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The Use of Equivalent Source Models for Reduced Order Simulation in Room Acoustics

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Motivation

❑ Room Acoustics with Source of Finite Size

Available techniques are not suitable for a fast and accurate room acoustics simulations of finite-size sources

Sound prediction of a flat screen TV:

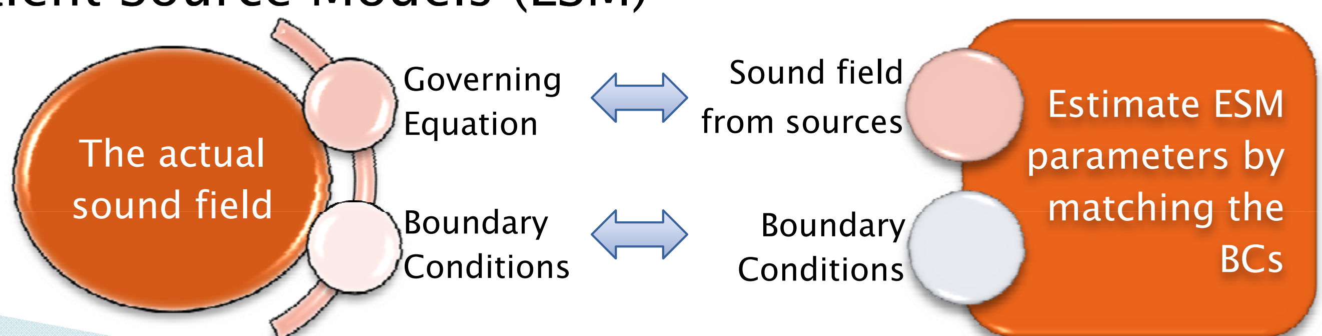
- Not point sources
- Arbitrary room shapes
- Fast

Traditional Methods:

- Ray Tracing
- Image Source Model (hybrid)
- BEM/FEM

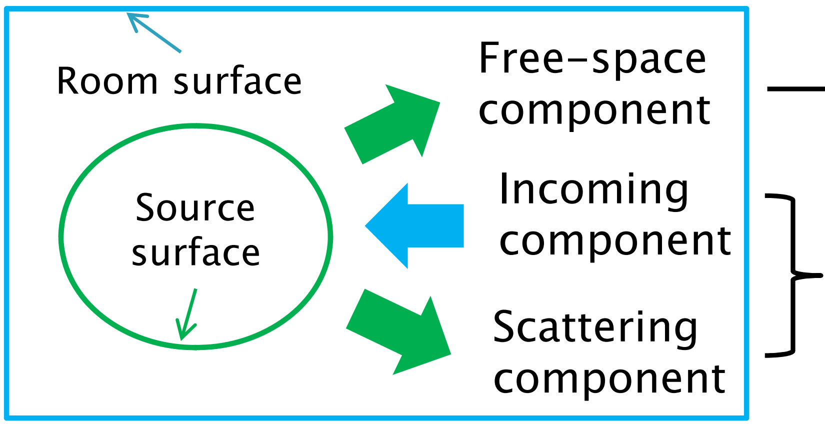
❑ The Use of Equivalent Source Models (ESM)

ESM in General:



ESM in Room Acoustics

❑ Sound Field Components in Room Acoustics



Pressure: P_f, **Particle velocity:** u_f

Room component: (P_r, u_r)

Total sound field:

$$\begin{cases} p_t = p_f + p_r \\ u_t = u_f + u_r \end{cases}$$

❑ Free-space ESM vs. Room Acoustics ESM

Free-space ESM		Room Acoustics ESM
• Outgoing waves only	Source type	• Both incoming and outgoing waves
• Pressure of the total sound field (sampled at a measurement surface)	Boundary condition	• Impedances of room component (sampled at both source and room surfaces)

Boundary Conditions

- BC of the Room Components in Room Acoustics
  - **BC on source surface, Γ_1 , (admittance β_1):**

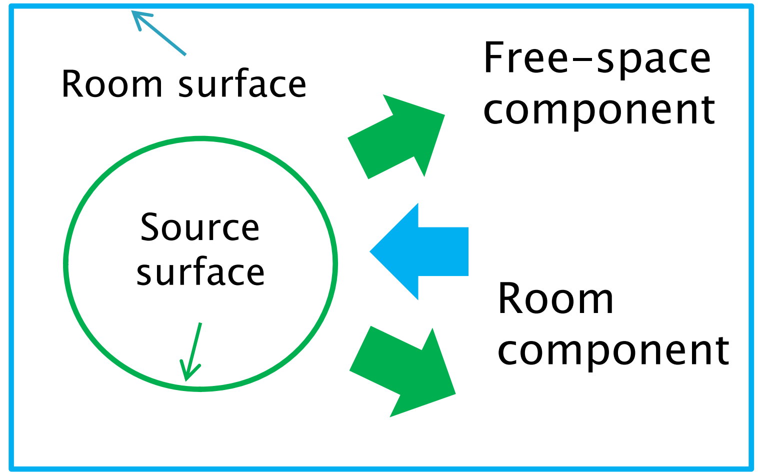
    Total: $\beta_1(x)p_t(x) + u_0(x) = u_{nt}(x)$

    Free-space: $\beta_1(x)p_f(x) + u_0(x) = u_{nf}(x)$

    In-vacuo driving velocity on the source surface. (source characteristics)

  - **BC on room surface, Γ_2 , (admittance β_2):**

    Total: $\beta_2(x)p_t(x) = u_{nt}(x)$



Total sound field:

$$\begin{cases} p_t = p_f + p_r \\ \vec{u}_t = \vec{u}_f + \vec{u}_r \end{cases}$$

Boundary conditions used in the room acoustics ESM:

$$\begin{cases} \beta_1(x)p_r(x) - u_{nr}(x) = 0 & x \in \Gamma_1 \\ \beta_2(x)p_r(x) - u_{nr}(x) = u_{nf}(x) - \beta_2(x)p_f(x) & x \in \Gamma_2 \end{cases}.$$

The problem of room acoustics:

Given: p_f, u_f; Calculate: p_r, u_r

Construction of ESM in General

❑ General Procedure of Constructing Room Acoustics ESM

After choose the type of equivalent sources:

$$\begin{cases} p(x) = \sum_{i=1}^{N} g_i(x, y_i) Q_i, \\ \vec{u}(x) = \dfrac{1}{j\omega\rho_0} \sum_{i=1}^{N} \vec{\nabla} g_i(x, y_i) Q_i, \end{cases}$$

Location in the sound field

Location of each source

- $g_i(x, y_i)$ – sound field of source with unit strength
- Q_i – the source strength

Evaluate p and u_n at a number of sampling locations on both source and room surfaces

Matrix form used to estimate the ESM parameters:

$$\begin{bmatrix} B_1 A_p^{(1)} - A_{u_n}^{(1)} \\ B_2 A_p^{(2)} - A_{u_n}^{(2)} \end{bmatrix} \vec{Q} = \begin{bmatrix} 0 \\ \vec{u}_{nf} - B_2 \vec{p}_f \end{bmatrix}$$

Boundary condition on source surfaces

Boundary condition on room surfaces

$$\begin{cases} B_1 = diag(\beta_1(x_1), \beta_1(x_2), ..., \beta_1(x_{M_1})), \\ B_2 = diag(\beta_2(x_{M_1+1}), \beta_2(x_{M_1+2}), ..., \beta_2(x_M)) \\ (A_p^{(1)})_{ij} = g_j(x_i, y_j), \quad (A_{u_n}^{(1)})_{ij} = \dfrac{1}{j\omega\rho_0} \partial_n g_j(x_i, y_j), \\ (A_p^{(2)})_{ij} = g_j(x_{M_1+i}, y_j), \quad (A_{u_n}^{(2)})_{ij} = \dfrac{1}{j\omega\rho_0} \partial_n g_j(x_{M_1+i}, y_j), \end{cases}$$

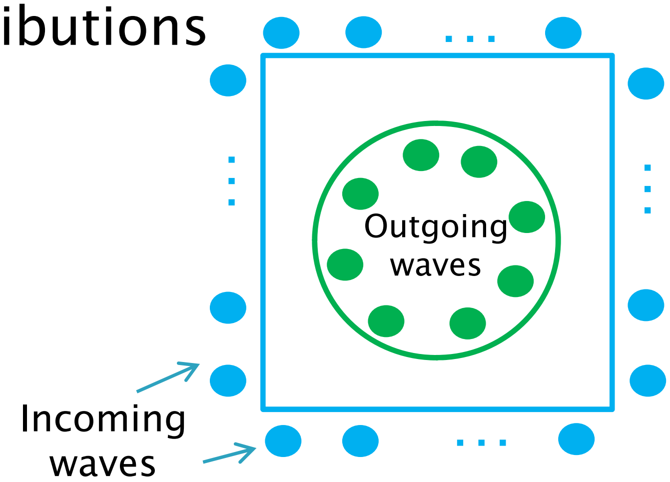
$\boxed{\text{Construct a specific ESM}}$

(find the sound field expression)

Two Types of ESMs

- ❑ Room Acoustics ESM Using Monopole Distributions
  - Monopoles are distributed inside the source surface (outgoing wave) and outside the room surface (incoming wave)
  - Sound field expression (2D):

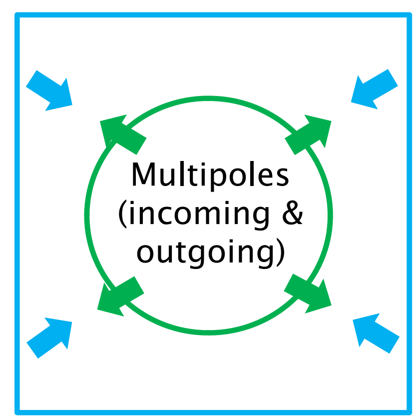
$$g(x, y) = \frac{j}{4} H_0^{(1)}(kr)$$



- ❑ Room Acoustics ESM Using Monopole Distributions
  - Multipoles of monopole, dipole, quadrupole, … n th order, … (both incoming and outgoing)
  - Sound field expression (2D):

$n = 0$: $$P_0^{out}(x, y) = \frac{j}{4} H_0^{(1)}(kr), \quad P_0^{in}(x, y) = \frac{j}{4} H_0^{(2)}(kr)$$

n th order: $$\begin{cases} P_{Sn}^{out} = S_n P_n^{out} = S_n R_n (P_0^{out}) \cdot \vec{v}_1 \cdot \vec{v}_2 \ldots \cdot \vec{v}_n \\ P_{Sn}^{in} = S_n P_n^{in} = S_n R_n (P_0^{in}) \cdot \vec{v}_1 \cdot \vec{v}_2 \ldots \cdot \vec{v}_n \end{cases}, \quad R_n = \nabla^{\otimes n}$$



$\otimes$ – tensor outer product

$\cdot$ – tensor inner product

Details on next slide

Sound field of Multipole Sources

❑ Sound field expression for multipole sources (2D):

Sound field of order $n+1$ source and order n source can be related by directional derivative:

$$P_{Sn+1}(\overrightarrow{X}\,|\,\overrightarrow{X_0},\omega) = d < \nabla P_{Sn}(\overrightarrow{X}\,|\,\overrightarrow{X_0},\omega), \vec{v}_{n+1} >$$

$$= d < \left[\frac{\partial P_{Sn}(\overrightarrow{X}\,|\,\overrightarrow{X_0},\omega)}{\partial x_0}, \frac{\partial P_{Sn}(\overrightarrow{X}\,|\,\overrightarrow{X_0},\omega)}{\partial y_0} \right]^T, \vec{v}_{n+1} >$$

Dipole: $P_{S1} = Sd_1 < \nabla P_0, \vec{v}_1 >,$ $\quad [\frac{\partial P_0}{\partial x}, \frac{\partial P_0}{\partial y}]^T$

Dipole strength: $S\,d_1$

Quadrupole: $P_{S2} = Sd_1 d_2 \vec{v}_2^{\,T} R_2 \vec{v}_1,$

$$\begin{bmatrix} \frac{\partial^2 P_0}{\partial x^2} & \frac{\partial^2 P_0}{\partial x \partial y} \\ \frac{\partial^2 P_0}{\partial y \partial x} & \frac{\partial^2 P_0}{\partial y^2} \end{bmatrix}$$

Quadrupole strength: $Sd_1 d_2$

Order n:

$$P_{Sn} = Q_n g_n$$

Source strength:

$$Q_n = Sd_1 d_2 \ldots d_n$$

$$g_n = R_n(P_0) \bullet \overrightarrow{v_1} \bullet \overrightarrow{v_2} \ldots \bullet \overrightarrow{v_n},$$

$$R_n = \nabla^{\otimes n}$$

$\otimes$ – tensor outer product

$\bullet$ – tensor inner product

Decomposition of the Multipoles

- For a source of order n ($n > 1$), there could be infinitely many types because of the orientation vectors.

Which source orientations should be included in the model?

Even more nonlinear if the orientations are unknown.

Orientations' effect is just a linear combination of components in R_n.

A source of arbitrary orientation can be decomposed into several standard source configurations.

Components in R_n are differentiations with respect to x,y different number of times.

Derivative does not depend on the sequence of differentiation

The number of independent components is less than 2^n.

- How many independent elements?

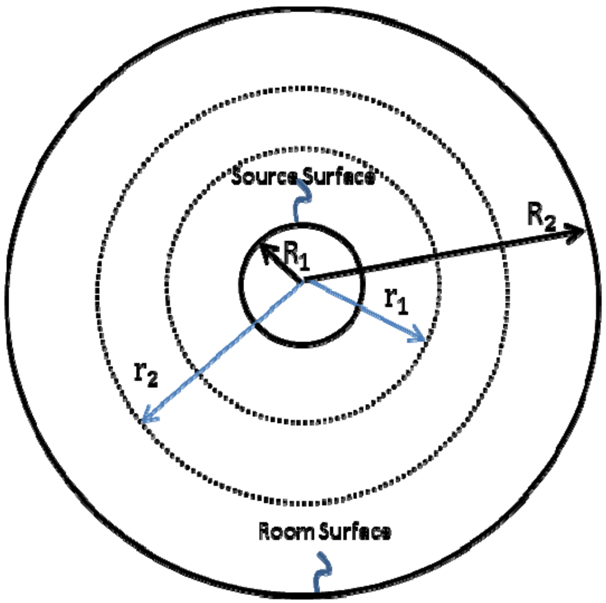
n th order source, in *r*-D space :

$$N(n,r) = \begin{cases} 1 , & n = 0 \\ C_{n+r-1}^{n} , & n > 0 \end{cases}$$

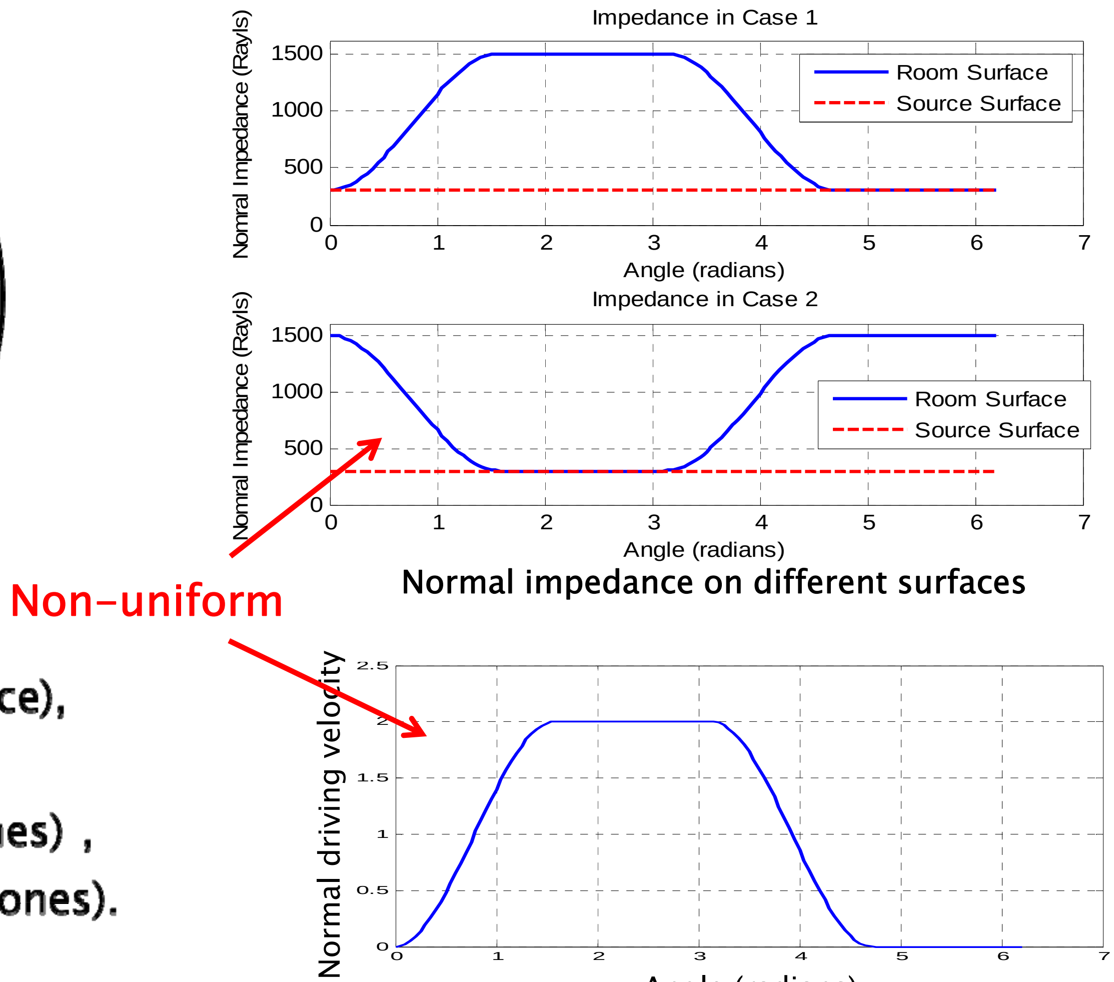
- Can be numerically enumerated for an arbitrary *n*.

Simulation Setup

- Simulation in a 2 dimensional room (circular geometry)



Geometry of the room



Normal impedance on different surfaces

In-vacuo Driving velocity on the source surface

- $R_1 = 0.5$ m (source surface),
- $R_2 = 2$ m (room surface),
- $r_1 = 1$ m (100 microphones) ,
- $r_2 = 1.5$ m (100 microphones).

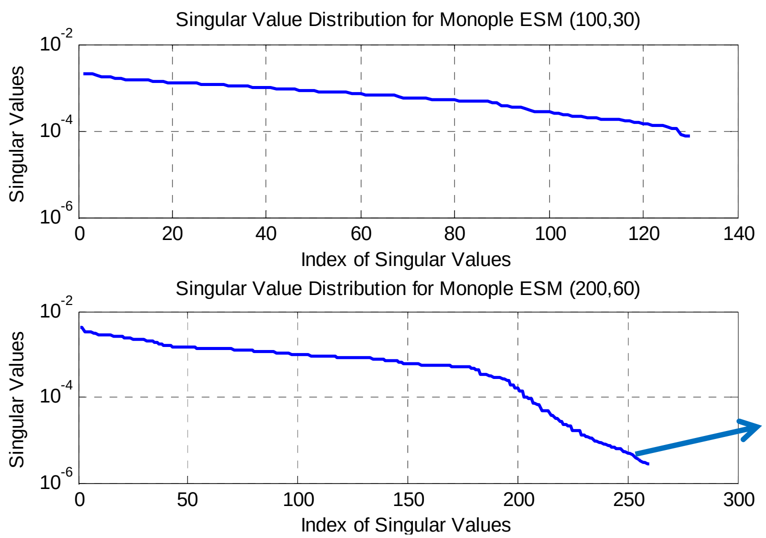
Models Used in Simulations

❑ Different models used in the simulation

Type of ESM	Number of Parameters
Multipole ESM (order up to 3)	10
Multipole ESM (order up to 6)	28
Monopole ESM (outside: 100; inside: 30)	130
Monopole ESM (outside: 150; inside: 45)	195
Monopole ESM (outside: 200; inside: 60)	260
Monopole ESM (outside: 1000; inside: 300)	1300

- Result from a Boundary Element Model was used as the true sound field.

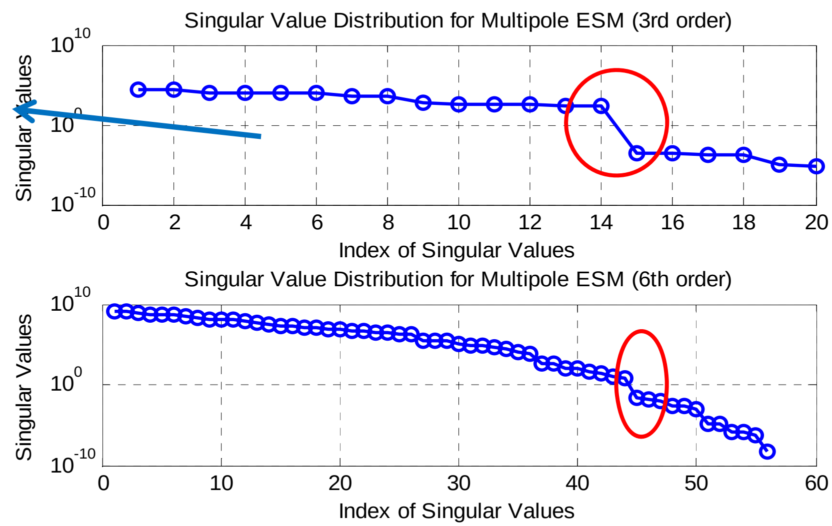
❑ Choice of regularization techniques



Singular value distribution in monopole ESMs

Use TSVD for multipole ESM

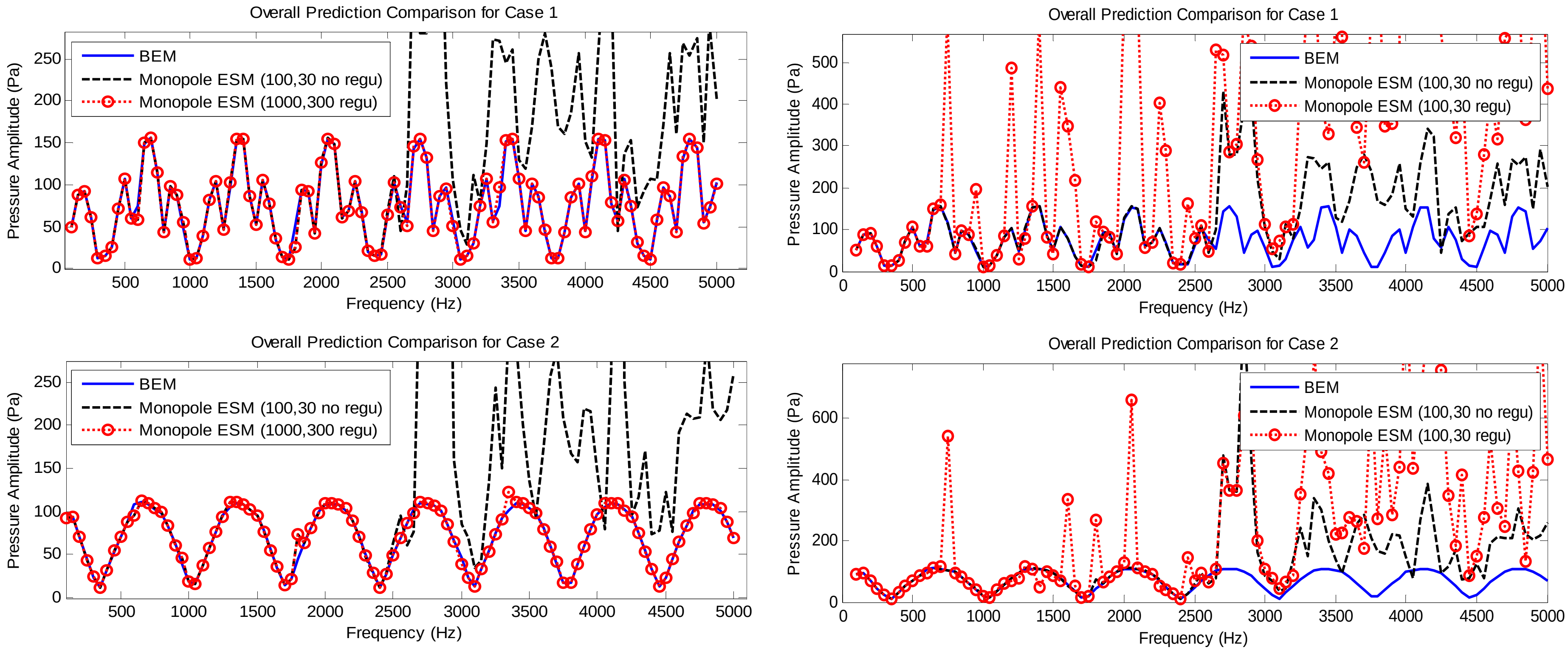
Use GCV for monopole ESM



Singular value distribution in multipole ESMs

Analysis of Results

- Results from monopole distribution ESMs



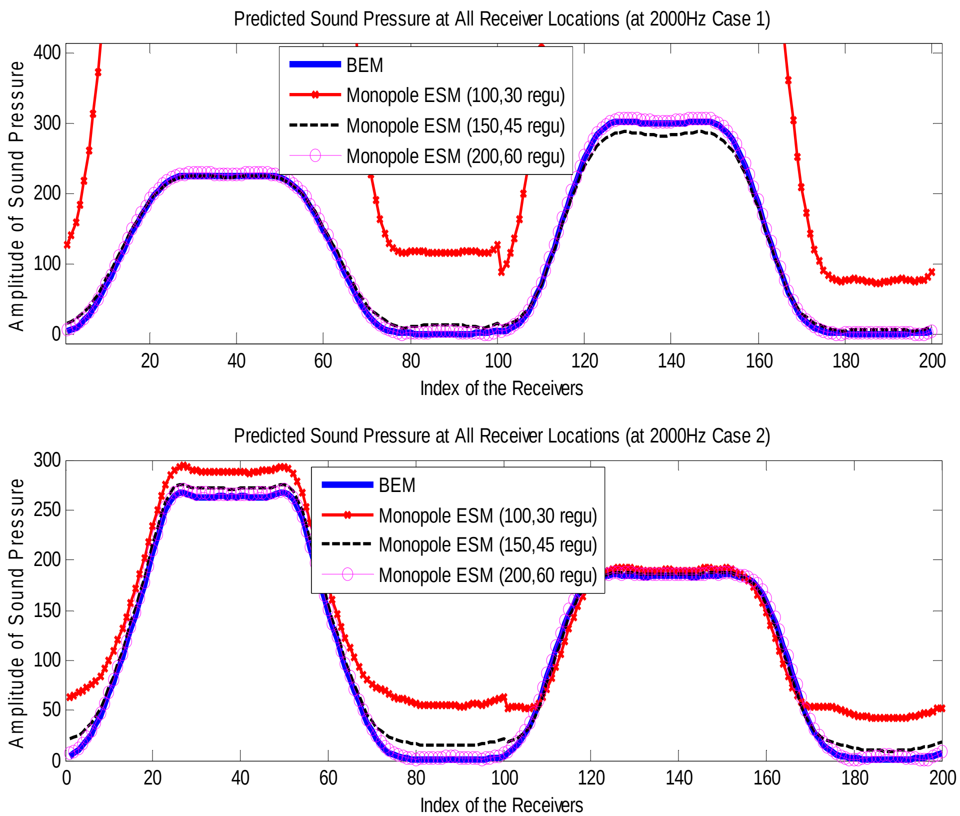
Spatially averaged prediction from monopole ESMs

Spatially averaged prediction from monopole ESMs

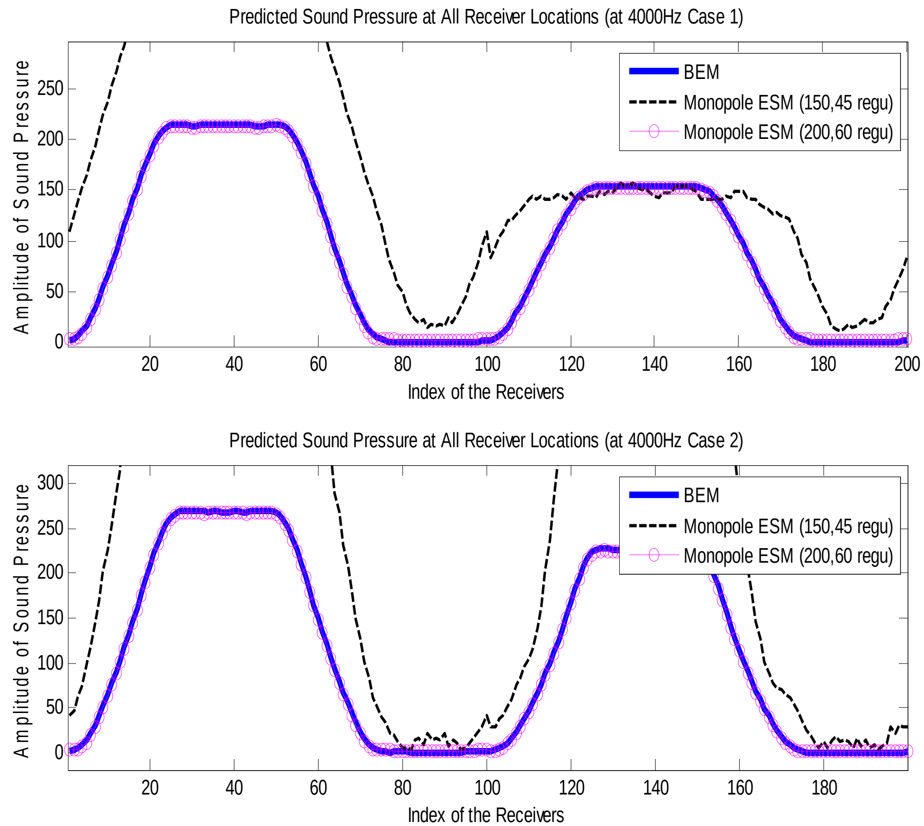
- Accurate (up to 5000 Hz) if there are a large number of monopoles and with regularization.
- Regularization may cause instabilities.

Analysis of Results

❑ Results from monopole distribution ESMs



Prediction from monopole ESMs at 2000 Hz



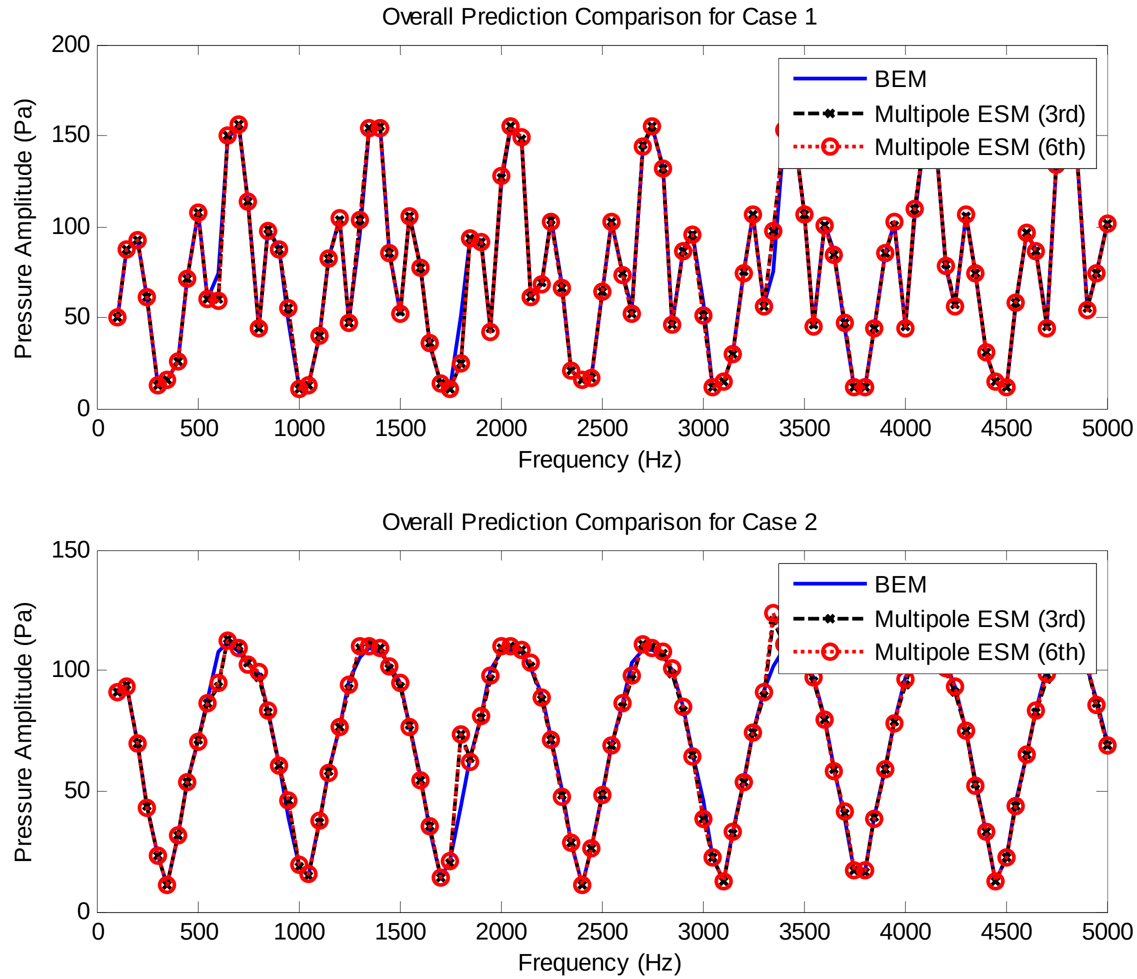
Prediction from monopole ESMs at 4000 Hz

- A monopole ESM requires at least 260 model parameters to achieve accurate predictions.

Analysis of Results

❑ Results from multipole ESMs

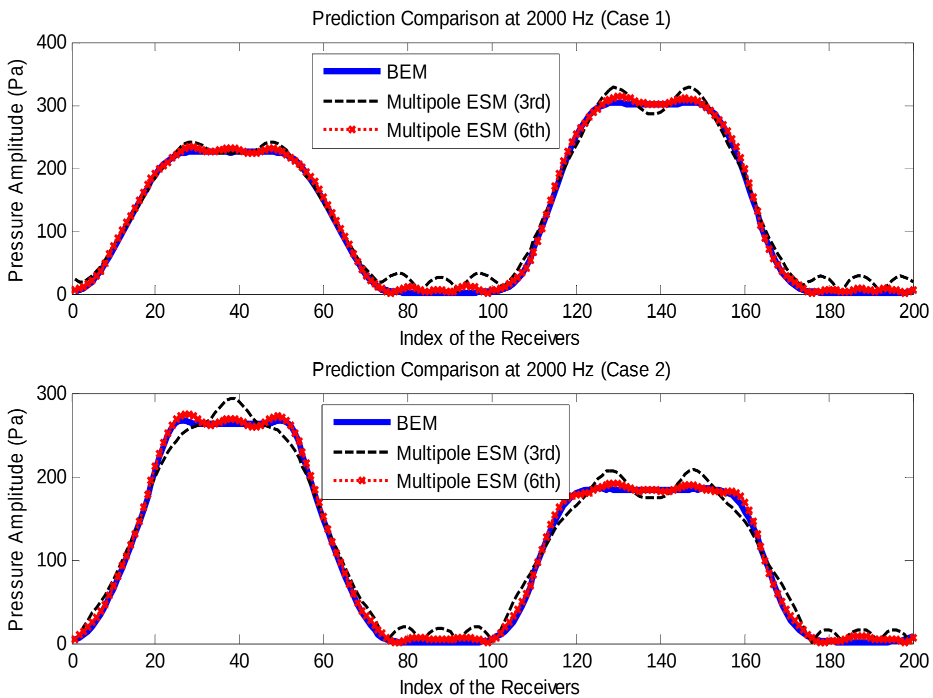
- Regularization does not cause instabilities (more robust).
- The spatially averaged predictions are similar with different multipole orders (accurate up to 5000 Hz).
- Multipole ESM requires much fewer number of model parameters than monopole ESM



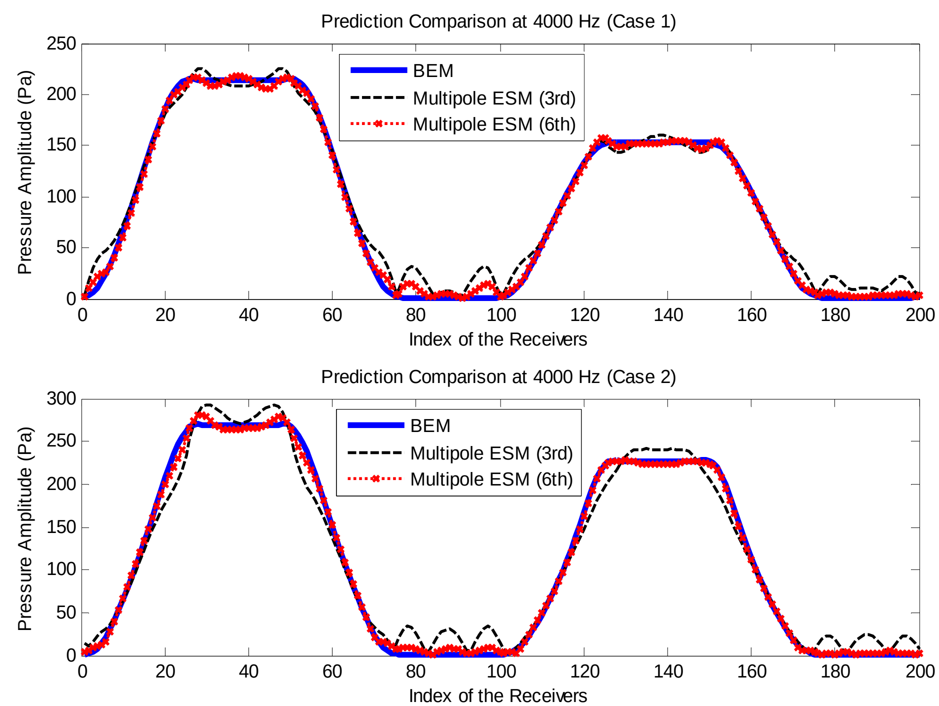
Spatially averaged prediction from multipole ESMs

Analysis of Results

❑ Results from multipole ESMs



Prediction from multipole ESMs at 2000 Hz



Prediction from multipole ESMs at 4000 Hz

- Increase of multipole order improves the prediction oscillation in space.
- It is flexible to balance the computational effort and the prediction accuracy. (choose an appropriate truncation order

Conclusions

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- Equivalent source models are constructed for room acoustics simulations with finite-size source, arbitrary geometry and non-uniform surface normal impedances.
- In room acoustics ESMs, both outgoing and incoming waves should be included, and the impedance boundary conditions for the room component sound field are used for parameter estimation.
- Both monopole ESMs and multipole ESMs can achieve accurate performance up to 5000 Hz, but the multipole ESM requires fewer model parameters and is more robust.
- When using multipole ESM, there is a flexible balance between the computational effort and the prediction accuracy, controlled by choosing an appropriate truncation order.

Thank You!