WAVE PROPAGATION INTO THE SPINAL CAVITY: A 1D MODEL WITH COAXIAL COMPLIANT TUBES

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Introduction

One–Dimensional models have been used to simulate pulse waves propagation in the spinal cavity and the interactions between CSF, blood and the spinal cord. Some adopted compliant coaxial configurations but neglected the fluid's viscosity [1, 2] while others took into account CSF viscosity but simplified the cavity as one equivalent distensible tube [3]. Previous studies in the inviscid coaxial configuration have shown that the confinement reduces the wave propagation speed of the compliant part by a factor equal to the square root of the area parameter, i.e. the ratio of the tubes cross-sectional areas, when the dura is considered rigid.

Methods

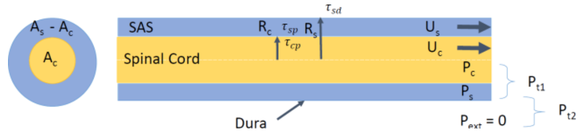

Figure 1: Schematic configuration of the 1D model.

Here we use one-dimensional modelling of the spinal compartment in the coaxial configuration (as seen in Figure 1) while considering CSF as a viscous fluid. For the spinal cord (SC), the governing equations are:

$$\frac{\partial A_c}{\partial t} + \frac{\partial (U_c A_c)}{\partial z} = 0 \tag{1}$$

$$\frac{\partial U_c}{\partial t} + U_c \frac{\partial U_c}{\partial z} + \frac{1}{\rho}\frac{\partial P_c}{\partial z} = \frac{2\sqrt{\pi}}{\rho\sqrt{A_c}}\tau_{cp} \tag{2}$$

And for the dura:

$$\frac{\partial A_s}{\partial t} + \frac{\partial U_s (A_s - A_c)}{\partial z} + \frac{\partial (U_c A_c)}{\partial z} = 0 \tag{3}$$

$$\frac{\partial U_s}{\partial t} + U_s \frac{\partial U_s}{\partial z} + \frac{1}{\rho}\frac{\partial P_s}{\partial z} = \frac{-2\sqrt{\pi}}{\rho(A_s - A_c)} \left(\tau_{sd}\sqrt{A_s} - \tau_{sp}\sqrt{A_c}\right) \tag{4}$$

The variables (A, U, P) are respectively the cross-section, the average velocity and the pressure. The subscript c stands for the spinal cord variables and the subscript s for the subarachnoid space (SAS) variables. The wall shear stresses τ are shown in Figure 1.

Results

Using linear relationships between the transmural pressures and the cross-sections and a steady approximation for the wall shear stresses, we can obtain all the variables time-space evolutions in the SC and SAS. For example the Figure 2 shows the wall shear stresses time evolution for a realistic spinal canal geometric configuration and flow rate cranial excitation.

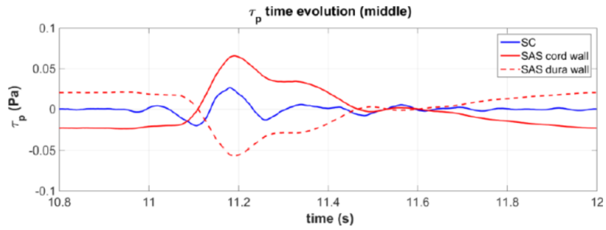

Figure 2: Wall shear stress time evolution at the middle of the spinal canal.

Discussion

Concomitant to the area parameter, the viscous shear stresses developed at the different walls are involved in the dynamics of the system. They impact the coupled wave velocity and therefore the coupled distensibility as well as the wave attenuation due to the interaction between the contents of the spinal cavity. The addition of the viscous nature of the fluids induces a viscous attenuation whose effect depends also on the area parameter and the Womersley number.

Although our modelling is nonlinear and the coupled system of equations is solved numerically we also consider the linear case and obtain a pressure damped wave equation similar to the so called telegrapher's equation. The pressure damping coefficient expression shows analytically how the area ratio and the shear stresses developed at the different walls are coupled.

References

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Acknowledgements

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