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## Modeling Reciprocating Air Compressor using Energetic Macroscopic Representation

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## ABSTRACT

This article introduces the energetic macroscopic representation (EMR) modeling of reciprocating air compressor. EMR has been introduced recently for research development of electromechanical systems based on action reaction causality principle, which organizes the subsystems according to the integral causality. The graphical modeling readability, modularity, structural and functional characteristics toward control structure development are presented. The EMR model of a four-stroke reciprocating engine is divided into a multitude of simple subsystems. Each subsystem describes an elementary type of conversion, several of these blocks may occur in a single module. Calculation principles: mass and energy balances developed for different subsystems, which finally lead to the overall EMR model presented here allow us to highlight the interactions of the electromechanical, thermodynamical, hydraulic and pneumatic processes occurring simultaneously in an air compressor.

## 1. INTRODUCTION

Reciprocating compressors are used throughout industry due to its importance. Considerable effort has been devoted by many researchers toward the development of computer models for this type of machinery (Stadel et al., 1974) which give a representative description of this type of work. In general, a cycle of operation of a high speed compressor is a complex and complicated phenomenon taking a short period of time.

As regards the modeling, their interest is double: firstly, as it is the easiest transfer of knowledge to the other users. In some literature, models of compressors are found (Eung et al., 1985 and Karnopp et al., 2000), based on the same equations and theoretical backgrounds than the previous models. This article highlights the understanding of the significant extent. The model describes the thermodynamic system through the use of the Bond Graph approach. Using the proposed overall picture of the considered system, a graphical approach is proposed to describe complex systems, Energetic Macroscopic Representation (EMR) has readability and modularity, EMR reveals the structural and functional characteristics in agreement with physical (e.g. Hiss et al., 2008). Both Bond Graphs and EMR are obvious model multiphysics, but there are nevertheless differences between them. Bouscayrols is based on energy and data flow and uses a uniform representation while with the latter specific applications are used to each power component increment. As a matter of fact, it shows the different physical domains crossed by the system and discriminates between the accumulation of potential energy and the release of energy.

making a difference. Furthermore, in Bond Graphs, all subsystems are used derivative relationships which violate the causality principle. The EMR oriented to control. Then any derivative relationships can be applied to some concatenation of Bouscay (2010) EMR has developed since 2002 systems from a macroscopic point of view; it allows the design thanks to its control feature. Control Structure (MCS) deduced from EMR using inversion structures (PCS) by applying simplification rules leads to a control and an energy management time.

Therefore, a complex system of several energy sources and several energy models by the EMR tool with a clear readability. Moreover, the control of implemented, thanks to the appropriate control tools.

As in other graphical representation methodologies, the product of the causality and reaction pair) is the instantaneous power (Chen et al., 2009) which is the energy connected. Though EMR has been used to describe electrochemical systems, possible to extend it to new energy domains. Expanded to electrochemical domains, EMR has allowed describing many devices such as mechanical systems, electrical over hybrid systems.

Until now, EMR is rather scarce. This paper is divided into two sections: some basic knowledge and some considerations regarding EMR application. In the first part, each physical phenomenon taken into account is described and analyzed. In the second part, their combination including the electrical part: mechanical part and valves described. The model is obtained, putting together all EMR subparts. Energetic Macroscopic Representation on the proposed model is set to the simulation results and the friction compensation is added to the discussion of the ob-

## 2. EMR DESCRIPTION

### 2.1 Short Presentation of the Graph Model

As mentioned before, EMR model is a first-order physical compression product of an actuator variable with the corresponding reaction variable. It is a variable speed at a constant power. Thus, it has been extended EMR by adding in order to describe a physical system. The EMR model inversion element by element control structure; it is called the Maximum Control Structure (MCS) and theoretical values are measurable. In practice, it is not always the case (PCS) can be deduced from the MCS by applying simplification rules. Bouscay (2010) and Chen (2007, 2009) papers, only the EMR part is presented. The generalized EMR pictograms used in the presented model.

Table 1. Extended Energetic Macroscopic Representation blocs

Energetic Macroscopic Representation (EMR) blocs

Pair of action a		Two parallel black lines pointing in opposite directions
Source of energy		Light green oval with red rim
Energy conversion Same domain		Orange square with red rim
Energy conversion Different domain		Orange circle with red rim
Energy accumulation		Orange rectangle with red rim and red diagonal
Coupling device (device exchanging energy) Same domain		Overlapping orange squares with red rim
Thermofluidic causality (device exchanging energy and liquid)		Orange squares with red rim

22. Adaptation of EMR to fluidic systems

The EMR methodology is based on the hypothesis that energy exchange can be described by a set of reaction pairs (Bousca, 2005). However, Chrenko (2009) has shown that this does not hold true all the time.

22.1. Causality between action and reaction in the EMR representation method (Bousca, 2005). This is a reasonable assumption as long as there is a relation between the action and reaction leading to the power. This relation has not necessarily to be a product. This is in analogy to the work of Chrenko (1993) who gives the possibility to handle the parameters more freely. For example, Chrenko (2009) uses mass flow or temperature and enthalpy flow as parameters in EMR.

22.2. Representation of multiple domains of EMR contain the fact that all systems are composed of small parts, which follow the conservation laws, a Kirchhoff law or a first order differential equation form. For most systems this subdivision can be obtained. However, in some cases this is not possible.

domains cannot be detached, they can form by be described time-energetic domain. gas flows cannot be detached. The element has two parameter pairs (2006), can element can be called EMR analogy - through Graph Chenko(2009) proposes that equation cannot be separated into the two energetic both parameter pairs used, see

Fig.1. Representation of a multiport in EMR.

### 3. DESCRIPTION OF AIR COMPRESSOR BEHAVIOR

A schematic of the compressor in Figure 2 is divided into three subsystems, namely:  
 A. slider mechanism,  
 B. cylinder head,  
 C. valves,  
 Each subsystem is modeled independently and in the submodel its arrangement of control system.

Fig.2. Compressor schematic

#### 3.1 Slider mechanism

The slider mechanism (h) was derived as:

$$m \frac{d^2x}{dt^2} + R \frac{dx}{dt} = \frac{R^2 \sin^2 \alpha \cos \alpha}{(L^2 - R \sin^2 \alpha)^2} \dot{\theta}^2 \quad (1)$$

This transformer modulus shows the detailed piston speed  $\dot{x}$ , as well as the torque  $\dot{\theta}$ , and piston force  $F$ .

$$m(\dot{q}) = \frac{V}{\dot{w}} \frac{\dot{\theta}}{F} \quad (2)$$

Figure 3 Crank and piston mechanism

Plus, the relation between the volume change and piston displacement is:

$$\dot{V} = \dot{A}_p \dot{x}_p \quad (3)$$

One may represent the device by the EMR of Figure 3.

Figure 4 EMR of crank and piston mechanism

### 3.2 Cylinder head

Considering the cylinder volume as a control volume that can transport mass a out port, but flow can go in either direction at either port. Finally we can volume expansion that we can assume that one pressure, one temperature, one entire internal volume of the control volume.

Writing the mass and energy equations for the mentioned control volume,

$$\frac{d}{dt} E = \dot{m}_i h_i - \dot{m}_e h_e - \dot{q} \quad (4)$$

$$\frac{d}{dt} m = \dot{m}_i - \dot{m}_e \quad (5)$$

Here is the internal energy of the control volume, and we assume that the g Even though the EMR representation is not a true one, but as the causality and therefore accepts flow input on all arrows. It then integrates these flows V. And finally the accumulator operates on these variables through appropriate output. The constitutive equations are

$$T = \frac{1}{c_v} \frac{E}{m} \quad (6)$$

$$T = \frac{m R T}{V} \quad \theta = \frac{m R}{V c_v} \quad \frac{E}{m} = \left( \frac{F}{c} \right) \quad (7)$$

Thus,  $\dot{E}_i$ ,  $\dot{Q}$  and  $\dot{V}$  are prescribed (causally), then the thermodynamic accumulator constitutive laws considered in the principals described in section 2. The EMR of the cylinder head is shown in Figure. 5.

The rate of heat transfer between the gas and the cylinder head wall is modeled

$$\dot{Q}_w = H_c A_c (T_g - T_w) \quad (8)$$

Where  $H_c$  is the overall heat transfer coefficient,  $A_c$  is the cylinder head surface area,  $T_g$  is the instantaneous gas temperature, and  $T_w$  is the instantaneous wall temperature. Regarding the heat transfer, given by Helber [9]

$$H_c = 2.4 \sqrt{P_c T_g} \sqrt{V_r} \quad (9)$$

Where  $P_c$  and  $T_g$  are instantaneous cylinder gas pressure and temperature, and  $V_r$  is the cylinder volume.

Figure 5. EMR of Cylinder head

### 3.3 Suction and discharge valves

The valves are assumed to be treated as instantaneous sections. In a plane,  $A$  is the area of the valve,  $\dot{m}$  is the mass flow rate through the valve as a function of valve area, temperature, and downstream pressure, computed using the well known relation

$$\dot{m} = A \sqrt{\frac{2k}{(k-1)R_u}} \sqrt{\frac{p}{T}} \left( \frac{p}{p^*} \right)^{\frac{k}{k-1}} \left( \frac{T}{T^*} \right)^{\frac{1}{k-1}} \quad (10)$$

Where  $k$  is the ratio of specific heats and  $R_u$  is the universal gas constant.

And for

$$p \leq p_{crit} \left( \frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad (11)$$

The flow is choked. And the mass flow is independent of the downstream pressure.

The transported mass is related with the mass flow

$$\dot{E}_n = \dot{m} \dot{V} \quad (12)$$

Considering the principals described, the EMR representation of the valve is shown

Figure 6: EMR of Valves

### 3.4 Complete EMR Model

The three previously developed submodels are assembled to show how the complete EMR displays in one simple diagram. The diagram illustrates the interaction of the single stage basic lumping process and the system structure. The EMR is also analyzed in a directed graph and causalities. The advantages of the EMR techniques are clearly illustrated in complicated mechanical machines can be modeled by using a general procedure. Finally the model for the inlet and the exhaust are specified by a logic expression. For example; for

$$\begin{aligned} \text{If } "e180 \text{ and } "d, & \quad = \quad (13) \\ \text{Otherwise,} & \quad = 0 \end{aligned}$$

This will keep the inlet pressure inside the cylinder is less than the atmospheric pressure. The logic can be defined for exhaust valve.

The equations mentioned until now can be delivered to any explicit equations we have used MATLAB Simulink for simulation. An integration algorithm will move step to next.

Figure 7: Complete EMR

## 4. SIMULATION RESULTS

The model equations are modeled in MATLAB Simulink® for simulation. Figure 8 and 9 show pressure and temperature of the compressor cylinder and the volume of the compressor cylinder.



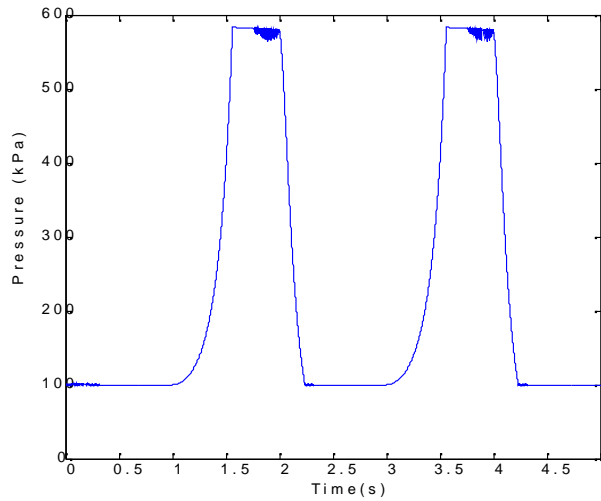


Figure 8 Simulated Pressure in the cylinder.

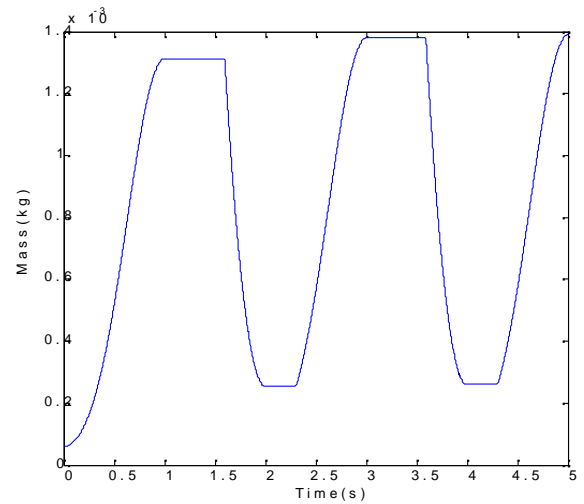


Figure 9 Simulated mass in the cylinder.

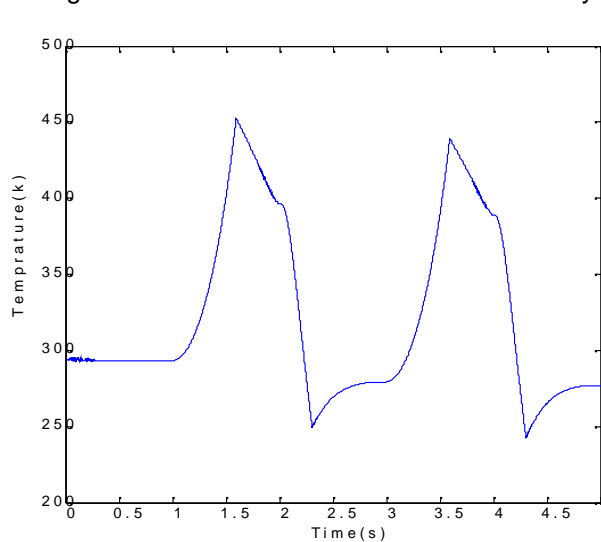


Figure 10 Simulated Temperature in the cylinder.

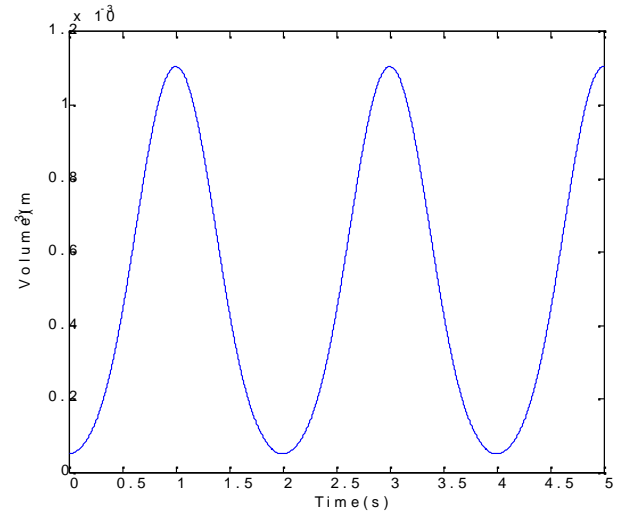


Figure 11 Simulated volume of the cylinder.

To see if the developed model of the complex geometry and description of the phenomenon, we have compared it to FEM by COMSOL. The procedure of Element modeling is described by the literature and a schematic diagram of both models has been shown in figure 12. The general agreement of the results comes from the difference in accuracy of the transfer in defining the geometry of valves.

Since the FEM model is more complicated, it can better predict the behavior at tr

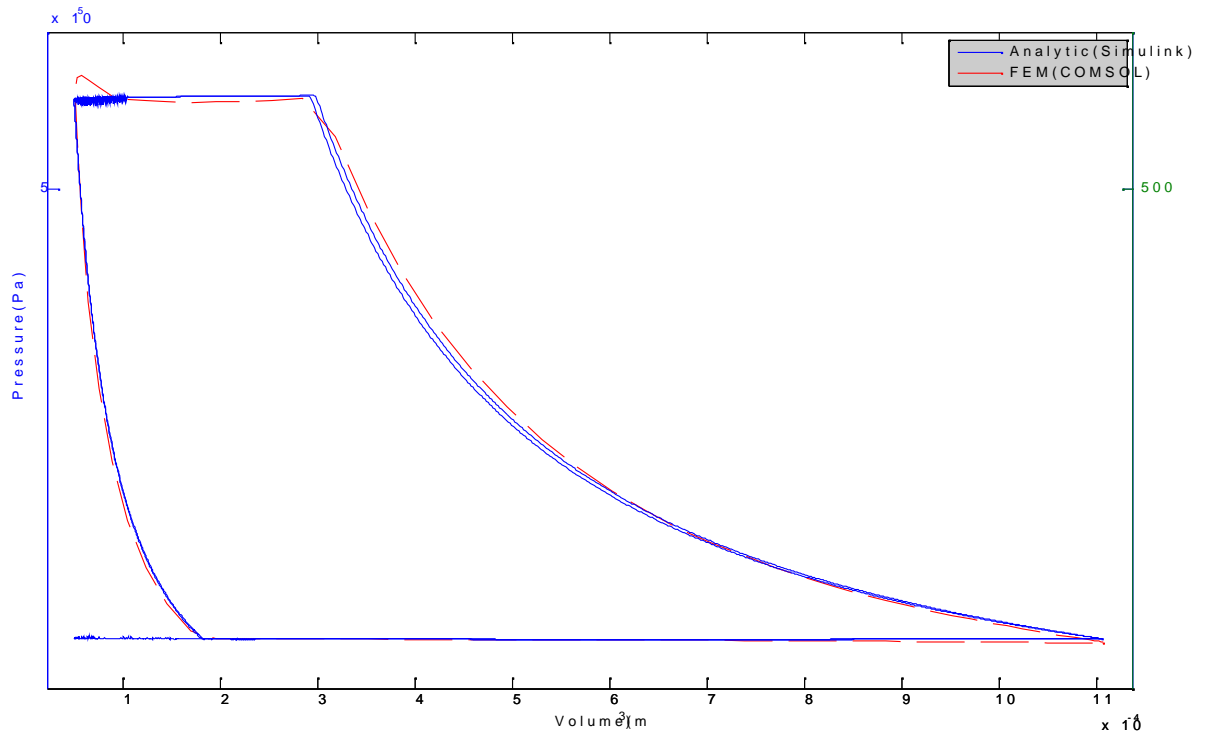


Figure 2 P-V diagram of FEM and analytical model.

## 5. CONCLUSIONS

In this paper the EMR approach for representing a compressor is presented. It is shown that Pseudo-RM is a very powerful tool for modeling of convective (open) thermodynamic systems. Pseudo-RM combination with EMR have pushed physical modeling forward to complex machines or systems which can be modeled in a short time and without any enhancement. Besides, the simple model developed here shows a good agreement with computationally expensive Finite Element Model.

## NOMENCLATURE

The nomenclature is at the end of this table following format:

$\dot{m}$	Mass flow	(kg/s)	Subscripts	
$\dot{E}$	Enthalpy flow	(J/K)	s	suction
$\dot{V}$	Volume flow	(m <sup>3</sup> /s)	i	inlet
T	Temperature	(K)	e	exit
p	Pressure	(Pa)	E	Exhaust
$\omega$	Rotational speed	(Hz)		
$\tau$	Torque	(N.m)		
A	Surface area	(m <sup>2</sup> )		
$C_p$	Isobaric heat capacity	(N.m/kg.K)		

k	Heat capacity ratio	(-)
$J_{\omega}$	Flywheel rotary Inertia	( $kgm^2$ )
R	crank radius	(m)
L	connecting rod length	(m)

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