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Quantitative analysis of the disorder broadening and the intrinsic gap for the $\nu = \frac{5}{2}$ fractional quantum Hall state

N. Samkharadze,[1] J. D. Watson,[1,2] G. Gardner,[2] M. J. Manfra,[1,2,3] L. N. Pfeiffer,[4] K. W. West,[4] and G. A. Csáthy[1,*]
[1]*Department of Physics, Purdue University, West Lafayette, Indiana 47907, USA*
[2]*Birck Nanotechnology Center Purdue University, West Lafayette, Indiana 47907, USA*
[3]*School of Materials Engineering and School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana 47907, USA*
[4]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*
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We report a reliable method to estimate the disorder broadening parameter from the scaling of the gaps of the even and major odd denominator fractional quantum Hall states of the second Landau level. We apply this technique to several samples of vastly different densities and grown in different molecular beam epitaxy chambers. Excellent agreement is found between the estimated intrinsic and numerically obtained energy gaps for the $\nu = 5/2$ fractional quantum Hall state. Furthermore, we quantify the dependence of the intrinsic gap at $\nu = 5/2$ on Landau-level mixing.

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Disorder plays an important role in the formation of the fractional quantum Hall states (FQHSs) observed in the two-dimensional electron gas (2DEG).[1,2] While qualitative aspects of the effect of the disorder have been appreciated early on,[1] the quantitative effect of the disorder on the properties such as the energy gap of the FQHSs remains poorly understood.

Currently significant effort is focused on the FQHS at the Landau-level (LL) filling factor $\nu = 5/2$.[3–22] This state does not belong to the sequence of FQHSs described by the theory of weakly interacting composite fermions (CFs)[23,24] and, therefore it may have exotic quantum correlations which are not of the Laughlin type.[23,25] It is believed that the $\nu = 5/2$ FQHS arises from a p-wave pairing of the CFs described by either the Pfaffian[26–29] or the anti-Pfaffian[30–32] wave function.

Agreement between the measured energy gap $\Delta_{5/2}^{\rm meas}$ and that from numerical studies is a necessary condition for an identification of the $\nu = 5/2$ FQHS with the Pfaffian wave function.[33–41] Gaps in numerical studies are always calculated in the absence of any disorder and they must be therefore compared to the measured gaps extrapolated to zero disorder, also called the intrinsic gap $\Delta^{\rm int}$. While the effect of disorder on the gap is small for the most prominent FQHS at $\nu = 1/3$,[42] it is quite large at $\nu = 5/2$.[4] Hence a quantitative knowledge of the gap suppression by the disorder and of the intrinsic gap plays a significant role in the identification of the nature of the exotic FQHSs in the second LL.

Three different methods have been used so far to obtain the intrinsic gap at $\nu = 5/2$ but, to date, they have not yielded consistent results. A scaling of the measured gaps of the even denominator FQHSs (Ref. 35) and an estimation using the quantum lifetime[13] found good agreement between experimental and numerical intrinsic gaps. However, extrapolations of $\Delta_{5/2}^{\rm meas}$ to infinite mobility[15,18] and a recent estimation from the quantum lifetime[17] found an intrinsic gap three times smaller than expected. This situation calls for a reexamination of the extraction of $\Delta_{5/2}^{\rm int}$ from the measurements.

We adopt the method of quantifying the effect of the disorder using the even denominator FQHS (Ref. 35) and propose a method using the two odd denominator FQHSs at $\nu = 2 + 1/3$ and $2 + 2/3$. We find that, within experimental error, these two methods give consistent results in samples of very different densities and grown in different molecular beam epitaxy (MBE) chambers. The intrinsic gaps $\Delta_{5/2}^{\rm int}$ found are in excellent agreement with gaps calculated from numerics which include the effects of Landau-level mixing (LLM) and finite extent of the wave function. Our results strongly indicate that the paired-state Pfaffian is the correct description of the $\nu = 5/2$ FQHS. We also show that $\Delta_{5/2}^{\rm int}$ cannot be reliably obtained from the quantum lifetime or from extrapolation of $\Delta_{5/2}^{\rm meas}$ to infinite mobility. From the dependence of the intrinsic gap of the $\nu = 5/2$ FQHS on LLM we find that the $\nu = 5/2$ FQHS becomes unstable beyond a threshold value of the LLM parameter $\kappa_{\rm th} = 2.9$.

There are two GaAs quantum-well samples used in this study. Sample A grown at Princeton has a well width of 56 nm, a density $n = 8.30 \times 10^{10}$ cm^{-2}, and mobility $\mu = 12 \times 10^6$ cm^2/V s. Sample B grown in a newly built MBE chamber at Purdue has a width of 30 nm, a density $n = 2.78 \times 10^{11}$ cm^{-2}, and mobility $\mu = 11 \times 10^6$ cm^2/V s. Both wells are flanked by $Al_{0.24}Ga_{0.76}As$ barriers with the Si donors placed symmetrically from the well at 320 and 78 nm, respectively. Samples are mounted in a ^{3}He immersion cell described in detail in Ref. 43.

Figure 1 shows the longitudinal R_{xx} and transverse R_{xy} resistances as function of the magnetic field B in the second LL (i.e., for $2 < \nu < 3$) for the two samples. The $\nu = 5/2$ FQHS is fully quantized in both samples; this state in sample A occurs, to the best of our knowledge, at the lowest magnetic field of 1.37 T yet reported.[15–17] Other FQHSs also develop. Notably, sample B has a fully quantized $2 + 2/5$ FQHS and an incipient $2 + 3/8$ FQHS, hallmarks of the highest-quality samples.[6,14] We note that the mobility of sample B is approximately a factor of 3 lower than that of other samples exhibiting similar higher-order FQHSs.[6,14]

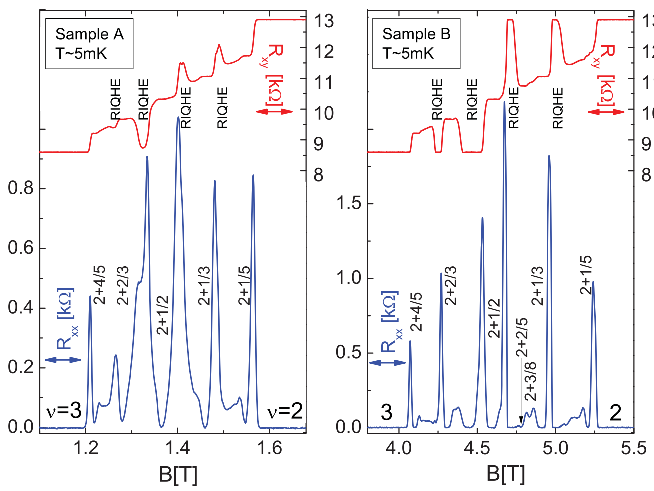

FIG. 1. (Color online) Magnetotransport data in the region of the upper spin branch of the second Landau level. We mark the filling factors ν of the observed FQHSs and the reentrant integer quantum Hall states (RIQHSs). Note the vastly different densities of the two samples.

Figure 2 shows the Arrhenius plots of R_{xx} for selected FQHSs observed in the second LL of sample A. The $\Delta^{\rm meas}$ extracted using $R_{xx} \propto \exp(-\Delta^{\rm meas}/2T)$ are shown in Table I. Since in this work we will analyze the gaps of the $\nu = 5/2$, $7/2$, $2+1/3$, and $2+2/3$ FQHSs, in Table I we also consider samples for which the gaps for these four FQHSs are available.[14,15] For the sample in Ref. 14, $\Delta^{\rm meas}_{7/2} = 240$ mK.

In order to estimate the intrinsic gap $\Delta^{\rm int}_{5/2}$ for the $\nu = 5/2$ FQHS, an extrapolation of $\Delta^{\rm meas}_{5/2}$ to infinite mobility has been used recently.[15,18] We argue that such an extrapolation is inherently inaccurate. Indeed, our sample B shows unusually large gaps in spite of a modest mobility $\mu = 11 \times 10^6$ cm^2/V s and, therefore, it is quite a bit off from the extrapolation done in Refs. 15 and 18. We conclude that, as previously noted,[17–21] the intrinsic gap does not directly correlate with the mobility.

The influence of the disorder on the gaps can be understood within the framework of a widely used phenomenological model[44] according to which the quantized energy levels of the 2DEG are broadened by the disorder into bands of localized states of width Γ. In this model the disorder broadening parameter Γ relates the measured and the intrinsic gaps

$$\Delta^{\rm int} = \Delta^{\rm meas} + \Gamma. \tag{1}$$

This model was instrumental in the analysis of the gaps of the FQHS in the lowest LL[42,45–47] in terms of Laughlin's wave function[25] and Jain's CF theory[23] and we will use it for the FQHSs of the second LL.

We turn our attention to an independent extraction of Γ from the measured data. As mentioned above, Γ has been estimated from the quantum lifetime τ_q. The B-field dependence of the envelope of the Shubnikov-de Haas oscillations at a fixed temperature contains the $\exp(-\pi/\omega_C\tau_q)$ multiplicative factor from which τ_q and $\Gamma_{\rm SdH} = \hbar/\tau_q$ is extracted.[48] Here ω_C is the cyclotron frequency. The values found are summarized in Table II.

Γ can also be found from a scaling of $\Delta^{\rm meas}$ of the even denominator FQHS at $\nu = 5/2$ and $7/2$ with the Coulomb energy $E_C = e^2/\epsilon l_B$.[35] Here $l_B = \sqrt{\hbar/eB}$ is the magnetic length. Particularly, by assuming that the intrinsic gap of the 5/2 and 7/2 is affected by LLM the same way, $\Delta^{\rm int} = \delta^{\rm int} E_C$ was found with the same adimensional intrinsic gap $\delta^{\rm int}$. Therefore $\Gamma_{\rm even}$ is extracted from $\Delta^{\rm meas} = \delta^{\rm int} E_C - \Gamma_{\rm even}$ equation as the negative intercept of the measured gaps of 5/2 and 7/2 FQHS versus E_C. Such an analysis is shown in Fig. 3. As seen in Table II and discussed in Ref. 15, $\Gamma_{\rm even}$ obtained this way may differ significantly from $\Gamma_{\rm SdH}$, by as much as one order of magnitude.

In order to resolve this discrepancy we introduce a third method of extracting Γ from the gaps of the odd denominator states $\nu = 2 + 1/3$ and $2 + 2/3$. Recently we reported that the equation $\Delta^{\rm meas} = \hbar e|B_{\rm eff}|/m_{\rm eff} - \Gamma_{\rm odd}$ describes the gaps of the $\nu = 2 + 1/3$ and $2 + 2/3$ FQHSs in the second LL.[14] Here $B_{\rm eff} = 5(B - B_{\nu=5/2})$ is the effective magnetic field after flux attachment from the CF theory.[23,24] This result was interpreted as being suggestive of Laughlin-correlated $\nu = 2 + 1/3$ and $2 + 2/3$ FQHSs.[14] We use the equation above to extract $\Gamma_{\rm odd}$ for the four analyzed samples. Fits to the data are shown in Fig. 3. $\Gamma_{\rm odd}$ is the intercept of the fits and the vertical scale and its values are listed in Table II.

We found that the disorder broadening terms $\Gamma_{\rm odd}$ and $\Gamma_{\rm even}$ have similar values in each sample. Typical errors in $\Delta^{\rm meas}$ of $\pm 5\%$ for gaps above 100 mK and of $\pm 10\%$ below

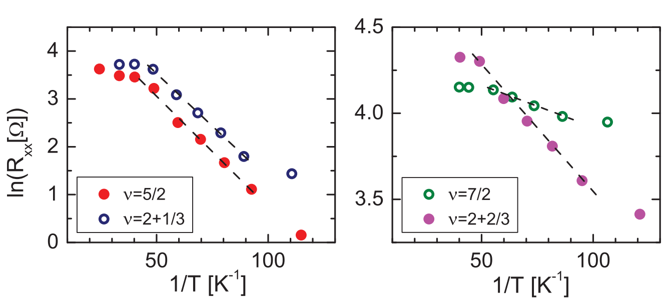

FIG. 2. (Color online) Arrhenius plots used for the extraction of the activation gaps in sample A.

TABLE I. Energy gaps $\Delta^{\rm meas}$ in units of mK for our samples.

Sample	$\Delta^{\rm meas}_{5/2}$	$\Delta^{\rm meas}_{7/2}$	$\Delta^{\rm meas}_{2+1/3}$	$\Delta^{\rm meas}_{2+2/3}$
A	88	10	81	27
B	446	120	497	240

TABLE II. Parameters of samples considered. n is in units of 10^{10}/cm^2, Γ and $\Delta^{\rm int}_{5/2}$ are in Kelvin.

Sample	n	w/l_B	$\Gamma_{\rm SdH}$	$\Gamma_{\rm even}$	$\Gamma_{\rm odd}$	$\Delta^{\rm int}_{5/2}$	$\delta^{\rm int}_{5/2}$
A	8.3	2.56	0.24	0.42	0.35	0.47	0.0080
B	27.8	2.52	2.04	1.65	1.55	2.04	0.019
Ref. 14	30	2.61	1.55	1.50	1.62	2.12	0.019
Ref. 15	16	2.55	0.23	1.16	1.01	1.33	0.016

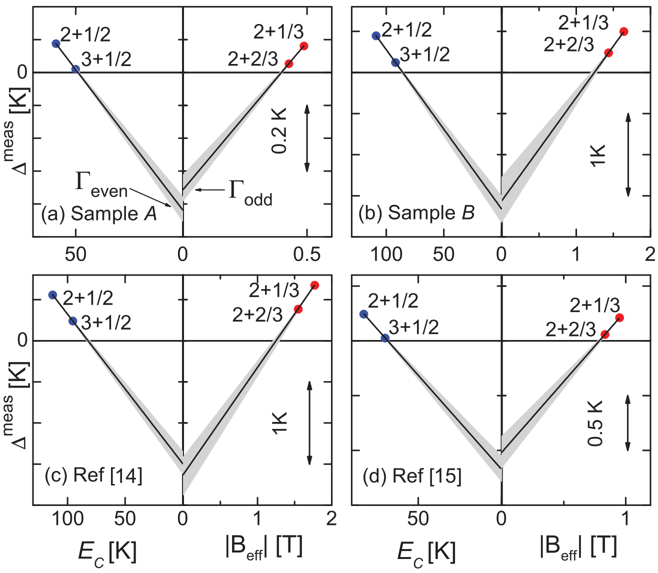

FIG. 3. (Color online) Comparison of the two methods of estimating the disorder paramater Γ for the four analyzed samples. The measured gaps of the even denominator FQHSs extrapolate to Γ_{even} at $E_C = 0$, while those of the $\nu = 2 + 1/3$ and $2 + 2/3$ FQHSs to Γ_{odd} at the vanishing absolute value of B_{eff}. The gray shadow is the estimated error for Γ's.

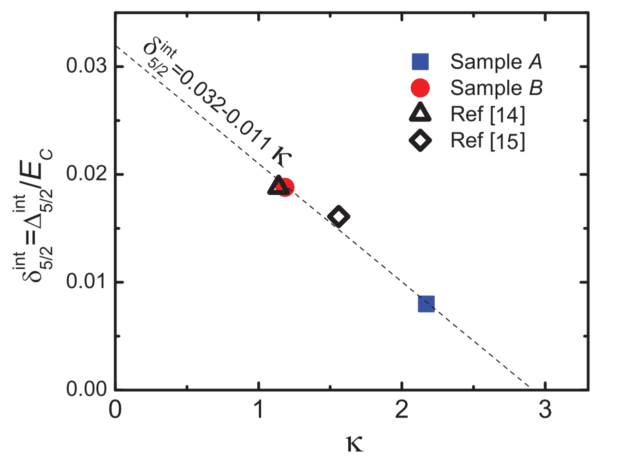

FIG. 4. (Color online) Intrinsic gaps at $\nu = 5/2$ as function of the LLM parameter κ. The dotted line is a linear fit through the data.

100 mK result in measurement errors in Γ, shown as a shadow in Fig. 3, of $\pm 12\%$. We conclude therefore that, within the errors, the even denominator FQHSs at $\nu = 5/2$ and 7/2 and the two strongest odd denominator FQHSs at $2 + 1/3$ and $2 + 2/3$ yield *the same disorder broadening* in samples grown in different chambers and covering a wide range of densities and mobilities. We note that the same disorder broadening for the above FQHSs described by different theories is possible as they all originate from the same type of CFs. Indeed, the $2 + 1/3$ and $2 + 2/3$ FQHSs can be understood from motion of flux-two CFs at a finite B_{eff}, while the 5/2 and 7/2 are due to paired flux-two CFs at $B_{\text{eff}} = 0$.

Γ_{SdH} and Γ_{odd} determined from odd denominator FQHSs in the second LL are not equal. This shows that level broadening is governed by different mechanisms for the low-field Shubnikov-de Haas oscillations and for the high-field second LL physics. Γ_{SdH} is therefore not expected to be relevant in determining the intrinsic gaps of FQHS in the second LL, including the $\nu = 5/2$ FQHS. A similar conclusion has also been reached for the FQHS of the lowest LL centered around $\nu = 1/2$.[45–47]

The experimentally derived $\Delta^{\text{int}}_{5/2}$ estimated from Eq. (1), together with the corresponding adimensional $\delta^{\text{int}}_{5/2} = \Delta^{\text{int}}_{5/2}/E_C$, are found in Table II. For Γ we used the average of Γ_{even} and Γ_{odd}. The comparison of the experimental and numerically estimated intrinsic gaps must be performed at the same extent of the LLM[35–37] and of finite sample width[38–41] as quantified by the LLM parameter $\kappa = E_C/\hbar\omega_C$ (Ref. 49) and adimensional width of the quantum well w/l_B, respectively. We find that $\delta^{\text{int}}_{5/2} = 0.019$ listed in Table II for sample B and that from Ref. 15 is only 19% larger than 0.016, the value calculated from exact diagonalization for similar sample parameters.[35] Also, $\delta^{\text{int}}_{5/2}$ =0.0080, 0.019, 0.019, and 0.016, the values we find in samples A and B and Refs. 14 and 15, compare well with the values 0.014, 0.018, 0.018, and 0.016 we extract from a recent exact diagonalization study.[17] We note that, while sample A and that from Ref. 15 do not have the same width as that in Ref. 17, the previous comparison is meaningful because of the relatively small contributions of finite width effects.[17] We conclude that the intrinsic gaps we find are in excellent agreement with the numerically obtained gaps for four samples of very different densities and mobilities, and which were grown in different MBE chambers. These experimental results, when combined with numerical results,[33–41] strongly support the Pfaffian description of $\nu = 5/2$ FQHS.

The data shown in Table II allows us to study the dependence of the intrinsic gap obtained from measurements on LLM. For a meaningful comparison of gaps in Fig. 4 we plot $\delta^{\text{int}}_{5/2}$ as function of the LLM parameter κ. The four samples listed in Table II have different widths w, but have very similar adimensional widths w/l_B at $\nu = 5/2$, and therefore the gap suppression seen in Fig. 4 is solely due to LLM. We find a decreasing $\delta^{\text{int}}_{5/2}$ with an increasing κ, which is consistent with expectations.[17,36] By assuming a linear dependence for the limited range of κ accessed we find $\delta^{\text{int}}_{5/2}(\kappa = 0) = 0.032$ at no LLM. This value compares well with $\approx$0.030, the numerically obtained gap in the ideal 2D limit.[17,33–35,39]

From our data we also see that $\delta^{\text{int}}_{5/2}$ extrapolates to zero at $\kappa_{\text{th}} = 2.9$ threshold. We conclude that the $\nu = 5/2$ FQHS should not develop for $\kappa > \kappa_{\text{th}}$ or, equivalently, for electron densities lower than $n_{\text{th}} = 4.4 \times 10^{10}$ cm^{-2} even in the limit of no disorder. This result could explain the absence of the $\nu = 5/2$ FQHS in 2D hole samples[50–52] in which, due to the enhanced effective mass of the holes, values of κ lower than 3 have not been achieved.

Finally we note that the dependence of $\Delta^{\text{meas}}_{5/2}$ on the density in an undoped heterojunction insulated gate field effect transistor (HIGFET) sample has recently been fitted to $\Delta^{\text{meas}}_{5/2} = \alpha E_C - \tilde{\Gamma}$, where α and $\tilde{\Gamma}$ are variables.[21] The equation is very similar to the one we used and one could mistakenly think that α is the intrinsic gap. However, in Ref. 21, α is forced to be a constant of the fit. As discussed earlier and also shown in Fig. 4, $\delta^{\text{int}}_{5/2}$ is a strong function of LLM and, therefore, E_C.[17,36,37,49] The intrinsic gap is therefore expected to change with the density. We conclude that, based on the theory, the density-independent constant $\alpha = 0.004\,26$ is *not* expected to be the intrinsic gap of the $\nu = 5/2$ FQHS and that $\tilde{\Gamma}$ is *not* the same disorder broadening as Γ we found in this Rapid Communication.

In summary, we have demonstrated that the disorder broadening can be reliably extracted from Δ^{meas} of the four major FQHSs at $\nu = 5/2$, $7/2$, $2 + 1/3$, and $2 + 2/3$ for samples over a wide range of densities and grown in different MBE chambers. The obtained intrinsic gap of the $\nu = 5/2$ FQHS was found to be in an excellent agreement with numerical results, lending therefore strong support to the Pfaffian description of the $\nu = 5/2$ FQHS.

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*gcsathy@purdue.edu

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