_{t-1} equal and opposite in the Δi_t equation of whatever SVAR is estimated i.e. $\Delta^2 y_t$ must be how the endogenous variable y_t enters into the Δi_t equation. If it does appear in this form then Δy_{t-1} can be used as an instrument for $\Delta^2 y_t$. If one identifies the nominal and real shocks in this way then we have one more instrument than needed for the Δi_t equation, as there are now four available- the two lagged ECM terms, Δy_{t-1} and the residuals from the first equation. It is the design of the model that produces this extra instrument however.

Based on the above analysis we can see that the distinguishing approach between what we advocate and what Gali does is that we do not need short-run restrictions to estimate the Δi_t equation. One of these, that $b_{23}^{0*} + b_{24}^{0*} = 0$, can be tested, and it is easily rejected at the 5% level of significance (a $\chi^2(2)$ test statistic value of 5.88). But fundamentally the problem is that the SVAR which Gali uses is inconsistent with his basic assumptions about the nature of the data.

6 Concluding Remarks

This paper considers the implications of the permanent/transitory decomposition of shocks for the identification of structural models when one or more of the structural shocks are permanent. It provides a simple and intuitive generalization of the work of Blanchard and Quah (1989), and shows that structural equations for which there are known permanent shocks must have no error correction terms present in them. This insight can be used to construct suitable instruments for the estimation of the structural parameters. The usefulness of the approach is illustrated by re-examinations of the structural identification of the monetary model of Wickens and Motta (2001), and the influential IS-LM model of Gali (1992). The general results provided in this paper are particularly relevant to a number of DSGE models with more than one permanent shock that have been recently advanced in the literature.

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