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## Particle-hole asymmetry of fractional quantum Hall states in the second Landau level of a two-dimensional hole system

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We report the unambiguous observation of a fractional quantum Hall state in the Landau level of a two-dimensional-hole sample at the filling factor  $\nu = 8/3$ . We identified this state by a quantized Hall resistance and an activated temperature dependence of the longitudinal resistance and found an energy gap of 40 mK. Notably, the particle-hole conjugate state at filling factor  $\nu = 7/3$  in our sample did not develop down to 6.9 mK. This observation is contrary to that in electron samples, in which the  $7/3$  state is typically more stable than the  $8/3$  state. We present evidence that the asymmetry between the  $7/3$  and  $8/3$  states in our hole sample is due to Landau-level mixing.

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In a two-dimensional electron system (2DES) subjected to a perpendicular magnetic field  $B$ , the Coulomb interaction between the charge carriers leads to the emergence of prototype many-body ground states unknown in any other condensed matter system. A well-known example is the series of fractional quantum Hall states (FQHSs) of the lowest Landau level<sup>1</sup> (LL) that develop at Landau-level filling factors  $\nu$  of the form  $m/(2m \pm 1)$ , where  $m$  is an integer. Extensive experimental and theoretical investigations<sup>2</sup> have established that the parent FQHSs are described by Laughlin's wave function<sup>3</sup> while the series of FQHSs of the lowest LL can be described in the framework of Jain's weakly interacting composite fermion model.<sup>4</sup>

Fractional quantum Hall states also form in the second LL (i.e.,  $2 < \nu < 4$ ) but, in contrast to their lowest-LL counterparts, the nature of these states is not well understood. Of these, the  $\nu = 5/2$  even-denominator FQHS has attracted significant attention<sup>5-21</sup> as it is believed to arise from a  $p$ -wave pairing of composite fermions described by the Moore-Read-Pfaffian wave function.<sup>22</sup> With increasing sample quality, an increasing number of odd-denominator FQHSs have been observed in the second LL.<sup>5-21</sup> For the  $\nu = 7/3$  FQHS, the most prominent of these states, recent numerical work finds evidence of Laughlin correlations.<sup>23-25</sup> However, other authors find the  $\nu = 7/3$  FQHS to be either exotic, with a wave function that is distinct from Laughlin's wave function,<sup>26-28</sup> or on the borderline between the Laughlin and exotic non-Abelian states.<sup>29</sup> Results of recent energy-gap measurements<sup>13</sup> and of experiments probing the back-propagating neutral modes<sup>21</sup> for the  $\nu = 7/3$  and its particle-hole conjugate  $8/3$  FQHS in high density 2DES are consistent with these states being of the Laughlin type. Experiments on lower-density 2DES in tilted magnetic fields, however, yielded surprising and as yet unexplained dependence of the energy gap at  $\nu = 7/3$  on the in-plane

magnetic field.<sup>16,20</sup> The nature of the odd-denominator FQHS in the second LL remains to be elucidated.

Fractional quantum Hall states can be probed by varying the Landau-level mixing (LLM).<sup>30</sup> Since the cyclotron energy greatly exceeds the Coulomb energy at large magnetic fields, the excited Landau levels can be neglected and the energy gap of the FQHS is therefore solely determined by the Coulomb energy. At low magnetic fields, at which the Coulomb energy exceeds the cyclotron energy, the gap is influenced by the higher Landau levels and therefore mixing of the Landau levels due to the Coulomb energy has to be considered.<sup>30</sup> In the lowest LL, LLM is known to reduce the energy gaps of the FQHSs.<sup>30-32</sup> In contrast, in the second LL, LLM is not yet fully understood but it is expected to have a more profound effect. Theoretical work on the  $\nu = 5/2$  Pfaffian found that LLM can lift the degeneracy of the Pfaffian and its nonequivalent particle-hole conjugate anti-Pfaffian.<sup>33-40</sup> It may induce a transition from the Pfaffian to the anti-Pfaffian state,<sup>33-36</sup> or it may enhance the  $\nu = 5/2$  Pfaffian.<sup>38</sup> Alternative possibilities are a linear superposition of the Pfaffian and anti-Pfaffian and spatially randomized domains of Pfaffian and anti-Pfaffian controlled by the disorder.<sup>37</sup> Similar ideas should also apply for exotic odd-denominator states in the second LL which are degenerate at vanishing LLM.<sup>26,27,41,42</sup>

We have studied the FQHS of the second LL at extremely large LLM, which is realized in a two-dimensional hole sample (2DHS). Indeed, due to the larger effective mass of holes as compared to that of electrons in GaAs, LLM is enhanced in  $p$ -doped samples as compared to  $n$ -doped samples with the same density.<sup>43</sup> We report the first unambiguous observation of a FQHS in the second LL of a 2DHS at  $\nu = 8/3$ . This was possible because of the combination of progress in the growth of exceptional-quality carbon-doped 2DHSs<sup>44,45</sup> and achievement of ultralow-charge carrier temperature.<sup>46</sup> The

$8/3$  FQHS has an energy gap of 40 mK and, remarkably, its particle-hole symmetric pair at  $\nu = 7/3$  does not develop down to the lowest temperatures of 6.9 mK. This observation is contrary to that in electron samples where the  $\nu = 7/3$  FQHS is typically more robust than the  $\nu = 8/3$  FQHS. Our data shows that the absence of the  $7/3$  state down to the lowest temperatures is unlikely to be caused by a spin transition and we conclude, therefore, that it is most likely a LLM effect.

The two samples used in this study were cleaved from the same wafer, which is a carbon-doped 20-nm wide GaAs-AlGaAs quantum well grown on the high-symmetry surface (100) of GaAs. We chose a carbon-doped 2DHG grown on (100) over silicon-doped samples grown onto (311)A because of its simpler band structure, more isotropic conduction in the absence of a magnetic field, and superior hole mobilities achieved at similar densities.<sup>44,45,47</sup> After illumination with red light, the first sample had a density of  $6.2 \times 10^{10} \text{ cm}^{-2}$  and mobility  $2.7 \times 10^6 \text{ cm}^2/\text{Vs}$  at 6.9 mK. The second sample was thinned down to 100  $\mu\text{m}$  in order to change the carrier density by back-gating. Eight Ohmic contacts were prepared on the perimeter of these 4 mm  $\times$  4 mm pieces from indium-zinc alloy.

Magnetotransport measurements at ultralow temperatures were performed at an excitation of 2 nA in a custom-designed Oxford 400- $\mu\text{W}$  dilution refrigerator. At mK temperatures, poor thermal contact often results in a saturation of the effective charge-carrier temperature at a value higher than that of the refrigerator. In order to mitigate this, the sample was soldered onto sintered silver electrodes that were immersed into a liquid He<sup>3</sup> bath.<sup>14,46</sup> The temperature is measured by monitoring the magnetic-field-independent viscosity of the He<sup>3</sup> with a quartz tuning fork immersed into the same He<sup>3</sup> bath.<sup>46</sup> Since we cannot measure the temperature of the charge carriers directly, we monitor a transport feature which depends strongly on  $T$ . For this purpose we have chosen the  $\nu = 2$  integer quantum

Hall state shown in Fig. 1. As seen in Fig. 2(b), the width of the  $\nu = 2$  plateau does not saturate but instead changes very rapidly with decreasing  $T$ . We therefore believe that the temperature of our charge carriers follows that of the He<sup>3</sup> bath to the lowest temperatures.

Figure 1 shows the longitudinal resistance  $R_{xx}$  and Hall resistance  $R_{xy}$  of the ungated sample at a bath temperature of  $T = 6.9 \text{ mK}$ . The terminal filling factor at the largest  $B$  fields is  $\nu = 1/3$  (not shown), the same as in the 2DHS grown on the (311)A surface.<sup>43</sup> The energy gap at  $\nu = 1/3$   $\Delta_{1/3} = 1.74 \text{ K}$  exceeds by 16% the 1.5 K value reported in a 2DHS with a similar density  $6.55 \times 10^{10} \text{ cm}^{-2}$  grown on (311)A surface,<sup>47</sup> demonstrating the exceptionally high quality of this sample. We also observe a large number of fully quantized FQHSs in the lowest LL, such as the ones at  $\nu = 4/3, 5/3,$ <sup>45,48–50</sup> and, notably, at  $\nu = 7/5, 8/5$ .

Figure 2(a) shows details of the second-Landau-level transport between filling factors two and three. We observe a well-developed FQHS at  $\nu = 8/3$  signaled by a Hall plateau quantized better than 0.2% as referenced to the  $\nu = 2$  plateau. The Arrhenius plot of Fig. 2(c) reveals an activated behavior with a gap  $\Delta_{8/3} = 40 \text{ mK}$ . The deviation from the activated law at the lowest  $T$  seen in Fig. 2(c) is commonly reported in transport data and is thought to be due to a change from thermally activated conduction to hopping. In our sample we do not observe any features<sup>50,51</sup> at  $\nu = 5/2$  and, unlike in

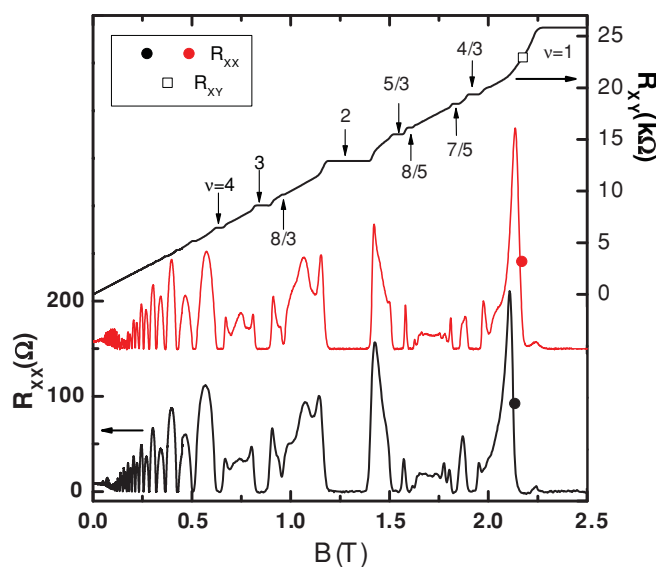


FIG. 1. (Color online) Magnetotransport data of the ungated sample at 6.9 mK. The two  $R_{xx}$  traces are measured along perpendicular directions and show the absence of a strong anisotropy even at finite  $B$  fields.

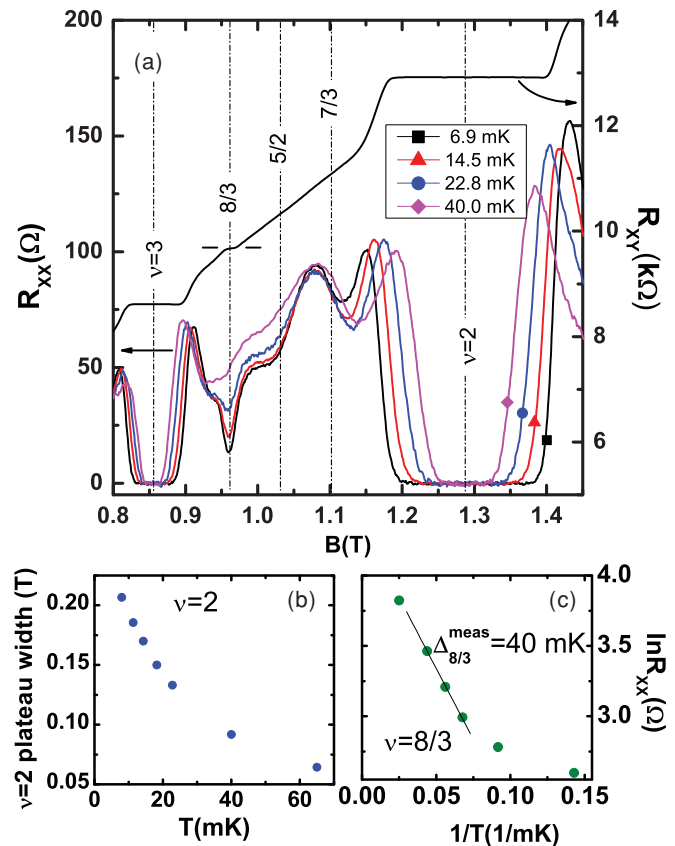


FIG. 2. (Color online) (a)  $T$  dependence of  $R_{xx}$  and  $R_{xy}$  at 6.9 mK in the second LL region. (b) The  $T$  dependence of the width of the  $\nu = 2$  plateau. (c)  $T$  dependence of  $R_{xx}$  at  $\nu = 8/3$  shows an activated behavior.

higher-density carbon-doped 2DHS,  $R_{xx}$  at  $\nu = 7/2$  and  $11/2$  is isotropic.<sup>50</sup>

In Fig. 2(a) we also observe a broad minimum in  $R_{xx}$  centered around  $1.13 T$ , but this minimum is not accompanied by any discernible features in  $R_{xy}$ , therefore we conclude it is not a signature of a FQHS at  $\nu = 7/3$ . Another broad feature in  $R_{xx}$  with no signature in  $R_{xy}$  is also seen at  $\nu = 8/3$  above  $40$  mK, a temperature at which the  $\nu = 8/3$  FQHS does not survive. A similar broad feature in  $R_{xx}$  at  $\nu = 8/3$  has been reported in Ref. 52 at  $100$  mK and in Ref. 53 at  $50$  mK in a tilted  $B$  field, but without mentioning a quantized  $R_{xy}$  plateau or an activated transport. Therefore, those features can hardly be ascribed to a FQHS. We thus report the unambiguous observation of a FQHS in the second LL of a 2DHS at  $\nu = 8/3$ .

It is remarkable that the  $\nu = 8/3$  FQHS develops at the very low  $B$  field of  $0.96 T$ , at which no FQHS of the second LL has been reported in either 2DHS or 2DES. Hence, the possibility of a spin transition has to be considered which is identified by a gradual decrease, followed by an increase, of the gap as either the in-plane  $B$  field or the density is varied.<sup>52–54</sup> In order to avoid possible anisotropic stripe phases induced by the tilted field observed in 2DES,<sup>20</sup> we investigated the response of the states to back-gating. In spite of the  $\nu = 8/3$  FQHS being adversely affected by the degrading of the sample due to processing, we still discern an inflexion point in  $R_{xy}$ . As seen in Fig. 3, in the  $8.77$  to  $5.15 \times 10^{10}$  cm<sup>2</sup> density range we do not observe a strengthening of the  $\nu = 8/3$  FQHS or an emergence of the  $\nu = 7/3$  FQHS. Thus, in the density range accessed we do not observe a spin transition for either the  $8/3$  or the  $7/3$  FQHS.

The effective mass of 2D holes in GaAs can be larger by a factor of five compared to that of electrons. As a consequence, LLM is enhanced by the same factor in 2DHSs as compared to 2DESs at a given density.<sup>43</sup> The strength of the LLM is encoded into the LLM parameter  $\kappa$  defined as the ratio of the Coulomb and cyclotron energies.<sup>30</sup> Using an effective mass  $m_{\text{eff}} = 0.39 m_e$  for our carbon-doped 2DHS,<sup>55</sup> we find  $\kappa = 14.8$  at  $\nu = 8/3$ . This value is one order of magnitude larger than  $\kappa = 1.6$ , the largest LLM parameter at which  $\nu = 8/3$  FQHS has been previously reported in a 2DES.<sup>9,11,20</sup> Thus the  $\nu = 8/3$  FQHS in our 2DHS develops in the limit of extremely strong LLM.

By ruling out the possibility of a spin transition in the density range accessed, we surmise that the different relative strength of the  $8/3$  and  $7/3$  FQHS in electron and hole samples is likely caused by LLM. LLM is known to break particle-hole symmetry<sup>33–40</sup> and it may change the relative strength of the  $7/3$  and  $8/3$  FQHS. A known example of particle-hole asymmetry for FQHS in the second LL thought to be induced by LLM is that of the well-developed  $\nu = 12/5$  (but missing particle-hole conjugate  $\nu = 13/5$ ) FQHS in electron samples.<sup>13,14,18</sup>

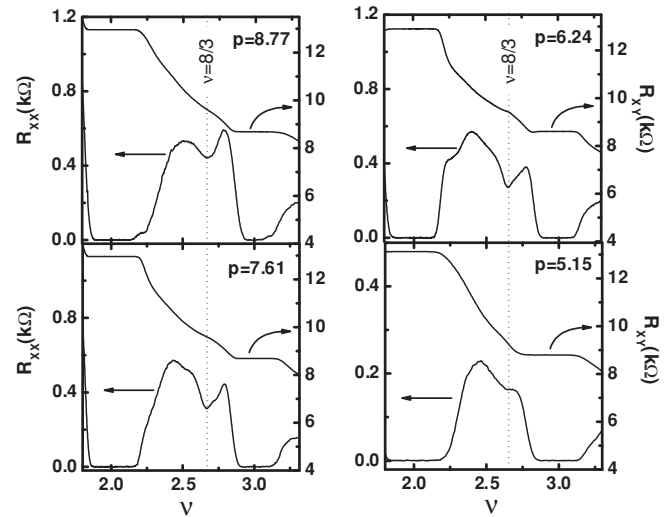


FIG. 3. Density dependence of the magnetoresistance of the back-gated sample at  $6.9$  mK. Densities are indicated in units of  $10^{10}$  cm<sup>-2</sup>.

The well-developed FQHS at  $\nu = 8/3$ , together with no transport signature at  $\nu = 7/3$  in our 2DHS, is in stark contrast to the observations in 2DESs, in which the gap of the  $7/3$  FQHS is found to be larger than that of the  $8/3$  FQHS in the vast majority of reports.<sup>6–13</sup> An exception is presented in Ref. 18. We have argued that this unexpected violation of the particle-hole symmetry in the second Landau level in our 2DHS must be due to LLM. In the absence of a thorough understanding of the details of the particle-hole symmetry violation, the nature of the  $8/3$  FQHS in our 2DHS remains unresolved, but our data hints toward a possible change of the nature of the  $8/3$  and (or)  $7/3$  FQHS with increasing LLM.

In summary, we found a quantized FQHS at  $\nu = 8/3$  in the second LL and at  $\nu = 7/5$  and  $8/5$  in the lowest LL of a 2DHS. An interesting unexplained feature of our data is the absence of the  $7/3$  FQHS, which we conjecture is a result of the particle-hole symmetry-breaking effects due to strong LL mixing. Our results in the 2DHS together with results in 2DESs point toward a need for theoretical models which include such symmetry-breaking terms.

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