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Risk pooling via unidirectional inventory transshipments in a decentralized supply chain

Emel Arikan and Lena Silbermayr*

Department of Information Systems and Operations, WU Vienna University of Economics and Business, Vienna, Austria

We study risk pooling via unidirectional lateral transshipments between two locations under local decision-making. Unidirectional transshipments can be applicable when cost structures and/or capabilities differ between locations, and it is also a common practice in dual channel supply chains with online and offline sales channels. We show that such a system cannot be coordinated only with varying transshipment prices. The transshipment receiver orders more and the transshipment giver orders less than the respective optimal centralised order quantities. In order to remove this discrepancy, we suggest horizontal coordination mechanisms by introducing a leftover subsidy for the location providing the transshipments or a shortage subsidy for the location receiving transshipments as well as a combination of shortage and leftover subsidy. Further, we evaluate the impact of network structure by comparing the equilibrium order quantities and profits under the uni- and bidirectional systems as well as a system without transshipments. Since demand correlation is a critical aspect in risk pooling we provide a detailed numerical study to discuss its impact on our findings.

Keywords: one-way transshipments; decentralisation; horizontal coordinating contracts; risk pooling; demand correlation

1. Introduction

Risk pooling via transshipments, i.e. serving excess demand at one location with excess inventory at another location, is a well-known concept in inventory management routinely performed in a variety of industries (Dong and Rudi 2004). It increases overall profitability of the supply chain through balancing locations' demand with available inventories by shipping excess stock to locations that face stockouts. In many practical situations, however, an asymmetric structure in the supply chain can be observed such that transshipments can be performed only unidirectional.

For example, in the online to offline supply channel studied by Seifert, Thonemann, and Sieke (2006) and He, Zhang, and Yao (2014) the traditional retailer can transship to the online shop. The other way around is not possible, since in-store customers in contrast to online customers are not willing to wait a certain shipment time. Unidirectional transshipments may also occur when it is not profitable for one location to sell excess stock to another location. Reasons can be high differences between the locations in shortage cost, size and/or proximity to transport hubs (see e.g. Axsäter 2003; Dong, Xu, and Evers 2012; Kranenburg and Van Houtum 2009; Olsson 2010). There might also exist a pre-specified redistribution route of transshipping items among the locations implying unidirectional transshipments (see Bouma and Teunter 2014, 2016). In order to offer transshipments a location must also enable transparency and invest in information systems to share information on inventories. If one location is not willing to do so, this can also imply unidirectional transshipments (Liang et al. 2014). Another example is the humanitarian supply chain within the network of the United Nation Humanitarian Response Depot. Humanitarian organisations that are members of the network can store relief items in the depots of the network that can be shared with each other in an asymmetric way, i.e. member can benefit from transshipments while stocking their own relief items outside of the network (Toyasaki et al. 2017). Unidirectional transshipments can also be interpreted by one-way product substitution in the inventory system if only the higher quality product may be substituted by a lower quality product that is not in inventory (Axsäter 2003).

Typically in the literature it is assumed that inventories are managed centrally (see e.g. Ahmadi, Torabi, and Tavakkoli-Moghaddam 2016; Bouma and Teunter 2016; Dong and Rudi 2004; Nakandala, Lau, and Shum 2017; Smirnov and Gerchak 2014; Tagaras 1989). In practice, however, transshipments often take place with independent locations maximising their own profits (Rudi, Kapur, and Pyke 2001). Motivated by practical examples, we assume that the different locations are not owned by one firm, i.e. they make decentralised decisions where each location reacts optimally in terms of the other location's given inventory level. Thereby, for example, a transshipment price can be determined that coordinates the decentralised supply chain for bidirectional transshipments (see e.g. Rudi, Kapur, and Pyke 2001).

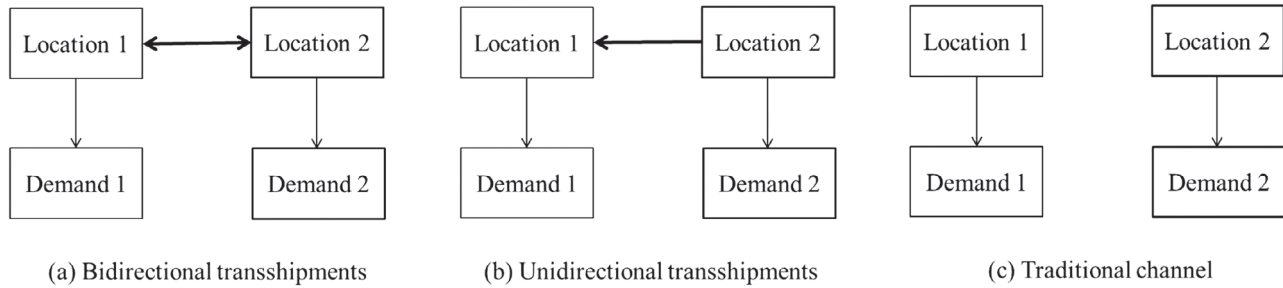*Corresponding author. Email: lena.silbermayr@wu.ac.at

Figure 1.   Illustration of decentralised supply chain structures under consideration.

In this paper, we study the risk pooling effect of unidirectional transshipments with two independently owned and operated locations selling identical products. Although unidirectional transshipments can be considered as a special case of bidirectional transshipments if the model assumptions are general enough (see e.g. Tagaras and Cohen 1992), following Axsäter (2003) we reduce the complexity of the model of mutual beneficial transshipments by removing the unneeded transshipment link. This allows us to derive analytical results specifically on unidirectional transshipments and to provide a direct comparison to bidirectional transshipments and to the no pooling case where each location fulfils their demands separately (see Figure 1). Our focus is on industries with high demand uncertainty, long lead times and short selling season, i.e. newsvendor type models.

In particular, the aim of this paper is to discuss the performance of unidirectional transshipments compared to bidirectional transshipments and no transshipments under decentralised decision-making. Further, we analyse how the locations choose inventory levels compared to the centralised (joint profit maximised) solution without any explicit coordination mechanism. We find that the induced asymmetry of unidirectional transshipments in terms of risk sharing between the locations excludes coordination in an unidirectional transshipment system. Thus, for such an asymmetric risk pooling situation, an increased balance of risk sharing between the locations would be necessary in order to minimise the difference to the centralised system. We propose a combination of transshipment price and leftover and/or shortage subsidy that induce both locations to choose centralised inventory levels. As another objective of the paper is to discuss the impact of demand correlation – a critical issue in inventory pooling – we therefore explore how correlation affects the inventory levels and profits under unidirectional transshipments.

This work is structured as follows. Section 2 reviews relevant literature. In Section 3 we introduce the models and discuss the existence of Nash equilibria and when the transshipment prices can coordinate the supply chain. In Section 4 we introduce contracts with leftover or shortage subsidy that can coordinate the unidirectional supply chain. Numerical examples are provided in Section 5, and conclusions are summarised in Section 6.

2. Literature

There is extensive literature on risk pooling via inventory transshipments. For a review of research on lateral transshipments and its classifications we refer to Paterson et al. (2011). An overview on supply chain coordination with contracts is given in Cachon (2003).

In decentralised systems most literature has focused on bidirectional transshipment in newsvendor-related supply chains. These papers all assume that transshipments are mutually beneficial among all retailers. Rudi, Kapur, and Pyke (2001) analyse coordinating transshipment prices and how this affects the optimal inventory orders at each location in a decentralised supply chain with two locations and bidirectional transshipments. Hu, Duenyas, and Kapuscinski (2007) analyse a two location inventory model similar to Rudi, Kapur, and Pyke (2001) and show that coordinating transshipment prices do not always exist. They derive sufficient and necessary conditions for the existence of coordinating transshipment prices such as e.g. symmetric locations. Lee and Park (2016) extend the transshipment models of Rudi, Kapur, and Pyke (2001) and Hu, Duenyas, and Kapuscinski (2007) by considering uncertain capacity of the supplier. They find that the sufficient condition for the existence of coordinating transshipment prices is more restrictive under supply capacity uncertainty and limitation than in the case of infinite capacity.

Li, Sun, and Gao (2013) discuss the coordination problem of preventive bidirectional lateral transshipments between two independent locations and propose a bidirectional revenue sharing contract to coordinate the system. Zou, Dresner, and Windle (2010) consider a two-location inventory model with transshipments in a competitive environment. Rivalry intensity

Table 1. Notation.

r_i	Selling price at location i
c_i	Purchasing cost at location i
s_i	Salvage value at location i
p_i	Shortage penalty cost at location i
r_{ij}	Transshipment price per unit transshipped from i to j, paid by j
c_{ij}	Transshipment cost per unit transshipped from i to j, paid by i
Q_i	Order quantity at location i
D_i	Random demand at location i
ρ	Coefficient of correlation between D_i and D_j
$F_X(\cdot)$	Cumulative distribution function of X
$f_X(\cdot)$	Probability density function of X

is assessed through a customer's switching rate investigating the impact of the switching rate on the performance benefits from transshipments. Zhao et al. (2016) study coordinated transshipment prices in a new e-commerce model, an online-to-offline market including a revenue share between online and offline retailer. Li and Li (2017) discuss the impact of bargaining power in a two-echelon supply chain consisting of one manufacturer and two symmetric retailers with bidirectional transshipments between them.

Literature focusing on unidirectional transshipments in a decentralised supply chain structure, however, is rare. Dong, Xu, and Evers (2012) study a multi-level framework considering a contract manufacturer and two inventory locations which differ in scale and scope such that transshipments are performed only unidirectional to analyse information asymmetry within the context of transshipments. Toyasaki et al. (2017) consider bidirectional and unidirectional transshipment of relief items in a decentralised humanitarian supply chain under correlated demands. However, since they consider a supply chain network in the non-commercial setting their model shows significant differences to the commercial setting in terms of cost and price parameters. For example, in their network transshipments between channels are based on a borrowing and loaning system without any transshipment price. Further, they consider backup prices for emergency orders from an additional echelon in the system that can coordinate the supply chain.

Seifert, Thonemann, and Sieke (2006) studies unidirectional transshipments integrating direct and indirect sales channels through a traditional retail store and a virtual store under decentralised decision-making. They analyse how the supply chain of a single manufacturer and multiple identical retail stores can be coordinated considering a combination of wholesale price, inventory subsidy and transfer payment. He, Zhang, and Yao (2014) study a dual channel supply chain with unidirectional transshipment policies between retailer and manufacturer (online shop) under endogenous and exogenous transshipment prices. The setting in both papers is somewhat different to our horizontal setting as they consider unidirectional transshipments between different echelons (i.e. a vertical setting with manufacturer and retailer). In these studies, unidirectional and bidirectional transshipments are not compared and their analyses are restricted to the assumption that the online shop serves different customer segments than the traditional retail store such that demands are not correlated. Our objective is to discuss the efficiency of unidirectional transshipments by comparing it with bidirectional transshipments, no transshipments and the centralised solution, propose horizontal coordination mechanisms and to investigate how demand correlation affects the results.

3. Model

Consider a single period model with two locations $i, j = 1, 2$ (throughout the paper, when using this indexing, we assume $i \neq j$). They face random demands D_i with continuous marginal distribution function F_{D_i} and the two demands can be correlated. The marginal and joint distributions of demands are common knowledge. We assume that the distributions are twice differentiable and strictly increasing on their supports. The product is sold at a selling price r_i per unit, purchased at c_i and salvaged at s_i, where $r_i > c_i > s_i$. Any unmet demand incurs a shortage penalty cost of p_i per unit. Transshipments from i to j incurs a transshipment cost c_{ij} per unit where the transshipment price is r_{ij} per unit. The main focus of this paper is on the unidirectional system where transshipments are only allowed in one direction, from location 2 to location 1 (see Figure 1(b)). However, for benchmarking we will also summarise the main results of bidirectional transshipments (Figure 1(a)) and a traditional channel with no risk pooling (Figure 1(c)).

We assume that $c_i < c_j + c_{ji}$, $s_i < s_j + c_{ji}$, and $r_i + p_i < r_j + p_j + c_{ji}$. These conditions guarantee that it is not beneficial to always purchase and/or salvage through the other location, and to sell to the other location instead of own customers. In order to assure that transshipments are profitable from location 2 to 1 the transshipment price should be set

within a specific range, i.e. $r_{21} \in [s_2 + c_{21}, r_1 + p_1]$ where $s_2 + c_{21} < r_1 + p_1$. To ensure mutually profitable transshipments in the bidirectional case, we additionally assume that $r_{12} \in [s_1 + c_{12}, r_2 + p_2]$, where $s_1 + c_{12} < r_2 + p_2$ (see Hu, Duenyas, and Kapuscinski 2007; Rudi, Kapur, and Pyke 2001). The complete notation is summarised in Table 1.

3.1 *No risk pooling*

If no transshipments at all are possible the problem reduces to the classical newsvendor problem. The expected profit function is concave and the optimal order quantity of location 1, Q_1^{NV}, is defined by the critical ratio $cr_1 = \frac{r_1 - c_1 + p_1}{r_1 - s_1 + p_1}$, such that $F_{D_1}(Q_1^{NV}) = cr_1$, and similarly for location 2.

3.2 *Risk pooling via transshipments*

We assume transshipments from location 2 to 1 are always possible. For given order quantities Q_1 and Q_2, transshipments from 2 to 1 is $T_{21} = \min((D_1 - Q_1)^+, (Q_2 - D_2)^+)$. Transshipments from 1 to 2, T_{12}, are only possible in the bidirectional setting and defined similarly, while in the unidirectional setting $T_{12} = 0$. Sales at location 1 are $S_1 = \min(D_1, Q_1) + T_{21}$, leftovers are $L_1 = (Q_1 - D_1 - T_{12})^+$, and unsatisfied demand is $P_1 = (D_1 - Q_1 - T_{21})^+$. Following Huang, Zhou, and Zhao (2011), we define $D_1^e = D_1 + (D_2 - Q_2)^+$ as the effective demand for location 1. It includes the initial demand at location 1 and the unsatisfied demand at location 2. Similarly, $D_1^n = D_1 - (Q_2 - D_2)^+$ is the net demand at location 1, which is the initial demand minus the part which can be satisfied by location 2. Note that $F_{D_1^e}(Q_1) \leq F_{D_1}(Q_1) \leq F_{D_1^n}(Q_1)$. These can be defined similarly for location 2.

When the two locations make their ordering decisions locally, in a decentralised manner, then the expected profit at location 1 is

$$\Pi_1(Q_1, Q_2) = E(r_1 S_1 + (r_{12} - c_{12})T_{12} - c_1 Q_1 - r_{21}T_{21} + s_1 L_1 - p_1 P_1), \tag{1}$$

and defined similarly for location 2.

Since the transaction between the locations is defined only by a per unit transshipment price we call this simple contract price-only contract.

If the ordering decisions for the two locations are centrally made the total expected profit is

$$\Pi^C(Q_1, Q_2) = E \sum_{i=1}^{2} (r_i S_i - c_{ij}T_{ij} - c_i Q_i + s_i L_i - p_i P_i). \tag{2}$$

3.2.1 *Bidirectional transshipments*

The setting with bidirectional transshipments has been studied in detail by Rudi, Kapur, and Pyke (2001) and Hu, Duenyas, and Kapuscinski (2007). In this section we briefly present their main results.

Hu, Duenyas, and Kapuscinski (2007) show that the expected profit under decentralised decision-making is concave in the order quantities. The first-order condition characterising the optimal order quantity of location 1 $Q_1^b(Q_2)$ is

$$(r_1 - c_1 + p_1) - (r_{21} - r_{12} + c_{12})F_{D_1}(Q_1) - (r_1 - r_{21} + p_1)F_{D_1^n}(Q_1)$$
$$- (r_{12} - c_{12} - s_1)F_{D_1^e}(Q_1) = 0. \tag{3}$$

It can be defined similarly for the optimal order quantity of location 2, $Q_2^b(Q_1)$. By checking the best response functions, specifically by evaluating $\partial Q_1^b(Q_2)/\partial Q_2$ and $\partial Q_2^b(Q_1)/\partial Q_1$ it can be shown that under bidirectional transshipments, there exists a unique Nash equilibrium (Q_1^b, Q_2^b).

If the ordering decisions for the two locations are centrally made, then the total expected profit is concave (Rudi, Kapur, and Pyke, 2001) and the first-order conditions characterising the optimal order quantity Q_1^{bC} for location 1 is

$$(r_1 - c_1 + p_1) - (c_{12} - r_2 + c_{21} + s_2 - p_2)F_{D_1}(Q_1) - (r_1 + p_1 - c_{21} - s_2)F_{D_1^n}(Q_1)$$
$$- (r_2 + p_2 - c_{12} - s_1)F_{D_1^e}(Q_1) = 0, \tag{4}$$

and can be defined similarly for location 2.

Rudi, Kapur, and Pyke (2001) show that the decentralised system can be coordinated by appropriately set transshipment prices. However, Hu, Duenyas, and Kapuscinski (2007) provide examples which show that such coordinating prices may not exist in several cases. Especially with increasing asymmetries in the economic parameters for the two locations, coordination of bidirectional transshipments may not be possible by varying the transshipment prices.

3.2.2 *Unidirectional transshipments*

Under the unidirectional setting transshipments are only allowed from location 2 to location 1, i.e. $T_{12} = 0$. Total system-wide profits could be maximised if the ordering decisions are made centrally. Total expected profit under central decision-making is concave, and the optimal order quantities Q_1^{uC} and Q_2^{uC} satisfy the first-order conditions

$$(r_1 - c_1 + p_1) - (c_{21} + s_2 - s_1)F_{D_1}(Q_1) - (r_1 + p_1 - c_{21} - s_2)F_{D_1^n}(Q_1) = 0, \tag{5}$$

$$(r_2 - c_2 + p_2) - (r_2 - r_1 + c_{21} + p_2 - p_1)F_{D_2}(Q_2) - (r_1 - c_{21} - s_2 + p_1)F_{D_2^e}(Q_2) = 0. \tag{6}$$

If the ordering decisions are made in a decentralised manner expected profits are concave, and the optimal order quantities $Q_1^u(Q_2)$ and $Q_2^u(Q_1)$ satisfy the first-order conditions

$$(r_1 - c_1 + p_1) - (r_{21} - s_1)F_{D_1}(Q_1) - (r_1 - r_{21} + p_1)F_{D_1^n}(Q_1) = 0, \tag{7}$$

$$(r_2 - c_2 + p_2) - (r_2 - r_{21} + c_{21} + p_2)F_{D_2}(Q_2) - (r_{21} - c_{21} - s_2)F_{D_2^e}(Q_2) = 0. \tag{8}$$

The existence of the Nash equilibrium is guaranteed by the concavity of the expected profit functions. The Nash equilibrium can be obtained by solving Equations (7) and (8) simultaneously. Further, in the following proposition we show the uniqueness of the equilibrium.

PROPOSITION 3.1   *Under unidirectional transshipments, there exists a unique Nash equilibrium* (Q_1^u, Q_2^u).

Please refer to Appendix 1 for the proofs. By comparing Equation (3) with Equations (7) and (8) we can conclude the following.

COROLLARY 3.2   *The best response function under unidirectional transshipments is smaller than the best response function under bidirectional transshipments for location 1, and the opposite holds for location 2, i.e.* $Q_1^u(Q_2) \leq Q_1^b(Q_2)$ *for all* Q_2 *and* $Q_2^u(Q_1) \geq Q_2^b(Q_1)$ *for all* Q_1.

Further, we can evaluate the impact of the network structure on equilibrium order quantities as follows.

PROPOSITION 3.3   *The order quantity of location* 2 *(transshipment giver) is always larger in the unidirectional setting compared to the bidirectional setting. On the other hand, the optimal order quantity of location* 1 *(transshipment receiver) is always smaller in the bidirectional setting compared to the unidirectional setting, i.e.*

(i) $Q_2^u \geq Q_2^b$ *and* $Q_1^u \leq Q_1^b$,
(ii) $Q_2^{uC} \geq Q_2^{bC}$ *and* $Q_1^{uC} \leq Q_1^{bC}$

Intuitively, the transshipment giver 2 orders more in the unidirectional case since it cannot receive any goods from location 1 in case of shortages. On the other hand, the transshipment receiver 1 orders more than in the unidirectional case since it does not have the chance of selling its leftovers to location 2. Additionally, location 1 decreases its order quantity further because it is expecting higher leftover supply at location 2, consequently higher chance for transshipment.

Unlike the bidirectional setting, the unidirectional system cannot be coordinated with a transshipment price-only contract. By comparing Equations (7) with (5), and (8) with (6) we obtain the following proposition.

PROPOSITION 3.4   *Under unidirectional transshipments, the optimal order quantity of location* 2 *(transshipment giver) is smaller in the decentralised system compared to the centralised system, i.e.* $Q_2^u \leq Q_2^{uC}$. *On the other hand, the optimal order quantity of location* 1 *(transshipment receiver) is larger in the decentralised system compared to the centralised system, i.e.* $Q_1^u \geq Q_1^{uC}$.

In the decentralised setting the transshipment giver orders less than would be optimal under centralised decision-making. This is because he bears the full risk of leftover supply; hence, he is more conservative and orders less. The transshipment receiver, on the other hand, orders more than under centralised control as the chance for transshipments is decreased under decentralised decision-making. Therefore, when transshipments are only allowed in one direction, the system cannot be coordinated by varying transshipment prices. This is similar to the setting under a wholesale price contract in a two-echelon system or to the bidirectional transshipment setting of Hu, Duenyas, and Kapuscinski (2007) with asymmetric parameters between two locations. Limiting transshipments in only one direction causes extreme asymmetry in the system. Therefore, we next suggest simple and easy to implement mechanisms to enable coordination in this unidirectional setting.

4. Coordinating unidirectional transshipments

In this section we propose simple contracts with a combination of transshipment price and a leftover and/or shortage subsidy. Under any of these contracts location 2 would have an incentive to increase the order quantity and as a reaction location 1 would order less. Therefore the discrepancy between the decentralised and centralised order quantities, as discussed in Proposition 3.4, may be avoided and the supply chain profit can be maximised.

4.1 *Coordination with leftover subsidy*

Under this contract location 1 pays a subsidy of τ^L per unit of leftover in location 2, i.e. the transshipment receiver shares the risk of leftover supply with the transshipment giver. Note that Seifert, Thonemann, and Sieke (2006) suggest a similar contract for vertical supply chain coordination in a two echelon system where transshipments occur between manufacturer and retailer and the contract includes also the wholesale price.

The expected profits with the leftover subsidy mechanism are

$$\Pi_1^L(Q_2, Q_1) = E(r_1 S_1 - r_{21} T_{21} - c_1 Q_1 + s_1 L_1 - p_1 P_1 - \tau^L L_2), \tag{9}$$

$$\Pi_2^L(Q_2, Q_1) = E(r_2 S_2 + (r_{21} - c_{21}) T_{21} - c_2 Q_2 + (s_2 + \tau^L) L_2 - p_2 P_2). \tag{10}$$

The feasible range for transshipment prices has to be modified to account for the leftover subsidy, $r_{21} \in [s_2 + c_{21} + \tau^L, r_1 + p_1]$. In this range both profit functions are concave and the optimal order quantities are uniquely defined by the first-order conditions

$$(r_1 - c_1 + p_1) - (r_{21} - s_1 - \tau^L) F_{D_1}(Q_1) - (r_1 - r_{21} + p_1 + \tau^L) F_{D_1^n}(Q_1) = 0, \tag{11}$$

$$(r_2 - c_2 + p_2) - (r_2 - r_{21} + p_2 + c_{21}) F_{D_2}(Q_2) - (r_{21} - c_{21} - s_2 - \tau^L) F_{D_2^e}(Q_2) = 0. \tag{12}$$

By using the same line of arguments as in Proposition 3.1, we can conclude the following.

PROPOSITION 4.1   *Under unidirectional transshipments with leftover subsidy, there exists a unique Nash equilibrium (Q_1^L, Q_2^L) for every contract (r_{21}, τ^L).*

The total centralised system profit is the same with or without leftover subsidy and the optimal centralised order quantities (Q_1^{uC}, Q_2^{uC}) are found through Equations (5) and (6).

PROPOSITION 4.2   *A unique combination of transshipment price and leftover subsidy (r_{21}^{L*}, τ^{L*}) can coordinate the unidirectional system where*

$$r_{21}^{L*} = (r_1 + p_1) \left(1 - \frac{F_{D_2^e}(Q_2^{uC})}{F_{D_2}(Q_2^{uC})}\right) + (c_{21} + s_2) \frac{F_{D_2^e}(Q_2^{uC})}{F_{D_2}(Q_2^{uC})}, \tag{13}$$

$$\tau^{L*} = r_{21}^{L*} - (c_{21} + s_2). \tag{14}$$

We can show that conducting unidirectional transshipments with such a contract is always beneficial for location 2 compared to not engaging in collaboration via transshipments.

COROLLARY 4.3   *In the unidirectional system, the expected profit for location 2 with a leftover subsidy contract is always larger than the expected profit without risk pooling, i.e. $\Pi_2^L(Q_2, Q_1) \geq \Pi_2^{NV}(Q_2)$ for all Q_2 and Q_1.*

However, we cannot conclude the same for location 1 which we further discuss in Section 5.2 with a numerical example.

4.2 *Coordination with shortage subsidy*

Under this contract location 2 pays a subsidy of τ^S per unit of shortage in location 1, i.e. the transshipment giver shares the risk of shortages with the transshipment receiver. The expected profits with the shortage subsidy mechanism are

$$\Pi_1^S(Q_2, Q_1) = E(r_1 S_1 - r_{21} T_{21} - c_1 Q_1 + s_1 L_1 - (p_1 - \tau^S) P_1), \tag{15}$$

$$\Pi_2^S(Q_2, Q_1) = E(r_2 S_2 + (r_{21} - c_{21}) T_{21} - c_2 Q_2 + s_2 L_2 - p_2 P_2 - \tau^S P_1). \tag{16}$$

The feasible range for transshipment prices has to be modified as, $r_{21} \in [s_2 + c_{21}, r_1 + p_1 - \tau^S]$. In this range both profit functions are concave and the optimal order quantities are uniquely defined by the first-order conditions

$$(r_1 - c_1 + p_1 - \tau^S) - (r_{21} - s_1) F_{D_1}(Q_1) - (r_1 - r_{21} + p_1 - \tau^S) F_{D_1^n}(Q_1) = 0, \tag{17}$$

$$(r_2 - c_2 + p_2) - (r_2 - r_{21} + p_2 + c_{21} - \tau^S) F_{D_2}(Q_2) - (r_{21} - c_{21} - s_2 + \tau^S) F_{D_2^e}(Q_2) = 0. \tag{18}$$

PROPOSITION 4.4 *Under unidirectional transshipments with shortage subsidy, there exists a unique Nash equilibrium* (Q_1^S, Q_2^S) *for every contract* (r_{21}, τ^S).

PROPOSITION 4.5 *A unique combination of transshipment price and shortage subsidy* (r_{21}^{S*}, τ^{S*}) *can coordinate the unidirectional system where*

$$r_{21}^{S*} = (r_1 + p_1)\frac{1 - F_{D_1^n}(Q_1^{uC})}{1 - F_{D_1}(Q_1^{uC})} + (c_{21} + s_2)\frac{F_{D_1^n}(Q_1^{uC}) - F_{D_1}(Q_1^{uC})}{1 - F_{D_1}(Q_1^{uC})}, \tag{19}$$

$$\tau^{S*} = r_1 + p_1 - r_{21}^S. \tag{20}$$

COROLLARY 4.6 *In the unidirectional system, the expected profit for location 1 with a shortage subsidy contract is always larger than the expected profit without risk pooling, i.e.* $\Pi_1^S(Q_1, Q_2) \geq \Pi_1^{NV}(Q_1)$ *for all* Q_2 *and* Q_1.

Therefore, conducting unidirectional transshipments with a shortage subsidy contract is always beneficial for location 1 compared to not engaging in collaboration via transshipments. However this might not hold for location 2.

4.3 *Coordination with leftover and shortage subsidy*

When the contract terms include either a leftover or a shortage subsidy we can show that the contract can be beneficial for one of the two locations. Potentially if we design the contract with three terms (r_{21}, τ^L, τ^S) we can achieve coordination such that both parties are better off compared to a no-transshipment setting. Expected profits under such a combination contract are

$$\Pi_1^{LS}(Q_2, Q_1) = E(r_1 S_1 - r_{21} T_{21} - c_1 Q_1 + s_1 L_1 - \tau^L L_2 - (p_1 - \tau^S) P_1), \tag{21}$$

$$\Pi_2^{LS}(Q_2, Q_1) = E(r_2 S_2 + (r_{21} - c_{21}) T_{21} - c_2 Q_2 + (s_2 + \tau^L) L_2 - p_2 P_2 - \tau^S P_1). \tag{22}$$

The feasible range for transshipment prices has to be modified as, $r_{21} \in [s_2 + c_{21} + \tau^L, r_1 + p_1 - \tau^S]$. In this range both profit functions are concave and the optimal order quantities are uniquely defined by the first-order conditions

$$(r_1 - c_1 + p_1 - \tau^S) - (r_{21} - s_1 - \tau^L)F_{D_1}(Q_1) - (r_1 - r_{21} + p_1 - \tau^S + \tau^L)F_{D_1^n}(Q_1) = 0, \tag{23}$$

$$(r_2 - c_2 + p_2) - (r_2 - r_{21} + p_2 + c_{21} - \tau^S)F_{D_2}(Q_2) - (r_{21} - c_{21} - s_2 + \tau^S - \tau^L)F_{D_2^e}(Q_2) = 0. \tag{24}$$

PROPOSITION 4.7 *Under unidirectional transshipments with combined leftover and shortage subsidy, there exists a unique Nash equilibrium* (Q_1^{LS}, Q_2^{LS}) *for every* (r_{21}, τ^L, τ^S).

Since we have three terms for coordinating two decisions there are multiple coordinating contracts, but for a given transshipment price we can represent the coordinating subsidy terms uniquely.

PROPOSITION 4.8 *For a given transshipment price* r_{21}, *a unique combination of leftover and shortage subsidy* (τ^{S*}, τ^{L*}) *can coordinate the unidirectional system where*

$$\tau^{S*} = \frac{(F_{D_1^n} - F_{D_1})(r_{21}F_{D_2} - (c_{21} + s_2)F_{D_2^e} - (r_1 + p_1)(F_{D_2} - F_{D_2^e}))}{F_{D_2^e}(1 - F_{D_1}) - F_{D_2}(F_{D_1^n} - F_{D_1})}, \tag{25}$$

$$\tau^{L*} = (r_1 + p_1 - r_{21} - \tau^S)\frac{(F_{D_2} - F_{D_2^e})}{F_{D_2^e}}. \tag{26}$$

All the probabilities have to be evaluated at (Q_1^{uC}, Q_2^{uC}).

Although we can represent the coordinating subsidy terms for every transshipment price, some of these combinations might turn out to be infeasible. A coordinating contract is feasible only if the transshipment price r_{21} satisfies $s_2 + c_{21} + \tau^L \leq r_{21} \leq r_1 + p_1 - \tau^S$. Since we can fix one of the contract parameters and change the other two accordingly, we can design a contract with e.g. $\tau^L = 0$ and set τ^S and r_{21} as defined in Proposition 4.5. This would give a feasible contract. Hence, there is at least one contract such that the combination of transshipment price, leftover and shortage subsidy coordinate the system. So if we design a contract with three parameters including the transshipment price, we can always find a feasible coordinating contract. For a feasible coordinating contract we should set $r_{21}^{L*} \leq r_{21} \leq r_{21}^{S*}$ which are defined in (13) and (19).

Next we show that such a contract can be strictly beneficial for both locations whenever there is a chance of utilising transshipments, i.e. $E(T_{21}) > 0$. There is no possibility to utilise transshipments only if the demands are perfectly correlated

and the order quantity of location one is sufficiently large. For example, for identical demands, if $Q_2 \leq Q_1$ transshipments cannot be an option when $\rho = 1$. For nonidentical demands, $\rho = 1$ implies that $D_2 = aD_1 + b$ with probability one for some constants $a > 0$, and b. In that case, if $Q_2 \leq aQ_1 + b$ transshipments can never be an option. On the other hand if $\rho = 1$ and Q_1 is not sufficiently large, transshipments can take place for some demand realisations. When the demand correlation is different than one, there is always a positive probability that the demand in location one is below its inventory level and the demand in location two is larger than its inventory level, which leads to transshipments. This might obviously not hold true if the order quantities are set at the bounds of random demand, e.g. Q_2 is equal to the lower bound of D_2 or Q_1 is equal to the upper bound of D_1, in case it is relevant. In the following, for ease of representation, we do not explicitly mention these boundary situations. First we summarise this discussion in Lemma 4.9 which lead to the following result.

LEMMA 4.9 $E(T_{21}) = 0$ if $\rho = 1$ and $F_{D_1}(Q_1) \geq F_{D_2}(Q_2)$, otherwise $E(T_{21}) > 0$.

PROPOSITION 4.10 *With a combination of leftover and shortage subsidy there exists at least one feasible coordinating contract which is beneficial for both locations.*

Proof. Let's define the differences $\Delta_1(Q_1, Q_2) = \Pi_1^{LS}(Q_1, Q_2) - \Pi_1^{NV}(Q_1, Q_2)$ and $\Delta_2(Q_1, Q_2) = \Pi_2^{LS}(Q_1, Q_2) - \Pi_2^{NV}(Q_1, Q_2)$ for a coordinating combination contract as

$$\Delta_1 = E((r_1 + p_1 - r_{21})T_{21} - \tau^{L*}((Q_2 - D_2)^+ - T_{21}) + \tau^{S*}((D_1 - Q_1)^+ - T_{21})) \qquad (27)$$

$$\Delta_2 = E((r_{21} - c_{21} - s_2)T_{21} + \tau^{L*}((Q_2 - D_2)^+ - T_{21}) - \tau^{S*}((D_1 - Q_1)^+ - T_{21})). \qquad (28)$$

Unless $\rho = 1$ and $F_{D_1}(Q_1) \geq F_{D_2}(Q_2)$, $\Delta_1 + \Delta_2 = (r_1 + p_1 - c_{21} - s_2)E(T_{21}) > 0$ for all (Q_1, Q_2) that is, total system profit under a LS contract is always larger than the total newsvendor profits without transshipments. This means at least one of the differences is always positive. Remember that a contract is feasible and coordinating if $r_{21}^{L*} \leq r_{21} \leq r_{21}^{S*}$. When $r_{21} = r_{21}^{L*}$, we know from Corollary 4.3 that $\Delta_2 \geq 0$, but Δ_1 might be negative. We also know that if we increase r_{21} to r_{21}^{S*}, Δ_1 becomes positive. So, when we increase r_{21} within the feasible (coordinating) range Δ_2 starts positive and might become negative, and Δ_1 might start negative and becomes positive. Since we assume continuous distribution functions, the two differences are also continuous, so they should cross at least once at a specific point. Since $\Delta_1 + \Delta_2 > 0$, the two differences should strictly be larger than zero at the crossing point.

When $\rho = 1$ and $F_{D_1}(Q_1) \geq F_{D_2}(Q_2)$, $T_{21} = 0$ for all realizations of demands, and $\Delta_1 = \Delta_2 = 0$. In this case, there is no contract which can improve the system profit. Although the two locations cannot make strictly positive benefit, they are also not worse off than the standard newsvendor setting. $\square$

Note that this result does not depend on any assumptions about random demands. The two previously discussed contracts with either one of the subsidy terms are beneficial for both locations only in a limited range of correlations (see Section 5.2). Under the combination contract we can find coordinating terms which are beneficial for both locations for every correlation level.

4.4 *Impact of demand correlation*

Although it is not possible to derive general monotonicity results with respect to demand correlation, in the following we study the impact of two extreme cases on optimal order quantities. The results depend on the symmetry of cost parameters and their impact on the critical ratios *cr* (see Section 3.1) of the two locations. When the correlation coefficient is one, then the optimal order quantities under unidirectional transshipments are equal to the optimal newsvendor quantities if the cost parameters are symmetric or if they are asymmetric such that $cr_2 \leq cr_1$. Otherwise, the optimal order quantity of location 1 (2) under unidirectional transshipments is less (more) than the optimal newsvendor quantity. On the other hand when correlation coefficient is minus one, then the optimal order quantity of location 1 (2) under unidirectional transshipments is less (more) than the optimal newsvendor quantity independent of the cost parameters. This relation holds both for the centralised setting and the decentralised setting. The following proposition summarises these results.

PROPOSITION 4.11

  (i) *If $\rho = 1$ and $cr_2 \leq cr_1$ then $Q_1^u = Q_1^{uC} = Q_1^{NV}$ and $Q_2^u = Q_2^{uC} = Q_2^{NV}$.*
 (ii) *If $\rho = 1$ and $cr_2 > cr_1$ then $Q_1^{uC} \leq Q_1^u \leq Q_1^{NV}$ and $Q_2^{NV} \leq Q_2^u \leq Q_2^{uC}$.*
(iii) *If $\rho = -1$ then $Q_1^{uC} \leq Q_1^u \leq Q_1^{NV}$ and $Q_2^{NV} \leq Q_2^u \leq Q_2^{uC}$ if $cr_2 > cr_1$.*

The implications for the contract terms are as follows: When $\rho = 1$ and $cr_2 \leq cr_1$ the decentralised order quantities are equal to the centralised ones. Hence there is no need for any subsidy terms, or we can say both terms τ^{S*} and τ^{L*} have to

be zero. When $\rho = 1$ and $cr_2 > cr_1$ or $\rho = -1$, at least one of the terms has to be positive for coordination, and the optimal order quantity of location 1 (2) turns out to be less (more) than the optimal newsvendor quantity.

5. Numerical examples

To obtain insights into the impact of general demand correlation on order quantities and the performance of unidirectional transshipments we consider the three supply chain structures (a), (b), and (c) shown in Figure 1 and also compare it to a centralised distribution. We use simulation to derive centralised order quantities, Nash-equilibria and expected profits. Optimal order quantities are obtained by simulating 100,000 correlated demand pairs to obtain point estimates of expected profits for given order quantities and solve the underlying optimisation problems.

To avoid the chance of negative product demand we assume that individual demands are gamma distributed $\Gamma(k, \theta)$ with $k = 4$ and $\theta = 25$. Hence, mean demands are 100 and coefficient of variations 0.5. The dependence among individual demands is captured by correlation coefficient $\rho \in (-1, 1)$. We are interested in the effect of both positive and negative demand correlation on the risk pooling benefit and the performance measures in the supply chain under unidirectional transshipments. For example, positive demand correlation can be caused if both demands are strongly affected by common external conditions such as economic conditions or weather changes. Negative demand correlations, on the other hand, may rather be caused through competition.

For our base parameters we adopt the price and cost parameters from Rudi, Kapur, and Pyke (2001) as follows. The selling price for each firm $r_i = 40$, purchasing cost per unit $c_i = 20$, salvage value $s_i = 10$, shortage penalty cost $p_i = 0$ and transshipment cost $c_{ij} = 2$. Note that in this case based on our assumptions we vary the transshipment price $r_{ij} \in [12, 40]$ in the price-only contract, since otherwise transshipments would not be profitable. The numerical experimentation will focus on the scenarios of symmetric locations, but we also discuss the impact of asymmetric shortage penalty cost with $p_1 > p_2$ on the results.

5.1 *Price-only contract*

First, we analyse the risk pooling effect of unidirectional transshipment for the transshipment price-only contract. The figures are based on the scenario of symmetric locations where a coordinating transshipment price for bidirectional transshipments exists.

#### 5.1.1 *Impact of transshipment price and demand correlation on response functions*

Figure 2 shows the order quantity response functions of the two locations under bidirectional and unidirectional transshipments for low, medium and high transshipment prices. The response functions decrease in the other location's order quantity, since it reduces the chance of transshipment to the other location while increasing the chance of receiving excess stock from the other location (see Proposition 3.1).

As discussed in Proposition 3.2, comparing response functions of the unidirectional case with those of the bidirectional case we see that $Q_1^b(Q_2) \geq Q_1^u(Q_2)$ and $Q_2^b(Q_1) \leq Q_2^u(Q_1)$. Consequently, the transshipment receiver's (location 1) order quantity is lower while the transshipment giver's (location 2) order quantity is higher under unidirectional transshipment than under bidirectional transshipments. However, since the risk pooling effect of the transshipments decreases with increasing demand correlation ρ, also the discrepancies between bidirectional and unidirectional response functions and equilibria decrease with increasing demand correlation. Note that as correlation coefficient approaches 1 the risk pooling effect disappears and the order quantities approach the newsvendor solutions.

In Figure 2(a) and (b) where $r_{ij} = 12$ location 2 has no benefit from transshipment in the unidirectional transshipment case, hence, its optimal order quantity is equal to the newsvendor solution. For location 1, on the other hand, it is profitable to order less than the newsvendor solution as Q_2 increases, since cheap transshipment units are available if $\rho < 1$. Figure 2(c) and (d) show the case where both locations benefit from transshipments in the unidirectional case; location 1 benefits through received transshipments and location 2 through reduced leftovers. Consequently, both locations order more than in (a) and (b) respectively. In Figure 2(e) and (f) where the transshipment price is high, unidirectional transshipments are only slightly beneficial for location 1, i.e. it orders close to the newsvendor solution.

5.1.2 *Impact of transshipment price and demand correlation on equilibrium quantities and profits*
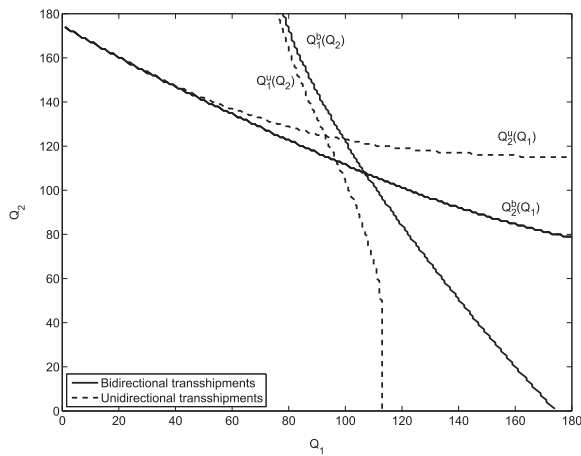
Figure 3 shows the comparison of decentralised and centralised optimal order quantities for the bidirectional and unidirectional supply chain structures. We see that in the bidirectional case the intersection of centralised and decentralised order quantities,
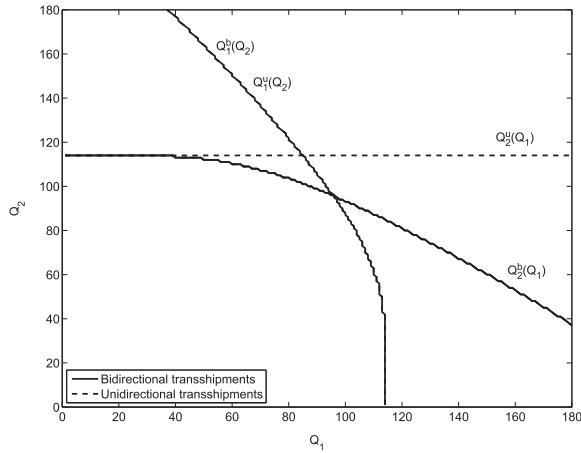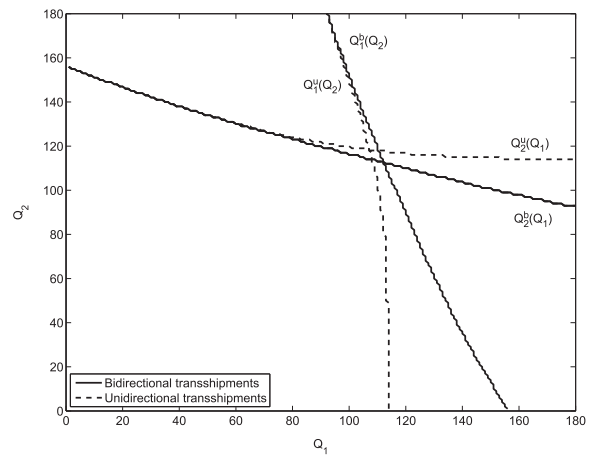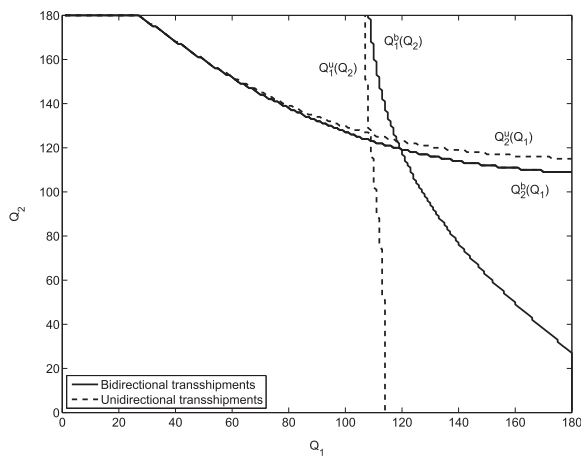
(a) $r_{21} = r_{12} = 12, \rho = -0.7$
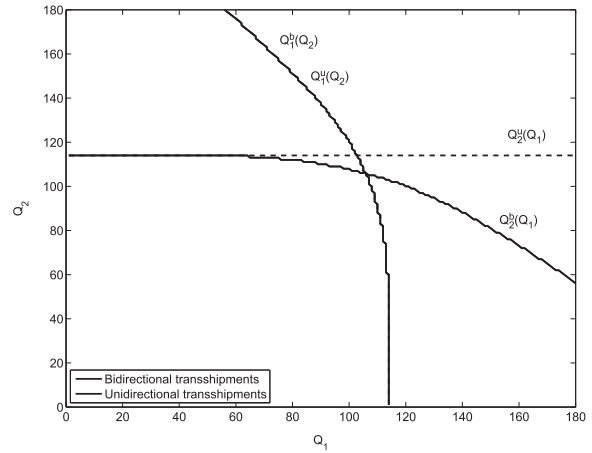
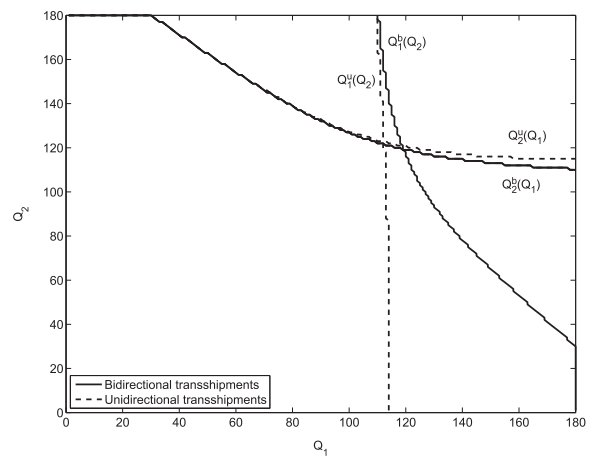(b) $r_{21} = r_{12} = 12, \rho = 0.7$

(c) $r_{21} = r_{12} = 24, \rho = -0.7$

(d) $r_{21} = r_{12} = 24, \rho = 0.7$

(e) $r_{21} = r_{12} = 36, \rho = -0.7$

(f) $r_{21} = r_{12} = 36, \rho = 0.7$

Figure 2. Response functions and equilibria under bi- and unidirectional transshipments.
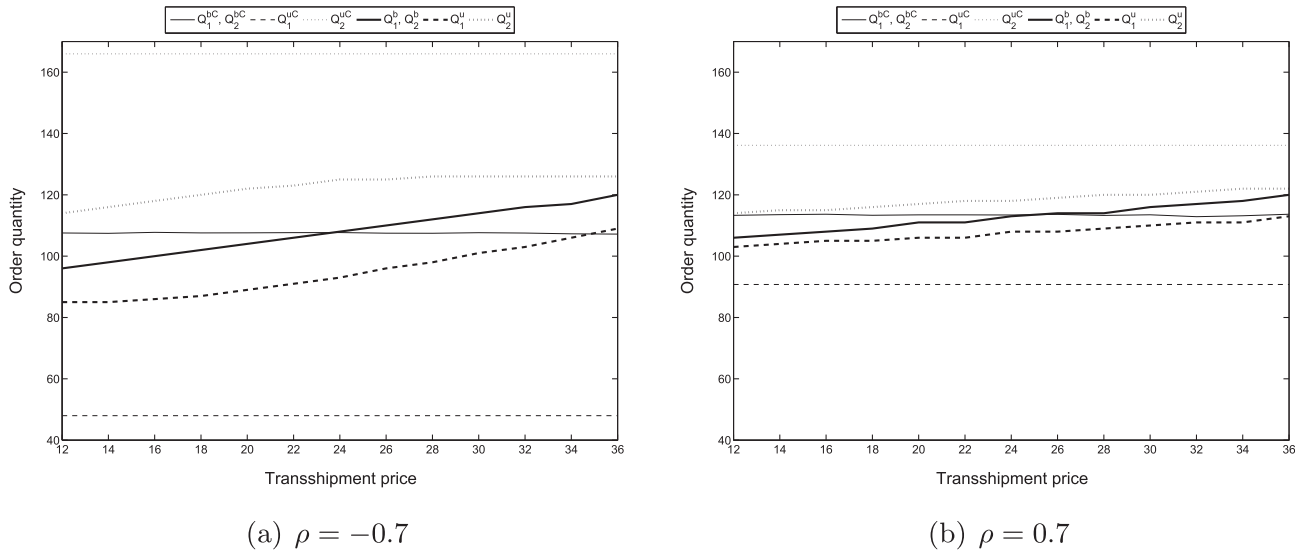
(a) $\rho = -0.7$ (b) $\rho = 0.7$

Figure 3. Optimal order quantity comparison.

i.e. the coordinated transshipment price increases as demand correlation increases (see also Rudi, Kapur, and Pyke 2001, Figure 5). Note, as discussed in Hu, Duenyas, and Kapuscinski (2007) for the case of asymmetric retailers the coordinating transshipment price only exists for a small range of parameters since the existence is only guaranteed if the optimal newsvendor quantities are both nongreater or nonsmaller than the optimal centralised order quantities. For example, we find that if $p_1 = 5$ and $p_2 = 0$, a coordinating transshipment price in the bidirectional case only exists for $\rho < -0.4$.

In the unidirectional case, the transshipment price cannot coordinate the system. Under decentralised control location 1 is ordering significantly more than under centralised control. Location 1 cannot expect to receive as many transshipment as under centralised control, because location 2 is more conservative with respect to order quantities than under centralised control, i.e. it orders less since he bears the full risk of leftover supply. As Figure 3 indicates, the differences of decentralised and centralised order quantities in the unidirectional case decrease with increasing demand correlation.

Figure 4 illustrates the impact of increasing demand correlation on the profit increase from uni- or bidirectional transshipments, i.e. $\Delta\Pi_i^{(u) \text{ or } (b)} = (\Pi_i^{(u) \text{ or } (b)} - \Pi_i^{NV})/\Pi_i^{NV}$ for low, medium and high transshipment prices. The profit increase is always positive, even under high demand correlation. The efficiency of unidirectional transshipments can be significant for both locations simultaneously, especially for a medium transshipment price (see Figure 4(b)).

5.2 Coordinating contracts

Next, we analyse the risk pooling effect of unidirectional transshipment for the coordinating contracts discussed in Section 4.

Figure 5(a) shows the set of transshipment price and leftover subsidy (r_{21}^{L*}, τ^{L*}) given in Equations (13) and (14) that coordinates unidirectional transshipments varying with respect to ρ. Figure 5(b) illustrates the set of transshipment price and shortage subsidy (r_{21}^{S*}, τ^{S*}). Although not significant, both r_{21}^{L*} and τ^{L*} increase as shortage cost of location 1 increases. Further, both r_{21}^{L*} and τ^{L*} decrease with increasing ρ since the risk pooling effect decreases and, as a consequence, the optimal order quantity of location 1 (location 2) increases (decreases). Unlike under the leftover subsidy contract, r_{21}^{S*} increases and τ^{S*} decrease with increasing ρ in the shortage subsidy contract, i.e. a decrease in the risk pooling effect implies that the transshipment price should increase while the shortage subsidy should decrease. Note that as ρ approaches 1, the resulting subsidy terms τ^{S*} and τ^{L*} approach zero (Proposition 4.11).

The impact of increasing demand correlation on the locations individual profits is shown in Figure 6. While the leftover subsidy contract is always beneficial for location 2 (see Corollary 4.3), it may be suboptimal for location 1 (the transshipment receiver). Location 1 would not take this contract for large ρ since the risk pooling benefit would be overweighted by the subsidy that he has to pay for location 2s leftovers. For the parameters used in Figure 6(a) this corresponds to a coefficient of correlation $\rho = 0$. However, it changes based on the problem setting, for example, it increases with increasing transshipment cost. Conversely to the leftover subsidy, the shortage subsidy contract is always beneficial for location 1 (see Corollary 4.6), while it is suboptimal for location 2 if ρ increases (see Figure 6(b)).
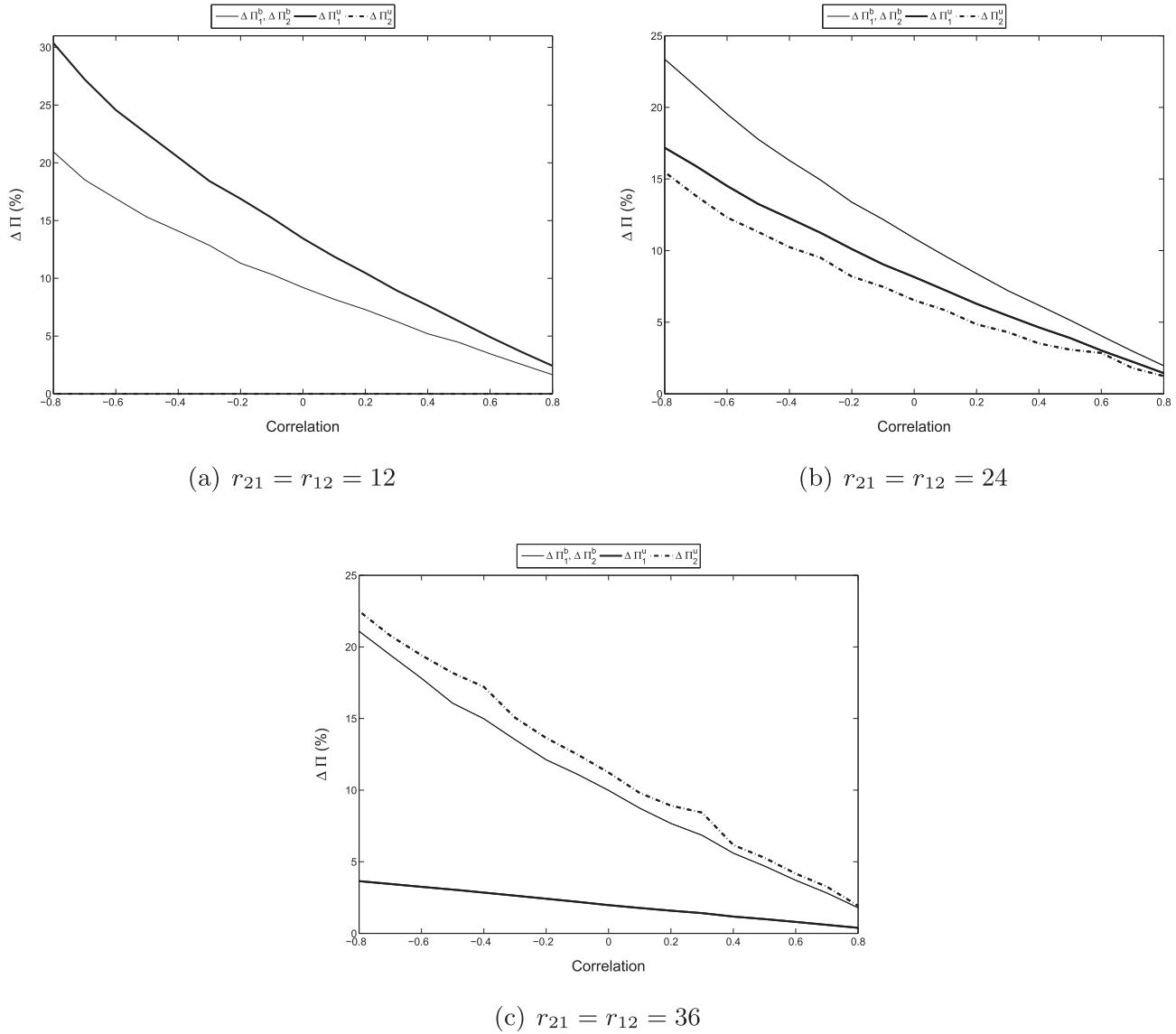
(a) $r_{21} = r_{12} = 12$

(b) $r_{21} = r_{12} = 24$

(c) $r_{21} = r_{12} = 36$

Figure 4. Profit increase from transshipments varying demand correlation ρ.

Finally, Figure 7 illustrates the impact of increasing demand correlation on the individual profits of the transshipment receiver and giver and the total profit compared to the newsvendor profits. Figure 7(a) and (b) show the results of all three coordinating contracts leftover subsidy, shortage subsidy and a combination of leftover and shortage subsidy for different transshipment prices if feasible, in case that both locations would take the contract (i.e. $\Delta\Pi_i \geq 0$ for $i = 1, 2$). For comparison we also illustrate the results of a price-only contract for different transshipment prices, which does not coordinate the supply chain. From Figure 7(c) we see that the benefit from coordination is significant, especially if the demand correlations is not that high. Further, a combined contract with leftover and shortage subsidy is a reasonable contract that coordinates the supply chain and that is optimal for both locations for all ranges of demand correlation (Proposition 4.10). The other coordinating contracts may not allow both locations to benefit from coordinating transshipments, especially when the risk pooling effect is small (i.e. demand correlation is high). However, as correlation becomes highly positive, it is known that risk pooling cannot contribute to the system profit significantly (see Figure 7(c)). Therefore, it will depend on the demand correlation between the locations but also other factors like bargaining power which contract to choose in the specific setting.
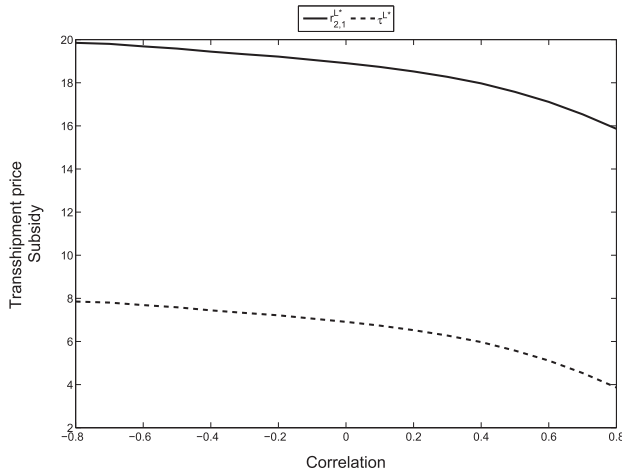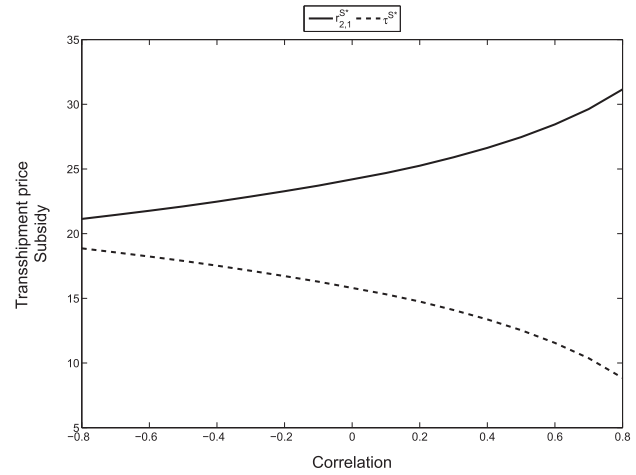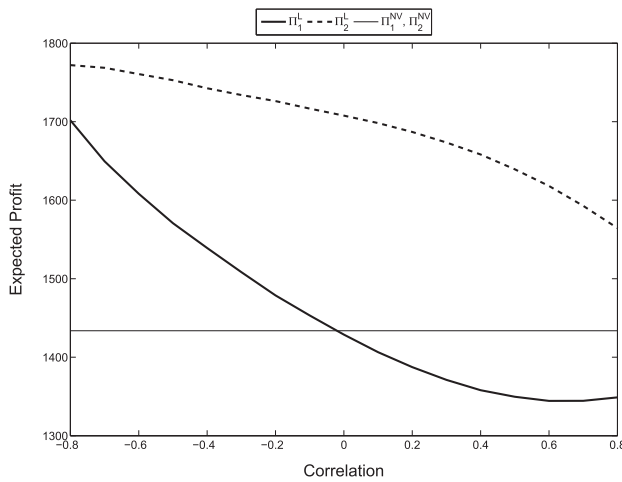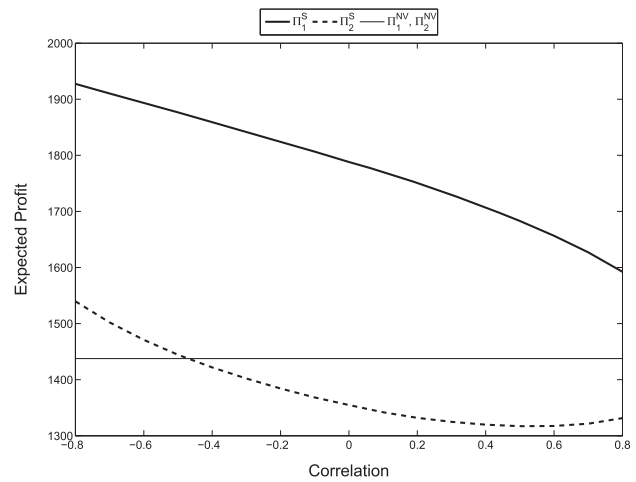
(a) Leftover subsidy contract   (b) Shortage subsidy contract

Figure 5. Coordinating transshipment price and leftover subsidy (a), shortage subsidy (b) varying demand correlation ρ.

(a) Leftover subsidy contract   (b) Shortage subsidy contract

Figure 6. Comparison of newsvendor profit and profit under coordinating leftover subsidy contract (a) and shortage subsidy contract (b) varying demand correlation ρ.

6. Extension to multiple locations

In this section we present a brief extension to a setting with $N \geq 2$ locations, some of which are transshipment givers and the others are transshipment receivers. Locations as $i = 1, ..., G$ are transshipment givers and the locations $j = 1, ..., R$ are transshipment receivers, where $G + R = N$.

Analysing such a system with more than two decentralised agents is known to be a nontrivial task (Huang and Sošić 2010; Rudi, Kapur, and Pyke 2001; Shao, Krishnan, and McCormick 2011). First of all an allocation rule has to be defined for assigning the leftovers to retailers with shortages. The amount of total shortages which can be potentially covered by transshipments is $TS = \sum_{j=1}^{R}(X_j - Q_j)^+$, and the amount of total leftover stocks which can be used for transshipments is $TL = \sum_{i=1}^{G}(Q_i - X_i)^+$.

For simplicity we assume symmetric locations in terms of their cost and demand parameters, and we adopt a proportional allocation rule suggested by Huang and Sošić (2010). Under this rule the quantity that locations j receives via transshipments
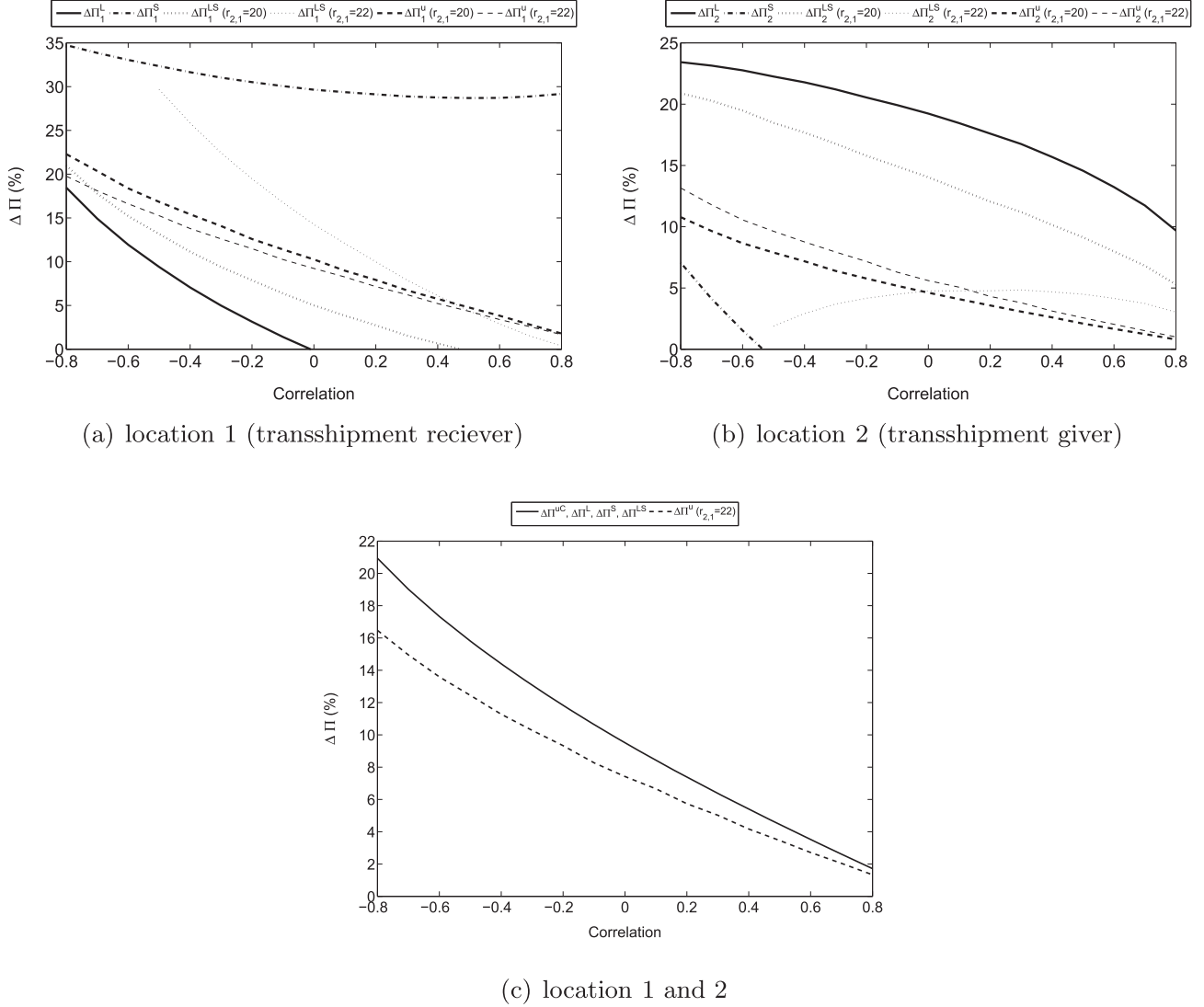
(a) location 1 (transshipment reciever)

(b) location 2 (transshipment giver)

(c) location 1 and 2

Figure 7. Profit increase compared to newsvendor profit for different contract types varying demand correlation ρ.

is $TI_j = \min(TL, TS)(X_j - Q_j)^+ / TS$. The first term, $\min(TL, TS)$, is the total amount which can be exchanged through transshipments. This term is multiplied with the proportion of locations j's shortages to the total shortages. Similarly, the quantity that locations i transships out is $TO_i = \min(TL, TS)(Q_i - X_i)^+ / TL$. Sales for transshipment givers and receivers are $S_i = \min(D_i, Q_i)$ and $S_j = \min(D_j, Q_j) + TI_j$, leftovers $L_i = (Q_i - D_i)^+ - TO_i$ and $L_j = (Q_j - D_j)^+$ and unsatisfied demands $P_i = (D_i - Q_i)^+$ and $P_j = (D_j - Q_j)^+ - TI_j$.

We assume transshipment prices and transshipment costs do not depend on i and j, otherwise, we need to keep track of which giver sends to which receiver, so $r_{ij} = r_t$ and $c_{ij} = c_t$ for all i, j. The expected profits incurred by location i (giver) and j (receiver) are

$$\Pi_i^{G,R}(\mathbf{Q_i}, \mathbf{Q_j}) = E\left(r_i S_i + (r_{ij} - c_{ij})TO_i - c_i Q_i + (s_i + \tau^L)L_i - p_i P_i - \tau^S\left(\sum_j P_j\right)/G\right), \qquad (29)$$

$$\Pi_j^{G,R}(\mathbf{Q_i}, \mathbf{Q_j}) = E\left(r_j S_j - r_{ij}TI_j - c_j Q_j + s_j L_j - \tau^L\left(\sum_i L_i\right)/R - \left(p_j - \tau^S\right)P_j\right). \qquad (30)$$

$(\mathbf{Q_i}, \mathbf{Q_j})$ is the vector of order quantities. Note that if $\tau^L = \tau^S = 0$ we have the price-only contract.
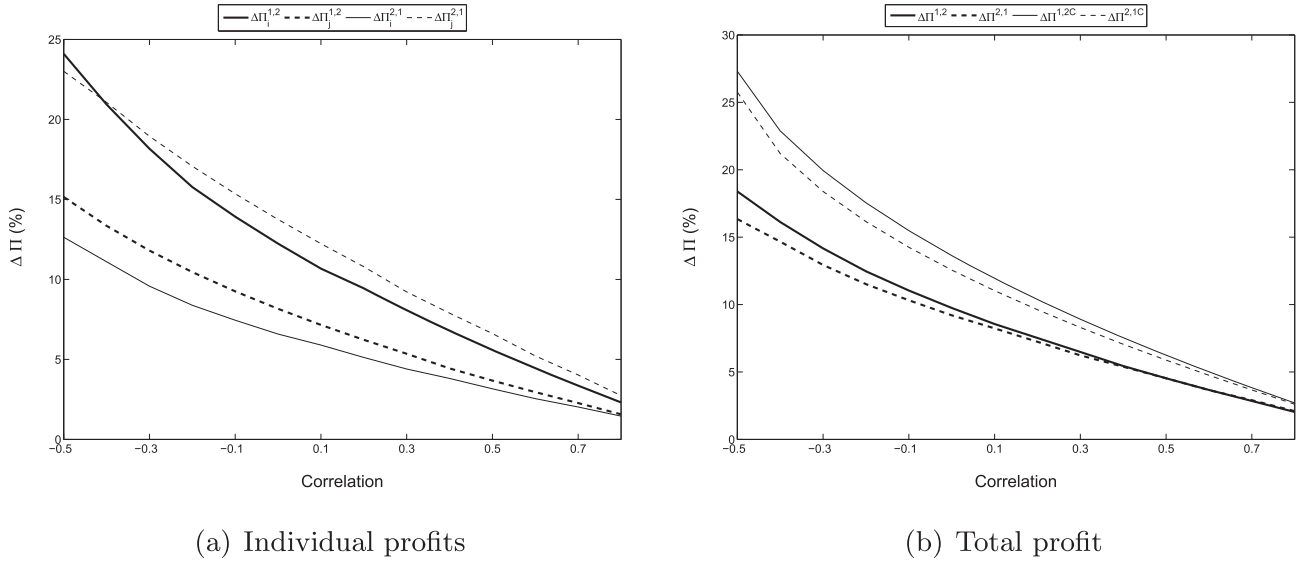
Figure 8. Profit increase from transshipments with price only contract compared to newsvendor varying demand correlation ρ, $N = 3$, $r_t = 24$.

The total centralised profit is

$$\Pi^{G,RC}(\mathbf{Q_i}, \mathbf{Q_j}) = E\left(\sum_{j=1}^{R}(r_j S_j - c_j Q_j + s_j L_j - p_j P_j) \right.$$
$$\left. + \sum_{i=1}^{G}(r_i S_i - c_i Q_i + s_i L_i - p_i P_i) - c_t \min(TL, TS) \right). \tag{31}$$

Since the locations are symmetric, under the proportional allocation rule, the transshipment givers have the same equilibrium order quantities $Q_i^{G,R}$, $Q_i^{G,RC}$, and consequently the same expected profit. Similarly, all transshipment receivers have the same equilibrium quantities $Q_j^{G,R}$, $Q_j^{G,RC}$ and expected profit.

We numerically investigate a system with $N = 3$ for the case of (i) $G = 1$ giver and $R = 2$ receivers and (ii) $G = 2$ givers and $R = 2$ receiver. From Figure 8 we see that the analytical results of Section 3 also hold for more than two locations. That is, the price-only contract does not coordinate the supply chain (see Figure 8(b)). However, as illustrated in Figure 8(a), unidirectional transshipments are beneficial for all locations, i.e. transshipment givers and receivers. These benefits are decreasing in demand correlation. Further, they depend on the number of locations that are givers and receivers, respectively. For example, if G increases the profit of i decreases as i's transshipment opportunities increase, while the opposite holds for j. Comparing Figure 8(a) with Figure 4(b) we can directly see the impact of N on the locations' profits for a fixed transshipment price, i.e. the individual profits of givers and receivers increase with N and the number of transshipment givers G. Note that we assume symmetric demand correlation between the individual locations. As we have to ensure that the correlations matrix is positive semi-definite we can only analyse cases with $\rho \geq -0.5$ for $N = 3$.

Our numerical experiments for $N > 2$ also confirm that the coordinating contracts of leftover subsidy, shortage subsidy and a combination of leftover and shortage subsidy might not be beneficial for all location as discussed for $N = 2$ in Section 4. However, also for $N > 2$ we are able to find feasible coordinating contacts which are beneficial for all locations (Proposition 4.10). This is illustrated in Figure 9: for a τ^L around 3 the supply chain is coordinated and taking this contract is beneficial for giver i and receiver j.

7. Conclusion

In this paper, we consider optimal transshipment policies with two locations in a decentralised setting when transshipments can be performed only unidirectional taking into consideration the positive and negative demand dependence between the locations. We analyse the performance of unidirectional transshipments and compare it with the performance of both, a
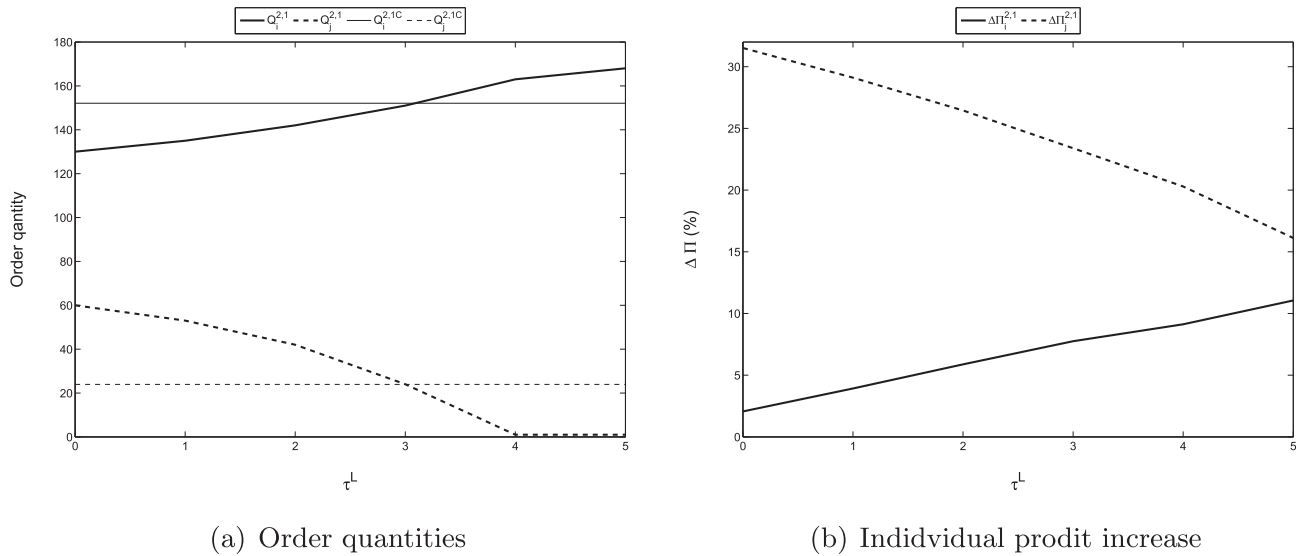
(a) Order quantities   (b) Indidvidual prodit increase

Figure 9. Order quantities and profit increase from contract with a combination of leftover and shortage subsidy varying τ^L, $N = 3$, $G = 2$, $R = 1$, $\tau^S = 10$, $r_t = 19$, $\rho = 0$.

traditional channel and a channel with bidirectional transshipments. In general, we observe that the differences between uni- and bidirectional systems and the traditional channel decrease as demand correlation increases. Further, we find that in comparison with the traditional channel the benefits of unidirectional transshipments can be significant for both the transshipment giver and receiver simultaneously, especially for medium transshipment prices.

For a system with unidirectional transshipments, we prove the existence of unique Nash equilibrium and show that the decentralised system cannot be coordinated by varying the transshipment price. Compared to the centralised system, the transshipment receiver orders too much while the transshipment giver orders too few. In order to enable coordination in this unidirectional setting we propose simple and easy to implement mechanisms by including a leftover and/or shortage subsidy to the contract. A subsidy on leftover inventories of the transshipment giver or a subsidy on shortages of the transshipment receiver allows for coordination in the system, i.e. gaining the highest supply chain profits. Our results show that one of the locations may prefer not taking this contract, especially when demands are highly positively correlated such that the potential benefits of risk pooling are small. We show that coordination can be also achieved through another subsidy contract that combines both leftover and shortage subsidy and that this combined contract is beneficial for both locations and every correlation level. Moreover, we demonstrate the relevance of coordinating unidirectional transshipments, especially when demand correlation is not that high.

Finally, we propose potential extensions to this work. An interesting research opportunity is to study possible mechanisms for fairly sharing the benefits of coordination under unidirectional transshipments. Additionally, other tools, such as options contracts and revenue sharing agreements, can be studied in this setting or a cooperative game theoretic framework could be analysed.

Disclosure statement

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Appendix 1.

Proof of Proposition 3.1 By implicit differentiation of (7) and (8) we can derive

$$\frac{\partial Q_1^u(Q_2)}{\partial Q_2} = -\frac{(r_1 - r_{21} + p_1)g_{D_1^n}(Q_1)}{(r_{21} + s_1)f_{D_1}(Q_1) + (r_1 - r_{21} + p_1)f_{D_1^n}(Q_1)} \tag{A1}$$

and

$$\frac{\partial Q_2^u(Q_1)}{\partial Q_1} = -\frac{(r_{21} - c_{21} - s_2)g_{D_2^g}(Q_2)}{(r_2 + p_2 - r_{21} + c_{21})f_{D_2}(Q_2) + (r_{21} - c_{21} - s_2)f_{D_2^g}(Q_2)}, \tag{A2}$$

where $f_{D_2^e}(Q_2) = \partial F_{D_2^e}(Q_2)/\partial Q_2$ and $g_{D_2^e}(Q_2) = \partial F_{D_2^e}(Q_2)/\partial Q_1$, and similarly for D_1^n. From the definitions of effective and net demand, we know that $F_{D_2^e}(Q_2) \leq F_{D_2}(Q_2)$ and $F_{D_1}(Q_1) \leq F_{D_1^n}(Q_1)$. Using these properties it can be shown that $f_{D_2^e}(Q_2) \geq g_{D_2^e}(Q_2)$ and $f_{D_1^n}(Q_1) \geq g_{D_1^n}(Q_1)$. As a result, it follows that $-1 \leq \partial Q_2/\partial Q_1 \leq 0$ and $-1 \leq \partial Q_1/\partial Q_2 \leq 0$. That is, the slopes of the best response functions are less than one, which is sufficient for a unique Nash equilibrium. $\qquad\square$

Proof of Corollary 3.2   The left-hand side of Equation (3) is larger than that of Equation (7) for all (Q_1, Q_2). Hence, the left-hand side of Equation (7) evaluated at $Q_1^b(Q_2)$ is smaller than zero. Since it is a decreasing function of Q_1 in order to satisfy the equality at zero, $Q_1^u(Q_2)$ has to be set larger than $Q_1^b(Q_2)$, i.e. $Q_1^b(Q_2) \geq Q_1^u(Q_2)$. Following the same line of argument it can be shown that the opposite holds true for Q_2, i.e. $Q_2^b(Q_1) \leq Q_2^u(Q_1)$. $\qquad\square$

Proof of Proposition 3.3   We look at the first-order conditions of the unidirectional system at the optimal bidirectional equilibrium order quantities. For part (i) the left-hand side of Equations (7) and (8) evaluated at (Q_1^b, Q_2^b) turns out to be strictly smaller and greater than zero, respectively. Note that both equations are decreasing in Q_1 and Q_2. Additionally, for (7), the impact of a change in Q_1 is stronger compared to the impact of Q_2, i.e. $\frac{\partial^2 \Pi_1^u}{\partial Q_1 \partial Q_2} \leq \frac{\partial^2 \Pi_1^u}{\partial (Q_1^u)^2}$ which also leads to $-1 \leq \frac{\partial Q_1^u(Q_2)}{\partial Q_2} \leq 0$ as discussed in Proposition 3.1. As a result when Q_1 decreases and Q_2 increases then the left-hand side of Equation (7) increases. A similar analysis follows for (7). Therefore, in order to satisfy the two first-order conditions simultaneously Q_1 has to be decreased, i.e. $Q_1^u < Q_1^b$ and Q_2 has to be increased, i.e. $Q_2^u > Q_2^b$. We can conclude similarly for part (ii). $\qquad\square$

Proof of Proposition 3.4   Note that the left-hand side of Equation (7) evaluated at the optimal centralised order quantities (Q_1^{uC}, Q_2^{uC}) turns out to be greater than or equal to zero, while the same for Equation (8) is smaller than or equal to zero. The rest follows similar to Proposition 3.3, in order to satisfy the two first-order conditions simultaneously Q_1 has to be increased and Q_2 has to be decreased. $\qquad\square$

Proof of Proposition 4.2   It is sufficient to look for the transshipment price and leftover subsidy which would make the two locations order the optimal centralised order quantities. We can conclude by equating the left-hand sides of Equations (11) and (12) with (5) and (6), respectively. $\qquad\square$

Proof of Corollary 4.3   The difference between the profits of location two with transshipments under leftover subsidy and without transshipments is $\Pi_2^L(Q_1, Q_2) - \Pi_2^{NV}(Q_1, Q_2) = (r_{21} - c_{21} - s_2)E(T_{21}) + \tau^L E(L_2)$ which is larger than or equal to zero since we assume $r_{21} \geq c_{21} + s_2$. $\qquad\square$

Proof of Propostion 4.11   When $\rho = 1$, we have $D_2 = aD_1 + b$ with probability one for some constants $a > 0$, and b. For $Q_2 \leq aQ_1 + b$, $F_{D_1^n}(Q_1) = F_{D_1}(Q_1)$ and $F_{D_2^e}(Q_2) = F_{D_2}(Q_2)$ since there is no chance of making transshipments. On the other hand for $Q_2 \geq aQ_1 + b$, there is a positive probability of having transshipments which implies $F_{D_1^n}(Q_1) > F_{D_1}(Q_1)$ and $F_{D_2^e}(Q_2) < F_{D_2}(Q_2)$.

For part (i), the costs are symmetric, or they are asymmetric such that $cr_2 \leq cr_1$, then $Q_2^{NV} \leq aQ_1^{NV} + b$. Then $F_{D_1^n}(Q_1^{NV}) = F_{D_1}(Q_1^{NV}) = \frac{r_1 - c_1 + p_1}{r_1 - s_1 + p_1}$ and $F_{D_2^e}(Q_2^{NV}) = F_{D_2}(Q_2^{NV}) = \frac{r_2 - c_2 + p_2}{r_2 - s_2 + p_2}$. This implies, the first-order conditions (5) and (6), and (7) and (8) are satisfied at (Q_1^{NV}, Q_2^{NV}).

For part (ii), the costs are asymmetric such that $cr_2 > cr_1$ then $Q_2^{NV} > aQ_1^{NV} + b$. This implies that (5) and (7) are negative and (6) and (8) are positive at (Q_1^{NV}, Q_2^{NV}), i.e. Q_1 should be decreased and Q_2 should be increased to satisfy the first-order conditions.

When $\rho = -1$ it is never simultaneously possible that $F_{D_1^n}(Q_1) = F_{D_1}(Q_1)$ and $F_{D_2^e}(Q_2) = F_{D_2}(Q_2)$. We always have either $F_{D_1^n}(Q_1) > F_{D_1}(Q_1)$ or $F_{D_2^e}(Q_2) < F_{D_2}(Q_2)$, or both inequalities hold at the same time. The rest follows as for part (ii). $\qquad\square$