INSTRUCTIONAL STRATEGIES FOR HIGH SCHOOL ELL REFUGEE STUDENTS IN MATHEMATICS

by

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DEDICATION

To my parents, thank you for your everlasting love.

To my sister and brother, thank you for your encouragement and support.

To my husband, thank you for your love and patience.

To my friends, thank you for your smiles and kind words whenever I needed.

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ABSTRACT

This study investigates instructional strategies used in high school mathematics classrooms to learn the way teachers modify their lessons to teach late-entry ELL students, including refugees with little or no formal education. Two separate ELL classes, two mainstream classes with mixed groups of ELL students and monolinguistic students, and one mainstream class without late-entry ELL students are observed for this study. Both qualitative analysis regarding teaching English language while using mathematics content, specifically vocabulary acquisition, and quantitative analysis as to how teachers spend their instructional time are used to present the findings of this study. The findings provide a forum for a further conversation with respect to ELL-oriented modification to serve late-entry ELL students' unique academic needs and the discussion on appropriateness of separate ELL mathematics classes.

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LIST OF ABBREVIATIONS

AP	Advanced Placement
AOA	Ague of Arrival
BICS	Basic Interpersonal Communicative Skills
CALP	Cognitive Academic Language Proficiency
DOE	(New York City) Department of Education
EL	English Leaner
ELL	English Language Learner
ESSA	Every Student Succeeds Act
L1	Frist Language
L2	Second Language
NCES	National Center for Educational Statistics
NNS	Non-Native Speaker
NS	Native Speaker
NYSED	New York State Education Department
OCR	Officer for Civil Rights
SIFE	Students Interrupted Formal Education
SLA	Second Language Acquisition
TL	Target Language

CHAPTER ONE: INTRODUCTION

The current ELL (English Language Learner) population in the classrooms of the United States is growing significantly, yet the importance of providing appropriate instruction that meets their needs still tends to be neglected (Thomas & Collier, 2001). ELL students must be granted access to the highest quality of education possible, as President Obama stated on December 10, 2015, “every child, regardless of race, income, background, the zip code where they live, deserves the chance to make of their lives what they will” while signing the Every Student Succeeds Act (ESSA) (U.S. Department of Education, 2015, December, 10). Back in 1951, Oliver Brown, a Topeka, Kansas resident, sued the city school board because his eight-year-old daughter, Linda, had to take a bus to go to a segregated school 21 blocks away even though a Whites-only school was located in their neighborhood. The local judges ruled in favor of the school board (McGrane, 2004). With the Civil Rights movement, many people raised their voice to show the ‘separate but equal’ doctrine established in 1896 by *Plessy v. Ferguson* was unfair. Finally, on May 17, 1954, the Supreme Court unanimously agreed that 'separate but equal' had no place in the education system and reversed the Brown Decision (McGrane, 2004). Since then, more accomplishments in the field of public school education have been made, including "the Elementary and Secondary Education Act of 1965, especially Title I, which together with Title VI of the Civil Rights Act of 1964, "assured equal educational opportunity" (McClure, Wiener, Roza, & Hill, 2008).

However, the work is still not yet complete when it comes to equitable and quality education, particularly for ELL students.

Extensive debates regarding the effectiveness of inclusion versus separation have been taking place in the field of ELL education (Platt, Harper & Mendoza, 2003; Reeves, 2004). Advocates of civil rights "allude to the individual liberties that stem from the Bill of Rights of the U.S. Constitution" with some concerned about the possibility of discrimination by separating students (Platt et al., 2003, p. 107). The Office for Civil Rights (OCR, 1991) stipulates, "discrimination results when a district fails to provide needed services to English language learners" (as cited in Platt et al., 2003). Academic mathematical vocabulary is critical to fully understand mathematical concepts, yet high school ELL students with low English proficiency are still asked to learn high school level mathematics in English. Therefore, these students, who do not have enough formal language skills, need to learn in classes where both English and mathematics are taught effectively. A first step is to investigate current practices in high school mathematics classrooms.

Studying mathematics in English, a foreign language

My experience as an international student from South Korea has assisted me in developing empathy and compassion towards ELL students; I myself encountered similar experiences learning mathematics in a foreign language. However, it would not be fair to compare my own experience to refugee secondary students. I came to the United States voluntarily at the age of 30 with formal education and work experience. I did not have traumatic experiences prior to my arrival in the States. Since I was a non-traditional ELL undergraduate student, it was predictable that I would struggle when faced with the task

of studying mathematics in a foreign language. I remembered very little mathematics from my high school years in South Korea, I had very poor English vocabulary, and I had never used a graphing calculator before I came to the United States. Even with all my challenges, I still enjoyed learning mathematics, but it was frustrating knowing that much of my difficulties were because of my lack of language skills. My personal experience studying mathematics made me think about students' readiness in academic language skills, particularly when it comes to studying mathematical concepts. I learned the importance of using appropriate and precise mathematical vocabulary in studying mathematics as a prospective mathematics teacher in the United States.

Others, like refugee students, have a lack of basic mathematical understanding, sometimes even a lack of arithmetic skills, when they are introduced to secondary mathematics content. Mainstream mathematics classes are very challenged to meet the unique needs of ELL students when students do not have a good foundation in mathematics.

Barrow (2014) claims that it is a myth that ELL students do not struggle with mathematics since it is universal. Inversely, she states that mathematics can be more complex and challenging to many ELL students because it requires studying academic language. Employing thoughtful strategies can help ELLs with learning mathematics. It will be interesting to observe how, and when, teachers teach academic vocabulary in their math classes.

The growth of the ELL population

The National Center for Educational Statistics (NCES) demonstrates a steady increase in the percentage of ELL students in public schools in the United States; “The

percentage of public school students in the United Stated who were ELLs was higher in school year 2014-15 (9.4 percent, or an estimated 4.6 million students) than in 2004-05 (9.1 percent, or an estimated 4.3 million students) and 2013-14 (9.3 percent, or an estimated 4.5 million students)" (NCES, March 2017). Thomas and Collier (2001) predict that 40 percent of the school-aged population will be ELLs by the year 2030 (as cited in DelliCarpini and Alonso, 2014). Previously, large ELL populations were concentrated in a few states, but today almost all states' ELL populations are growing. Thus, American schools have recently been experiencing more cultural and linguistic diversity. One reason for the increasing ELL populations that bring linguistic and cultural diversity into classrooms across various states can be explained by understanding refugees' resettlement. The President, in consultation with Congress, determines the numerical ceiling for refugees each year. According to the American Immigration Council (2015), for Fiscal Year 2016, the proposed ceiling was 85,000 refugees. The numerical ceiling and the number of refugees admitted for the past three consecutive years have been approximately the same. Though there have been some changes in numbers each year, an average of about 70,000 refugees have come to the United States annually since 2009 (American Immigration Council, 2015). The vast majority of refugee students fall in the ELL category; with the increase of the refugee population, our classrooms have become culturally and linguistically more diverse.

Kersaint, Thompson, and Petkova observe that (2009), "Not all ELL students are the same" (p. 22). All ELL students deserve a good education, but the needs of ELL students vary based on their immigration status and their educational experiences in their home countries. Ogbu and Simons (1998) assert the importance of classifying minority

students depending on how they came to the United States. Ogbu's (1978) cultural ecological theory of minority school performance requires understanding the differences between voluntary and involuntary minorities; the same reasoning should to be applied to classify ELLs. In fact, ELLs include, "permanent residents, naturalized citizens, legal immigrants, undocumented immigrants, refugees, and asylees" (Kersaint et al., 2009). For example, some high school ELL students attended school back home, prior to their arrival to the United States, while others students who were born and raised in refugee camps may not have received any formal education.

High School Refugee Students

Socioeconomic disparity between the refugee students and other ELL students is a contributing factor to their poor academic performance. Law and Eckes (2000) write, “An immigrant leaves his homeland to find greener grass. A refugee leaves his homeland because the grass is burning under his feet” (p. 86). The 1951 Refugee Convention defines that a refugee is someone who, “owing to a well-founded fear of being persecuted for reasons of race, religion, nationality, membership of a particular social group of political opinion, is outside the country of his nationality, and is unable to, or owing to such fear, is unwilling to avail himself to the protection of that country” (The UN Refugee Agency, UNHCR. n. d.). Both refugee students and immigrant students are ELLs; however, while economic immigrants choose migration in hopes for a better future, refugees have little choice in the matter. This discrepancy easily extends to their socio-economic gap, which is also an important factor to be considered in the field of education. Many high school refugee students are pressured to have a job to support their

family and thus struggle with their schoolwork because they cannot focus on their learning.

"ELL students are a highly heterogeneous and complex group" (National Council of Teachers of English, 2008, p. 1). While some may focus on only the complexity of the varying levels of English language skills ELLs have, immediate attention needs to be given to refugee students because of the traumatic events they experienced in the past. For example, according to school counselors I met and ELL teachers I interviewed, some of refugee students are still suffering from tragic experiences such as witnessing the death of family members, exposure to violence including physical and/or sexual assault, etc. These students need to have a classroom environment in which they are treated with additional care and sensitivity.

Some refugee students lack the structure set forth within a formal education system. While some attended schools in the refugee camp before arriving in the United States, they are not well prepared to attend public schools in the United States. The new school system, including culture, curriculum, and transportation, may be different from what they are used to. Many of them do not know how to use simple technology such as calculators. Some refugee students may not have note-taking skills. Though they are old enough to attend high school, they may not have the years of experience in schooling that one would have by attending elementary through middle school. Their lack of formal schooling not only affects their adjustment in American schools, but also their learning of school subject content.

Many refugee students may fall into a category of SIFE (Students with Interrupted Formal Education), which is a subgroup of ELLs. While the New York State Education

Department (NYSED) and the New York City Department of Education (DOE) were identifying “a subpopulation of ELL students with distinct needs who face additional challenges in school” in 1996, they found that these ELL students had “large gaps in their education and were therefore significantly behind their peers” (Advocates for Children of New York, 2010, p. 8). “Adolescent SIFEs are the most challenged because they are placed in middle or high school but lack literacy in their first language and have attained low proficiency” (Roy-Campbell, 2012, p. 187).

Some ELL students are just beginning to learn English and some have already developed substantial proficiency in English. (LaCelle-Peterson & Rivera, 1994). We need to be aware that students have distinct academic needs depending on their English proficiency. Cummins (2000) says it is a misconception that there is no need for bilingual education since young students can quickly acquire fluency in the target language (p. 24). Several times in his years of studies, Cummins observed that despite the rapid growth in conversational fluency, it takes at least five years for them to improve their academic language to the same level of native-speakers (2000, p. 16)

High school refugee students who recently arrived in the United States do not stay in the classroom long enough to catch up with their native English speaking peers due to their high Age of Arrival (AOA). Young ELL students might have a better chance to be on a par with their peers in mainstream classes after several years of schooling; however, high school refugee students do not have enough time to reach grade level English skills before their graduation from high school.

Besides the relatively short period of education, high school refugee students have additional issues regarding low proficiency in academic vocabulary in their first language

compared to other ELL students with a high school level education in their home country. According to Cummins' interdependent theory (1979), the proficiency in L1 (first language) plays a critical role in learning L2 (second language); ELLs studying L2 are categorized as proficient, partial, and limited, and ELL students' L1 proficiency substantially impacts their acquisition of L2 skills, especially in academic language that requires cognitive comprehension. He also reports that, students who acquire oral L2 skills can learn academic L2 as long as they have proficiency in L1. High school refugee students with limited to zero formal school experience will struggle to learn high school subject content in English because many of them do not have a chance to reach proficiency in their L1. According to teachers interviewed, many of refugee ELL students have often been exposed to multiple languages; they can use multiple languages for communication, but their vocabulary for abstract concepts or formal language in L1 can be limited.

The Case of Boise

Boise is a refugee-friendly city; the Boise City Council voted to adopt “a resolution highlighting the city’s long-standing role as a welcoming community and a community of refuge for those fleeing violence and persecution from conflicts around the globe” (Sewell, 2017, January 31). “The success of Boise’s refugee resettlement program has attracted international attention,” and a group of professionals, officers, and volunteers from Germany visited Boise to learn how the city has been working with the refugee community, how public schools integrate refugee students and how resettlement offices work to find jobs for refugees (Kruesi, 2016, April 23).

According to the Office of Refugee Resettlement (2015), 817 refugees arrived in Idaho in 2012, and since then, the number of refugees has been increasing. In 2013 Idaho had 920 new refugees and another 978 refugees in 2014. These numbers may look small but they present a great influx of refugees into Idaho considering the ratio of new refugees to the total population is 1 to 1,600. The ratio of new refugee arrivals to total population in Idaho for the year of 2014 ranks third in the U.S. after North Dakota and South Dakota. In Idaho, Boise and Twin Falls are the two host cities for refugee resettlement. Recent Syrian resettlement can be used to show the increasing refugee population in the relatively small city of Boise, Idaho. "118 Syrian refugees have settled in Boise alone, according to the State Department, while New York has only accepted nine refugees, while Los Angeles has only taken in 45" (as cited in Qazvini, 2016). In addition to Idaho being a part of Obama's refugee resettlement program (Qazvini, 2016), Boise's relatively "low cost of living, and long reputation as a welcoming location" bring more refugees into Boise (Hotakainen, 2016, September 16).

Because of this, it provides an excellent opportunity to research ELL classroom environments, especially classrooms with many SIFEs. Most ELL students in high school go to mainstream classes to learn mathematics, and they get additional support such as study skill classes and tutoring programs. However, "the "sink or swim" approach is not effective for newly arrived students who do not have adequate level of literacy in either their native language or English" (Gil & Bardack, 2010, p. 11). Fortunately, some local high schools offer ELL mathematics classes targeting newly arriving ELL students with limited academic English skills, which means students have an option to go to separate ELL math classes instead of mainstream. One school studied offers a two-year program

(It is called “Bridge Program”) while the other school in this study offers four years of separate ELL math classes (It is called “ELCI”, English Language and Cultural Immersion).

Research question

When I visited a local high school for a Service Learning project during my bachelor's degree, the situation I noticed with the refugee students reminded me of my own challenges. This allowed me to have a special connection with the high school refugee students. Several serendipitous encounters with the students inspired me to visit their classroom again as a volunteer for the duration of that academic year. I can understand refugee students' struggles with learning mathematics because I went through a path akin to their own, along which I was required to learn something with inadequate language skills. My interaction with both ELL math teachers and refugee students led me to consider how mathematics should be taught to meet these students’ unique needs. As a graduate student, I began by investigating different teachers' instructional practices in teaching mathematics to high school refugee students who have difficulties in handling formal English language instruction.

This thesis is a case study of three high school math teachers who teach both ELL mathematics and mainstream mathematics classes. I expect that ELL mathematics classes that mainly target high school refugee students with limited English skills are taught differently from mainstream mathematics classes. The following are my research questions.

- How do high school mathematics teachers modify and differentiate their lessons for high school refugee ELL students?

- How do high school mathematics teachers use vocabulary acquisition as an instructional strategy in their class?
- Do high school mathematics teachers spend more instructional time teaching vocabulary in mathematics class targeting ELL students?

CHAPTER TWO: LITERATURE REVIEW

The purpose of this literature review is to provide background and content on three primary components: ELL Education, Language Acquisition, and Math Education for ELL students. In summary, all students, including ELL students with their unique needs, should have accessible and high-quality lessons to be successful in class. Knowing that direct vocabulary acquisition is more effective than indirect vocabulary acquisition (Maki & Chow Voon Foo, 2015), and that ELL students can learn English through content-based classes (Schleppegrell, Achugar & Oteiza, 2004), teachers' roles become critical in teaching both English and mathematics to ELL students. Teachers can link conversational language to formal English in mathematics class. More research about mathematics education for ELL students, especially high school students, needs to be done.

ELL Education

ELL Learning Environment at Schools

Harklau (1999) and Olsen (1997) describe the ELL learning environment as "substandard, limited to low-level, remedial course work meant to compensate for students' limited language skills" (as cited in Callahan, 2005, p. 309). When ELL education focuses on teaching English and delays teaching academic content, ELL students are often moved to the periphery, being both physically and pedagogically neglected in academic discussions and lectures (Adger & Peyton, 1999; Katz, 1999). Katz (1999) also argues that putting students in low track courses or remedial programs

not only restricts students' access to academic and linguistic content but also influences teachers' perceptions of students, causing them to expect little of their students academically when they are placed in such classes (Olsen, 1997, Valenzuela, 1999).

ELL students experience less content coverage compared to mainstream students (Parrish, Linquanti, Merickel, Quick, Laird, & Esra, 2002). According to Callahan (2005), science classes for ELL students are not college preparatory and, instead, are textbook-based lecture based while mainstream classes meeting college entry requirements involve more hands-on laboratory work, experiments, and active exploration. She also notes that only a few ELL students (15% of 355 students) had taken one or more college preparatory science courses. Many researchers find that students enrolled in non-college-preparatory curricula find school boring, undemanding, and disengaging (Oakes & Lipton, 1999); and that low expectations from teachers leads to students' low performance (Chunn, 1989; Fritzberg, 2001; Gamoran, 1989). It is critical for teachers to understand that modification for ELL students does not mean lower expectations for the students because those expectations negatively influence students' performance. The effects that expectations can have on performance—called the Pygmalion effect—has been researched and documented in a variety of fields. (High Expectations, 2013).

Equity in Educational Programs

In order to accomplish educational equity for ELL students, ELL students have: "access to the full range of content knowledge that is valued by the school, community and society," "participation in meaningful interaction with challenging subject matter, with classmates, and with teachers," and "continued development of their native language

abilities to the greatest possible extent" (LaCelle-Peterson & Rivera, 1994, p. 60). ELL programs are ideal to meet students' academic needs if implemented effectively. However, modified instruction for ELL programs often translates to less linguistically and academically stringent classes than mainstream courses. As a result, ELL students may fail to acquire the academic and linguistic competency to meet their grade level standards (Callahan, 2005, p. 306). According to the U.S. Department of Education (2015), ELL students have lower achievement in college and career readiness; "Non-ELs participated in Advanced Placement (AP) programs at a rate that was two-and-a-half times that of ELs (5% and 2% respectively)"; a lower percentage of ELs (65%) than non-ELs (69%) had access to a full range of math and science courses; "In SY 2011-12, 59 percent of ELs received a regular high school diploma within four years of starting ninth grade for the first time" while national average was 80 percent.

Meaningful participation is critical in learning; students retain 90% of what they do and say together, while they retain 10% of what they read, 20% of what they hear, 30% of what they see, 50% of what they see and hear, 70% of what they say. (Stice 1987, as cited in Oros, 2007, p. 295). Advocates for both 'inclusion' and 'separation' ELL programs argue why one is more beneficial for ELL students than the other (Platt, Harper, & Mendoza, 2003). Inclusion advocates argue that we cannot achieve equity in education as long as we separate ELL students from mainstream classes and that ELL students benefit from rich discourse and conversation in mainstream classes. However, many ELL students fear being ridiculed by other classmates and are not engaged in class conversation actively (Duff, 2001). Portelli (2011) writes, "Today, no one would dare argue against inclusive education" (p. 8), however, this does not imply all classrooms are

inclusive nor that inclusive classes meet students with unique needs currently. “Despite concerns that separation is inherently discriminatory,” Platt, Harper, and Mendoza (2003) remind us that “separation of students for specialized instruction is warranted in order to achieve educational goals, provided that service in the separate environment facilitates equal access to the curriculum in a timely and effective manner” (p. 109).

Schools must offer a supporting structure for all students including ELLs. Furthermore, ELL students’ families and community members' encouragement is essential to motivate ELL students to stay in school (Wong-Fillmore, 1990). LaCelle-Peterson and Rivera (1994) claim that ELL students' continued development of their first language benefits both ELL individuals and the nation in a highly interdependent global community. Multilingual community members can be great assets to their nation. Some researchers argue ELL students should continue to receive instruction through their native language in order to develop full bilingualism (Hakuta, 1986).

Content-based ELL Programs

“From the fourth grade onward, sometimes earlier, students are not ‘just learning to read’ anymore, they are being required to read to learn” (Law and Eckes, 1990, p. 147). They must have the requisite reading fluency and skills and the background knowledge required to gather information from a text. Because many of our non-English-speaking students join American schools with poor reading skills and many gaps in the kind of background knowledge that we take for granted with our mainstream students, these requirements pose special problems for teachers of older students. The problems are even more acute for those junior high and high school teachers who specialize in one subject.

Several researchers claim that the educational goal of ELL students should not be restricted to reaching a certain level of proficiency in English, and suggest teaching content rather than waiting for ELL students to reach a minimal level of proficiency (Law & Eckes, 1990; Collier, 1992). ELL students should not wait to learn content in English because it takes longer for ELL students to acquire formal academic English proficiency than social conversational skills, and academic proficiency surges through using English in academic settings (LaCelle-Peterson & Rivera, 1994). DelliCarpini and Alonso (2014) also emphasize the importance of having content-specific language skills in order to participate in meaningful conversation in class.

Hands-on Activities to Bridge the Gap

Even though it can be challenging for ELL students to be successful in school, there are ways in which students can succeed in content area classes. When students follow instructions in class and understand subject matter, the subject matter becomes the "vehicle for language acquisition" (Law & Eckes, 1990, p. 151). The appropriate use of hands-on activities in class bolsters students' engagement and leads to students' learning for both mainstream classes and ELL classes.

Hands-on activities can benefit students, especially tactile learners; tactile learners may end up in low-ability sections partly because many teachers will only appeal to one learning style (O'Neil, 2001, p. 236). ELL students also need to be exposed to several ways of teaching the content since they may not follow the verbal instructions and lectures in classes; hands-on activity can benefit ELL students whether they are tactile learners or not. ELL students, like all students, can also benefit from well-organized activities. As Dewey (1963) said, not all experiences are educative; it is critical for

teachers to use appropriate hands-on activities for their lesson to reinforce students' learning. Since ELL students are still working on learning the English language, it is necessary to create lessons that are accessible to ELL students and to adjust instructions in a way that ELL students can easily follow.

The "Chèche Konnen project" in Cambridge, Massachusetts provides a good example of teaching ELL students effectively by using subject content (Rosebery, Warren & Conant, 1989). The project is designed to teach science to middle school and high school ELL students for a year where the students learn science by "doing" it, like "practicing scientists" (Rosebery, Warren, & Conant, 1989). *Chèche Konnen* means "search for knowledge" in Haitian Creole and the goal of this project is to provide ELL students access to science learning through engagement in authentic science practices. As a result of the Chèche Konnen project, researchers learned that students with no education in science were able to develop analytical skills, which allowed them to use scientific discourse to analyze hypotheses (Rosebery, Warren, & Conant, 1992). As Sylvan (2013) asserts ELL students learn English by using English just like kids learn how to ride a bicycle by riding one. Especially "*late*-entry secondary ELL students have a very short window of time" to learn both English language and academic content, and "the most appropriate place to learn language is in context – and in school, the context is the learning of academic content" (p. 21). Sylvan (2013) claims that actively "doing" is essential and "watching" or "waiting" is not good since it does not give ELL students an opportunity to use language actively.

Important Role of Teachers

Mainstream teachers may not have enough understanding of ELL students' academic needs or a sufficient sensitivity toward ELL students due to the lack of their professional development in ELL education (Batt, 2008). Even if more professional development opportunities are offered to in-service teachers and prospective teachers, it will take a considerable amount of time for the majority of mainstream teachers to develop the understanding of ELL education required to allow them to differentiate their lessons for ELL students.

It is important to provide accessible content to ELL students by differentiating lessons, however, what is no less important is that teachers have a positive relationship with ELL students. “A relationship with a single teacher is unlikely to determine a student's entire academic trajectory, but a series of negative relationships cannot help but diminish a student's academic self-concept” (Callahan, 2005, p. 308). Negative or indifferent attitudes from even one teacher may negatively influence not only ELL students’ academic success but also their successful adjustment in a new country.

Teachers' Roles in Teaching Vocabulary

Teachers should be aware of ELL students’ backgrounds to create lessons that meet their needs. Teachers should use explicit vocabulary instruction and connect new words to students’ prior knowledge and experiences (Haynes & Zacarian, 2010). ELL students make connections among symbols, words, and meanings in other languages and cultural contexts (Crandall, Jaramillo, Olsen & Peyton, 2002). In light of his research, Stahl (1986) concludes that teaching words directly could substantially improve text comprehension. Stahl’s three principles of vocabulary acquisition are: give both context

and definitions, encourage “deep” processing, and give multiple exposures. Since teachers have limited time to teach, they must make a series of choices regarding what vocabulary they need to emphasize, which leads to the need to narrow down the number of words. Stahl (1986) suggests teachers could skip some words if the words are not applicable to either of the following questions, “Is it important for understanding the text?” and “Is it a word the student is likely to run into again and again?” Teachers need to make good choices on both what has to be taught and which methods are appropriate depending on their students’ level.

Challenges faced by ELL Teachers and Mainstream Teachers

Platt, Harper, and Mendoza (2003) find that ESL/bilingual teachers are increasingly assigned to mainstream classrooms in Florida districts. Formerly self-contained ESL classroom teachers become resource persons who support curriculum teachers, spend less time working with students directly, and become a “jack-of-all-trades” in school, dealing with other duties such as “record keeping regarding compliance with the Consent Decree, staff development, tutoring training, or curriculum adaptation” (p. 128). Platt et al. (2003) are concerned that ESL teachers' curricular and methodical expertise is either lost or diluted for distribution to the general educators who often lack fundamental knowledge in ELL education and how to implement their understanding in class.

According to Menken and Antunez (2001), fewer than one-sixth of teacher preparation programs offered specific course work related to working with ELLs to mainstream teachers (as cited in DelliCarpini and Alonso, 2014, p. 159). They also state that only five states (Arizona, California, Florida, New York, and as of 2011,

Pennsylvania) have adopted teacher certification requirements including explicit coursework for teaching ELLs. As DelliCarpini and Alonso (2014) say, "every teacher is an ESL teacher" since many mainstream teachers have ELL students in their classes; however, they are not prepared for teaching ELL students (p. 159).

While 42 percent of teachers had ELL students in their classroom, only 12.5 percent of teachers received more than eight hours of professional development for teaching ELL students (Harper & Jong, 2004) and seventy-seven percent of content-area teachers had no coursework or professional development for teaching ELL students (DelliCarpini & Alonso, 2014). Above all, what is most concerning is the apathy amongst teachers. We have a continuously increasing population of ELL students, but schoolteachers are not well prepared to teach them in their mainstream classes. According to McKinney (2008), several researchers echo that overall teachers take a favorable view of inclusion unless they don't work directly with the included special education students (Lee-Tarver, 2006; McGregor & Vogelsberg, 1998; Scruggs & Mastropieri, 1996; Youngs & Youngs, 2001). Relatedly, Karabenic and Noda (2004) link the similar tendency in ELL education; "teachers held mostly favorable attitude regarding ELLs in the classroom, yet did not want to have the ELL in their own classroom" (as cited in McKinney, 2008, p. 53). Since ELL students spend about 80% of school days with mainstream class teachers (Dong, 2002), mainstream teachers' awareness of ELL culture and attempt to modify their lessons for ELL students are essential for ELL students' academic success.

Efforts have been made to educate mainstream teachers about ELL education. For instance, "State university teacher education programs in Florida have begun to prepare

all new teachers for the roles of ESL/ bilingual specialists" (Platt et al., 2003, p. 128). Davison (2001) highlights the "tension between the philosophical base of the ESL field which emphasizes diversity and complexity, and the demands of the mainstream educational agenda for commonality, simplicity, and homogeneity" (p. 29). ESL classes need to provide challenging subject matter effectively and mainstream classes need embrace to ELL students' special academic needs. ELL teachers worry their roles might be neglected by mainstream education reforms, and mainstream teachers struggle with teaching ELL students whom they are not prepared to teach.

Teacher Collaboration

DelliCarpini and Alonso (2014) argue that "it is no longer acceptable for teachers of other subjects to have little to no knowledge of the issues related to the education of ELLs" and contend the importance of teacher collaboration (p. 175). ELL students' needs are met better when collaborative practice between English language teachers and content teachers occurs (Wertheimer **&** Honigsfeld, 2000). Teacher collaboration has a variety of formats: co-teaching, co-planning lessons, sharing lesson plans and materials, professional visits to each other's classroom, expert consultations, and small discussions among teachers during lunch break, and even a small chat in the hallway (DelliCarpini & Alonso, 2014).

Teacher collaboration is challenging because of time, the culture of isolation, teacher positioning, and ESL teachers' knowledge of content (DelliCarpini, 2009), however, what is needed most is a conceptual shift of teachers. Mainstream teachers need to change the way they think about collaboration with ELL teachers. A high school biology teacher commented; "I am not exactly sure how I could work with the ESL

teacher in my school since she doesn't know science" (as cited in DelliCarpini & Alonso, 2014, p. 171). Mainstream teachers' appreciation of ELL teachers' fundamental knowledge and understanding of ELL education seems to be indispensable.

One of the challenges ELL teachers face to facilitate teacher collaboration is their lower status**.** Arkoudis (2006) reports that teaching ELL "is perceived as being lower in the subject hierarchy of the school" because ELL is positioned as "a strategy-driven and does not have the same authority as subjects such as mathematics and science within the secondary school curriculum" (p. 417). She argues that this institutionalized positioning of the subject needs to be changed to develop collaborative practices between teachers, and emphasizes the importance of training of ELL teachers regarding how to gain "epistemological authority within the mainstream curriculum" (p. 429). Teacher collaboration should take place in a way to benefit both English language teachers and content teachers.

Co-planning followed by co-teaching might be very challenging. However, it is still worthwhile because it benefits both students and teachers. Daisey and José-Kampfner (2002) share a great example of successful collaboration between math and language art teachers by examining collaborative projects combining math instruction, writing, and storytelling. At an urban American school, recently emigrated students from Puerto Rico and Mexico studied biographical stories about successful Latina mathematicians and engineers. To ensure students' understanding, the goal of the final project is to build "student self-esteem through expanding the range of available role models" (p. 579). This storytelling activity plays an important role in student engagement; moreover, this example shows the strength of teacher collaboration in ELL

education and the positive role teachers have in influencing students' attitudes about gender roles as well.

DelliCarpini and Alonso's study (2014) shows an example of teacher collaboration between mathematics/science teachers and ELL teachers. All 25 in-service or pre-service STEM teachers and 8 ELL teachers who teach mathematics or sciences in a sheltered program for high school level have a positive attitude towards the teacher collaboration, however, they don't know how to engage in such practices at the beginning of the study. These 33 in-service or pre-service teachers took graduate level course work that is co-developed and co-taught by a TESOL professor and a mathematics education professor. The content teachers show optimistic changes in beliefs and knowledge related to working with ELLs in the mainstream classroom and ELL teachers understand better in their role of integrating language and content effectively.

Language Acquisition

Vocabulary Acquisition

Second Language Acquisition (SLA) refers to learning a language subsequent to learning their first one (L1) as young children, and to the process of learning that language. Although the additional language (L2) may be the third or fourth to be acquired, the term 'second language' is still used to describe the target language (TL) (Saville-Troike, 2006). There are two ways to learn a target language: informal L2 learning and formal L2 learning. For example, if individuals are exposed to an L2 speaking environment daily, they will 'pick up' their target language, which is considered informal learning. Formal L2 learning takes place in the classroom through specialized language instruction.

Children's language acquisition has sparked arguments over nature versus nurture; after a long debate, most researchers agree that language acquisition occurs "through interplay of biology and environmental factors," however, the way nature and nurture coordinate to influence this complex process is still open to research (Mahoney, n.d., para 2). The term, 'Language Acquisition' can mean either the natural development of language or the process of 'discovering' it depending on the researchers' intention in research about L1. Researchers need to take time to define the terms they use and use the terms carefully in second language research (Foster-Cohen, 2001). Some researchers (Katz, 1981; Bever, 1983) used the term 'acquisition' to contrast the word, 'development' since they see children's vocabulary acquisition as a process of nurture (as cited in Foster-Cohen, 2001). Krashen's (1985) dichotomization of the two terms, 'acquisition' versus 'learning' is also used to support researchers in second language acquisition who argue about the importance of defining and using appropriate terms. Krashen (1985) distinguished learning from acquisition; learning is conscious in SLA (Second Language Acquisition) while acquisition is unconscious (Saville-Troike, 2006; Foster-Cohen, 2001). The two terms, learning and acquisition, have the same meaning in SLA in general and the two terms are used interchangeably. The field of education for L2 is "labeled SLA (second language acquisition) rather than SLL (second language learning) or SLD (second language development), without the intention to restrict it to the logic-of-acquisition perspective" (Foster-Cohen, 2001).

In this paper, the term 'vocabulary acquisition' refers to a direct instructional strategy, which implies the view that individuals can learn a language through specialized language instruction.

Vocabulary

The core vocabulary in every language includes function words, a limited set of terms that carry primarily grammatical information; *the, that, this* as determiners, *to, of, for* as prepositions, *and, but* as conjunctions, *I, it he, she, you* as pronouns, and *is, was, be, have, has, had* as auxiliary verbs. Compilations of the fifty most commonly used words in English including *yeah, oh, it's, that's don't, know, like,* and *think* are also core vocabulary (McCarthy &Carter, 1997). Vocabulary is the component of language involved with words and their meanings, and two words, vocabulary and lexicon, are used interchangeably. "Vocabulary is the most critical level of L2 knowledge for all learners to develop, whether they are aiming primarily for academic purposes or interpersonal competence" (Saville-Troike, 2006, p. 138). Vocabulary acquisition pertains to how people expand the number of words they understand when they learn a new language, and this includes both first and second language acquisition.

BICS and CALP

Basic Interpersonal Communicative Skills (BICS) and Cognitive Academic Language Proficiency (CALP) are important to understand why many ELL students still struggle in classes even though they seem to have good daily conversation skills (Cummins, 2000). The initial distinction between CALP and BICS came from discussions with school psychologists who were concerned about potential bias; they pointed out the implications of conflating conversational fluency in English as L2 with proficiency in English academic language (Cummins, 1984). According to Cummins (2000), "The term (conversational language) refers to nonacademic language and usually is the first type of language acquired by second language learners" (as cited in Coggins,

Kravin, Coates, & Carroll, M. 2007, p. 1). While Cummins argued that this misunderstanding between language proficiency in daily conversation and the ability to handle formal academic English causes some ELL students to prematurely exit their ELL programs, Callahan (2005) drew attention to the opposing situation; she warned that ELL students can get stuck in a "vicious cycle" if the requirements to exit are too rigid (p. 306).

Direct Vocabulary Acquisition

There are two types of vocabulary acquisition; direct learning happens when students are taught vocabulary for a specific purpose, while indirect learning occurs by naturally hearing or reading new vocabulary in their daily lives (Haynes and Zacarian 2010). One of the misconceptions about ELL students' learning is exposure and interaction will result in their learning (Harper and Jong, 2004). Haynes and Zacarian (2010) claim that ELLs do not learn much vocabulary from indirect learning such as hearing or reading during their daily life since English is not spoken often at home and it is difficult to understand much of the daily conversation around them with their limited English skills. When ELL students were taught for a certain purpose using direct vocabulary acquisition, they were able to learn new words faster. Though there is no single best method of vocabulary instruction identified (Beck and McKeown, 1991), “many methods to increase vocabulary knowledge have resulted in more words learned than otherwise occurred during normal incidental learning opportunities” (Baker, Simmons, & Kameenui, 1995, p. 16).

Behaviorist theory of second language acquisition expects “ELLs will make mistakes in their second language based on the previously learned patterns in their first

language"; Repetition and pattern drills as instructional strategies based on behaviorist theory are very advantageous, however, other methods need to be used as well in order to learn complex structures in sentences and communication abilities. (Carr et al., 2009, p. 27) Both the acquisition of L1 and the acquisition of L2 involve constructive and social processes (Krashen, 1985; Long, 1985, Snow, 1977, Vygotsky, 1978). Creative Constructionist Theory is based on the application of the innatist theory of first language acquisition to second language learning, Krashen (1982) has developed a theory of second language learning consisting five of hypotheses: acquisition-learning hypothesis, monitor hypothesis, natural order hypothesis, input hypothesis, and affective filter hypothesis. Krashen's Input Hypothesis (1982) assumes that we learn a language by understanding messages, which is comprehensible input. The formula i + 1, input plus one, means that a language learner can learn by exposure to a target language when he/she has known some structure of the language already.

In contrast to Krashen's Input Hypothesis, Swain's Output Hypothesis (1985) asserts a language learner needs to be pushed to produce output to learn a language; output is part of the learning mechanism, not just producing what they have learned. Swain argues that comprehensible input alone is not enough to learn a language, and emphasizes the importance of a learner's awareness of what he/she knows already and what he/she does not know. The recognition of his/her restrictions occurs when he/she fails to remember linguistic forms or structures that are needed to produce output in a target language. Swain claims, when a language learner experiences their linguistic deficiencies, he/she may try to fill the gaps actively (as cited in Russell, 2014, p. 26). Krashen (2009) publishes a follow-up to his work (1982); it is critical to put forth an

effort to inform learners about the process of language acquisition for their continuous improvement on their own instead of suggesting the use of a form of deception which students acquire vocabulary or learning subject matter as a result of getting comprehensive input without knowing the subconscious process.

Long's Interaction Hypothesis (1985) emphasizes the importance of oral interaction and communication in SLA. The input-interaction model theory amalgamates aspects of Krashen's Input Hypothesis and Swain's Output Hypothesis and proposes that language acquisition occurs through interactions between either native speakers (NS) and non-native speakers (NNS) or between NNS and NNS (VanPatten & Williams, 2015). According to interactionist theory, NS "can be responsive to the needs of language learners by modifying their language with the explicit goal of making it comprehensible" and NNS can "negotiate their own comprehension by asking for clarification or repetition" (Carr et al., 2009, p. 29).

According to Gass and Torress (2005), "There is no theory or approach to SLA that does not recognize the importance of input" and there has been rising interest in the role of integration in SLA; they investigate the effect of input and interaction, more precisely, effectiveness depending on the order of input and interaction. The group in which students have a one-on-one interaction with a researcher followed by in-class input session shows a significant learning effect. Since the opposite order, 'input + interaction' group does not show any significant learning effect, the effectiveness of 'interaction + input' group is not the result of repeating the content using two different approaches. Besides, materials and procedures for two groups were identical except for ordering the treatment sessions.

According to the Critical Period Hypothesis, learning language becomes much more difficult and effortful after passing an ideal time window, linked to biological age, to acquire language in a linguistically rich environment. Older L2 learners need more structured methods and lessons in learning L2. Young children are able to learn a second language without explicit instruction, while adults need explicit instruction and much guided practice. Adolescents fall somewhere in between (Carr et al., 2009, p. 27)

Traditional Vocabulary Acquisition

The traditional, and common, way of teaching vocabulary acquisition is based on teacher-led instruction where the teacher explains definitions of words and gives some examples of how the words can be used in sentences. There is no research saying that the method of vocabulary acquisition for native English speakers does not work for ELLs, however, several researchers found that teaching words from a list of dictionary definitions is not effective (Haynes and Zacarian, 2010). Learning the meaning of a definition can be challenging, even for some native English-speaking students. This kind of learning will be especially difficult for many ELL students because they might not have enough vocabulary to comprehend the definitions of new words. “Vocabulary teaching methods which gave only definitional information about each word’s meaning did not appear to significantly improve comprehension, nor did methods which only gave students one or two exposures to the word” (Stahl, 1986, p. 665).

Alternative Instructional Methods for Vocabulary Acquisition

Baker et al. (1995, p. 19) share some alternative vocabulary teaching techniques such as semantic mapping/features analysis (Bos & Anders, 1990), keyword (Baumann & Kameenui, 1991), and the computer-assisted method (Reinking & Rickman, 1990) in

their analysis of vocabulary acquisition. Semantic mapping involves taking words that children know the meaning of and adding to them to-be-learned words. This can be done using a map diagram, physically displaying the relationships discussed. For example, teachers might discuss concepts such as “alone” and “lonely” first, then discuss how the word, “solitude” is being alone without being lonely (Stahl, 1986). Semantic mapping/feature methods have students predict the relationships among concepts or to answer cloze-type sentences using a relation matrix.

The keyword method has students construct a visual image to connect to the target word. This visual image helps students recall the vocabulary in the future. Pressley, Levin, and McDaniel (1987) used the word “carlin” as an example of using the keyword method by relating the word to the visual image of an old lady driving (as cited in Baker et al, 1995). ‘Driving’ helps students remember part of the spelling of the word, car-, and the image of the old lady represents the meaning of the word, as carlin is a noun meaning an old lady.

Computer-assisted interventions seem to be attractive for three reasons: less direct teacher time than teacher-led instruction, the potential to individualize instruction, and the potential to systematically integrate important instruction within a lesson framework such as systematic review or instructional scaffolding (Baker et al., 1995). One example of computer-assisted intervention is students learning vocabulary by using computer program through pre-test, lessons about words they should learn, and reviews (Johnson, Gersten, & Carnine, 1987). Computer-assisted intervention in the form of computer-mediated texts provide students immediate access to the definition of words on a computer screen (Reinking & Rickman, 1990).

All the alternative methods of vocabulary acquisition are more effective than normal incidental learning opportunities (Baker et al., 1995). However, there is not evidence that alternative methods and techniques are more effective than traditional methods, which include oral recitation and memorization of concise context-related meaning. A single best method of vocabulary instruction has not been identified (Beck and McKeown, 1991).

Timing for Vocabulary Acquisition within Lessons

Vocabulary Acquisition in ELL classes is essential. However, when determining the most appropriate timing during the lesson, opinions start to diverge. According to Haynes and Zacarian (2010), there are two opposite opinions as to pre-teaching vocabulary; "those in favor of it feel that the rhythm of a lesson is broken if vocabulary words are explained during the reading of text, whereas those against it feel that they are teaching out of content if they introduce new vocabulary before the lesson begins" (p. 56). Haynes and Zacarian support the former; in light of the reality that ELL students might not be able to understand the lessons without knowing vocabulary and key concepts, their argument may seem to be reasonable. However, some have different views. For example, Coggins et al. (2007) claim that "front-loading vocabulary" is not as productive in mathematics and argue that math classes are different from language arts classes, therefore, "going over new key vocabulary at the beginning of a math lesson is usually not productive" (p. 25). Mathematics vocabulary words often represent the main idea of an entire chapter or unit of study; thus, it would be difficult to teach vocabulary itself without introducing mathematical concepts students are about to learn. However, getting familiar with vocabulary such as spelling and/or pronunciation of a word at the

beginning of the lesson can still helpful for ELL students' learning and reviewing vocabulary at the end of the lesson can reinforce their learning. Timing of teaching vocabulary should be flexible depending on the nature of the subject and a group of students in class.

Trusler (2016) summarizes a team discussion among educators regarding three different approaches to teaching vocabulary in a timely manner at 2016 TESL Ontario conference in Toronto. The first approach, teaching vocabulary at the beginning of the lesson, has several advantages: "student can focus on vocabulary alone and then focus on reading," "the reading is easier to understand," "there is less of need to stop and look up words," "students have a handy list for later review," and "there is more exposure to new words" (para. 3). The benefits of the second approach, teaching vocabulary during the lesson include: "students may be able to infer the meaning of new words from context," "students learn a keyword as a specific part of speech in a sentence," "students realize they don't need to know every word to understand the gist of reading," and "they can apply this to reading material outside of the classroom where there won't be a list of vocabulary words" (para. 4). The third approach is saving all vocabulary discussion and tasks until the end of the lesson. Four benefits result from teaching vocabulary at the end of the lessons: "the vocabulary task might make more sense because students have seen it in context," "student can see how well they inferred the meaning of new words from context by completing a task and getting a score," "they will see which words they know well and which need more practice," and "they can go back and see the word in context to help them if they still can't work out the meaning" (para. 6). Trusler and other educators' conclusion is having the best of all three approaches.

Math Education for ELL students

Mathematics in ELL education

Mathematics is an under-researched topic in ELL education, perhaps because of a misguided conception that Math is less difficult for ELL students because it is based on a language of numbers (Janzen, 2008). Language is a critical issue in teaching mathematics; most of the content is taught through oral language, students do not gain a significant portion of their knowledge from reading textbooks, and teachers usually do the majority of the talking in classrooms (Veel, 1999, as cited in Janzen, 2008).

Both students and teachers should use math language, and teachers' instructions need to assist students to progress from daily conversational language to the more academic mathematics language (Scheppergrell, 2007). This means that mathematics educators who teach ELL students need to have a good understanding about the linguistic characteristics of the target language and have techniques that assist ELL students in linking daily conversational English and formal mathematics language (Ron, 1999). Ron (1999) also mentions that several mathematics terms have different meanings in daily conversational English, explaining why teaching mathematics vocabulary in class is important; "learning mathematical language can be only acquired in school and not through conversational interaction" (as cited in Janzen, 2008, p. 1017). Khisty and Viego (1999) suggest ELL math teachers use mathematics terminology consistently and clearly and require their students to use math vocabulary in the same way, and state that this is the way to promote mathematical thinking processes.

Linking Conversational Language and Mathematics Vocabulary

Ms. Wilson's class introduced in Coggins et al. (2007) shows a great example of how math classes could be taught to expand students' academic vocabulary. Her students are asked to sort out shapes on the table; while they are categorizing shapes, students describe the given shapes using their own words. For example, a parallelogram can be seen as a racing car, a circle can be described as a baseball, and students realize that not every shape is pointy or has a straight line. Instead of learning the definition of a polygon and the characteristics of each given shape, students have a chance to talk about what they think about each shape and discuss how they should categorize them first. Ms. Wilson uses appropriate mathematics terms, but also encourages discussions with daily conversational words. Ms. Wilson is conscious of the language that she uses and anticipates possible breaks in understanding. Ms. Wilson's example demonstrates so many important teaching strategies that build daily conversational English as well as mathematical vocabulary and concepts at the same time. Coggins et al. (2007) write "math and language goals are intertwined" (p. 7). If math teachers were to work in conjunction with English teachers to create class activities and assignments based on shared lesson materials, this may better help the students fully understand both English and Math.

ELL students are expected to learn "everyday language" and "academic language" in English at the same time. Conversational language plays an important role in the mathematics classroom (Coggins et al., 2007). People can learn better by participating in discussions. If the students do not understand the teacher's instructions, a hands-on activity is nothing more than "playing." Thus even math teachers need to be

able to teach English to their students. Coggins et al. (2007) introduce a list of sample sentence structures students can use to participate in class discussion.

> "I notice that…"
>
> "I agree with (name) that there are…"
>
> "I'd like to build on (name's) idea…"
>
> "I don't understand what (name) meant when she said…" (p. 8)

When teachers create an atmosphere where students share their thoughts in class, students can use mathematics vocabulary appropriately and their conversations **re**inforce their learning mathematics content.

Additional Advice for ELL Mathematics Teachers

Coggins et al. (2007) claimed, "it's not a good idea to emphasize key words in math problems" (p. 26). It might be beneficial to emphasize some key words in a reading context, but in reading "key phrases such as 'in all,' 'how many', and 'less,' students often fail to focus carefully on the meaning of a story problem, on the mathematical structure implied and what operation actually makes sense" (p. 26). Some mathematics teachers have their students highlight certain key words in math problems, however, Coggins et al. (2007) warned that "A word focus also leads to a general lack of individual skill at interpreting story problems, identifying the underlying mathematical relationships, and making plans for a solution." We should distinguish the difference between keywords and mathematical vocabulary. Both need to be addressed in math classes for ELL students.

Summary of Literature Review

The effort to ensure equity in education has been made through the Bilingual Education Act in 1968, mandating that schools provide bilingual education programs, the No Child Left Behind Act (NCLB) in 2002, and the Every Student Succeeds Act (ESSA) in 2015. Schools, researchers, and educators' efforts to have enhanced programs for ELL students have continued. ELL students learn English language effectively via direct vocabulary acquisition, exposure to English is not sufficient. Direct vocabulary acquisition is needed because knowing precise words concretizes learning new concepts, deeper conceptual understanding, and more effective communication (Allen, 1999; Haynes & Zacarian, 2010).

Sheltered English instruction and content-based ESL are the two most common models in ELL education. In a sheltered program, English speaking teachers used diverse adaptation in their teaching for ELL students who are not yet fluent in English but who can grasp grade-level content standards and concepts, and ELL students are expected to have enough English skills to join a mainstream class in a short period of time, one year. On the other hand, content based ELL program provides separated ELL classes where content learning and language development are combined. (Alvermann, Phelps, & Gillis, 2007, p. 43 - 45). The effort to seek an improved ELL model that works well for each school and district should be continued incessantly because of the complexity and diversity in ELL population schools experience these days.

CHAPTER THREE: METHODOLOGY

There are two local high schools providing separate ELL math classes while other high schools provide only mainstream math classes for ELL students in the Boise area. Grand High School and Baker High School attract many refugee students since they offer classes aiming to meet ELL students' unique academic needs. Both schools' separate ELL math classes are full of newly arriving refugee students with limited English skills and lack of basic study skills. Baker High School has a two-year separate ELL program for newcomers. On the other hand, Grand High School has a full four-year separate ELL program as an option. Investigating three mathematics teachers' classes and learning how they teach high school ELL students, including refugee students with lack of academic English, is my focus in this research.

The following are my research questions.

- How do high school mathematics teachers modify and differentiate their lessons for high school refugee ELL students?
    - How do high school mathematics teachers use vocabulary acquisition as an instructional strategy in their class?
    - Do high school mathematics teachers spend more instructional time teaching vocabulary in mathematics class targeting ELL students?

Participants

Data was collected from two local high schools in Idaho with a large population of refugee students. Both schools selected for this research were good representatives of

high schools with a high population of ELL students, or more precisely, refugee students. Baker High School has 191 registered ELL students that are refugees; this does not include a handful of refugees that came when they were young and are not considered ELL. Other high schools in the same school district have 79 and 54 respectively. Grand High School is located in a neighboring district and is well known for having the largest ELL refugee student population in that district. Both schools' refugee population is large enough to be provided with ELL mathematics classes that deliver content-based instruction modified for ELL students. All of the students in ELL mathematics classes were high school refugee students who recently started attending schools in the United States; they did not go to elementary and middle schools in the United States. Baker High School offers a two-year separate ELL program while Grand High School offers a four-year program. Many newly arriving high school refugee students go to one of the two schools. Students may transfer to other schools if they so desire.

The participants for this study were three high school mathematics teachers from the two different schools. Initially, emails were sent to invite math teachers who teach ELL students to participate in this research project, then face-to-face conversations followed to provide more details about the research. Three mathematics teachers in two districts were selected for data collection.

Alex had three ELL mathematics classes and three mainstream mathematics classes at Grand High School. Bob taught three ELL mathematics classes and three mainstream mathematics classes at Baker High School. Cody taught mainstream mathematics classes with a mixture of native English-speaking students and ELL students at Baker High School.

Some ELL students in Cody's mainstream math classes had studied in Bob's separate ELL math classes, while some had not previously attended any separate ELL math classes. The name of the two high schools and three teachers are pseudonyms, and the three teachers' gender is treated as male in an effort to ensure confidentiality and to avoid readers' gender bias.

Table 1. Classes to be observed

	Mainstream math class	**Separate ELL math class**
Alex at Grand High School	No ELL students, all native English speaking students	All ELL students (Four-year separate ELL math class)
Bob at Baker High School	Mix of native English-speaking students and ELL students	All ELL students including refugee students who recently came to the United States (Two-year separate ELL math class)
Cody at Baker High School	Mix of native English-speaking students and ELL students	

Data Collection from Class Observations

This research is a case study of three high school mathematics teachers teaching ELL students including refugee students; observations of the five classrooms (in Table 1) were conducted to collect data on how each teacher's instruction actually occurred in

their classrooms. The main focus of this research was to investigate how teachers modify and differentiate their lessons to teach this at-risk population. Teaching mathematics vocabulary is necessary to teach mathematics; close observation about what, how, and when vocabulary acquisition takes place was carefully recorded as data. Instructional time was coded to find if more instructional time for vocabulary acquisition was provided in ELL math classes than in mainstream classes.

The instructional time for each of the five classes participating in this research was audio-recorded on two separate days. These audio-recordings were used to determine how much instructional time was spent on vocabulary acquisition in math classes for refugee students, what they were taught, how they were taught, and when extra vocabulary acquisition occured for students with limited English skills apart from learning typical mathematical concepts and mathematics vocabulary. Some pictures of the teachers' work on the whiteboard or Smart board were taken. Field notes were taken during the observation to support the audio-recordings, which were transcribed. Table 2 is a field notes template.

Table 2. Field note template

Time	Activity	Note / Comments	"additional instructional time for teaching English" OR "teaching mathematical vocabulary"

The goal of the class observation was to investigate how the participants taught their classes and to see if they made any changes for their ELL mathematics classes; the comparison of these teachers and the comparison between their mainstream math class and their ELL math classes were noted. Both qualitative and quantitative analysis took place. How participants modified their lessons for ELL students was analyzed qualitatively while how much additional instructional time spent for vocabulary acquisition for English language learners was analyzed quantitatively. In the process of transcribing the audio-recorded lessons, the time spent by activities was measured and calculated as percentages of overall classtime. There were five codes used to classify classroom activity: Announcement & Class Routine, Teaching, Break, Quiz, and Individual work. The data from each observed lesson were used, and two consecutive lessons were combined to determine the percentage of each category and the

subcategories in teaching time. For example, Alex's first lesson for ELL math class and Alex's second lesson for ELL math class were combined to calculate time spent by activities (in seconds) to get the percentage of each category.

The teaching time was coded into five subcategories: EL-friendly vocabulary Acqusition, EL-friendly Modification, Teaching Math Content, Other, and Interruption and Discipline. Any effort to make a lesson more EL-friendly was considered either EL-friendly vocabulary acquisition or EL-friendly modification. From the interviews with the three teachers, slow pace and repetition were expected in separate ELL math classes. EL-friendly vocabulary acquisition was the time spent by a teacher using any ELL teaching strategies, techniques or any extra effort to teach new English words or mathematics vocabulary to ELL students. Using mathematics vocabulary that students need to learn for their math lesson or giving just the definition of a term was not counted as a modification for EL-friendly vocabulary acquisition; it was counted as teaching Math Content. The EL-friendly modification category was the time spent by a teacher revisiting any basic math skills or concepts that most high school students are expected to know from their previous schooling. Some teachers spent extra instructional time teaching certain content or skills in ELL math classes which most American high school students would know already, and teachers attempting to revisit such "easy" content or teach it for the first time in an ELL class would be counted as an EL-friendly modification. The observed lessons were in the middle of curricular units, so students learned some new words while reviewing several vocabulary words they learned at the beginning of the unit they were studying. The category, Other, was still a part of the instructional time but the teacher and students had either a conversation or activity that

was not related to their learning. The last subcategory for teaching time was Interruption and Discipline; this was time where the lesson was interrupted by students' misbehavior or a teacher had to discipline the class or an individual student in the middle of his instruction.

While analyzing Alex's class conversation data qualitatively and quantitatively, it was evident that he used flash cards and warm-up problems at the beginning of the lesson for both his mainstream and ELL math classes, and this activity captured a considerable amount of his teaching time. It would be absurd that his flash card and warm-up activity fell into either EL-friendly category or teaching mathematics content solely. The way he utilized flash cards and warm-up problems and the nature of those two tools were considered a combination of EL-friendly approach and teaching math content. As such, a new category named Flash Cards & Warm-up was deemed necessary for Alex's classes.

Data Collection from Interviews with Participants

Initial interviews took place before the observations in order to understand the teacher's background, such as: how long they had been teaching mathematics; how long they had been teaching ELL students in schools; how they were assigned to teach ELL students; if they saw themselves primarily as secondary math teachers and/or ELL teachers when they were teaching ELL students. Also, any additional comments they had about mathematics education for ELL students were recorded.

The second interview occurred after the observations to learn if, and how, the participants modified their lessons, what kind of activities they used to improve students' English skills, and what their greatest challenges in teaching ELL students were. The following interview questions were written by the researcher. The questions are used for

interviews are provided in Appendix A. Most of the questions were open-ended, and the interviews were audio-recorded and transcribed. The participants might have their own opinions on inclusion and separation in ELL education. There might have been dissimilarity between the participants' views regarding separate ELL mathematics classes and what their school district or school ELL program implements. Nonetheless, it will be meaningful to hear from these teachers who work with these students closely.

CHAPTER FOUR: RESULTS

Introduction

Initial interviews were done before the observations of the three high school mathematics teachers, Alex, Bob, and Cody, to understand their background. A second interview followed the class observations to learn their thoughts on how they modify or differentiate their lessons to help ELL students with learning mathematics and improving their English skills and their challenges in teaching this at-risk group.

Teacher Alex at Grand High School

Alex started as a paraprofessional who worked closely in math classes with ELL students who recently arrived in the United States. He is a certified high school teacher in both ELL education and secondary math and accepted a proposal by the school district to teach separate ELL math classes. This is his third year of teaching mathematics at Grand High School, where he currently teaches both separate ELL mathematics classes and mainstream classes with mixed groups. He is excited about the upcoming academic year when he will exclusively teach separate ELL math classes. Alex does not ask any questions that might be personal, such as how ELL students came to the United States, but he states that at least half of the 50 ELL students he is teaching this semester are refugee students. His ELL students are newcomers to the United States; their English skills and/or formal education backgrounds are very different from other ELL students who have been living in the United States for a while. Students in Alex's class are the most at-risk population at Grand High School when it comes to English skills and

mathematics skills. He thinks his job is teaching both mathematics and English so that eventually his ELL students can become successful in the United States, and teaching English using mathematics content is a part of this job. Alex would like to teach only ELL students if he has a choice, because "it is challenging, fun challenging, and very rewarding." He doesn't see any single greater challenge than teaching high school ELL students, because, he says, "there is a different challenge every day, but that's what makes things interesting!"

Teacher Bob at Baker High School

Bob is the most experienced teacher among the three teachers. He has been teaching 16 years in a various range of students' age, in a few different states, and half of his teaching career is mainly with ELL students. He is certified in both ELL education and mathematics. He is teaching about 115 ELL students, 75 percent of which came to the United States as refugees, while the rest are immigrants. His eventual goal of educating ELL students is "teaching his students regardless of subject area or age." He states that teaching his ELL students is very rewarding and emphasizes the importance of serving each ELL students' academic needs; one of his suggestions is to keep options open for them such as a separate mathematics class for only ELL students and an inclusion for ELL students who want to join mainstream classes. Bob mentions that the challenge is not teaching ELL students itself, and states "it actually is outside of themselves, some of them are politics that surrounds them, which is hard to overcome." Bob is not interested in teaching mainstream classes; he will teach only separate ELL mathematics classes next academic year and hopes to teach only these classes in the future.

Teacher Cody at Baker High School

Cody has been teaching mathematics for five years and experiencing inclusion of ELL students in general/mainstream math classes for four years. Thirty percent of his students are ELL. Half of them came to the United States as refugees and have had formal schooling experiences at a public high school for at least two years, and the other half are students of Hispanic descent who are learning English. His goal of teaching ELL students is helping them become successful in the United States. He is a certified mathematics teacher who happens to have a large group of ELL students. This is because his mathematics classes are relatively less challenging than other mathematics classes at Baker High School. He says teaching ELL students is "difficult but rewarding; there is a lot of correlation between ELL and Special Education students." Cody likes inclusion classes with a mixed group instead of having ELL students in separate classes since physical separation hinders ELL students' opportunities to expand their social group. He also mentions that "it is sad to watch students struggling with conceptual stuff due to the lack of formal education."

Overall Approach

This section discusses the three teacher participants' overall approaches to teaching newcomer ELL students. Their opinions about ELL education in general and their thoughts on modification and differentiation of their lessons, if any, to help ELL students with learning English and mathematics are presented.

Three Teachers' Opinions on ELL Education

All three teachers, Alex, Bob, and Cody, strongly agree that ELL students with limited English skills need to learn mathematics and English simultaneously. They think

teaching English first and having them wait to learn mathematics in English later in their academic journey is not ideal. Both Alex and Bob say ELL students with limited English skills benefit more in separated ELL classes than in mainstream classes while Cody says he is "indecisive", arguing that ELL students do get more help from separate ELL class, but the mainstream class has larger benefits. Alex says, "If students work with ELL students in class or help them, inclusion is better." However, that doesn't happen often. Bob thinks it is important to "have the flexibility within the district, at school, and in the classroom because there is no one right answer" to which model is better between inclusion and separation; however, he emphasizes that separation might be better especially "if you get through emotional layers and the language separation." Even though Cody supports the inclusion model, he restrictively advocates the inclusion of ELL students with "higher functioning students." However, he acknowledges the reality is that ELL students typically join mainstream mathematics classes with many low-performing students.

There is a discrepancy in opinions on ELL students' readiness to join mainstream classes. Alex points out that it really depends on ELL students' level of formal education before they come to the United States; however, he doesn't think they would be ready to jump into mainstream classes if they came to "a brand-new country with hardly any English as a 9^{th} grade student, it is just too big gap to join mainstream classes." Alex says, "It is all combined," since the lack of an educational background of some ELL students negatively affects their basic number sense and basic mathematics skills, they would have a hard time learning mathematics in mainstream classes. After a few years of

learning English, their English is good, but they may have missed out on learning mathematical concepts, and they tend to struggle and lag behind in mainstream classes.

Bob says, “Absolutely not, there is no way you can catch up to fifteen years of education in two years.” He mentioned “two years” since Baker High School offers only two years of separate ELL math classes to newcomers. Bob thinks the main hindrance to his ELL students’ learning is language, and people cannot learn anything without communication (knowing the language for the communication). He also adds that “communication might have been hindered by academics, by their brains, or by their heart; sometimes it’s the language of love they need.”

Cody is more positive on this; he thinks ELL students would be ready to join mainstream classes if they had been challenged in prior separate mathematics classes. He doesn’t think English language is the main hindrance because “students pick up English fairly well.” As Alex says, Cody also thinks the main hindrance for some ELL students who come to the United States as refugees is the lack of formal education.

Teachers’ Strategies in Differentiating Their Lessons for ELL

All three teachers think that modification is needed to teach ELL students in mathematics classes. Alex’s first response is “slower pace” and Bob also starts with, “My main and only modification is to go much slower and repeat multiple times.” Cody does not mention a slower pace, which is understandable since he is teaching general mathematics classes even though 30 percent of his students are ELL students.

Alex says that he teaches the same level of mathematics curriculum as he does in general/mainstream mathematics classes. However, Alex seems to put a zealous effort into having his teaching material and content accessible to his ELL students: he crosses

out sentences that are too wordy and boils them down to basics, he crosses too difficult mathematics questions off from their handouts or homework, and he replaces complex English with a vocabulary that is more accessible for ELL students. Alex also mentioned that he uses visual aids in class such as flash cards and graphic organizers. Bob teaches several vocabulary words, both math words and daily conversation words. Cody focuses on using mathematics terminology and uses story problems/application problems. Cody states that he highlights certain terms and tries to use different colors in writing hoping this helps ELL students with following his instruction.

Three Teachers' Vocabulary Acquisition in Mathematics Class

As stated during the initial interview, Alex and Bob consider themselves as both mathematics teachers and English teachers to their ELL students, while Cody identifies himself as their mathematics teacher. During the second interview, all three teachers claimed that it is very important to teach vocabulary in mathematics classes. Bob says, "Without vocabulary, using numbers only to do mathematics is against Common Core. And it is also against common sense." Both Alex and Bob say they teach new vocabulary and phrases all the time and repeatedly, while Cody says he usually starts with teaching new words at the beginning of the lesson, but new words can be taught at any time through the class.

Alex says he creates flash cards and uses them in class to highlight difficult or academic words, and then talks about them instead of just giving the definitions. Often Alex's students actively ask the meaning of words in class. Alex also has students create a 'graphic organizer' whose content varies according to the current lesson. During the interview he showed a few samples of graphic organizers containing the mathematics

keywords “PEMDAS”, for teaching the order of operations, and the definition of several terms used to describe polygons and other topics from geometry. Alex says he tries to find creative ways to organize mathematical concepts and new vocabulary to help his ELL students. Additionally, Alex shared a class activity in which ELL students fix any errors of word problems written by native English language speaking students in his general mathematics class. Sometimes ELL students are asked to write a word problem that is suitable for a simple math function, and the class works together to correct any errors on the whiteboard.

Bob creates a ‘word search’ puzzle at the beginning of each new unit, hoping ELL students familiarize themselves with the spelling of the words they are learning. Bob also encourages students to translate new English words into their own native language to have an anchor to their vocabulary. He uses target vocabulary repeatedly in class and tries to use all the synonyms of a new word. Bob also mentions that he uses gestures in class and has his students use some physical movement as well; one example he provided about the usages of gestures is moving their arm or index finger to show the idea of slope in a linear function. To present slope, ‘rise over run’ in Cartesian coordinates, students move their finger vertically to show the change of y-values, ‘rise’ and move their finger horizontally to show the change of x-values, ‘run’.

Cody says, “I teach vocabulary like a brand new language, almost they are like a baby.” He intentionally uses new vocabulary in conversations in class repeatedly so that the students can surmise the meaning of the words. He also mentions that he tries to give visual references by writing the word he is using on the whiteboard. Cody also mentions the importance of using proper terms in class and cites ‘numerator and denominator’ as

an example; he says, “top number and bottom number in fractions should not be used since that’s the ones students call forever instead of using appropriate terms.”

Activities to Help Improve ELL Students’ English Skills

Alex says he allows his ELL students to talk more than what he would normally accept in class, and encourages them to ask any questions. He also has students work in groups or in pairs so they can talk to each other in English.

Bob says he tries to build ELL students’ confidence by having them read English aloud in class. Bob says, “I wish I would allow them to work with each other more, but there is bit of control and trust issue. I don’t want students to help each other with full confidence that they are doing it right when, in fact, they are doing it wrong. And misleading their peers unknowingly, you know, causing great harm.”

Cody says, “Honestly, I don’t do much else to help them with English, but I talk with them.”

Teachers’ Suggestions to Improve ELL Education for New ELL High School Students

Alex thinks it is very important to educate high school teachers about ELL students’ situations and their needs. He suggests teachers should get an ELL endorsement or take some classes about ELL culture.

Bob thinks high school ELL students who recently came to the United States need to stay a full four or more years in high school instead of rushing to graduate in a few years. He points out that ELL students’ and American students’ graduation plans need to be different.

Cody shared a story about one of his ELL students during the interview; his student, who is a newcomer to the United States, was placed in his mathematics class,

and it turned out the content he teaches in class is too easy for her since she had taken several advanced mathematics classes and was very close to graduating from high school in her home country. Cody states appropriate placement for individual students is essential, and he proposes that teachers, administrators, and staff should help ELL students with finding classes that best suit their level.

Modification in Practices

Qualitative data analysis from a total of ten class observations of the three teachers is used to ascertain how they teach high school level mathematics to students with limited English skills.

Alex's Graphic Organizer

Ives and Hoy (2003) write, "Using an appropriately modified graphic organizer to teach higher-level mathematics skills may help students with relatively weak verbal skills and strong nonverbal reasoning skills to be more successful in mathematics" (p. 36). Alex shared his example of using a graphic organizer. Many of his students still struggle with simple word problems even after they have become very confident at adding and subtracting numbers. Alex thinks it is important to teach them several different ways to describe the four basic arithmetic operations using words. He uses a graphic organizer as part of a class activity; each student makes his or her own graphic organizer. Alex provides a blank piece of paper, and the students need to complete the blank cells of the table they create with the four operation symbols. First, Alex teaches keywords, and then the students create their own graphic organizer using these keywords. Figure 1 and 2 are what students are given at the beginning of this activity. Figure 2 is the material Alex teaches. Figure 3 is what each student would have at the end of this activity.

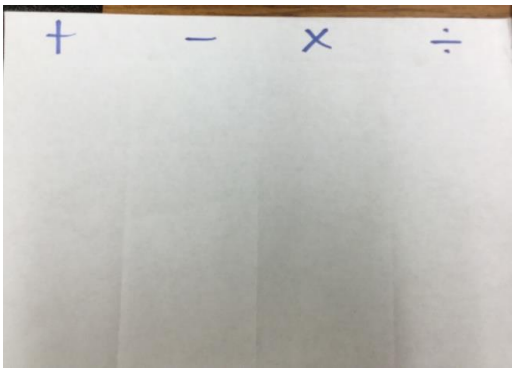
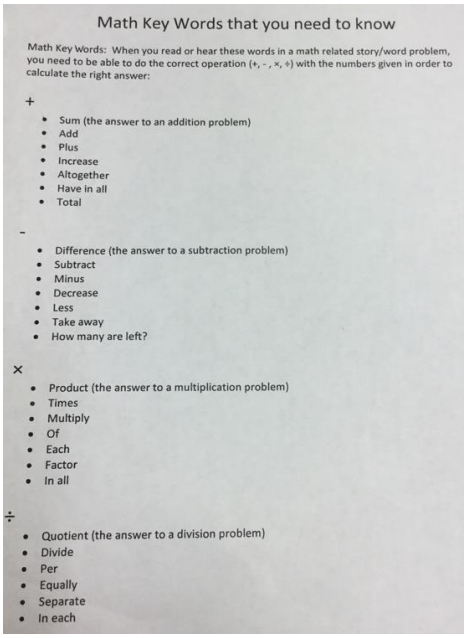


Figure 1. Folded blank paper that students use to create their own a graphic organizer



Math Key Words that you need to know

Math Key Words: When you read or hear these words in a math related story/word problem, you need to be able to do the correct operation (+, -, ×, ÷) with the numbers given in order to calculate the right answer:

+

- Sum (the answer to an addition problem)
- Add
- Plus
- Increase
- Altogether
- Have in all
- Total

-

- Difference (the answer to a subtraction problem)
- Subtract
- Minus
- Decrease
- Less
- Take away
- How many are left?

×

- Product (the answer to a multiplication problem)
- Times
- Multiply
- Of
- Each
- Factor
- In all

÷

- Quotient (the answer to a division problem)
- Divide
- Per
- Equally
- Separate
- In each

Figure 2. Key words handout provided

+	−	×	÷
• Sum (the answer to an addition problem) • Add • Plus • Increase • Altogether • Have in all • Total	• Difference (the answer to a subtraction problem) • Subtract • Minus • Decrease • Less • Take away • How many are left?	• Product (the answer to a multiplication problem) • Times • Multiply • Of • Each • Factor • In all	• Quotient (the answer to a division problem) • Divide • Per • Equality • Separate • In each

Figure 3. Key words in mathematics that students need to learn in Alex's class

Alex showed another example of using a graphic organizer when using the abbreviation, 'PEMDAS' to teach the order of operations during the interview (Figure 4).

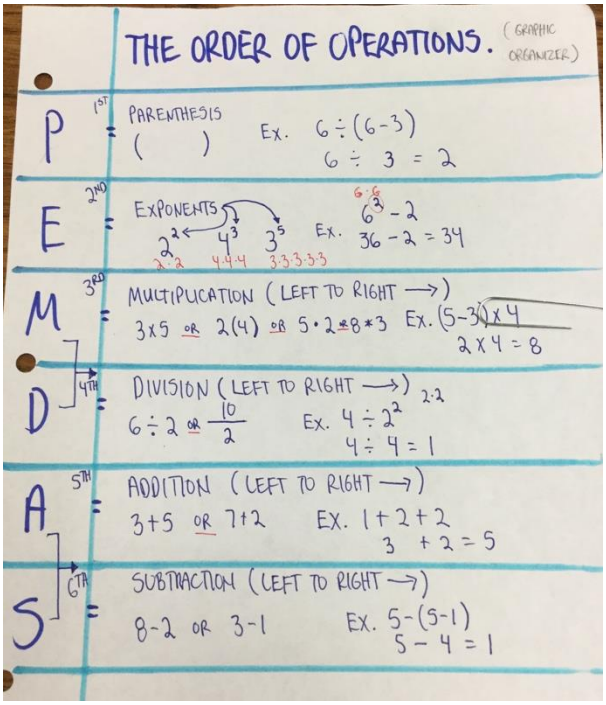


Figure 4. graphic organizer for the order of operation

The last example of Alex's graphic organizer is used in his Geometry unit. As Figures 5 and 6 show, students divide the paper into 8 rows and fold it in half. The front page is used for definitions of mathematics vocabulary, and the inner page contains the vocabulary term. The rows are cut so that the tabs can be revealed individually as in Figure 6.

As seen in Figures 5 and 6, the first row of the front page says *a quadrilateral with four congruent sides*. The student can read this and quiz himself or herself and then flip the tab to check the answer, *rhombus*. Alex also encourages the students to highlight important words or phrases. The back side of this graphic organizer also contains the summary of their lessons such as perpendicular lines, characteristics of similar triangles, the symbol of congruence, etc., so that his students can make further use of the blank spaces of the graphic organizer.

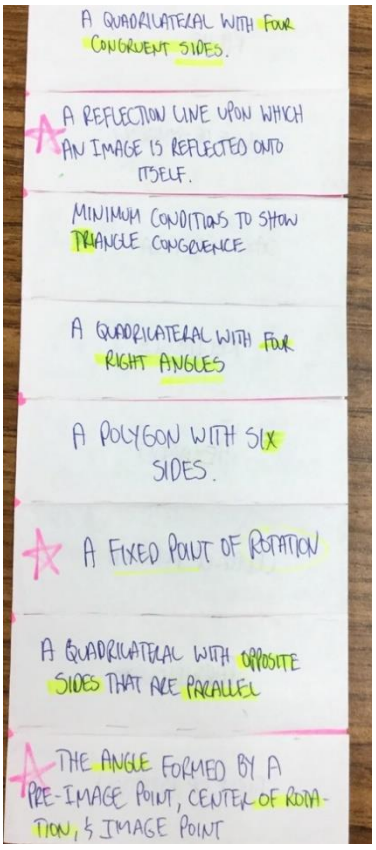


Figure 5. the front page of a graphic organizer for geometry lesson

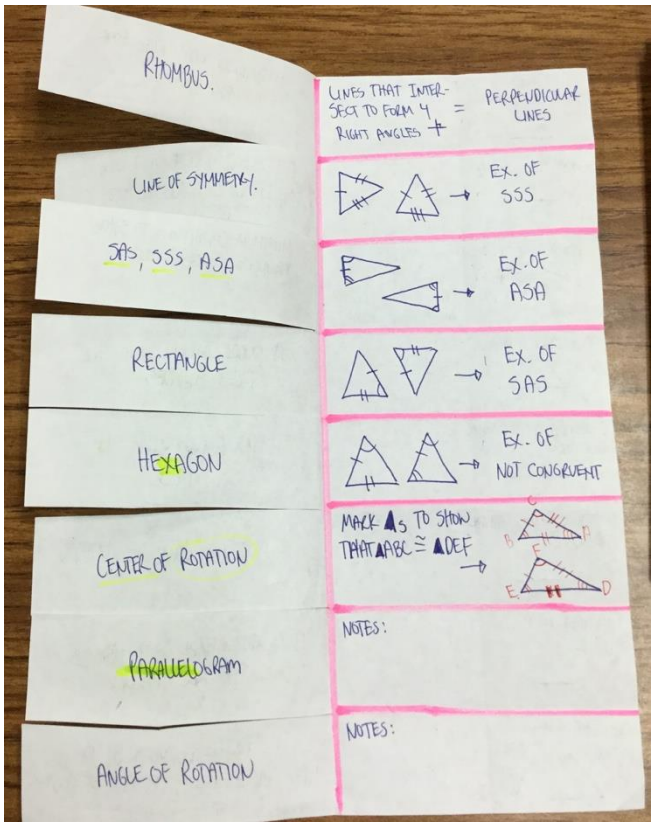


Figure 6. the inside of a graphic organizer has mathematics vocabulary of each definition

Alex uses graphic organizers to teach new mathematics vocabulary and to summarize the key points of the lessons, which seem to be very effective in his class. Another use of graphic organizers is a tool to give a kind of intervention for new ELL students who are still learning basic English while they are currently enrolled in high school mathematics classes. Alex uses graphic organizers to teach his students new words and help them with memorizing some basic facts in mathematics.

Discussion of Alex's Graphic Organizer

Karp, Bush, and Dougherty (2015) indicate the problem of rules many teachers use to teach mathematics in class. They exhort mathematics teachers to reconsider the use of a key word approach to solve world problems and the use of 'PEMDAS' to teach the order of operation. According to Clement and Bernhard (2005), "the use of key words subverts mathematical understanding, can lead to incorrect solution… because this method has limited utility, it can also prevent students from making appropriate generalization" (p. 364). However, a key word approach is still taught in many math classrooms. Keywords seem to work well in elementary classes, however, students start experiencing confusion when they enter into a higher level of mathematics with more complicated application problems. Many key words are commonly used in daily conversation, and students are expected to know how to use them in context. Students need to learn how to comprehend the context of word problems instead of memorizing the list of key words and applying the operation corresponding to their key word chart. However, ELL students require acquisition of basic mathematics key words before they reach the level of reading comprehension to think about why an overgeneralized key

words approach fails with some word problems. Therefore, Alex's graphic organizer for math keywords is properly selected for his ELL students.

Many students memorize the mnemonic phrase, 'Please Excuse My Dear Aunt Sally', and recall the order of operations: parentheses need to be done first, exponents need to be taken care of, and then multiplication, division, addition, and subtraction. As Karp et al. (2015) suggest, teachers should be careful about teaching this abbreviation; students need to understand they can do division before multiplication, and subtraction can be done prior to addition. More importantly, the first letter P of PEMDAS needs to be considered as a grouping symbol, and parentheses are one of many other math symbols such as "brackets, braces, square root, and horizontal fraction bar." There have been some changes in middle school and high school math classes regarding how to teach the order of operation without using PEMDAS; however, PEMDAS has been used for many years in math classes and is helpful for many students. Alex's graphic organizer is a memory aid, and his ELL students can learn the acronym, PEMDAS without memorizing the mnemonic phrase.

Math teachers can utilize graphic organizers not only to teach new words or simple facts in math but also to guide students in learning higher levels of mathematics, especially ELL students. Traditional instruction relies heavily on verbal instruction, which is not always the most effective way to teach ELL students. The advantage of using graphic organizers is that they "rely on visual/spatial reasoning skills more so than conventional teaching approaches" (Ives & Hoy, 2003, p. 41). According to Fisher, Frey, and Williams (2002), graphic organizers "provide students with visual information that complements the class discussion or text" and the students at a local high school

"consistently reported that the graphic organizer is the most helpful strategy" and their teachers also noted their students' vocabulary knowledge became "increasingly transportable across content areas" after using graphic organizers for vocabulary instruction (p. 71-72).

While ELL education is definitely not a part of Special Education, they do have something in common: students' individual needs should be served (accommodated), and special considerations must be made for their unique needs. Currently, Common Core emphasizes the importance of cultivating students' ability to verbalize their reasoning and knowledge in mathematics. However, many ELL students don't have enough academic English to demonstrate or articulate their knowledge in mathematics using words.

"Visual mathematics is not important only for some students – struggling or so called "visual" thinkers, nor is it only a prelude for abstract mathematics – visual mathematics is important for everyone, at all levels of mathematics" (Boaler, Chen, Williams & Cordero, 2016, p. 6) Graphic organizers are also effective for teaching higher level mathematics such as algebra. Ives and Hoy's (2003) work on using graphic organizers to teach how to solve systems of three linear equations with three variables deserves careful reading from mathematics teachers, especially those whose students are mainly ELL in high school level mathematics classes.

Alex's Geometry Lesson

Alex starts his lesson to review formulas and vocabulary by using flash cards, and students are asked to raise their hands to answer. Sometimes, students mumble or seem to have trouble pronouncing a word; Alex is patient enough to wait until they try first, then he repeats the words so everyone can hear them from him again. He not only repeats

words, but he also restates the meaning of the words, so students have enough review on targeting words. He uses ‘a straight line' and ‘linear’ together.

Alex: What does this symbol mean? Raise your hand.

Student: Congru… congruent.

Alex: Congruent.

Students: Congruent. (Everyone repeats after Alex)

A: What does congruent mean?

Student: They are the same.

Alex: Yes, they have the same or equal measurement.

After spending a few minutes reviewing vocabulary they have been studying, students get a scratch paper or small index card to do warm-up problems. Alex asks questions and students write down an answer on the paper; this is a daily routine, and it is an opportunity to review what they learned in the past. Students are asked to draw symbols such as ∥, ⊥, ≅ to show ‘parallel, perpendicular, and congruent.’ Alex says, “It looks like a straight line, looks like a number, eleven.” A student says, “It looks like a house?” and Alex replies, ‘Yeah, one of them kind of looks like a house, and one of them looks like a wave, maybe?” They review several formulae as well: slope formula, slope-intercept formula, standard formula, point-slope formula, distance formula. Even though there are many formulas, they quickly review using flash cards in advance; most students seem to have no trouble remembering each formula. When students seem confused, Alex reminds them of formulas by using an interesting nickname. ‘Rainbow thing’ is used for slope formula, and ‘a big square root thing’ is for distance formula.

After twenty minutes of lessons, the students have a four-minute break. Students seem to understand they get a second and third break when they work hard during the lesson. Alex mentions the possibility of a second and third break when they are still chatty and not getting back to their seat right away after Alex rings a small class bell.

Students are asked to state what a given quadrilateral is first, then prove or justify why a given quadrilateral is either rectangle, parallelogram, or a rhombus. Alex guides them to show how to justify two given straight lines are parallel to each other or perpendicular to each other by using their slopes in advance. The following images are from Alex's work for the class. Alex says, "My recommendation, find the slope, find the slope." Students seem to know how to find the slope, and they know when the slopes are the same, the two lines are parallel to each other. Figure 7 displays examples of the work Alex and his ELL students show.

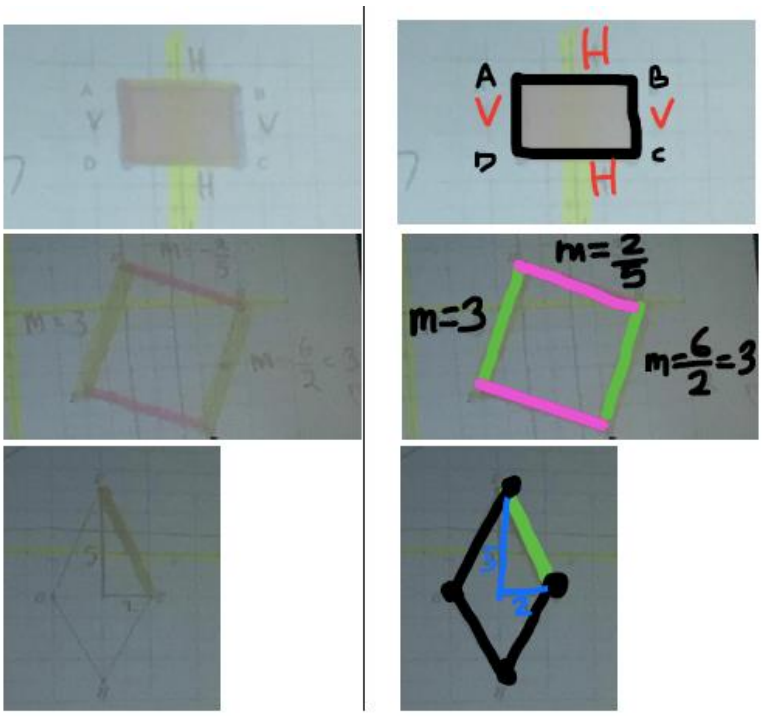


Figure 7. examples of Alex's class work and redrawn image for cliarity of piture

Alex's class progresses through a series of questions: "What shape this time?" "How do we know it's a rectangle?" "Are these blue lines horizontal or vertical?" "What does horizontal mean?" "How do we know they are right angles?" "If you put a rhombus and a rectangle together, what would you get?"

Alex asks the students to find the slopes of four sides of a given figure and reminds them of the word, negative reciprocal (opposite reciprocal).

Alex: Look at the slope of the green, look at the slope of the pink. What do you notice? They flipped and switched. Negative.

Student: Positive?

Alex: Negative re…?

Student: Recursive!?

Alex: Recipro…?

Students: (Laughing)

Alex: I know that's a hard one. That's why I want you to try it. Negative reciprocal. They flipped and switched. Well, if this flipped and switched with that, what is that?

Student: Ninety degrees.

Alex and his students draw a conclusion that the given figure is a rectangle because the slopes of green lines and pink lines are negative reciprocals of each other, which is evidence that a pink line and a green line intersect at 90 degrees. Students work on several quadrilaterals, and are challenged to prove or justify why each quadrilateral is a rectangle, a rhombus, a parallelogram, or a square. The student handout Alex uses to teach this lesson is attached in Appendix B, C, D, and E.

On the second observation day, Alex asks the students to find a slope of a line segment when two points on the line are given. When students say ‘negative’ when they were supposed to say ‘minus,’ Alex takes this as an opportunity to talk about the difference between negative and minus.

Alex: This is a good conversation. Listen very carefully. Negative one minus two. I know, I am not telling you to do the math, I am telling you how to say it. When it’s a number, you say, “a negative one.” When it’s in the sentence, you say, “minus two.” It’s not a negative one, negative two. It’s negative one minus two. Okay, what is the answer?

Student: Eight.

Alex: Remember, it’s a number.

Student: Negative.

Alex: Not minus.

Appendix F is the handout Alex uses in class. Alex talked about ordered pairs, and linear and exponential functions; they studied this several months ago, and questions 1, 2, and 3 below are used to review. Question 4 is used to review what they learned in their previous lesson. There is no new content introduced or taught; students review the same lessons using flash cards and a few example questions. Half of the class meeting time was used for taking a quiz, and the copy of the quiz is provided in Appendix G and H.

Discussion of Alex’s Geometry Lesson

Alex asked multiple questions to guide students to use the vocabulary they have been learning. He patiently waited to hear what students say, his students tried to

remember how to pronounce words. They shared a good environment where students seem to feel comfortable to try even though they are not certain about their own answers and/or their pronunciation of a new word they learned recently. Even when one of the students made a mistake, he was supposed to say 'reciprocal,' but he shouted another vocabulary they learned in the past, 'recursive' instead. Students laughed, but everyone was smiling even the student who made that mistake. The word 'congruent' was on the tip of their tongue, but everyone, including Alex, is patient until one student finally said the word successfully. This is one of the strong points of having a separate ELL math class as Alex mentioned during the interview; everyone is struggling with the English language.

Alex's questions required short answers and aimed to see if his students were following the instruction or to check if they remembered what they had learned. Thus, there were not long or deep conversations. It wouldn't be easy to maintain a long conversation regarding math with the students who have limited English. His lesson was very well organized and planned to guide the students to learn mathematics by connecting their algebra lessons several weeks ago and the current lesson in geometry.

Bob's Pythagorean Theorem Lesson I

Bob informs the students that he is going to teach a new, important, geometry lesson. Bob teaches this advanced lesson as extra to this group of students in a separate ELL mathematics class while the students in other separate ELL mathematics classes won't have this lesson because there are many brand new ELL students. The lesson is on the Pythagorean Theorem. In this particular class, Bob invests enough time to introduce new words to his ELL students. Instead of starting a lesson with $A^2+B^2=C^2$, he starts with

how to read the word by breaking it down into four syllables and has the students repeat after him several times. Once students are confident saying "Pythagorean," he introduces Pythagoras as a Greek mathematician in ancient history.

Bob: Okay, how do I say this? Break it down by syllables. Py-tha-go-rean. Four syllables. Pythagorean.

Students: Pythagorean. (Repeating after their teacher)

Bob: Pythagorean. Okay. This, when you say this part, it's almost like, it's like uh, Pua., like Puh, and then, tha, like, go, re-an. Pythagorean. Okay. Pythagorean. Okay, let's talk about this. Let's talk about Pythagorean.

Bob: So, Pythagoras. This guy, his name was Pythagoras.

Student: It's a name?

Bob: It's a name of a man. He lived hundreds (thousands) of years ago. You can probably Google it. And he was a Greek mathematician. And, back, you know, before, in history, when there was no phone, no internet, no cars, right? People sat around and thought. They did a lot of math. They thought about religion, right, so, when you have a lot of time, you think about things, right? And you try to solve problems, so, math, the reason Pythagoras came up with this formula is because when you have a building, and you're trying to build, let's say you're trying to build a house. Okay, this house is very tall, and it has many floors, how do you get up here?

Student: Stairs.

Bob: There's no elevator. There is no electricity. Stairs, you have to build stairs, right? And what if you build stairs, and they are not long enough. And you need to get up here, but you build stairs that are too short. That's bad, right? Then you waste time, and you waste wood, you waste stone, you waste a lot of resources. Okay, so, in history, when people need a thing, they would do all the math before they would build. You cannot build unless you have math, right? So, he had to find a way, to find out, what, what would the length of these stairs need to be so that I could go there? Okay. And do you put stairs way out here to go there? No, you would have to put the stairs, maybe like here, and reach up there, right? So, this part of the triangle is very special. These two parts here are also very special. And there is a relationship between those three sides, what angle is this?

Students: Ninety degrees.

Bob: Always. Right? Always. So, you have a wall. You have the floor, and this is right here, it is always ninety degrees. Okay. What type of triangle is that called?

Student: Right triangle.

Once students have practiced the pronunciation of a new word multiple times, Bob also asks two volunteers to read short sentences about right triangles and the hypotenuses and helps them with the pronunciation of the new words. Bob encourages ELL students to read out loud. It seems students volunteer actively to read sentences in class and use it as an opportunity to learn how to pronounce new words accurately. Even

though many times students get stuck on new words, they accept their teacher's corrections and bravely continue reading. Bob always thanks the students and gives positive comments after they read.

Bob pays attention to the details of the students' responses when he is presenting new information. When they make errors, he does not give explicit corrections. Instead, he restates the students' statements in the appropriate form. For example, when a student answers, "Five to the square," Bob restates, "Five squared. Very good." Bob seems to show his interest in learning ELL students' first language and ask for their help to remind him of the word, square root, in their languages, such as Arabic and Spanish.

Bob also teaches Pythagorean Triples, such as 3-4-5 or 5-12-13 and asks them to memorize several examples of Triples. When Bob notices some students do not know the meaning of 'double, he revisits the word 'triple' again to make sure everyone understands 'tri' means three and quadruple means to multiply by four. Bob shows the multiples of the Pythagorean triple, 3-4-5, still holds true to Pythagorean Theorem. Double of 3-4-5, which is 6-8-10, triple of 3-4-5, which is 9-12-15, and quadruple of 3-4-5, which is 12-16-20, all obey Pythagorean Theorem. Bob applies the Pythagorean Theorem to demonstrate the three Triples above until students agree.

Discussion of Bob's Pythagorean Theorem Lesson I

Klowss (2009) states that "Using history to teach mathematics, makes your lessons not only interesting but more meaningful to a large percentage of your students as they are interested in knowing the who, how and why about certain rules, theorems, formulas that they use everyday in class" (p. 328). The story about Pythagoras and how people would measure the land or the lengths of wood they would need to build stairs is

composed of simple English words and structure, which allow Bob's students to understand the information. There are two ways to use history in mathematics class, either telling small stories about math history or taking actual historical events and extracting the mathematics from them; both methods motivate and interest students (Klowss, 2009). Panagiotou (2011) states, "The value of history of mathematics in teaching has been pointed out for many years" and summarizes the main reason why history of mathematics contributes in teaching mathematics.

- History of mathematics can help students understand better the mathematical concepts, methods and proofs showing them how they were discovered and developed.
- History of mathematics can help students realize that mathematics is a human and dynamic activity influenced by social and cultural factors and is shaped according to the utilitarian and intellectual needs of each era.
- History of mathematics can help stimulate students' interest for learning and improve their perceptions of mathematics and attitudes towards it. (Panagiotou, 2011, p. 28)

Bob used simple words and careful pronunciation while telling a story, and drew a picture of a building with stairs on the whiteboard so the students could see how a right triangle was formed when people make stairs for buildings. According to Siu (2000), history of mathematics "not only does help in teaching the subject, but that in this age of "mathematics for all", history of mathematics is all the more important as an integral part of the subject to afford perspective and to present a fuller picture of what mathematics is to the public community" (p. 3). As Siu (2000) states, using history of mathematics does

not make students' scores higher overnight, but "it can make learning mathematics a meaningful and lively experience (p. 9).

Bob's short storytelling not only motivated his students to engage with the lesson, but it also helped them learn English. Bob facilitated better understanding by making his short story EL-friendly. I was able to witness another example of Bob using EL-friendly language to make the content more understandable to his students. Bob said, "Okay, so this word right here 'theorem' is a rule, it is truth, it is always true no matter what, even if you go to the moon, or you go to the sun, or you go to a different planet, it will still be a rule, it will be true. We can use this; the rule is like a law that you must follow."

Bob also explained the meaning of a new word by using a context familiar to his students. One example of this was the way he explained the word, 'substitute.' He asked, "When I plug in that number here, what's that called? It starts with S, like when Mr. Smith taught you last semester when I was gone." Instead of telling the students to substitute means 'plug in a number in the formula, such as $A^2+B^2=C^2$'," he drew an analogy with the word using the substitute teacher they had in the past. Bob also used the word 'substitute,' multiple times through the lesson and encouraged his students to use the word.

In the excerpt of his lesson presented earlier, Bob broke a new word, 'Pythagorean,' down into syllables and had the students say the word multiple times before he teaches the meaning. Bob was very consistent about it; he also used the same method for other new words. He said, "This is called hypotenuse. When you pronounce that, it's like 'hi', 'pu', 'te', 'news'." Richgels (2004) states that formal aspect of language include "sentence structure (syntax), the architecture of words and word parts

(morphology), word meaning and word choices (semantics), and the characteristic of interplay among sounds (phonology)" and phonemic awareness is undoubtedly important to beginning reading and writing achievement (p. 471, p. 475) Bob had students read aloud, and he chose a couple of students to read a short paragraph even if it is very short. He said it is important for them to hear their voice. According to Robertson, "To improve fluency in English, provide independent level texts that students can practice again and again, or read a short passage and then have the student immediately read it back to you" (n.d.).

Bob's Pythagorean Theorem Lesson II

Students are pointing out some of their math problems in the handout are missing a diagram:

Bob: What's missing? Picture? Do you like pictures better or just numbers?

Students: Pictures.

Bob: I like pictures too. That's the most important thing you need to remember about geometry, especially when you are learning two languages at the same time, math and English, okay? You need pictures; anywhere you can draw a picture, draw a picture, okay? What else is missing besides pictures? Like four, four what?

Student 1: Square?

Student 2: Centimeters? Or uh..?

Student 3: Inches?

Bob: You're missing a label, right? You can't just say, four! What? Four dollars? Four people? Okay, so I like Kamu's suggestion, let's go with centimeters. What letter do you want to assign to this side?

Student 1: A?

Bob: Sounds good.

Later in the lesson, Bob asks students to suggest another unit for the measurement of a length except for centimeters and one student says, "inch." Bob asks, "What's the abbreviation for inches?" and "What is the symbolic abbreviation?" Several students seem to know, they chant, "two." Bob replies, "Two, very good. It's the same number of syllables. So, for instance, five feet seven inches. One foot, one syllable, like one tick (mark). Inches, two ticks, two syllables, okay?"

Bob asks several questions so that his students can recall words they learned in his class, and he also makes several positive comments: "I love you guys, you sing math together for me, tell me Pythagorean Theorem, please," "What is this step called? It starts with S. (substitute)", "Excellent, my next step is, which operation?", "This is the longest part of this triangle, what's the name for that? What's the longest segment? It starts with 'h,' and it's kind of hard to say, uh? (hypotenuse)", "Yes, you're right, very smart!", "Good job," "Cool, your brains are so smart if you can handle that," and "You guys are wonderful."

Bob also demonstrates how students can use their cellular phones as calculators. Bob says, "What is this mode called?" One student says, "Scientific." Bob says, "Very good, Scientific. Now, on this phone, do I do button and then number? Or number then button?" He is using his iPhone, and students are using their own cellular phones and

learn they need to type number first and then square root sign to get an approximate value of square root of sixty-five. Bob says, “This one is an important one, you learn how to round, it’s called estimating. When teachers say, to the nearest tenth, okay? Some teachers say estimate. Some teachers use the word, round, round. Rounding it. It doesn’t make a lot of sense English wise, but we use it. So, if I want to round two dollars and thirteen cents, that will be two dollars and ten. For two dollars and sixteen cents, it would go up to twenty cents. Okay, that’s what we call ‘rounding your number’ or ‘estimating’.” Since they are reviewing how to find the length of a hypotenuse by applying the Pythagorean Theorem, they use their cell phones to find an estimated value of the square root of some number and round the value to nearest tenth multiple times.

Discussion of Bob’s Pythagorean Theorem Lesson II

Since some of the students do not know how to use a calculator or their phone as a calculator efficiently yet, Bob demonstrated how to use those tools to do their math work. Some of the students are not familiar with the square root symbol yet, and Bob seemed to try to help them see the value of a number with the square root of the symbol. For instance, students might not know how long the square root of 65 cm is, or the square root of 65 feet is. Students were told they should type the number 65 first and the symbol of square root, $\sqrt{\ }$, needs to follow. This might need to be changed for a different type of calculator, but it looks like all students use smartphones that work the same way as Bob’s cell phone. Bob also used his fingers and hands to show the length of cm and feet and taught them how to use little tick marks to present five feet seven inches, 5′ 7″. He might not need to teach those extra small lessons or tips in his mainstream math classes since most high school students probably know how to use their cell phone as a calculator, how

long one cm or one foot is, or what round means in mathematics classes. Bob is also teaching English words such as approximate, estimate, and scientific mode.

Students are taking notes, copying Bob's work from the board on their paper. Bob provides a packet containing several pages of worksheets. He uses his projector so that the whiteboard has the same image of the worksheet the students have. It is easier for the students to write down what he has on the whiteboard. Appendix J is the word search for a geometry unit, and it was a part of their packet for geometry unit. Students are asked to complete this as their homework. During the interview, Bob mentioned that creating a word search is the first thing he does for each unit. He provides several vocabulary words students need to learn, and students are asked to complete this word search assignment, and are encouraged to translate each English word into their first language to earn extra credit. He experienced criticism from his colleagues for using word searches in the past. However, he believes this is helping his students since students still need to become familiar with the alphabet and spelling in English. One of the criticisms he is aware of is that students in ELL programs are not only struggling with English vocabulary, their academic vocabulary in their first language is also limited, and this doesn't help ELL students with learning academic vocabulary in English. Some teachers who worked with Bob think translating academic English words into their first language won't help ELL students learn a new language. Bob argues that these people who shared their concerns didn't know how our brain works or didn't have the same experience that most ELL students go through in learning English as a foreign language. He claims that their vocabulary in ELL students' first language might not be advanced enough to translate all of the new academic vocabulary they face to learn in English, however, having some

anchor words in their first language still helps them understand and remember many things.

There has been a debate if word searches are a waste of instructional time, and some administrators forbid teachers to use word searches in class since research shows that crossword puzzles and word search puzzles have no educational value (Meier, 2008). From reading several comments from teachers (including ELL or ESL teachers), teachers who favor using word searches in the class claim that they use it to reinforce vocabulary and they think it helps develop students' visual acuity for recognizing English words. Another positive effect of using word searches is the sense of accomplishment when students complete them.

According to Danesi and Mollica (1994), *puzzleology* is "the study of puzzles and games in human cultures and has enjoyed a long-standing role in the educational domain," and they state that ,"In the area of second-language teaching, *puzzleogical* techniques such as crosswords, word searches, scrambled words, simulations, interactive games, board grams, etc. have now become intrinsic components of many approaches, and the choice of many teachers, as format for students to review and reinforce grammar, vocabulary, and communication skills" (p. 345). They claim *puzzlelogical* techniques serves "reinforcing structural and lexical knowledge" and "communication and functionality" (p. 346).

Some teachers claim that a crossword puzzle has educational value while they agree that word search has no or little educational value. According to Merkel (2016), "Crossword puzzles have been researched in connection with many facets of cognitive development" such as "analysis of the cognitive skills of crossword enthusiasts

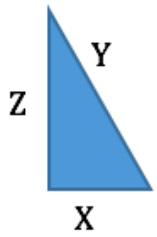
(Underwood, Deihim, & Batt, 1944), investigation of how the structure of words in the mental lexicon helps solve word fragments in which letters are either clustered or dispersed (Goldblum & Frost, 1988), the link between types of crossword puzzle clues and how the mind works (Nickerson, 2011), and crossword puzzles and lexical memory (Nickerson, 1977)" (as cited in Merkel, 2016, p. 902). In spite of efforts to value the efficacy of crossword puzzles in ELL education, Merkel (2016) concludes that there is no research on the efficacy of crossword puzzles for ELLs. The usage of a crossword puzzle for reviewing does not show any significant improvement in college students' Sociology course exam; furthermore, one of the two groups show a decrease in exam scores with use of crossword puzzles (Davis, Shepherd, & Zwiefelhofer, 2009).

The mixed opinion is that word searches or crossword puzzles may be useful activities to reinforce vocabulary or a particular skill when teachers use them appropriately in the lesson, and teachers see a value of those activities (Meier, 2008). Misuse has led to banning both types of puzzles in some instances. Research on both sides seems to be limited; some studies introduce or revisit games and puzzles as a teaching techinque in language classes, but there is no research showing immediate correlation between using word searches and learning vocabulary.

Cody's Special Right Triangle Lesson

Cody starts his lesson by reviewing a special triangle whose angles measure 30, 60, and 90 degrees.

Cody: Let's try a couple of these, real quick. Umm, we have 30, this is 90 *(he draws a right triangle on the whiteboard and labels the sides of a triangle using, X is the*



length of the short leg, Y is the hypotenuse, and Z is the long leg), how about this, this one? Let's just do Y first. What is Y? Is that the long leg or the hypotenuse?

Students: Hypotenuse.

Cody: Hypotenuse, so, what do I, what do I do with the short legs to get to the hypotenuse?

Students: Double it!

Cody: Double it, so Y equals…?

Student 1: What?

Cody: Two.

Student 2: What the heck?

Student 3: Could you write off the steps? As like you're saying it. Because I forget a lot to say it. Um, in order, like the process if we need the problem?

Cody: This is the process. Remember these ones? Remember last week, when we talked about these things, that is, uh, don't make it too difficult. Right? These are really, really simple, they are really simple processes. We just don't want to try make them overcomplicated.

Student: Okay.

Cody: I am saying… I didn't, I didn't mean to poke fun at you.

Student: No.

Cody: But I…, this is all we are gonna do, right? No matter what the number is, we will start with a short leg. So, start with a short leg.

Some students are chatting and using inappropriate language in the classroom. With Cody's gentle discipline, the students refocus; however, their confusion continues. Cody says, "Start with the short leg to find either hypotenuse or the long side" several times. One student asks, "What if we have to find the short leg?" and Cody answers, "We are gonna use these exact same steps, only in reverse" and shows an example. Students seem to understand what Cody meant by 'do the exact same steps, only in reverse.' Instead of multiplying by two, they understand that the length of the hypotenuse divided by two gets them the length of the short leg of the right triangle. Some students seem to be puzzled, yet most of the students seem to understand the long side is $2\sqrt{3}$. Some of the students who are actively engaged in the class do not seem to know that the ratio of the length of sides of this special right triangle is 1: $\sqrt{3}$: 2, in which the ratio is for "short leg, long leg, the hypotenuse" respectively. When Cody says, "(This is) Long leg. I would multiply by square root of 3 to find the long leg. So, I have to go the other way." Students know Cody expects them to divide, not multiply. However, students' answers are "divide by two" and "divide by three;" one student finally answers "square root of three."

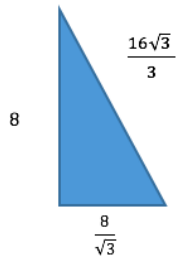
Cody: Divide by square root of three. So, I am just gonna do this. I am gonna do eight divided by square root of three. Do you like the square root of three on the bottom? Or what do you want to do? Multiply by square root of three, which in this case is gonna be eight square root of three over three. What's on the top?

Student 3: Eight square root of three.

Cody: So, now we know this is eight square root of three over three. What are we gonna do to get X? So, I have the short leg, so what can we do?

Student: Double.

Cody: Double to find the hypotenuse. So, how am I gonna double that? Do I have to double everything?



Student: Just eight. Eight. (Double of eight is) Sixteen. Sixteen square root of three.

Cody: Sixteen square root of three over?

Student: Square root of three.

Cody: Not square root of three. Just three. Right? That's right. We don't have to double one of them. We don't have to double everything. These ones you need to double. Sixteen square root of three over three. Not too bad? Feeling good about all of this?

Student: No.

Cody: No?

The lesson continues by demonstrating a special triangle with 45-45-90 degrees. Students' confusion on multiplying or dividing square root of two endures.

Discussion of Cody's Special Right Triangle Lesson

At the beginning of the lesson, one ELL student asked Cody to write down or organize the process of each step. However, Cody asked everyone not to make things complicated. This might be very simple for someone as he mentioned, but this could be a very complicated and difficult problem to some students, especially if they don't have a strong foundation in mathematics. Writing down a sequence of instructions might not be the best way to teach this lesson; as Cody stated, it might make things more complicated than they are. However, since many students are still struggling after repeating the same

strategy of "starting with the short leg, then either multiply or divide by either 2 or $\sqrt{3}$, depending on problems", it would be more productive to spend time talking about their confusion or about the reason some of their answers are wrong/incorrect. Maoto and Wallace (2006) argue that "there are different kinds of telling, including direct forms (explanations and demonstrations) and indirect forms (via questions, and the nature and structure of activities)" and claim that "the consequences of the norm of 'direct' telling, we believe, are too often passive and bored learners" (p. 68). When students do not understand information from direct forms, teachers should try to use indirect forms instead of repeating the same statements hoping they comprehend eventually.

It might be easier to teach students to follow certain instructions to solve math problems rather than to have students experience confusion and let them figure out where their own confusion originates. Cody didn't ask "why not?" or "why?" when students gave incorrect answers. Instead, he asked everyone if they agreed or disagreed. He said, "Not too bad? Feeling good about all of this?" several times in class when they found a final answer for a question. Even when one student says, "No" out loud, no additional comments or explanation took place. Teachers have a very limited time to teach a large amount of content; however, teachers' efforts to ask better questions in class is essential since good questions improve students' learning. Ostergard (2010) states teachers trying to ask higher-order questions in class experience increased dialogue about academic tasks from students. During the interview, Cody said, it is hard to watch some ELL students giving up already, however, it would be very frustrating for some students to experience that their numerous attempts to get help do not make any difference in class.

Cody mentioned the importance of using appropriate mathematics terminology and vocabulary in class. He used ‘the top number and the bottom number’ as a bad example of using inappropriate words and added that students would remember ‘the top number and the bottom number’ only if he doesn’t make enough effort to use mathematics vocabulary. However, Cody didn’t use ‘numerator and denominator’ at all while teaching this lesson. He also spoke without using a specific object which does not help students with their confusion. For instance, he said, “We don’t have to double one of them”, “We don’t have to double everything”, and “These ones you need to double.” Cody also said, “Go the other way” or “Do the exact same steps, only in reverse” when he meant to say, “Do division instead of multiplication” or “Apply inverse property of multiplication.” It was interesting to notice that students seemed to understand what Cody meant by reverse or “the other way” without getting confused with “apply steps in opposite order”; it looked like the phrases are often used to indicate “divide by.” As the teachers mentioned several times, vocabulary matters, especially for ELL students who have to learn English and mathematics at the same time.

Cody’s Application of Pythagorean Theorem Lesson

On Cody’s second observation day, students were given four application problems for the Pythagorean Theorem. The following is one of the examples of application problems they used in class.

> 3. Meteorologist Paul Windward and geologist Rhaina Stone are rushing to a paleontology conference in Pecos Gulch. Paul lifts off in his balloon at noon from Lost Wages, heading east for Pecos Gulch Conference Center. With the wind blowing west to east, he averages a land speed of 30 km/h. This will allow

him to arrive in 4 hours, just as the conference begins. Meanwhile, Rhaina is 160 km north of Lost Wages. At the moment of Paul's liftoff, Rhaina hops into an off-road vehicle and heads directly for the conference center. At what average speed must she travel to arrive at the same time Paul does?

Cody says there are many tough words here, but he didn't talk about any of the words with the students. Students didn't ask what meteorologist, geologist, or paleontology is. Cody starts reading the problem out loud and stops in the middle of the first sentence, saying "I will never be able to say this name" when he is about to read 'Rhaina.' Several ELL students try to correct Cody multiple times, but he decides to change this geologist's last name from Rhaina to Rhianna and comments that "I'm gonna just call her Rhianna since she is a really good singer."

The following is another example of word problems Cody uses in class.

4. A 25-foot ladder is placed against a building. The bottom of the ladder is 7 feet from the building. If the top of the ladder slips down 4 feet, how many feet will the bottom slide out? (It is not 4 feet.)

The students ask Cody if they can guess. Students have a few minutes talking about this problem with their classmates in groups. One student says, "If you push the ladder down four feet, then, it should extend four more feet to seven (feet). So, the bottom should be eleven feet." Cody replies, "That's an interesting guess, how many people do you think that sounds right?" Other students say it cannot be right since the problem states 'It is not 4 feet.' When the student who thinks the answer should be four feet was told that the answer cannot be four feet, he says an inappropriate word.

Cody: Language, Man!

Student: I said, Fa!

Cody: Ah, I think you said something else. Sorry, sorry, my bad.

This kind of short conversation takes place several times throughout the lesson and Cody tries to redirect students whenever they are off-task or talking loudly. Cody and students are working on this problem by applying the Pythagorean Theorem and conclude that when the ladder gets pushed down four feet, the bottom of the ladder will be moved eight feet on the ground which means the ladder is 15 feet away from the building. One ELL student asks if the top of the ladder slides down another four feet, then the bottom of the ladder on the ground will move another eight feet.

Cody: It wouldn't double this time. If we were given, uh, if it slides another four, it wouldn't slide another eight. It will change.

Student: When it slides four feet, it's eight. So, if it's four more feet, then, why not another eight (feet)?

Cody: Not necessarily. It's the way the triangle works. Okay. You guys are ready? Ready?

Student: No.

Cody: Let's do (question) number two. And then we call it good.

Student: That's college stuff. That's college classes.

Cody: College classes? Man, if you do this in college that means you are not getting credits for it. You're paying for it, but it doesn't count for your degree.

Student: Is it hard? College classes?

Cody: Oh, it's like this, we are only using, uh, it's like Calculus. So, similar stuff, just a different math. And nothing is really harder; it's just different. You have to know a little bit more stuff. Right? Are you ready? So, somebody had a right answer. But he didn't actually have the work. He had what I would call, copied work, which means he didn't actually write up the work, he tried to copy the work from somebody else's picture, which they actually copied from my work in the first period. So, it's really.., uh, you guys play a telephone game?

Student: Yeah! (Students want to play the telephone game.)

Cody: Play on the last day (of school), how about that? So, what's happening is one person is copying from another person, and they got a copy from another person, and later it looks like it's a right work, but everything is kind of in the wrong spot, and they are missing stuff, and if you're asking them what's going on, and they go, "I don't know, that's the work." Let me tell you, let's do this. Are you guys ready?

Student: Seniors' last day or our last day of school?

Cody: Senior's last day.

Student: Let's do our last day!

The lesson continues with one more application problem. The last three minutes of the lesson was spent introducing 'distance formula,' and very few students were engaged in learning something from a new section.

Discussion of Cody's Application of Pythagorean Theorem Lesson

As Cody mentioned during the interviews, he seemed to use several application mathematics problems. There was an evident increase of student participation in class on the second observation day compared to the first observation day where problems asked students to apply the Pythagorean Theorem without any context. Students were struggling with both lessons; however, students were certainly more engaged and stayed on focus during the lesson when they worked on word problems.

Since Cody said, "There are many tough words here," he seemed to be aware that his ELL students might not know several words in their handout. However, he didn't spend any time talking about those big words such as meteorologist, geologist, or paleontology. It could have been a good lesson had Cody spent some time talking about why someone who wrote this problem chooses Windward for the last name of this meteorologist, why Rhaina's last name is Stone, and what is Gulch and why the word, Gulch is selected for the place for paleontology. According to Merriam-Webster dictionary, the definition of gulch is "a deep or precipitous cleft." This little extra work of math teachers can help students learn more English through their math lessons.

A person's name in a word problem does not play an important role. However, Cody could have shown a little bit more effort to learn a new name especially since several ELL students seemed so eager to help their teacher with this particular name. Cody seemed to know everyone's name, was kind and friendly in the classroom, yet he could be more sensitive and willing to put greater effort into it when several students are persistent in teaching him a new name.

Leith, Rose, and King (2016) argue that word problems should be created to provide a context so students can apply the formulas they learn in class, but word problems need to be carefully selected for ELL students. As they pinpointed, sometimes a word problem "contains many linguistic pitfalls, which might interfere with your ELL students' understanding." The following is the example of word problems from a math textbook Leith et al. introduced in their paper.

> When a gymnast making a vault leaves the horse, her feet are 8 feet above the ground traveling with an initial upward velocity of 8 feet per second. Use the model for a vertical motion to find the time t in seconds it takes for the gymnast's feet to reach the mat. (Holliday & Cuevas, 2003, p. 499)

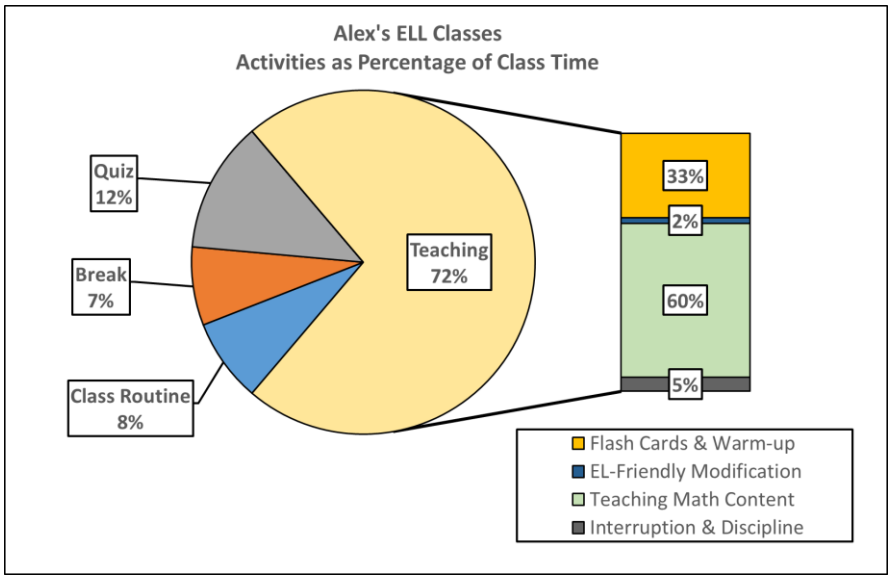
Some ELL students might not be familiar with the word, gymnast. The phrase, making a vault might need to be described or demonstrated with a visual aid. The word, 'feet' has two different meanings. The two distinct words, 'ground' and 'mat' are used for the same meaning. They claim that even though some ELL students know how to use formula for the vertical motion of a projected object: $h = -16t^2 + vt + s$, where h is the height in feet, t is the time in seconds, v is the initial upward velocity in feet per second, and s is the starting height of the object in feet," they might fail to demonstrate their knowledge and understanding because the context is not accessible to ELL students. Teachers and educators need to be careful to design application problems for tests so that ELL students can comprehend the context to demonstrate their understanding. The following is an example of a more EL-friendly version of the gymnastic problem suggested by Leith et al. (2016).

> Maria is a goalkeeper on a soccer team. During a soccer game, Maria kicks the soccer ball to the opposite side of the field. The ball is 1 meter above the ground when Maria kicks it. When she kicks it, the ball has an initial upward velocity of 8 meters per second. Use the model for a vertical motion to find the time t in seconds it takes for the ball to land on the ground at the opposite side of the field after Maria kicks it.

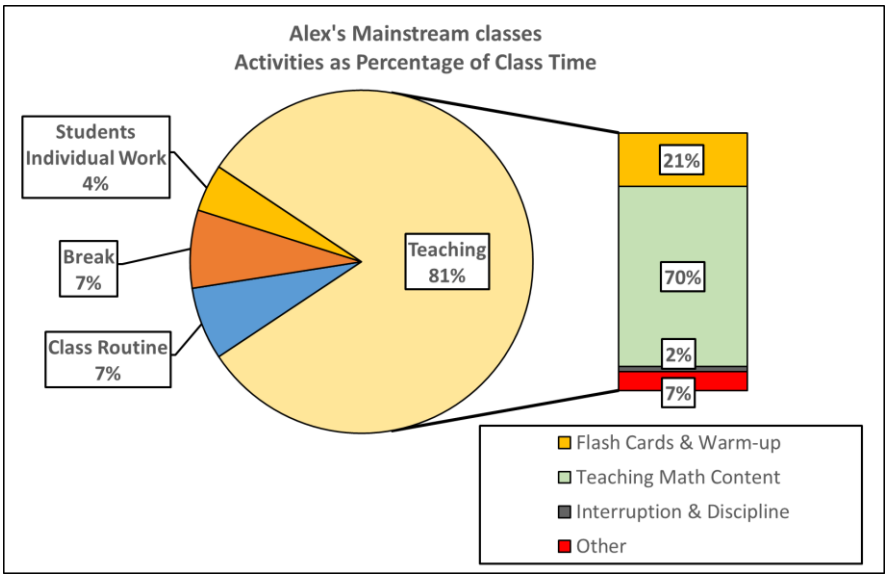
The modification they made is certainly easier for ELL students. Cody did not choose four word problems to test his students' understanding; he used them to teach how to apply the Pythagorean Theorem to solve application problems in class. His lesson could have been even better had his students taken advantage of those word problems to expand their vocabulary and knowledge about the world around them. One of the questions was about a giant California redwood tree; a photo of redwoods would make this class more exciting. Even though ELL students in Cody's class seemed to understand Cody's instruction fairly well, some of them were still struggling to comprehend what they read or did not know the meaning of some basic words, such as 'double.' His question was, "A giant California redwood tree, 36 meters tall, cracked in a violent storm and fell as if hinged. The tip of the once beautiful tree hit the ground 24 meters from the base. Researcher Red Woods wishes to investigate the crack. How many meters up from the base of the tree does he have to climb?" Cody restated what the question says, "They tell us that this is a giant redwood tree that is 36 meters tall. It broke over the storm, right? So, they want to know the tip of it 24 feet over? No, 24 meters. We know it broke somewhere here. They need to come up here and look at this to see it." In addition to Cody's restating, the question contains a picture that goes along with the text. Thus,

students seemed to understand the situation well. However, they never read the questions aloud, and Cody doesn't read the questions for the students either. Students could learn new words, such as ‘as if hinged,' ‘the base’ (of a tree). The two words, ‘ground’ and ‘base’ are used in one sentence together; this might not be clear for ELL students to understand. Even though the question with a picture was given to the students as Appendix D shows, Cody’s description of the problem was not clear. His lesson was interrupted multiple times due to some students’ misbehavior, however, Cody was able to manage to teach his lesson without getting engaged in the situation. A few warnings and his attempt to redirect the class seemed to work.

Time Spent in Teaching



Graph 1. Alex's ELL Class Instructional Time Spent by Activity



Graph 2. Alex's Mainstream Class Instructional Time Spent by Activity

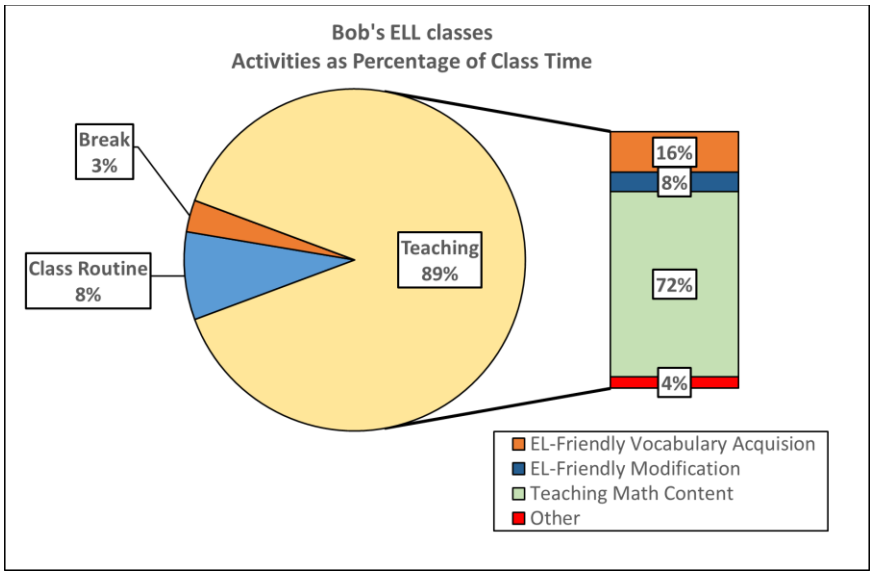
Alex started with flash cards to refresh students' memory in vocabulary and gave warm-up problems to review mathematics content they have been learning recently. Alex gave breaks during the lessons so that the students could move around and chat with their

friends in both the ELL and mainstream classes. Students either had short, multiple breaks or one short break and a longer break at the end of the lessons, depending on the day. Alex continuously used the target words through the lessons. The way Alex taught mathematics in ELL math classes was not different from how he taught his mainstream class. He repeated and retaught the same content several times, but he did not talk slowly, nor did he speak in short, straightforward sentences. His instruction was clear in both ELL mathematics class and mainstream mathematics class.

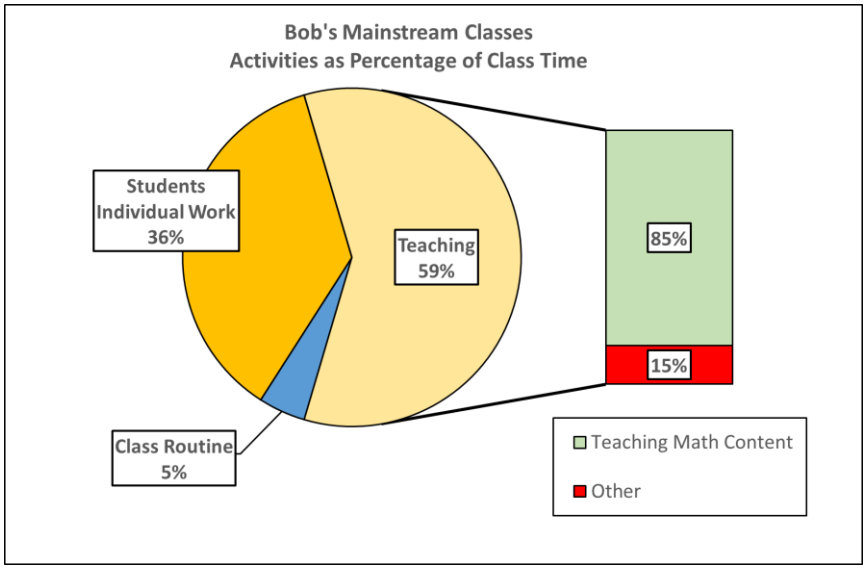
Alex's overall format in both ELL class and mainstream class was relatively similar, yet there was a noticeable difference in how he spent his instructional time. He spent 33% of his instructional time using flash cards and working on warm up problems in his ELL group while he spent 21% of his teaching time in his mainstream class. The ELL group had 12% more of his instruction time for the review at the beginning of the lesson because he used about five minutes to go over several flash cards to refresh and reteach mathematics vocabulary with them before he assigned daily warm up problems and he retaught several concepts while assigning warm up problems. With the mainstream students, he started with warm up problems immediately and used a few flash cards selectively when he went over the answers for the warm up problems. Alex spent 2% of his instruction time reteaching basic math skills, such as how to add and subtract with negative integers and how to simplify a fraction; these were not the content of the current module he was teaching. Five percent of Alex's instruction time was either interrupted by a misbehaving student or spent on discipline in the ELL class, while he spent 2% of teaching time in his mainstream class even though mainstream had several students who were off task and chatting with their classmates. Alex does not have ELL

students with low English proficiency in his mainstream class; he does not have any EL-friendly modification in his mainstream class. Alex spent 60% of his instructional time teaching mathematics content he planned to cover while 70% was spent in his mainstream. He spent 7% of his teaching time playing a game in his mainstream class; the game itself was not related to mathematics, but they played it as a whole class with his monitoring; it was not a break where students could move around. Students seemed to know how to play this game and had experience in playing as a class.

Besides revisiting basic math skills and spending more time reviewing math vocabulary using flash cards at the beginning of the lesson, Alex made another modification for the ELL students in that he waited with patience until they replied to his question or until they tried to pronounce a big word they had been learning, such as congruent, reciprocal, perpendicular, parallelogram, etc. Alex was patient with his students in both ELL Math class and mainstream class overall, however, he did not have to pause to wait until his mainstream class students recall and pronouce the vocabulary they are learning.



Graph 3. Bob's ELL Class Instructional Time Spent by Activity



Graph 4. Bob's ELL Class with Mixed Group Instructional Time Spent by Activity

While Alex's ELL and mainstream classes had reasonably similar patterns, Bob's ELL classes and mainstream classes were very different in spending class meeting time. Instructional time was coded using five categories: EL-friendly vocabulary acquisition, EL-friendly modification, teaching mathematics content, interruption & discipline, and other.

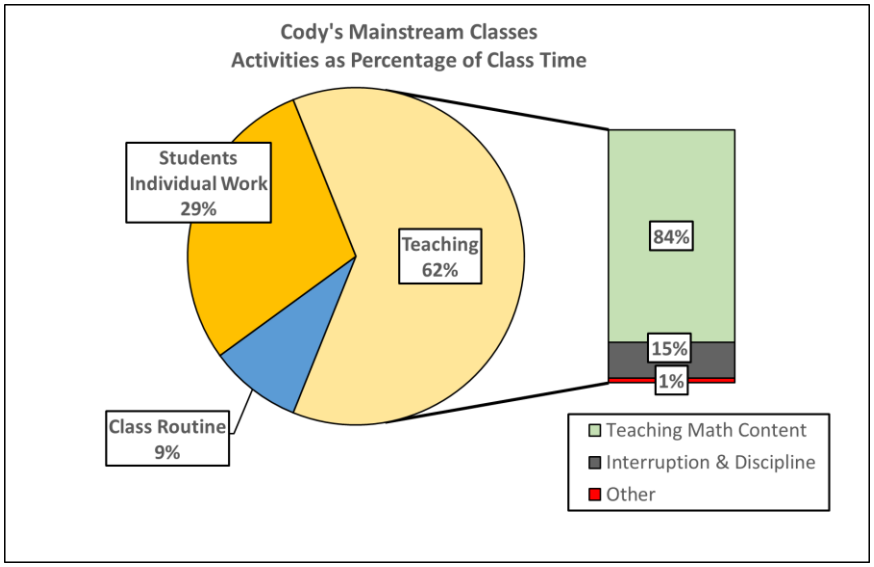
The EL-friendly vocabulary acquisition was the time Bob spent teaching any new Math vocabulary and focused on pronunciation as well as the meaning of the words and teaching basic words which most high school students would know, but ELL students might not know yet. The second category, the EL-friendly modification, was the time Bob spent to make his lesson more accessible to his ELL students. For example, Bob shared the history of Pythagoras, and the story he shared with his drawing was composed of basic English words. He used the word 'rule', 'formula', and 'law' to explain the theorem after teaching how to pronounce "Pythagorean." Bob also showed how to use their phone as their calculator so students could get an approximate value of a number such as the square root of 65. The time Bob spent teaching how to round decimal numbers to nearest tenth was included in the category of EL-friendly modification since he has a little chance to teach how to round decimal numbers to high school mainstream students because they would have learned. As he mentioned in ELL math class, Bob had several 'a lesson in a lesson.' Teaching ELL students what most American students would learn from their elemenetary and middle school curriculm in his high school ELL math class is consided as 'lesson in a lesson.' With the same reasoning, the time he spent to explain the words "rounding," "estimating," and "approximation" is coded under EL-friendly modification since many of his ELL students learned those words for the first time. This particular example of his teaching fell into the category of EL-friendly modification, not in EL-friendly vocabulary acquisition to avoid double counting since his primary focus was teaching how to round a decimal number. The abbreviation for inches and feet and how to use symbols for them is labeled as EL-friendly modification since he does not need to teach that for mainstream American high school students.

The third category, teaching mathematics content, was the time Bob taught math content; he modeled the Pythagorean Theorem by using small cube manipulatives, solved several questions using a whiteboard, and asked several short questions while he was showing how to get the final answer to math questions.

The last category, other, is the time Bob spent talking about something else that was not related to the math content such as the weather outside, the message from their student teacher last year, which is 4% of his instructional time. As Graph 3 shows, most of Bob's teaching time (72%) is devoted to teaching mathematics content and showing how to do math problems. EL-friendly vocabulary acquisition (16%) and EL-friendly modification (8%) are composed of 24% of the teaching time.

Like Alex, Bob did not give students any individual work time in his ELL math class during the two observations, while his mainstream class students had more than one-third of class meeting time (36%) to work on math problems individually after his instruction. Bob spent 89% of class time to teach in his ELL math class while 59% of the class time was used for instruction in his mainstream class. While Alex's students' individual work time was only 4% and the students' work was monitored by Alex, and the correct answers were given in a few minutes, individual students' work time in Bob's class was a longer period of time (36%). Bob's assigned work was more like homework that was encouraged to be completed in class; therefore, the close monitoring didn't occur, and the correct answers were not given at the end of the lesson. Bob gave 3% of his class time as a break in his ELL class. In the mainstream math class, he didn't give any breaks in the middle of the lesson, however, the students were able to chat with their peers as long as they completed their work during their individual work time.

For a mainstream class with mixed groups, Bob spent 59% of his whole class meeting time for instructional time. Eighty-five percent of the time of instruction was used to teach mathematics, and the last 15% was labeled “other;” Bob shared some personal stories about himself or students talked about something unrelated to the math topic. Even though Bob’s mainstream class is a mix of ELL students, who are still relatively new to the country, and American students who were born in the United States, the same or similar quality of EL-friendly modification he made in his ELL class did not occur in his mixed mainstream class. Bob talked very fast when he had a conversation with an individual student or made a joke, but his speech was relatively slow, and the sentence structures were simple and straightforward during the instructional time in both the mainstream and ELL class.

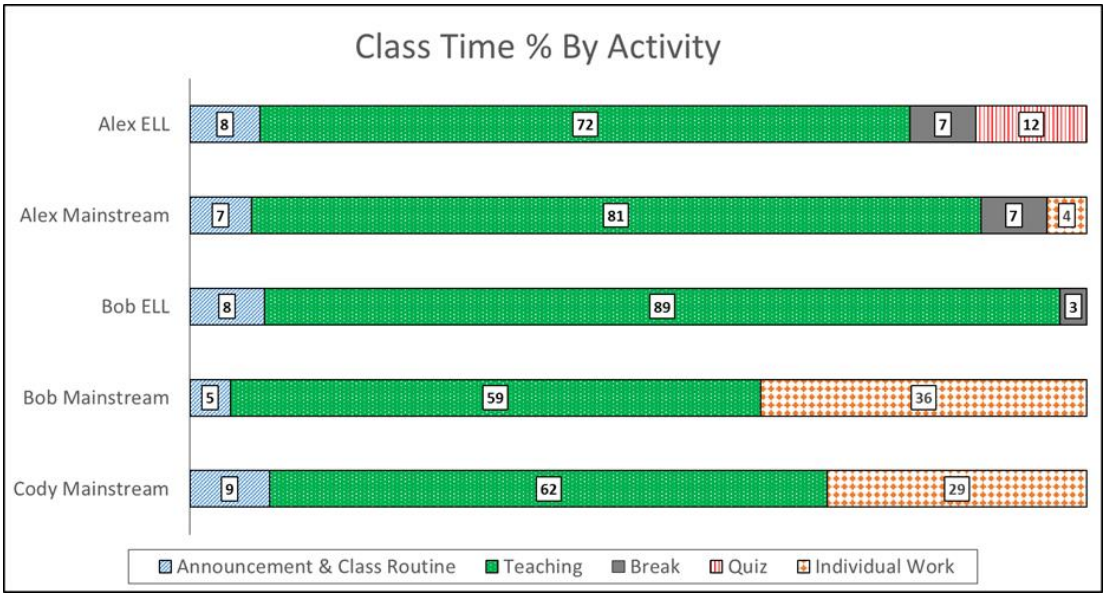


Graph 5. Cody’s Mainstream Class with Mixed Group Time Spent by Activity

As Graph 5 shows, Cody spent 9% of class time for class routine, 29% for students’ individual work with his close monitoring, and 62% for instructional time. Eighty-four percent of this teaching time was used to teach mathematics content.

However, Cody did not spend any time teaching vocabulary during the two days of observation. Cody gave a considerable amount of time to work on mathematics problems before he showed his work. He monitored students' work during individual work time and helped students when asked. Cody's lesson got interrupted several times by students' misbehavior, which was 15% of his instructional time. However, he managed to redirect students to focus on their work without causing more trouble for the whole class. Cody's first lesson was taught through a worksheet that did not have many English words and sentences. His second lesson was also taught using a worksheet containing several word problems, but there was no teaching of vocabulary. There was no modification made for ELL students, who are more than 30% of his students.

Graph 6 disaggregates class time by activity in percentage. shows how much time each teacher spent for their two days of lessons observed. Alex taught a module called



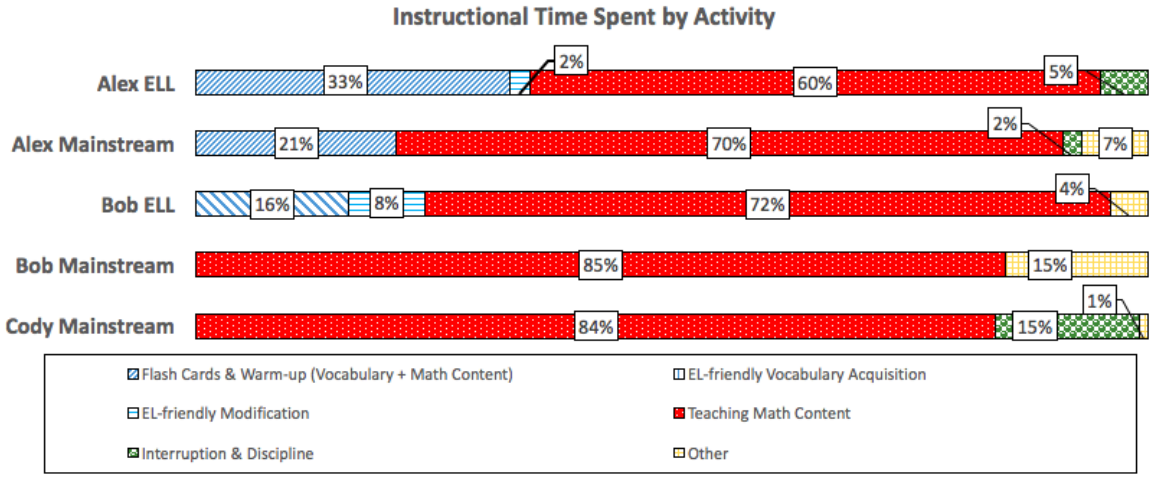
Graph 6. Class Time Spent by Activity Percentage

"Connecting Algebra and Geometry" in ELL math class while teaching a module named "Modeling Data" in the mainstream class. However, the format of the lessons in the two

classes was the same. As Graph 6 shows, Alex did not give students any individual work time in his ELL Math class while 4% of his mainstream class meeting time was spent for students to work on their own. Alex's ELL students had a quiz at the end of the second observed day (12%); this result shows that more teaching time (9% more) was spent in mainstream class.

Comparative Data Analysis

Graph 7 further disaggregates the time teaching by identifying specific instructional strategies. This allows us to see the differences between each teacher's activities and the amount of time they spent (in percentage) for instructional strategies.



Graph 7. Instructional Time Spent by Activity

Alex's instruction in his ELL math class and in his mainstream class were overall similar; he spent more time using flash cards to review mathematics vocabulary with ELL students. Both Alex and Bob did not give any individual work time for ELL students. They showed all the work on the board and waited long enough for the ELL students to copy their work onto their paper. Alex used flash cards several times during the lessons, whenever he needed to revisit mathematics vocabulary in the ELL math

class, so the students could see the spelling of the word multiple times. Alex used the same curriculum, high school math Common Core 1 for both ELL math classes and mainstream classes. The ELL math class seemed to be a few lessons behind, but the content they were learning was no different than the mainstream class content.

Alex had a paraprofessional in his class who sat next to the students who recently joined his class. The paraprofessional helped the new students so that they could follow Alex's instruction without getting lost. Alex's ELL math class was an excellent example of teaching ELL students both English and mathematics, and his class was mainly teaching mathematics content. He slowed down and repeated simple examples multiple times until they understood the steps or concepts. Many of the students were not fluent in English yet, but they seemed to have no trouble following his instruction. Alex had very well organized lesson plans targeting a few concepts and skills to teach for each lesson.

Bob's ELL class and mainstream class were very different with respect to the time spent in class. Bob spoke slowly and used short sentences in ELL classes while he made several jokes and talked quickly in his mainstream class. Bob was flexible to add to or modify the lessons depending on the levels of his ELL students. The ELL class I observed had students who had a good mathematics foundation compared to his other ELL classes where many brand new ELL students with little core competencies in schooling had joined recently. Thus, he was able to teach the Pythagorean Theorem to the ELL group observed even though it was not part of the curriculum initially. His other ELL math class did not have this same lesson. Bob made various EL-friendly modifications through the lessons. However, this modification happened only when he had separated the ELL students group even though his mainstream has several ELL

students. Since ELL students had gaps in mathematics, Bob had to teach many math skills and content while he was teaching the current topic, Pythagorean Theorem. He was trying to teach Pythagorean Theorem while he was explaining square root and how to round the decimal to the nearest tenths. This resulted in Bob spending much more instructional time in his ELL class while he spent much less time teaching in the mainstream class.

Cody also gave his students individual work time, and he walked around the students and helped everyone when they asked for help. Cody worked with many ELL students since they were actively seeking his help. During the interview, Cody mentioned that he didn't want to "pollute" what ELL students do with other teachers in English or ELL classes. As Cody stated, since he believed he didn't know much about how to teach ELL, he wanted to teach only mathematics which he knew how to teach. His class was a mainstream class with more than 30% of ELL students in it. His lesson was not modified for ELL students, and his class seemed to be the most difficult one to teach due to students' behavior such as inappropriate language and lack of motivation. This was what Cody was worried about the reality where ELL students have classes with the American students with low performance and discouraging behavior. He did not spend time nor put in the effort to make his lesson more accessible for ELL students, but he was very kind and patient with all of his students and willing to help ELL students when they asked for help. Cody's students spent two days working on one worksheet with application problems with teacher's help, and as Cody openly said, as long as they copy his work and make good notes on their paper, they will get full credit. Since many students were not

motivated, Cody encouraged students to pay attention in class and set a low expectation regarding completing their work.

Another significant difference Cody showed from the other two teachers, Alex and Bob, is Cody did not use gestures while both Alex and Bob used many gestures while teaching. Piaget (1959) states, “gesture plays an important role in learning, development, and communication of children”, and Koschmann and LeBaron (2002) add, “gestures assist students in coordinating their interaction” (as cited in Roth, 2001, p. 373, p. 381). Alex used his arms and hands every time he said ‘perpendicular’ or ‘parallel’ and he kept pointing the mathematics symbols while saying ‘congruent’ etc. Bob also used several gestures, he also used his arms, hands, and figures to show a short leg of a right triangle and the hypotenuses of a right triangle. Bob also used a hand gesture and a facial expression when he talked about approximated value for the final answer to a problem. Boaler et al. (2016) show the importance of visual thinking, especially finger representations, to all level of mathematics from a collaboration between a neuroscientist and mathematics educators.

Coggins et al. (2007) also advises teachers to let students have mixed use of language during students’ mathematics discussions and teach how to involve classmates with limited English proficiency in group work by using gestures, pictures, questions, and short clear statements. All three teachers used diagrams or drawings appropriately to help students understand mathematical concepts or the content that were taught and all of them used several questions through the lessons even though most of the questions were closed-ended generating a set of limited responses. The questions were used to see if students remember the previous lessons or if they recall vocabulary they need to use. Bob

used a few Spanish and Arabic words he knows, such as square root, about, multiplication, lesson; those words might not make substantial differences in their learning mathematics, however, he also encouraged students to continue learning academic words in ELL students' first language.

CHAPTER FIVE: CONCLUSIONS

In this chapter, research questions are answered based on the results of the study. The following are my research questions.

- How do high school mathematics teachers modify and differentiate their lessons for high school refugee ELL students?
    - How do high school mathematics teachers use vocabulary acquisition as an instructional strategy in their class?
    - Do high school mathematics teachers spend more instructional time teaching vocabulary in mathematics class targeting ELL students?

Moreover, any additional attempts of teachers to make their lessons accessible to ELL students are shared as well as any findings of this study that lead to presumable conclusions. Limitations of the study and recommendations for further research and conclusions are presented.

Discussion

Additional support from teachers in this study allows the students with low English skills to be able to learn high school level mathematics. The most extraordinary supports were a continuous review and supplemental lessons aiming to progress basic math skills or to develop fundamental mathematics concepts the students are expected to know to comprehend the content they are learning in class. Visual cues such as flash cards, graphic organizers, manipulatives, drawings/diagrams, and gestures are purposefully used. Several question/answer sessions and read aloud

are used to engage students to learn mathematics content or to remember new words' meanings and pronunciations. Well-equipped classrooms allowed these teachers to provide organized class notes so that students can copy what they see on the smartboard or projected screen. Teachers modified and differentiated high school mathematics curriculum when they had ELL students only; no ELL-oriented modifications were made in mainstream classes where several ELL students are attending.

Traditional Vocabulary Acquisition based on teacher-led instruction was mainly used in the ELL math classes. However, both Alex and Bob did not teach vocabulary by providing the definition of a word from a dictionary. Instead, they explained the meaning of words in a way the students can comprehend. One of the alternative instructional methods for vocabulary acquisition, semantic mapping method, is used to some extent in both Alex's and Bob's classes. "Semantic mapping is a process for constructing visual displays of categories and their relationships" (Dilek, 2013, p. 1533). ELL students in Alex's class created graphic organizers; however, the format and information for their graphic organizers were provided by Alex, and they were used more as a visual memory aid containing lists of vocabulary. Bob mentioned several words (law, rule, and formula) the students can relate to the target word (theorem). However, they did not create any visual display nor define the relationships between words. Both teachers spent a considerable amount of time in teaching vocabulary; however, what each teacher focused on was different. While Alex focused on repetition of using target vocabulary in class, Bob focused on teaching how to pronounce target vocabulary by breaking a word down into syllables. Bob especially

focused on phonology, and phonemic awareness; he said a word slowly pronouncing each syllable and had students repeat after him. Both ELL mathematics teachers used appropriate mathematics vocabulary in class, while the mainstream teacher (Cody) did not use mathematics terms and used ‘easy’ words. Both of the ELL math teachers taught vocabulary during the lesson when new words are presented on students’ handouts initially while teaching mathematics concepts instead of using front-loaded vocabulary instruction or saving the explanation of target words until the end of the lessons. Teaching mathematics vocabulary before teaching concepts might not be appropriate in teaching mathematics since it can be difficult to explain the meaning without explaining the content (Coggins et al., 2007).

According to Cummins (1981a), it takes at least two years for students to master BICS (Basic Interpersonal Communication Skills) in L2. Cummins (1981b) adds that older students learn faster than young students since common underlying proficiency in their L1 and L2 promote learning a second language (as cited in Collier, 1987, p. 619). It is well known that proficiency in L1 plays a critical role in learning L2 (Cummins, 1979), and this is why ELL students should be encouraged to continue learning their first language, not only for conversation but also academically, so they will have strong foundations of language skills.

One of the teachers in this study, Bob, shows effort to inspire his students to study CALP (Cognitive Academic Language Proficiency) in their L1 by assigning a word search puzzle for each unit; his students earn extra homework credits for translating mathematics vocabulary into their L1. No research demonstrates the educational efficacy of word search puzzles in teaching L2 to students at the secondary

level. However, students in Bob's ELL math classes use word search puzzle as a tool to improve academic vocabulary in their first language.

Teachers in this study spent more time teaching and reviewing vocabulary with ELL students than they did in mainstream classes. This result naturally arouses curiosity on how the teachers spare the extra time for focusing on vocabulary instruction for ELL students when the same amount of time was given for ELL math classes and mainstream ones. Students in mainstream classes have more individual work time while ELL students have very little or none. And, students in mainstream class spend some time engaging in non-math related activities such as playing a game at the end of the lesson or having little chats with their teachers and peers. Spending extra time for vocabulary instruction with ELL students can be seen as spending less instructional time in mainstream class. Students in mainstream classes have more time that does not require full attention to their teacher's instruction compared to ELL classes. This difference of the quantity of direct instructional time between ELL math classes and mainstream classes might be inevitable since both classes must provide the same amount of content despite ELL students' lack of skills in English or mathematics.

The ELL math teachers, Alex and Bob, in this study seem to provide comprehensible input for their students. Based on Krashen's view (1985), teachers' main role is to administer "comprehensible input by providing learners with listening and reading materials" (Zhang, 2009, p. 92). Teachers teaching ELL students mathematics in this study apply Krashen's well-known formula, i+ 1, by scaffolding new material and linking the 'new *i*' to what their students already know. The efforts to connect BICS to CALP, the attempts to give 'a lesson inside lesson,' and multiple

reviews of their previous lessons should be highly valued.

Teachers in this study appear to offer several opportunities for their ELL students to produce output in L2 in several different manners, such as pronouncing vocabulary, writing down or orally spelling vocabulary words, and completing teachers' sentences by recalling vocabulary they have been learning. In light of Swain's Output Hypothesis (1985), comprehensible input is not sufficient, and language acquisition occurs while learners construct output in a target language. When they experience failure to produce output, learners pay attention to the details to overcome the challenges. Constructing comprehensible output needs to occur even after ELL students reach beyond intermediate level of English proficiency because output may stimulate language learners to move from processing comprehension in L2 to reaching grammatical processing for accurate production. (Swain, 1995).

ELL students' interactions in speaking English in class seem to be limited to speaking with their teachers, and having monolingual peers in a mainstream class does not provide any better opportunities to interact with NS (Native Speakers). ELL students in this study have very limited interactions in educational manners with their peers in class; there was no group work or open-ended class conversation in ELL math classes. In ELL math classes, students speak English with their friends during the breaks due to the diversity in their first languages. In mainstream math class, students whose L1 are the same sit together, thus there are several subgroups based on cultural/linguistic background; they use only their L1 to talk to their peers in class. Advocates of inclusion of ELL students hope rich discourse and conversation in class help ELL students learn the English language. However, reality behaves differently, as

teachers mention in the initial interview, most NS do not talk with NNS.

The reality of no productive conversation between NS students and NNS students is unfortunate; however, this does not mean ELL students do not have interactions required to learn English. Conversation with other NNS still benefits them in learning the English language, and more importantly, they interact with their teachers in class. According to Long (1996), interaction facilitates SLA (Second Language Acquisition), and the conversational and linguistic modifications occur in such discourse, which learners experience negotiating for meaning. ELL students learn better when they have a conversational interaction before having input in class (Gass & Torress, 2005). The classes in this study left much to be desired since there were not enough interesting hooks at the beginning of the lessons or engaging conversation. Only one teacher shared a story about Pythagoras as a mathematician and philosopher in ancient history before teaching the Pythagorean theorem. It could be challenging to have an interesting conversation connecting the goals of the lessons in ELL math classes in which students' English is still very limited. However, ELL students in mainstream classes have been studying English more than a couple of years at least, and their BICS is good enough to have a conversation. Interesting hooks or discussions before the exposure to mathematics vocabulary and content might be helpful to motivate students to engage in their lessons.

White (1987) asserts that “what is necessary for learners is not comprehensible input but incomprehensible input,” and that incomprehensible input triggers negotiation that allows learners to acknowledge the insufficiency of their own rule system (as cited in Gass, 1997, p. 132). Gass (1997) claims that “negotiation is a

facilitator of learning" (p. 131). Both comprehensible and incomprehensible input are essential for L2 acquisition; either comprehensible input is needed to learn something new or incomprehensible input is used to trigger the negotiation and get learners' full attention to learning. Certified, experienced teachers in this study seem to provide good quality of content-based lessons in ELL math classes. Nonetheless, active conversation in class continued to be a demanding challenge.

Unfortunately, the mainstream classes studied did not provide any supportive instruction that could enhance ELL students' L2 acquisition. Therefore, students have a good chance to 'sink' instead of 'swim' when they are pushed to join the mainstream class. The National Commission on Teaching America's Future (1996) acknowledges that, writing "Lack of teacher knowledge and skills has often been cited as contributing to the lack of success in large-scale reform efforts, relating this lack of knowledge to teachers' inability to make significant improvements in the quality of instruction provided to students" (as cited in Aguirre-Muñoz et al., 2008, p. 316). It is not a coincidence that a mainstream mathematics teacher does not show any EL-friendly modification during the observation; the mainstream teacher does not have enough knowledge to modify his lessons for ELL students, as he admitted during the interview. Furthermore, he thinks he should not implement any ELL instructions because it might 'pollute' what ELL students learn from other classes.

Teachers who teach both ELL mathematics and the mainstream class in this study expressed their wish to teach ELLs only. They are certified in mathematics and English and have experience in teaching both groups. From the impressions during the interviews with teachers and the class observations, it is noticeable that students in the

ELL mathematics classes are more enthusiastic to learn mathematics and behave better in class compared to students in the mainstream classes. Bob did not spend time on discipline issues, while Cody, in his mixed mainstream class, spent 15% of his teaching time on discipline. Alex spent 5% and 2% of his teaching time in ELL class and mainstream class respectively. However, the level of noise and interruptions caused by students was more severe in Alex's mainstream class than in his ELL classes. The mainstream class, with many ELL students attending, seems difficult to teach since the class has two groups of students, ELL students with the extra language challenge and monolingual students with a lack of motivation or low performance in school. Unfortunately, this is the reality that high school ELL students with limited academic language skills face in mainstream classes. Their academic skills are not good enough yet to join advanced or 'regular' mainstream classes. Thus, they study with peers who have little or no interest in learning. This will continue to challenge teachers to modify their lessons for ELL students since their student group is already challenging enough. This issue might explain why the two ELL teachers in the study do not want to teach mainstream classes.

Limitations

Grand High School has a block schedule with 90 minutes per class every other day while Baker High School offers 50 minutes of class every day. Since Alex's lessons is longer than Bob's and Cody's lesson, Alex might have more chances to implement instructions supporting ELL students. The data sample is small with only three high school teachers as participants, and Cody has no separate ELL math class. However, Cody would be the typical example of secondary mainstream teachers who

had several ELL students in their class. Alex and Bob did not teach the same lesson in ELL math class and mainstream class. Alex's mainstream students were learning the statistics module while ELL students were finishing up their geometry module; thus direct comparison in presenting the content was challenging. However, the study was focused on seeking any evidence of EL-friendly modification and EL-friendly vocabulary acquisition and measuring their instructional time by activities.

Recommendations

The brand-new ELL students' performance in learning high school mathematics content collected in each separate ELL math class group and mainstream math class group through academic years would be useful in determining which model is more beneficial to ELL students' growth in learning mathematics. In light of the concerns about a lack of knowledge on ELL education shown by the mainstream teacher in this study and the suggestions of the two ELL and math teachers, it is important for secondary mainstream teachers to have professional development opportunities to learn about ELL education. It will be meaningful to research to what extent states or districts support mainstream teachers to get an appropriate education so that they can be prepared for inclusion of ELLs.

Besides scholars' work indicating the importance of teachers' collaboration between ELL and content subjects, the tangible performance such as lesson plans or curriculum created by researchers and educators needs to be published. By doing this, more mainstream teachers can try to modify their lessons to make them more accessible to ELL students.

Conclusions

In vivid contrast to concerns described by researchers (Harklau, 1999; Olsen, 1997), the high school ELL mathematics classes observed in this study are neither substandard nor limited to low-level. These classes not only compensate students' limited language skills but also provide appropriate grade-level mathematics content. Both ELL mathematics classes have the same curriculum as the mainstream classes that are taught by the same teacher. Since the two ELL math classes are designed to serve late-entry ELL students to the United States with very limited English skills, EL-oriented modification and differentiation in teachers' instruction were expected. Several modifications were observed throughout the lessons in ELL mathematics classes, while no distinct efforts to EL-friendly modification were made in mainstream classes with mixed groups of ELL students and monolinguals. The most apparent modification is repetition, and it was witnessed both in teaching mathematical content and target vocabulary.

It has long been known that vocabulary plays a critical role in learning a language (Alqahtani, 2015). Vocabulary acquisition in ELL mathematics classes in this study occurred throughout the lessons and the teachers spent more time to teach or review vocabulary in ELL mathematics classes than they did in mainstream classes. It is clear that much additional work is required before a complete understanding regarding the efficacy or appropriateness of teaching ELL students mathematics separately from the mainstream environment. However, it is still meaningful to listen to the teachers who work with high school ELL students with very limited English and document their EL-oriented lessons. With the growing population of ELL students and

the fact that such a stretched time is required for ELL students to reach the level of proficiency in using English, the preparation of mainstream teachers for teaching ELL students is unavoidable. Language acquisition, more precisely vocabulary acquisition, must take place in every class for every group of students regardless of their level of English including both multilingual and monolingual students.

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APPENDIX A

Appendix A.1 Initial interview questions

1. What is the official title of your class?
2. How many years have you been teaching? How many years have you been teaching ELL students? If you teach ELL math, how did you start teaching it at first? Did you apply for that position or did your school ask you to teach ELL math courses?
3. How many ELL students do you teach? Are they mostly refugee students?
4. Read each following statement a), b), c) and d) and e) and choose if you agree or disagree from the given options.
    - Strongly disagree
    - Disagree
    - Agree
    - Strongly Agree

a) I'd rather make comments during the interview.

b) ELL students with limited English skills need to learn mathematics and English at the same time.

c) ELL students with limited English skills need to focus on learning English first before learning mathematics.

d) ELL students with limited English skills benefit more in separated ELL classes than in mainstream classes.

e) ELL students with limited English skills benefit more in mainstream class than in a separated class environment.

5. Which statement do you agree with the most?

a) I teach separate ELL math classes. I see myself more as an ELL teacher in the class.

b) I teach separate ELL math classes. I see myself more as a math teacher in the class.

c) I teach separate ELL math classes. I am both a math teacher and an English teacher in the class.

d) I do not have ELL math classes. I have mainstream classes with some ELL students mixed in. I see myself more as an ELL teacher in the class.

e) I do not have ELL math classes. I have mainstream classes with some ELL students mixed in. I see myself more as a math teacher in the class.

f) I do not have ELL math classes. I have mainstream classes with some ELL students mixed in. I am both a math teacher and an English teacher in the class.

6. Which statements about teaching mathematics to ELL students do you agree with the most? You can choose more than one option.

a) My job is teaching mathematics.

b) My job is teaching English using mathematics content.

c) My job is teaching both mathematics and English.

d) My job is teaching ELL students how to become successful students in the United States.

e) Other (Please share any other thoughts you have)

7. Do you think newly arriving high school ELL refugee students will be ready to join mainstream math classes after attending a few years of separate ELL math classes?
8. Are you certified to teach secondary mathematics? Are you certified to teach ELL students?
9. Do you think it is necessary to modify or differentiate math lessons for refugee students with limited English skills? If so, how do you modify your lessons?
10. Do you have any comments on educating ELL students or refugee students from your own experience?

Appendix A.2 Second Interview Questions:

1. How do you teach new vocabulary in your class? What do you do to teach new words? What kinds of activities or strategies are used for studying new vocabulary in your class?
2. Do you think a lack of English skills is the main hindrance for refugee students learning mathematics? If not, what do you think the main hindrance is? What other hindrances do you think they have?
3. Which model is better to educate ELL students, inclusion or separation?
4. What do you think the purpose of having ELL mathematics classes exclusively for refugee students is? Do you think it is beneficial to the students?
5. How important is it to teach vocabulary in mathematics classes?
6. Are you familiar with vocabulary acquisition?
7. When do you teach new vocabulary or key phrases in your class?

    A) I teach vocabulary or key phrases at the beginning of the lesson.

B) I teach vocabulary or key phrases in the middle of the lessons.

C) I teach vocabulary or key phrases at the end of the lesson.

D) I teach vocabulary or key phrases when students ask for an explanation.

8. What other activities do you use to help improve your students' English in your math classes?
9. What do you suggest can be done to improve the education of high school refugee students with limited English skills?
10. What are your greatest challenges in teaching high school ELL students? Do you enjoy teaching this demographic? Why or why not? If you could choose only one, would you prefer to teach mainstream math classes or ELL math classes?

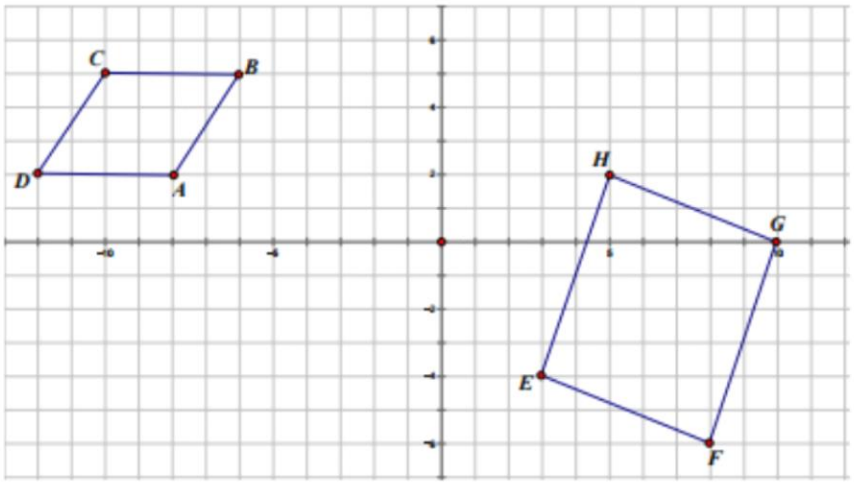
APPENDIX B

7.3 Prove It!

A Solidify Understanding Task

www.flickr.com/photos/safari_vacatio

In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.



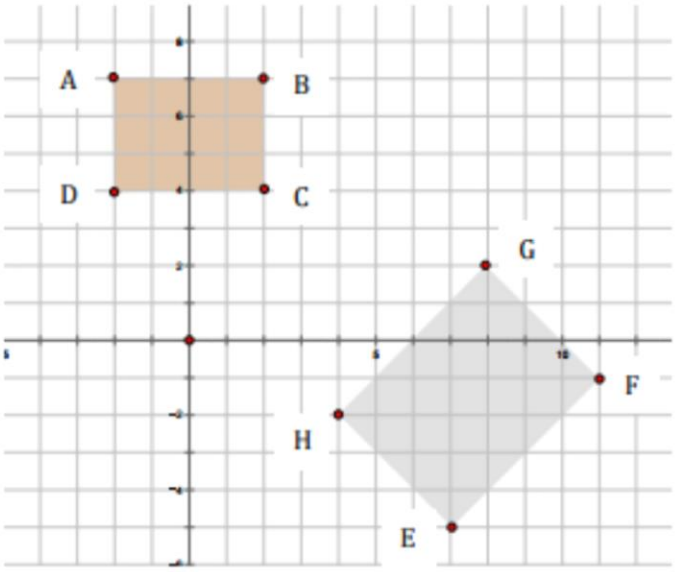
Is ABCD a parallelogram? Explain how you know.

Is EFGH a parallelogram? Explain how you know.

Students' handout for Alex's ELL Math Class

APPENDIX C

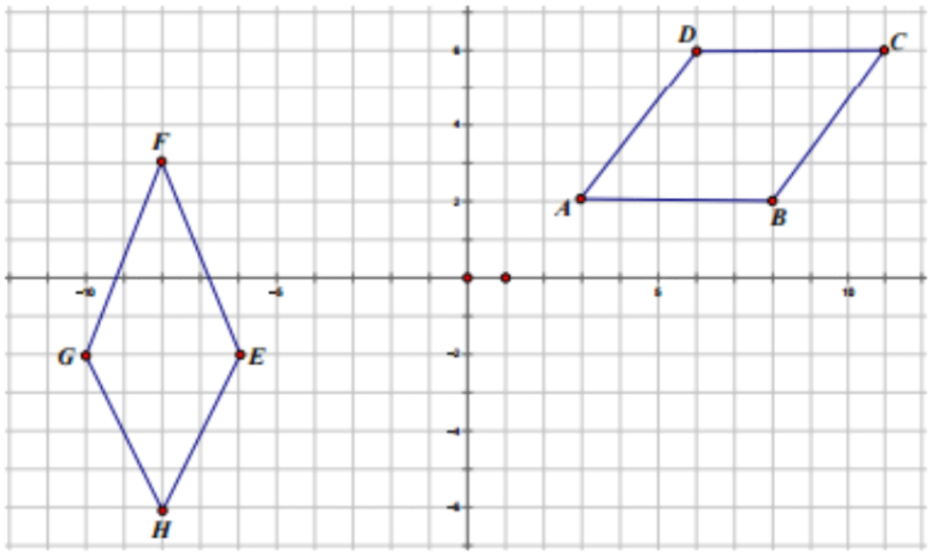
Students' handout for Alex's ELL Math Class



Is ABCD a rectangle? Explain how you know.

Is EFGH a rectangle? Explain how you know.

APPENDIX D



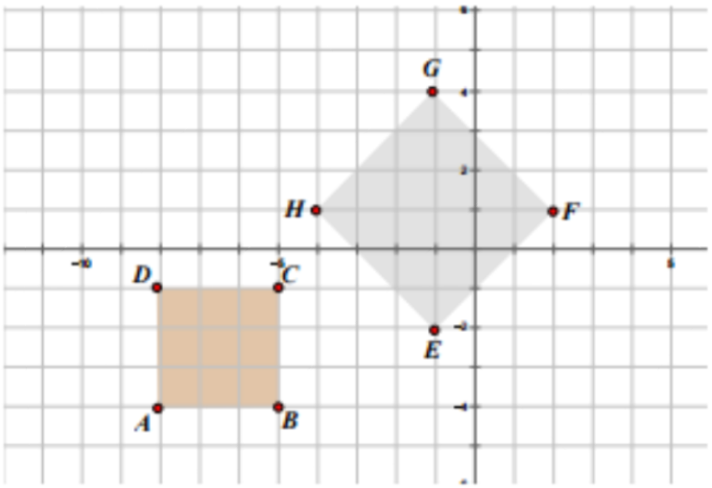
Is ABCD a rhombus? Explain how you know.

Is EFGH a rhombus? Explain how you know.

Students' handout for Alex's ELL Math Class

APPENDIX E

Students' handout for Alex's ELL Math Class



Is ABCD a square? Explain how you know.

Is EFGH a square? Explain how you know.

APPENDIX F

Questions for Review in Alex's ELL Math Class

Ready

Topic: Tables of value

Find the value of $f(x)$ for the given domain. Write x and $f(x)$ as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

3. $f(x) = 5^x$

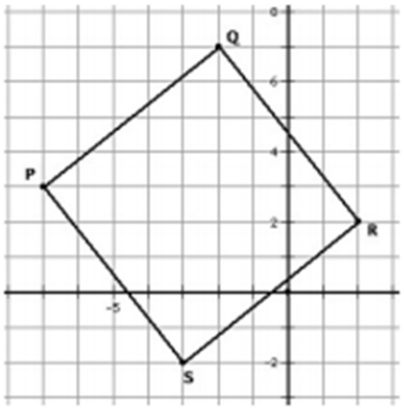
x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

Set

Topic: Characteristics of rectangles and squares

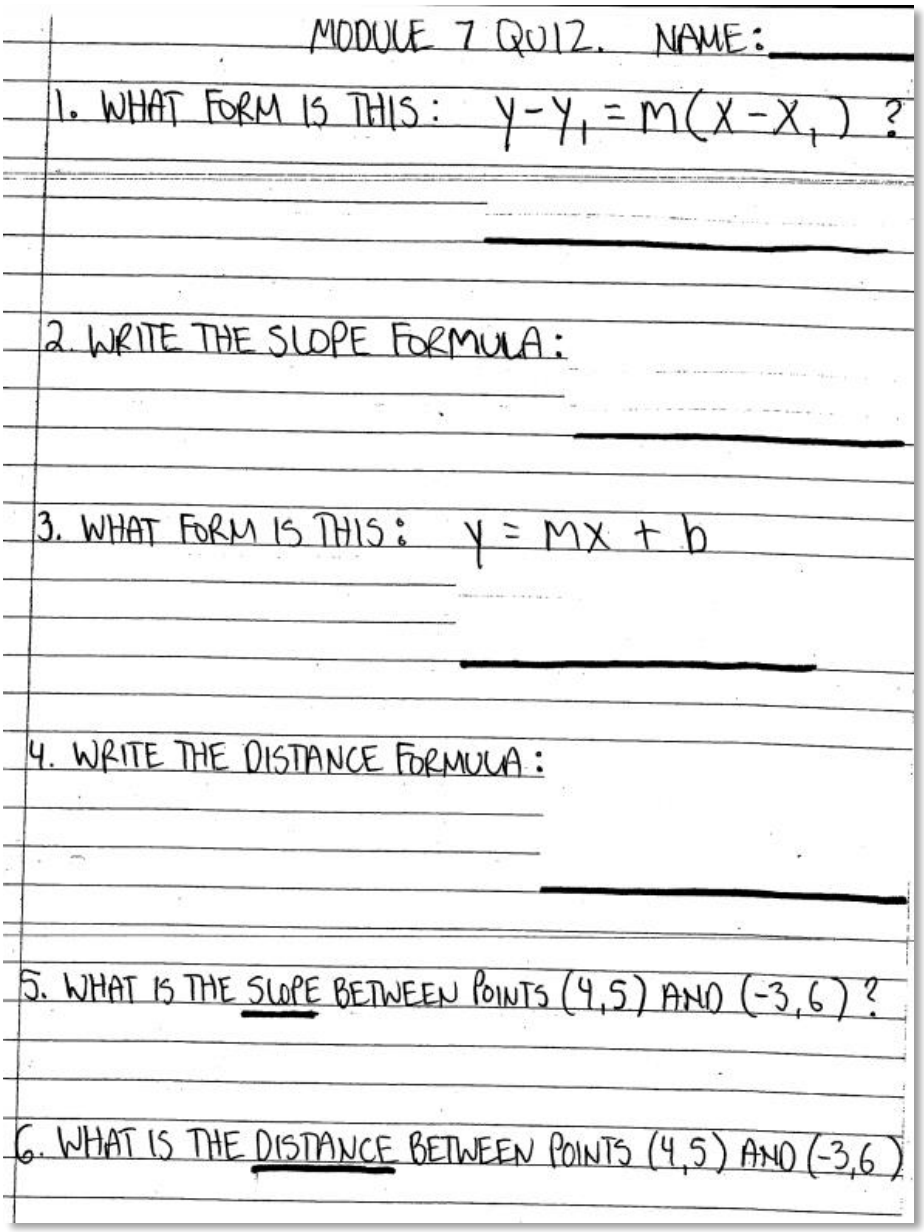
4a. Is the figure below a rectangle? (Justify your answer)

b. Is the figure a square? (Justify your answer)



APPENDIX G

Quiz (Part I) in Alex's ELL Math Class



MODULE 7 QUIZ. NAME: _____

1. WHAT FORM IS THIS: $y - y_1 = m(x - x_1)$?

2. WRITE THE SLOPE FORMULA:

3. WHAT FORM IS THIS: $y = mx + b$

4. WRITE THE DISTANCE FORMULA:

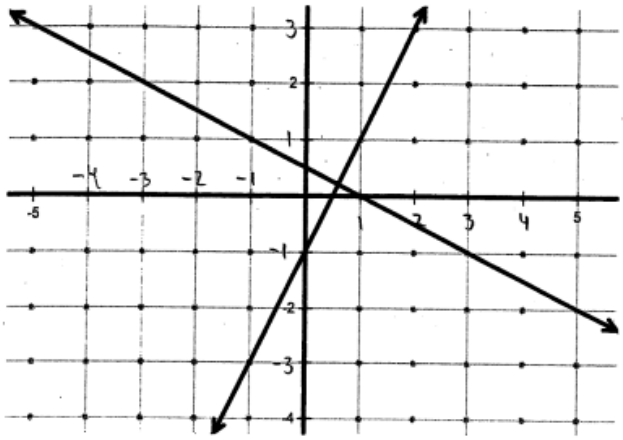
5. WHAT IS THE SLOPE BETWEEN POINTS (4,5) AND (-3,6)?

6. WHAT IS THE DISTANCE BETWEEN POINTS (4,5) AND (-3,6)

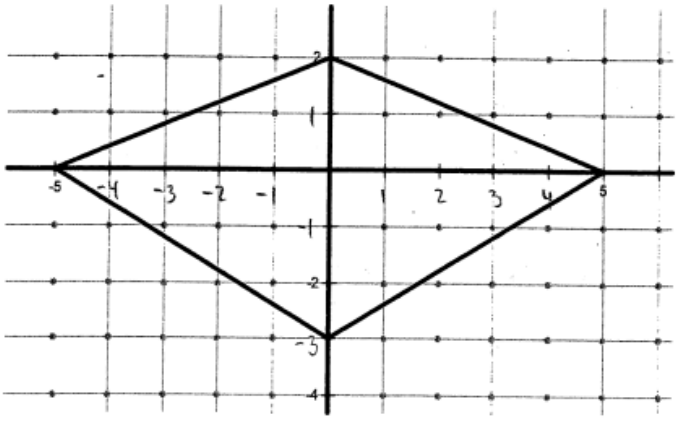
APPENDIX H

Quiz (Part II) in Alex's ELL Math Class.

7. Are the following lines perpendicular? SHOW WHY OR WHY NOT WITH MATH.

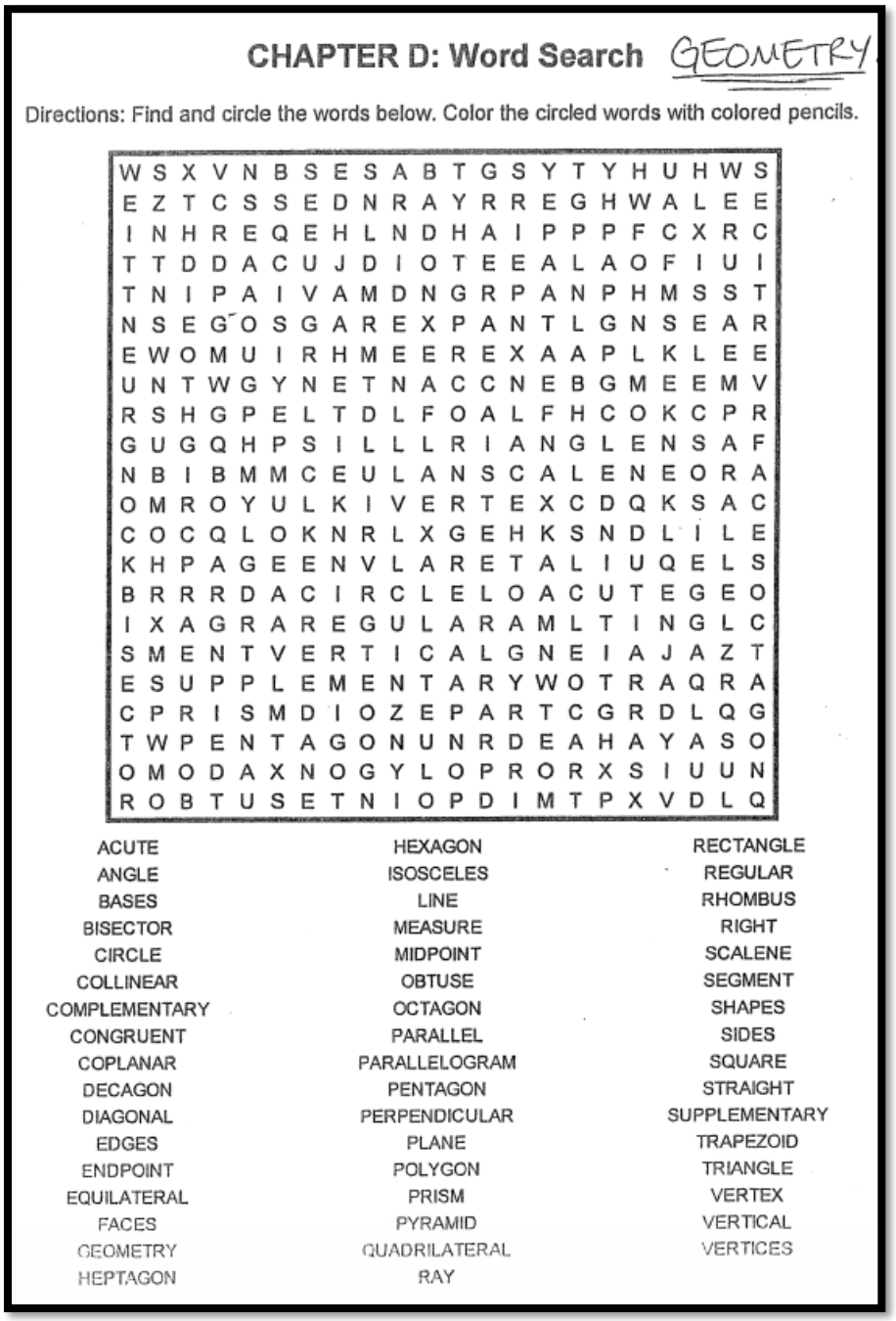


8. IS the following quadrilateral a rectangle, square, parallelogram, rhombus, or neither? SHOW ME HOW YOU KNOW WITH MATH.



APPENDIX I

Bob's Word Search



CHAPTER D: Word Search GEOMETRY

Directions: Find and circle the words below. Color the circled words with colored pencils.

W	S	X	V	N	B	S	E	S	A	B	T	G	S	Y	T	Y	H	U	H	W	S
E	Z	T	C	S	S	E	D	N	R	A	Y	R	R	E	G	H	W	A	L	E	E
I	N	H	R	E	Q	E	H	L	N	D	H	A	I	P	P	P	F	C	X	R	C
T	T	D	D	A	C	U	J	D	I	O	T	E	E	A	L	A	O	F	I	U	I
T	N	I	P	A	I	V	A	M	D	N	G	R	P	A	N	P	H	M	S	S	T
N	S	E	G	O	S	G	A	R	E	X	P	A	N	T	L	G	N	S	E	A	R
E	W	O	M	U	I	R	H	M	E	E	R	E	X	A	A	P	L	K	L	E	E
U	N	T	W	G	Y	N	E	T	N	A	C	C	N	E	B	G	M	E	E	M	V
R	S	H	G	P	E	L	T	D	L	F	O	A	L	F	H	C	O	K	C	P	R
G	U	G	Q	H	P	S	I	L	L	L	R	I	A	N	G	L	E	N	S	A	F
N	B	I	B	M	M	C	E	U	L	A	N	S	C	A	L	E	N	E	O	R	A
O	M	R	O	Y	U	L	K	I	V	E	R	T	E	X	C	D	Q	K	S	A	C
C	O	C	Q	L	O	K	N	R	L	X	G	E	H	K	S	N	D	L	I	L	E
K	H	P	A	G	E	E	N	V	L	A	R	E	T	A	L	I	U	Q	E	L	S
B	R	R	R	D	A	C	I	R	C	L	E	L	O	A	C	U	T	E	G	E	O
I	X	A	G	R	A	R	E	G	U	L	A	R	A	M	L	T	I	N	G	L	C
S	M	E	N	T	V	E	R	T	I	C	A	L	G	N	E	I	A	J	A	Z	T
E	S	U	P	P	L	E	M	E	N	T	A	R	Y	W	O	T	R	A	Q	R	A
C	P	R	I	S	M	D	I	O	Z	E	P	A	R	T	C	G	R	D	L	Q	G
T	W	P	E	N	T	A	G	O	N	U	N	R	D	E	A	H	A	Y	A	S	O
O	M	O	D	A	X	N	O	G	Y	L	O	P	R	O	R	X	S	I	U	U	N
R	O	B	T	U	S	E	T	N	I	O	P	D	I	M	T	P	X	V	D	L	Q

ACUTE
ANGLE
BASES
BISECTOR
CIRCLE
COLLINEAR
COMPLEMENTARY
CONGRUENT
COPLANAR
DECAGON
DIAGONAL
EDGES
ENDPOINT
EQUILATERAL
FACES
GEOMETRY
HEPTAGON
HEXAGON
ISOSCELES
LINE
MEASURE
MIDPOINT
OBTUSE
OCTAGON
PARALLEL
PARALLELOGRAM
PENTAGON
PERPENDICULAR
PLANE
POLYGON
PRISM
PYRAMID
QUADRILATERAL
RAY
RECTANGLE
REGULAR
RHOMBUS
RIGHT
SCALENE
SEGMENT
SHAPES
SIDES
SQUARE
STRAIGHT
SUPPLEMENTARY
TRAPEZOID
TRIANGLE
VERTEX
VERTICAL
VERTICES

APPENDIX J

Cody's handout for Application of Pythagorean Theorem Lesson

10.3 Application of Pythagorean Theorem

Name _____
Period _____

1. Amir's sister is away at college, and he wants to mail her a 34 in. baseball bat. The packing service sells only one kind of box, which measures 24 in. by 20 in. by 12 in. Will the box be big enough?

2. A giant California redwood tree, 36 meters tall, cracked in a violent storm and fell as if hinged. The tip of the once beautiful tree hit the ground 24 meters from the base. Researcher Red Woods wishes to investigate the crack. How many meters up from the base of the tree does he have to climb?

3. Meteorologist Paul Windward and geologist Rhaina Stone are rushing to a paleontology conference in Pecos Gulch. Paul lifts off in his balloon at noon from Lost Wages, heading east for Pecos Gulch Conference Center. With the wind blowing west to east, he averages a land speed of 30 km/h. This will allow him to arrive in 4 hours, just as the conference begins. Meanwhile, Rhaina is 160 km north of Lost Wages. At the moment of Paul's liftoff, Rhaina hops into an off-road vehicle and heads directly for the conference center. At what average speed must she travel to arrive at the same time Paul does?

4. A 25-foot ladder is placed against a building. The bottom of the ladder is 7 feet from the building. If the top of the ladder slips down 4 feet, how many feet will the bottom slide out? (It is not 4 feet.)