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ABSTRACT

In this paper we consider the problem of best sensor location for optimal state estimation for real time energy efficient optimal control. Although the basic idea can be applied to the joint sensor / actuator location problem, we focus on the sensor placement because of its immediate application to retrofits where sensors can easily relocated while the location of control inputs (diffusers, vents, etc.) are less movable. It is important to note that this approach does not require "time accurate high fidelity simulations" and has been successful in other disciplines such as structural control, control of chemical processes and fluid flow control. We show how these techniques can be applied to high performance building design and control. We present the theoretical basis for the method, discuss the computational requirements and use simple models to illustrate the ideas and numerical methods for solution. We then apply the method results to a multi-zone hospital suite problem.

1. INTRODUCTION

The use of sensor data to monitor spatial phenomena such as temperature and indoor air quality in a building is an essential ingredient in the design and control of high performance buildings. The selection of sensor and actuator hardware and solving the problem of optimal location of this hardware are crucial to efficient system design. It is important to consider the purpose of data collection and then determine how this data can be effectively employed to achieve the system design objectives. For example, given a set of sensors, the problem of determining optimal locations for detecting system failures is not the same as determining the optimal locations if the data is to be used for parameter identification. Consequently, both hardware selection and placement depend on the overall goals and performance requirements.

Determining sensor locations to optimize the performance of the state estimator is important for monitoring and for practical implementation of optimal feedback control laws (e.g. energy efficient controllers). We use distributed parameter control theory to formulate the optimal sensor location problem as an optimal control problem. The cost function is the trace of the state estimate covariance operator for the Kalman filter in Linear Quadratic Gaussian (LQG) optimal control design. The approach formulates the optimal sensor location problem as a distributed parameter system optimal control problem where the state equation is taken to be the Riccati partial differential equation defining the covariance operator. This framework has the benefit that spatial information (sensor location, temperature fields, etc.) occurs naturally and the covariance operator and its trace can be computed with slight modifications of standard numerical algorithms and tools. It is important to note that this approach does not require "time accurate high fidelity simulations" (Burns *et al*., 2008) and has been successful in other disciplines such as structural control, control of chemical processes and fluid flow control (Burns *et al*., 1998). We show how these techniques can be applied to high performance building design and control. The goal of this paper is to demonstrate how distributed parameter system (DPS) theory can be employed to formulate a rigorous optimal sensor placement problem and to provide a computational framework that allows for the development of efficient numerical algorithms.

2. PROBLEM FORMULATION

Let $\Omega \subseteq \mathbb{R}^3$ depict a bounded region such as the conference room in Figure 1 below with inflow inlets and vents in the ceiling. We assume that the room is configured so that the control inlet vents and outlet vents are fixed. The problem we consider is one of optimal sensor location. In particular, we wish to find the locations that provide measured outputs which can be used to optimally estimate the state of the thermal conditions inside the room. We shall use the Kalman filter as the state estimator.

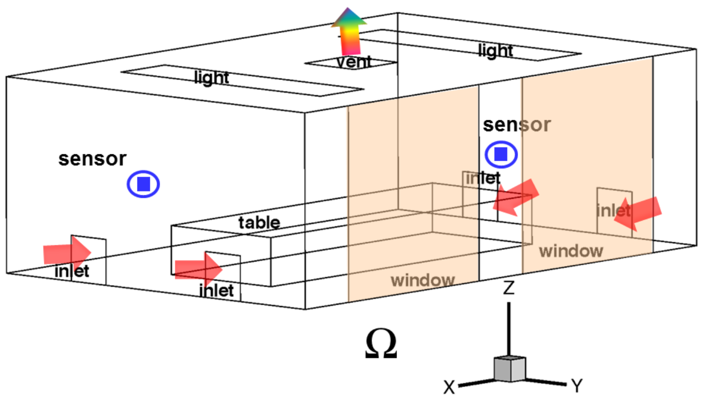

Figure 1: A Typical Conference Room with Sensors on the Walls

The equation that describes the temperature field $T(t,\vec{\mathbf{x}})$, is given by the partial differential equation

$$\tfrac{\partial}{\partial t}T(t,\vec{\mathbf{x}}) + v(\vec{\mathbf{x}})\bullet\nabla T(t,\vec{\mathbf{x}}) = \kappa\nabla^2 T(t,\vec{\mathbf{x}}) + \eta(t,\vec{\mathbf{x}}), \quad \vec{\mathbf{x}} \in \Omega, \tag{1}$$

where $v(\vec{\mathbf{x}})$ is the air flow field, $\eta(t,\vec{\mathbf{x}})$ is a time varying spatial disturbance field. It is typical (and physically meaningful) to assume that the disturbance is a spatially filtered random field given by

$$\eta(t,\vec{\mathbf{x}}) = [\mathcal{G}w(t,\cdot)](\vec{\mathbf{x}}) = \iiint_{\Omega_D} g(\vec{\mathbf{x}},\vec{\mathbf{y}})w(t,\vec{\mathbf{y}})d\vec{\mathbf{y}}, \tag{2}$$

where for each $\vec{\mathbf{x}} \in \Omega$, the kernel $g(\vec{\mathbf{x}},\cdot)$ is defined on a region $\Omega_D \subseteq \bar{\Omega}$ of the room. This assumption is very weak and implies that the disturbance locations (e.g. windows, doors) are known but the disturbance is random and unknown. The control inputs are defined through the boundary conditions and hence for simplicity we assume

$$T(t,\vec{\mathbf{x}})\,|_{inlets} = b_v(\vec{\mathbf{x}})u_v(t) \ \text{ and } \ \mathbf{n}(\vec{\mathbf{x}})\bullet(\kappa\nabla T(t,\vec{\mathbf{x}}))\,|_{\Gamma} = 0, \tag{3}$$

respectively. Here, $\Gamma = \partial\Omega - inlets$ is the remaining portion of the domain boundary and $\mathbf{n}(\vec{\mathbf{x}})$ is the unit outward normal. Since the optimal sensor location problem is independent of a particular control input, we can, without loss of generality, assume $u_v(t) \equiv u_v$ is constant. The advection velocity field $v(\vec{\mathbf{x}})$ is computed using the Navier-Stokes equations with parabolic inflow.

Assume that $\vec{\mathbf{q}} \in \partial\Omega$ is a location on a wall where one places a sensor so that the measured output has the form

$$y(t) = \mathcal{C}(\vec{\mathbf{q}})T(t,\cdot) = \iiint_{\Omega(\vec{\mathbf{q}})} \mathbf{c}(\vec{\mathbf{x}})T(t,\vec{\mathbf{x}})d\vec{\mathbf{x}} + n(t) \in \mathbb{R}^1, \tag{4}$$

and the effective radius of the sensor is defined by $\Omega(\vec{\mathbf{q}}) = \left\{\vec{\mathbf{x}} \in \bar{\Omega} : \|\vec{\mathbf{q}} - \vec{\mathbf{x}}\| < \delta\right\}$. Here, $n(t)$ is the sensor noise and the signal $y(t)$ represents a weighted average temperature, localized at the sensor location $\vec{\mathbf{q}} \in \partial\Omega$. In order to provide a framework that allows for a mathematical formulation of the optimal sensor location problem and to define the optimization algorithm, it is helpful to formulate this PDE control system as an abstract distributed parameter system (DPS) in an infinite dimensional state space.

We use standard notation for and let $X = L^2(\Omega)$ denote the Hilbert space of measureable functions $\varphi(\cdot):\Omega \to \mathbb{R}^1$ on Ω, satisfying $\iiint_{\Omega} [\varphi(\vec{\mathbf{x}})]^2 d\vec{\mathbf{x}} < +\infty$ and define the inner product and norm on X by

$$\langle \varphi(\cdot), \psi(\cdot) \rangle = \iiint_{\Omega} \varphi(\vec{\mathbf{x}})\psi(\vec{\mathbf{x}})d\vec{\mathbf{x}} \text{ and } \|\varphi(\cdot)\| = \sqrt{\iiint_{\Omega} [\varphi(\vec{\mathbf{x}})]^2 d\vec{\mathbf{x}}}, \tag{5}$$

respectively. The Sobolov spaces $H^1(\Omega) = \left\{ \varphi(\cdot) \in L^2(\Omega) : \nabla \varphi(\cdot) \in L^2(\Omega) \right\}$, $H_0^1(\Omega) = \left\{ \varphi(\cdot) \in H^1(\Omega) : \varphi(\cdot)|_{\partial\Omega} = 0 \right\}$ and $H^2(\Omega) = \left\{ \varphi(\cdot) \in H^1(\Omega) : \nabla^2 \varphi(\cdot) \in L^2(\Omega) \right\}$ are defined in Curtain and Zwart (1995). The state operator is the differential operator defined on the domain

$$D(\mathcal{A}) = H_0^1(\Omega) \bigcap H^2(\Omega), \tag{6}$$

by

$$\mathcal{A}\varphi(\vec{\mathbf{x}}) \triangleq \kappa \nabla^2 \varphi(\vec{\mathbf{x}}) - v(\vec{\mathbf{x}}) \cdot \nabla \varphi(\vec{\mathbf{x}}). \tag{7}$$

With this framework we can formulate the convection diffusion problem (1) - (4) as a DPS on $X = L^2(\Omega)$ by identifying $[z(t)](\vec{\mathbf{x}}) \triangleq T(t, \vec{\mathbf{x}})$ and suppressing the spatial variable to yield

$$\dot{z}(t) = \mathcal{A}z(t) + \mathcal{G}w(t) \in X = L^2(\Omega), \tag{8}$$

with output

$$y(t) = \mathcal{C}(\vec{\mathbf{q}})z(t) + n(t). \tag{9}$$

It is important to note that the operator $\mathcal{A}$ is a differential operator, while the disturbance to state operator $\mathcal{G}$ and the measured output operator $\mathcal{C}(\vec{\mathbf{q}})$ are integral operators. Based on the form of these operators, the theory of distributed parameter systems yields valuable and practical information about the structure of controllers and observers and provides the theoretical foundations for advanced numerical algorithms.

2.1 The Optimal Sensor Location Problem

By formulating the room control problem as a DPS in the form (8) – (9), one can apply the theory in papers (Burns and Rautenberg, 2011) and (Rautenberg, 2010) to construct optimal estimators of the temperature field using measured/sensed information alone. In particular, the DPS optimal Kalman filter is a state estimator of the form

$$\dot{z}_e(t) = \mathcal{A}z_e(t) + \mathcal{F}(\vec{\mathbf{q}})[y(t) - \mathcal{C}(\vec{\mathbf{q}})z_e(t)] \in X = L^2(\Omega), \tag{10}$$

where the estimator gain operator $\mathcal{F}(\vec{\mathbf{q}})$ depends on the location $\vec{\mathbf{q}} \in \bar{\Omega}$. The optimal estimator $\mathcal{F}(\vec{\mathbf{q}})$ is given by

$$\mathcal{F}(\vec{\mathbf{q}}) = \Sigma \mathcal{C}(\vec{\mathbf{q}})^*, \tag{11}$$

where Σ is the solution to the operator Riccati equation

$$\mathcal{A}\Sigma + \Sigma\mathcal{A}^* - \Sigma\mathcal{C}(\vec{\mathbf{q}})^*\mathcal{C}(\vec{\mathbf{q}})\Sigma + \mathcal{G}\mathcal{G}^* = 0. \tag{12}$$

Moreover, the solution $\Sigma = \Sigma(\vec{\mathbf{q}})$ is the state estimation covariance operator and the estimation error is given by

$$\mathbb{E}\left(\int_0^{+\infty} \left\| z_e(s, \vec{\mathbf{q}}) - z(s) \right\|^2 ds \right) = Tr(\Sigma(\vec{\mathbf{q}})), \tag{13}$$

where $\mathbb{E}(\mu)$ denotes the expected value of the random variable μ and $Tr(\Sigma(\vec{\mathbf{q}}))$ denotes the trace of the operator. Consequently, the optimal sensor location problem is to find an optimal location $\vec{\mathbf{q}}^{opt}$ such that $\mathcal{J}(\vec{\mathbf{q}}) \triangleq Tr(\Sigma(\vec{\mathbf{q}}))$ is minimized. Also, since there are physical limitations to possible location of the sensors, the sensor locations are constrained by $\vec{\mathbf{q}} \in \mathcal{Q}$ where $\mathcal{Q} \subseteq \bar{\Omega}$ is the set of admissible locations. In particular, we consider the problem:

Optimal Sensor Location Problem: Find $\vec{\mathbf{q}}^{opt} \in \mathcal{Q}$ such that

$$\mathcal{J}(\vec{\mathbf{q}}^{opt}) = Tr(\Sigma(\vec{\mathbf{q}}^{opt})) \leq Tr(\Sigma(\vec{\mathbf{q}})) = \mathcal{J}(\vec{\mathbf{q}}) \tag{14}$$

for all $\vec{\mathbf{q}} \in \mathcal{Q}$ such that $\Sigma(\vec{\mathbf{q}})$ satisfies the operator Riccati equation (12).

Note that this an optimization problem with equality constraint defined by the Riccati equation (12). Moreover, the DPS formulation allows for specific spatial information to be naturally included as part of the problem. It is interesting to note that this formulation is not new and may be found in (Omato *et al.*, 1978) among other places. However, what is new are advances in numerical algorithms for solving operator Riccati equations and in the computational power that allows for rapid implementation of these algorithms. Also, special techniques have been developed for solving (12) that specifically take advantage of the structure of the equations and the observation that the Riccai operator equation is, in fact, a non-linear partial differential equation. Finally, note that the Riccati equation (12) is time independent and hence one does not have to solve (simulate) the time dependent system (1) – (3) to evaluate the objective function $\mathcal{J}(\vec{\mathbf{q}}) = trace(\Sigma(\vec{\mathbf{q}}))$. In (Burns *et al*., 2008) new mesh independent numerical methods were developed specifically to solve (12) and the corresponding algorithms are very efficient.

In the next section we consider a simple example to illustrate the potential payoffs of using this formulation. In particular, we apply this approach to a 2D problem motivated by the hospital room presented in (Mendez *et al*., 2008).

3. A NUMERICAL EXAMPLE

We consider a 2D version of a hospital suite with one zone devoted to the bed area and the remaining zones are bath and dressing areas as depicted in the Figure 2 below. Also shown is a typical grid we used to solve for the advection velocity field $v(\vec{\mathbf{x}})$. There are two inlet vents and one outflow vent which is the only outflow when the door is closed.

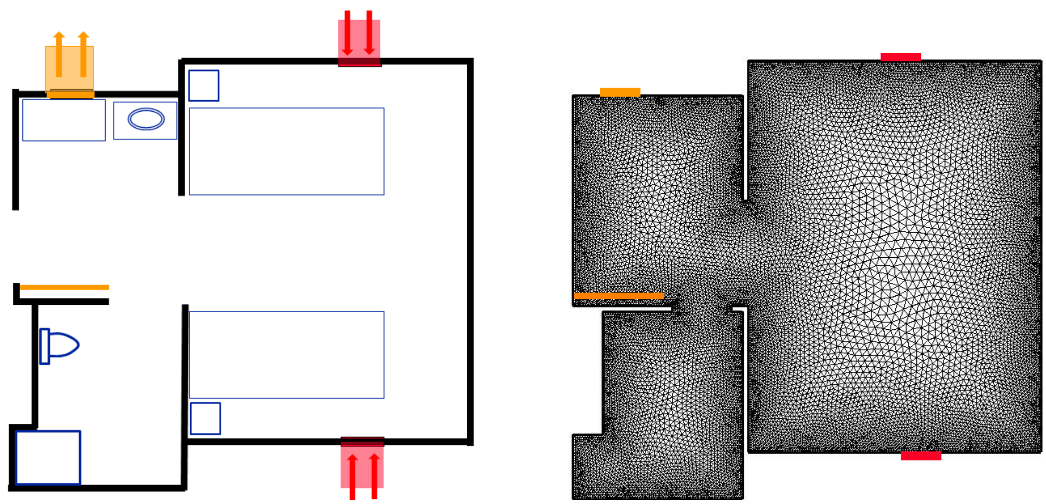

Figure 2: A Hospital Suite Problem with Grid

It is important to note that the air flow through the room can have a tremendous impact on sensing and control. In particular, if one were to assume a “well mixed flow” and formulate the sensor location problem, then the “optimal location” would in general not be valid for more realistic room operation. Figure 3 shows the advection velocity field $v(\vec{\mathbf{x}})$ for the cases with the door open and closed. In the model problem here, clearly the flow is not well mixed in either case. However, the flow in the room to the right has similar a structure and one might guess that if one limits the sensor location to the bed area, then the optimal location will not change dramatically. This indeed is true as demonstrated in the numerical examples below.

To illustrate the range of values for the cost function $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$, we assume that there is only one sensor and that the sensor can only be placed on a wall in the bedroom area on the right side of the suite. The spatial variation starts with the sensor located on the intersection of the upper wall in the bed area with the room divider on the left of the bed area. As $\vec{\mathbf{q}}$ moves along the upper wall to the right, down the right wall and back to the left of the lower wall, the cost $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$ is computed and plotted. For the computations here, we make no assumption about where the disturbance is located. In particular,

$$\eta(t,\vec{\mathbf{x}}) = [\mathcal{G}w(t,\cdot)](\vec{\mathbf{x}}) = \iiint_{\Omega} \delta_{\varepsilon}(\vec{\mathbf{x}}-\vec{\mathbf{y}})w(t,\vec{\mathbf{y}})d\vec{\mathbf{y}} \approx w(t,\vec{\mathbf{x}}), \tag{15}$$

where $\delta_{\varepsilon}(\vec{\mathbf{x}})$ is an approximation of the Dirac delta function so the disturbance operator is approximately the identity. This particular choice of disturbance means that we are assuming the minimal information about the spatial location and intensity of the random field.

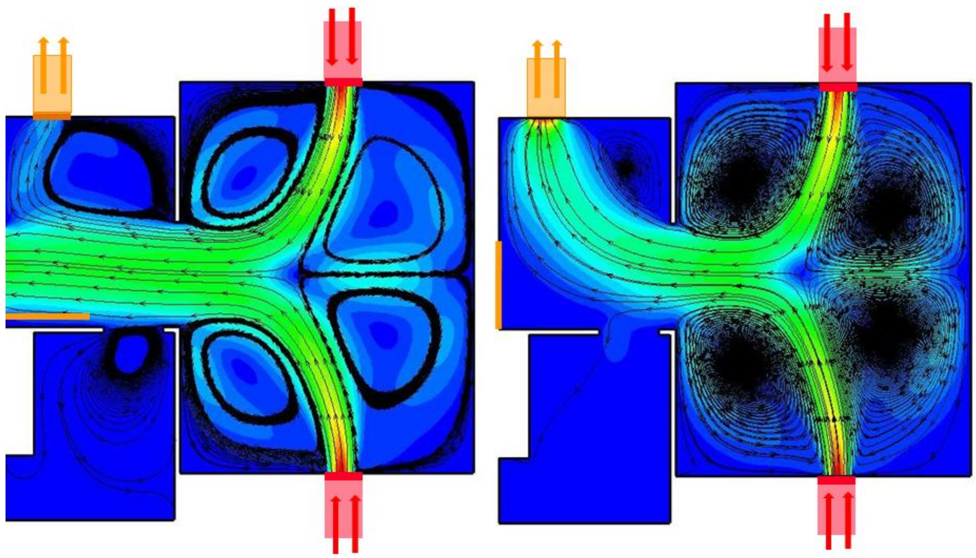

Figure 3: Flow Through the Suite Problem: Door Open and Door Closed

In Figure 4 we plot the values of $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$ as the sensor location $\vec{\mathbf{q}}$ moves around the wall in the right room. Here, "upper" refers to the upper wall, "right" refers to the right wall, "lower" is the bottom wall and "left" is the left wall of the bed area. Note that we also computed the cost $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$ for the case where the sensor is placed in the opening between the bed area and the left zone of the suite even though there is no wall there.

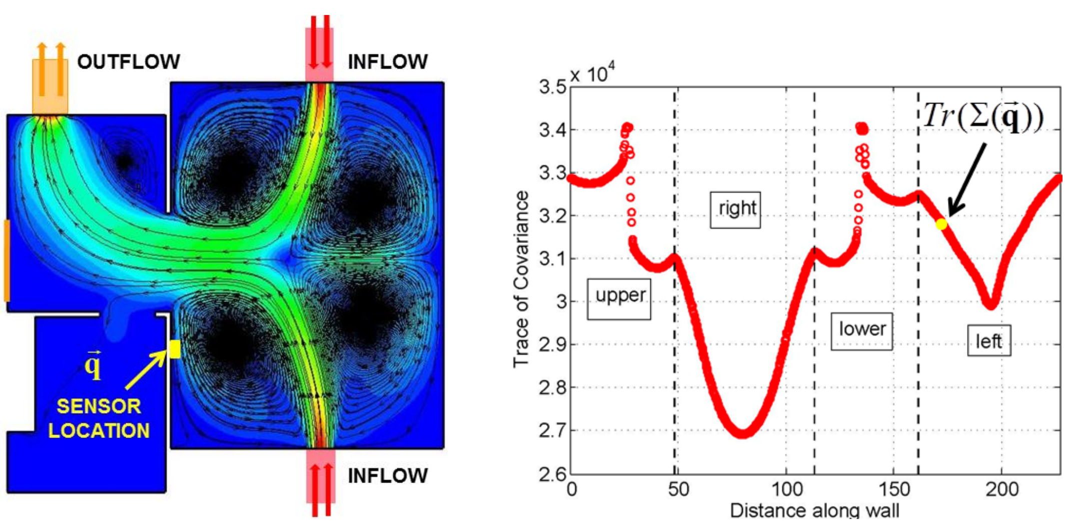

Figure 4: Plot of the Cost $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$ and for a Specific Location on Lower Left Wall

Observe that the optimal location is in the middle of the right wall and the absolute estimation error is approximately $\mathcal{J}(\vec{\mathbf{q}}^{opt}) \approx 2.68 \times 10^4$ which should be expected since the error is computed over all time and nothing is assumed about the location and intensity of the disturbances. The important point is that by placing a sensor in the optimal position, the total estimation error can be reduced by nearly 30% compared to the worst placement near the inlet vents. Also observe that as the sensor location moves across the inlet vents, there is sharp jump in the cost $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$. Elsewhere, the cost is relative smooth which implies a gradient based optimization scheme should work well if one avoids placing a sensor on inlet vents. In fact, `fminsearch` in the Matlab™ optimization toolbox easily found the global minimum for this test problem

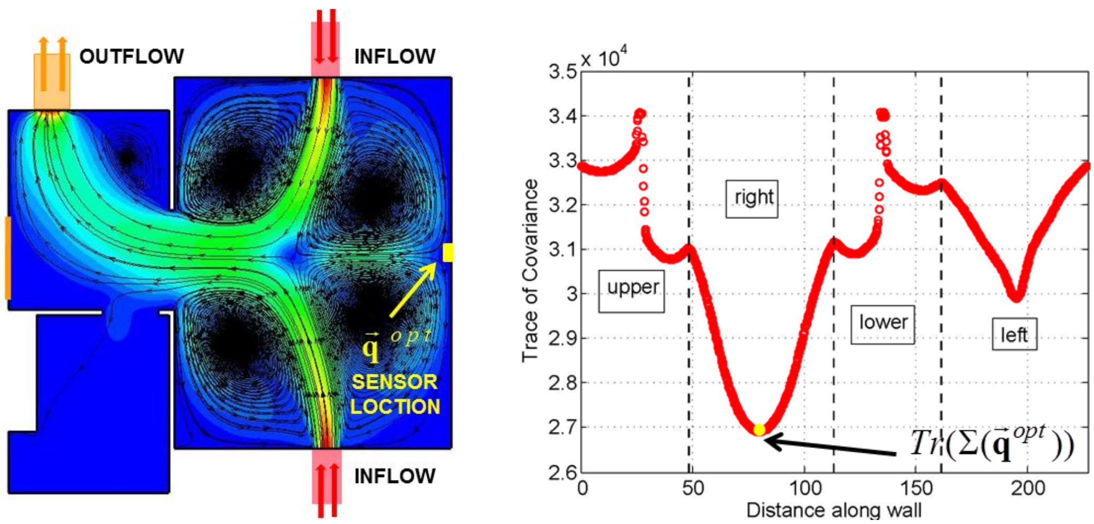

Figure 5: The Optimal Location on Right Wall and the Plot of the Cost $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$

As noted above, the case where the door is open is similar to the closed door case and the optimal location is essentially the same. However, as shown in Figure 6 the absolute estimation error is approximately $\mathcal{J}(\vec{\mathbf{q}}^{opt}) \approx 3.54 \times 10^4$ which implies the state estimator is less accurate when the door is open.

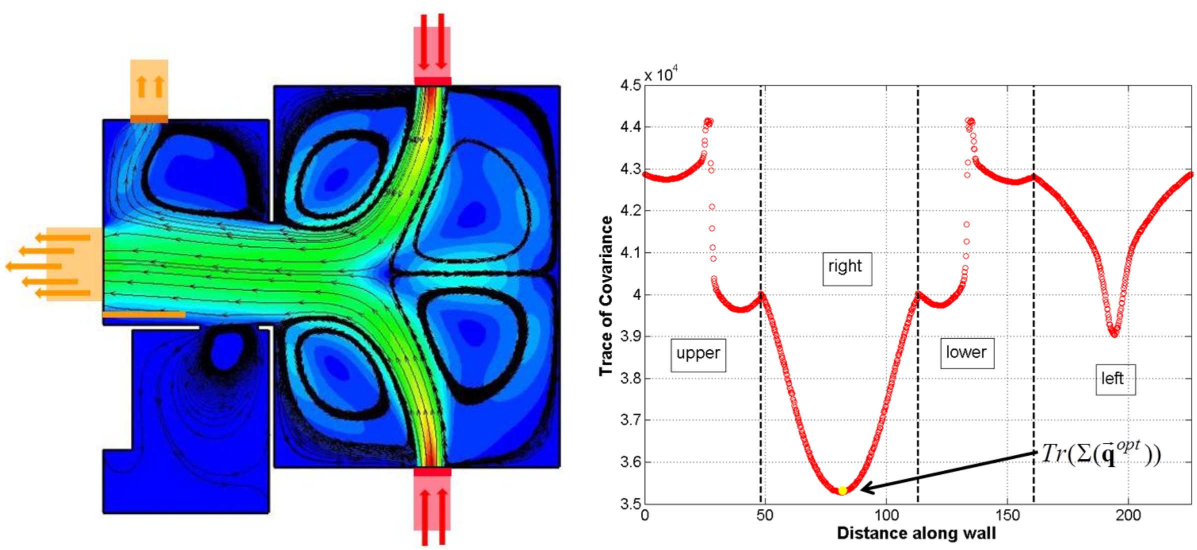

Figure 6: Plot of the Cost $\mathcal{J}(\vec{\mathbf{q}}) = Tr(\Sigma(\vec{\mathbf{q}}))$ for the Open Door Case

4. CONCLUSIONS

In this short paper we described an approach to the problem of optimally locating sensors to minimize the mean square error between a temperature field and an estimated temperature field based on localized sensed output alone. This is important since measurements in typical building systems often come from sensors that are spatially distributed and provide accurate measurements only in (small) local regions. We focused on the Kalman filter since it is optimal for a fixed sensor type and location. Another important application of the Kalman filter is in the development of practical feedback controllers when only partial sensed information is available. In addition, the basic method can be applied to problems where robustness of the controller is essential. Since buildings are highly uncertain dynamical systems, placing sensors for robust control and optimality of performance is a key to the operation of future high performance buildings.

The framework presented here can be rigorously justified and applies to a variety of control and estimation problems of the type common in the building sector. Finally, we are currently developing optimization algorithms to numerically solve the sensor placement problem using gradient based methods. Details of this work will appear in a complete long paper.

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