Comparison between the IDIM-IV method and the DIDIM method for industrial robots identification

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***Abstract*— This paper deals with two robot identification methods recently introduced. The first one is based on the use of the Inverse Dynamic Identification Model (IDIM) and the Instrumental Variable (IV). The second one is the Direct and Inverse Dynamic Identification Models (DIDIM) method, which is a closed-loop output error method minimizing the quadratic error between the actual and simulated joint torques. Both methods rely on the simulation of the Direct Dynamic Model (DDM). They are compared with a six degrees of freedom industrial robot. The experimental results show that the DIDIM method has the advantage of requiring less data preprocessing. Nevertheless, the IDIM-IV method appears to be more robust to modelling errors in the simulation which are not located in the identified dynamic model.**

I. INTRODUCTION

Industrial robots can be modelled thanks to the Inverse Dynamic Identification Model (IDIM) that has the advantage to be linear with respect to the dynamical parameters [1]. Therefore, linear estimation techniques seem the natural solution to address the identification of the dynamic models. In practice, the Least-Squares (LS), based on the IDIM, have been the usual robot identification methods for decades [1]. Since the robots have a double integrator behaviour, they must be operated in closed-loop. The issue that arises is that the regressors are correlated with the measurement noise due to the feedback. The methodology described in [2] deals with this correlation problem thanks to a careful filtering process.

Recently, the Instrumental Variable (IV) method has been introduced for robot identification [3]. That method has the advantage to be intrinsically robust to the correlation issue [4]. The method relies on the simulation of the Direct Dynamic Model (DDM) to generate the instrumental matrix needed for the estimation. Another technique relying on the DDM simulation has also been suggested in [5]. This technique called DIDIM, for Direct and Inverse Dynamic Identification Model, is an output error identification method. Both IDIM-IV and DIDIM methods have proven their efficiency to identify industrial robots [6], [7], [8].

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The aim of this paper is twofold. Firstly, we carefully detail the IDIM-IV and the DIDIM methods in order to make them clear for practitioners. Secondly, their performances are compared on experimental data from a six Degrees-Of-Freedom (DOF) industrial robot: the Stäubli® TX40.

This paper is organised as follows. After this short introduction, preliminaries introduce the model of the robot dynamic models and the notations. Section III summarizes the regular robot identification technique: the IDIM-LS method. The following section introduces and compares the IDIM-IV and the DIDIM methods. In Section V, the experimental results are exposed and discussed. Finally, concluding remarks are provided in Section VI.

II. ROBOT MODELLING

The Inverse Dynamic Model (IDM) of a rigid robot with n moving links is the expression of the $(n\times1)$ torque vector, $\boldsymbol{\tau}_{idm}$, as a function of the joint positions and their derivatives [1]. From the Newton's law or the Lagrangian equations, the following relation comes out

$$\boldsymbol{\tau}_{idm}(t) = \boldsymbol{M}\left(\boldsymbol{q}(t)\right)\ddot{\boldsymbol{q}}(t) + \boldsymbol{N}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)\right) \quad (1)$$

where $\boldsymbol{M}$ is the $(n\times n)$ inertia matrix; $\boldsymbol{N}$ is the $(n\times1)$ vector of centrifugal, Coriolis, gravitational, and friction torques; and $\boldsymbol{q}$, $\dot{\boldsymbol{q}}$, $\ddot{\boldsymbol{q}}$ are respectively the $(n \times 1)$ noise-free vectors of joint positions, velocities and accelerations.

According to [9], a joint j of an industrial robot has 14 standard parameters:

$$\begin{aligned}\boldsymbol{\chi}_j = [&XX_j \quad XY_j \quad XZ_j \quad YY_j \quad YZ_j \quad ZZ_j \\ &MX_j \quad MY_j \quad MZ_j \quad M_j \quad Ia_j \quad F_{v_j} \quad F_{c_j} \quad \tau_{off_j}]^T\end{aligned} \quad (2)$$

where XX_j, XY_j, XZ_j, YY_j, YZ_j and ZZ_j are the six components of the inertia matrix at the origin of frame j; MX_j, MY_j, MZ_j are the three components of the first moments; M_j is the mass of link j; Ia_j is the total inertia moment for rotor and gears of the actuator; F_{v_j} and F_{c_j} are respectively the viscous and Coulomb friction coefficients; τ_{off_j} is an offset parameter containing the asymmetry of the Coulomb friction with respect to the sign of the velocity and the current amplifier offset which supplies the motor.

Since some of those parameters have no effect on the dynamic model, while others are regrouped with linear relations, we obtain a $(b\times1)$ vector of base dynamic parameters: $\boldsymbol{\theta}$; see [10]. In addition, the IDM is linear with the base parameters and so we obtain the following linear relation

$$\boldsymbol{\tau}_{idm}(t) = \boldsymbol{\phi}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), \ddot{\boldsymbol{q}}(t)\right)\boldsymbol{\theta} = \boldsymbol{\phi}(t)\boldsymbol{\theta}, \quad (3)$$

where $\boldsymbol{\phi}$ is the $(n \times b)$ matrix of basis functions (also called observation matrix). Each element of $\boldsymbol{\phi}$ is a basis function of the body dynamics. Those basis functions can be nonlinear relations of the positions, velocities and accelerations.

Because of perturbations coming from measurement noise and modelling errors, the actual torque $\boldsymbol{\tau}$ differs from $\boldsymbol{\tau}_{idm}$ by an error $\boldsymbol{v}$. This usual definition of the IDIM is given by

$$\boldsymbol{\tau}(t) = \boldsymbol{\tau}_{idm}(t) + \boldsymbol{v}(t) = \boldsymbol{\phi}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), \ddot{\boldsymbol{q}}(t)\right)\boldsymbol{\theta} + \boldsymbol{v}(t). \quad (4)$$

The DDM is the expression of the joint accelerations as a nonlinear function of the states (positions and velocities) and the parameters, such as

$$\ddot{\boldsymbol{q}}(t) = \boldsymbol{M}\left(\boldsymbol{q}(t)\right)^{-1}\left(\boldsymbol{\tau}_{idm}(t) - \boldsymbol{N}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)\right)\right). \quad (5)$$

III. ROBOT IDENTIFICATION WITH LEAST-SQUARES

A. Filtering Process

In most applications, the available information is the $(n \times 1)$ measurement vector of the joint positions, $\boldsymbol{q}_m$. The joint velocities and accelerations have to be retrieved from this information in order to build the observation matrix $\boldsymbol{\phi}$ as described in [2]. $\boldsymbol{q}_m$ is firstly filtered to obtain $\widehat{\boldsymbol{q}}$. From this filtered position, the derivatives can be calculated with finite differences while avoiding noise amplification. The filter type and the cut-off frequency, ω_{f_q}, are selected such as $\left(\widehat{\boldsymbol{q}}, \widehat{\dot{\boldsymbol{q}}}, \widehat{\ddot{\boldsymbol{q}}}\right) \approx (\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$ in the range $[0, \omega_{f_q}]$. The filter, which is usually a Butterworth one, is applied in both forward and reverse directions to avoid lag introduction. The signals are indeed used thereafter to construct the nonlinear basis functions and those nonlinearities do not tolerate any phase shift. The rule of thumb for the cut-off frequency is $\omega_{f_q} \geq 5\omega_{dyn}$. The combination of the Butterworth filter and the central differentiation is referred to as the *bandpass* filtering process.

In practice, the torque is perturbed by high-frequency ripples: unmodelled friction and flexibility effects, which are rejected by the controller. Those ripples are removed prior to the identification with a parallel lowpass filtering of each basis function at the cut-off frequency $\omega_{F_p} \geq 2\omega_{dyn}$. The choice of ω_{F_p} may be involved to keep enough information while avoiding the high frequency noise. Since there is no more useful information beyond the cut-off frequency, the data are also re-sampled by keeping one sample over n_d. This combination of parallel filtering and re-sampling is referred to as the *decimate* process. After data acquisition and parallel filtering, we obtain

$$\begin{aligned}\boldsymbol{\tau}_{F_p}(t) &= F_p(z^{-1})\boldsymbol{\tau}(t) \quad (6)\\ &= \boldsymbol{\phi}_{F_p}\left(\widehat{\boldsymbol{q}}(t), \widehat{\dot{\boldsymbol{q}}}(t), \widehat{\ddot{\boldsymbol{q}}}(t)\right)\boldsymbol{\theta} + \boldsymbol{v}_{F_p}(t),\end{aligned}$$

with F_p the parallel filter applied to each element of the observation matrix $\boldsymbol{\phi}_{F_p}\left(\widehat{\boldsymbol{q}}(t), \widehat{\dot{\boldsymbol{q}}}(t), \widehat{\ddot{\boldsymbol{q}}}(t)\right) = F_p(z^{-1})\boldsymbol{\phi}\left(\widehat{\boldsymbol{q}}(t), \widehat{\dot{\boldsymbol{q}}}(t), \widehat{\ddot{\boldsymbol{q}}}(t)\right)$ and the error vector $\boldsymbol{v}_{F_p}(t) = F_p(z^{-1})\boldsymbol{v}(t)$.

B. Data Conditioning

The dynamic models previously described are continuous-time but the identification is performed from sampled data recorded at the frequency f_m. Furthermore, we focus on off-line identification that allows to regroup the recorded data. If n_m measurements are recorded during the experiment, after the re-sampling we have $N = n_m/n_d$ available sets of data. After the bandpass filtering and the decimation process, the following over-determined system is obtained

$$\boldsymbol{y}(\boldsymbol{\tau}) = \boldsymbol{X}\left(\widehat{\boldsymbol{q}}, \widehat{\dot{\boldsymbol{q}}}, \widehat{\ddot{\boldsymbol{q}}}\right)\boldsymbol{\theta} + \boldsymbol{\epsilon} \quad (7)$$

where $\boldsymbol{y}(\boldsymbol{\tau})$ is $(r \times 1)$ measurements vector built from the filtered torques $\boldsymbol{\tau}_{F_p}$; $\boldsymbol{X}\left(\widehat{\boldsymbol{q}}, \widehat{\dot{\boldsymbol{q}}}, \widehat{\ddot{\boldsymbol{q}}}\right)$ is $(r \times b)$ observation matrix built from $\boldsymbol{\phi}_{F_p}\left(\widehat{\boldsymbol{q}}, \widehat{\dot{\boldsymbol{q}}}, \widehat{\ddot{\boldsymbol{q}}}\right)$; $\boldsymbol{\epsilon}$ is the $(r \times 1)$ vector of filtered errors $\boldsymbol{v}_{F_p}$; $r = n \cdot N = n \cdot n_m/n_d$ is the number of rows in (7).

C. IDIM-LS Method

Once $\boldsymbol{y}$ and $\boldsymbol{X}$ available, the LS estimates of $\boldsymbol{\theta}$ are given by

$$\widehat{\boldsymbol{\theta}}_{LS}(N) = \left[\boldsymbol{X}^T\boldsymbol{X}\right]^{-1}\boldsymbol{X}^T\boldsymbol{y} = \boldsymbol{X}^{+}\boldsymbol{y}, \quad (8)$$

with $\boldsymbol{X}^{+}$ the Moore-Penrose pseudo-inverse. Without modelling errors, the LS estimator is consistent under the two conditions:

- $\boldsymbol{X}^T\boldsymbol{X}$ is full column rank;
- $\mathbb{E}\left[\boldsymbol{X}^T\boldsymbol{\epsilon}\right] = 0$;

with $\mathbb{E}[]$ the expectation operator. For closed-loop systems, the assumption that the observation matrix is not correlated with the error is not valid due to the feedback [11]. In practice, thanks to the appropriate filtering, the IDIM-LS predictor is still consistent provided that ω_{f_q} and ω_{F_p} are tuned accordingly to ω_{dyn}.

IV. ROBOT IDENTIFICATION WITH DIRECT DYNAMIC MODEL SIMULATION

A. IDIM-IV Method

The IV method consists in introducing an $(r \times b)$ instrumental matrix denoted as $\boldsymbol{Z}$. To be a valid instrumental matrix, $\boldsymbol{Z}$ must fulfil the following conditions

- $\boldsymbol{Z}^T\boldsymbol{X}$ is full column rank;
- $\mathbb{E}\left[\boldsymbol{Z}^T\boldsymbol{\epsilon}\right] = 0$.

The first condition means that the instrumental matrix must be well correlated with the observation one. This condition can be called the *instrument relevance* [12]. The second condition expresses the fact that the instrumental matrix must be uncorrelated with the error, which is known as the *instrument exogeneity*. Assuming that the two previous conditions hold, the consistent IV estimates are given by

$$\widehat{\boldsymbol{\theta}}_{IV}(N) = \left[\boldsymbol{Z}^T\boldsymbol{X}\right]^{-1}\boldsymbol{Z}^T\boldsymbol{y} \quad (9)$$

In the last decade, different IV solutions have been developed for closed-loop identification; see e.g. [13]. Although the IV method is an interesting alternative to the LS method for closed-loop identification of continuous-time models, the main issue is the construction of the instruments. There exist many ways of constructing the instrumental matrix [14]. For robot identification, it has been shown, in [3], that the simulation of the DDM provides a very convenient way to obtain the instruments. This simulation model contains the whole closed-loop and is referred to as the *auxiliary model*. From the simulation of this auxiliary model, noise-free simulated signals are retrieved and used to construct the instrumental matrix. The signals are noise-free since the only input is the reference trajectory which is perfectly known. The process is iterative because the simulation is based on the parameters previously identified. By noting the simulated signals with a subscript s, the instrumental matrix at iteration it is $\boldsymbol{\zeta}(t, \widehat{\boldsymbol{\theta}}_{IV}^{it}) = F_p(z^{-1})\boldsymbol{\phi}\left(\boldsymbol{q}_s(t, \widehat{\boldsymbol{\theta}}_{IV}^{it}), \dot{\boldsymbol{q}}_s(t, \widehat{\boldsymbol{\theta}}_{IV}^{it}), \ddot{\boldsymbol{q}}_s(t, \widehat{\boldsymbol{\theta}}_{IV}^{it})\right)$. The instrumental matrix can be viewed as an estimation of the noise-free part of the observation matrix. This IDIM-IV method also includes the parallel filter and the downsampling process. That method has the advantage to be less sensitive than the IDIM-LS method to an inappropriate filtering, as shown in [4]. The $(r \times b)$ instrumental matrix $\boldsymbol{Z}\left(\widehat{\boldsymbol{\theta}}_{IV}^{it}\right)$ is constructed from the matrices $\boldsymbol{\zeta}(t, \widehat{\boldsymbol{\theta}}_{IV}^{it})$, like $\boldsymbol{X}$ from $\boldsymbol{\phi}_{F_p}$. The iterative estimation of the parameters is computed with

$$\widehat{\boldsymbol{\theta}}_{IV}^{it+1} = \left[\boldsymbol{Z}(\widehat{\boldsymbol{\theta}}_{IV}^{it})^T \boldsymbol{X}\right]^{-1} \boldsymbol{Z}(\widehat{\boldsymbol{\theta}}_{IV}^{it})^T \boldsymbol{y}(\tau). \tag{10}$$

As explained in [3], the process is iterated until its convergence. The convergence criterion is based on relative variation of estimated parameters and the one of the estimation error $\boldsymbol{\rho}^{it} = \boldsymbol{y}(\tau) - X\widehat{\boldsymbol{\theta}}_{IV}^{it}$. Concerning the initialisation, the inertia parameters are usually initialised with Computer-Aided Design (CAD) values, whereas the others parameters are set to zero.

B. DIDIM Method

The two methods previously described rely on the linear regression based on measured or reconstructed signals, contained in $\boldsymbol{X}$. Another well-known family of system identification techniques is the one of Prediction Error Methods (PEM). The idea is to minimize the sum of square filtered differences between the measured robot outputs and simulated model outputs. In other words, the aim is to solve

$$\widehat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, J(\theta) = \underset{\theta}{\operatorname{argmin}}\, \frac{1}{2N} \sum_{k=1}^{N} \|\boldsymbol{e}_\tau(t_k, \boldsymbol{\theta})\|_2^2, \tag{11}$$

where $\boldsymbol{e}_\tau$ is the $(n \times 1)$ prediction error vector. In the system identification community, it is common to consider the position as output of the model [15], whereas it is common to use the input torque/force for robot identification. We thus talk about Closed-Loop Input Error (CLIE), since the prediction is based on the simulation of the whole closed-loop model. For the CLIE approach whose the DIDIM method is a specific case, the prediction error vector is defined by

$$\boldsymbol{e}_\tau(t_k, \boldsymbol{\theta}) = \boldsymbol{\tau}(t_k) - \boldsymbol{\tau}_s(t_k, \boldsymbol{\theta}), \tag{12}$$

where $\boldsymbol{\tau}_s$ is the $(n \times 1)$ vector of simulated joint torques. The problem (11) is usually solved thanks to nonlinear optimisation algorithms such as the gradient or Newton methods. Those methods are based on a first- or second-order Taylor's expansion of the criterion $J(\boldsymbol{\theta})$. The optimization is based on an iterative approach until convergence. For the DIDIM method, at the iteration $it + 1$, the parameters are updated using $\widehat{\boldsymbol{\theta}}_{DIDIM}^{it+1} = \widehat{\boldsymbol{\theta}}_{DIDIM}^{it} + \Delta\widehat{\boldsymbol{\theta}}^{it}$, where the parameters increment $\widehat{\boldsymbol{\theta}}^{it}$ depends on the considered optimisation algorithm. We focus on the Gauss-Newton (GN) algorithm. After data sampling and parallel decimation, the following over-determined system is obtained

$$\Delta\boldsymbol{y}(\boldsymbol{\tau}) = \boldsymbol{\Psi}^{it}\Delta\boldsymbol{\theta}^{it} + \boldsymbol{\epsilon}_{DIDIM} \tag{13}$$

where

- $\Delta\boldsymbol{y}(\boldsymbol{\tau})$ is the $(r \times 1)$ vector built from the sampling of $\boldsymbol{y}(\tau) - \boldsymbol{y}\left(\boldsymbol{\tau}_s\left(\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}\right)\right)$;
- $\boldsymbol{\Psi}^{it}$ is the $(r \times b)$ matrix built from the sampling of the $(n \times b)$ jacobian matrix $\boldsymbol{\Delta}_{\boldsymbol{\tau}_s, \widehat{\boldsymbol{\theta}}} = \frac{\partial \boldsymbol{\tau}_s}{\partial \boldsymbol{\theta}^T} |_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}}$;
- $\boldsymbol{\epsilon}_{DIDIM}$ is the sampling of $(\boldsymbol{o} + \boldsymbol{e}_{DIDIM})$ where $\boldsymbol{o}$ is the residual of the Taylor series expansion and $\boldsymbol{e}_{DIDIM}$ is the error vector $\boldsymbol{y}(\tau) - \boldsymbol{y}\left(\boldsymbol{\tau}_s\left(\widehat{\boldsymbol{\theta}}_{DIDIM}^{it+1}\right)\right)$.

Each element of the jacobian $\boldsymbol{\Delta}_{\boldsymbol{\tau}_s, \widehat{\boldsymbol{\theta}}}$ is an input sensitivity function which defines the variation of the input torque/force with respect to the parameter. Usually, in system identification, those sensitivities functions are numerically approximated because they are not exactly known. An exact calculation of the derivatives would be too involved due to the system nonlinearities. The main advantage of the DIDIM method is that, by using the IDM definition (1), the input sensitivities can be written (at any sampling time)

$$\begin{aligned}\boldsymbol{\Delta}_{\boldsymbol{\tau}_s, \widehat{\boldsymbol{\theta}}} &= \frac{\partial}{\partial \boldsymbol{\theta}^T}\left(\boldsymbol{\Phi}\left(\boldsymbol{q}_s, \dot{\boldsymbol{q}}_s, \ddot{\boldsymbol{q}}_s\right)\boldsymbol{\theta}\right)|_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}} && (14)\\ &= \boldsymbol{\Phi}\left(\boldsymbol{q}_s, \dot{\boldsymbol{q}}_s, \ddot{\boldsymbol{q}}_s\right)\\ &\quad + \frac{\partial}{\partial \boldsymbol{\theta}^T}\left(\boldsymbol{\Phi}\left(\boldsymbol{q}_s, \dot{\boldsymbol{q}}_s, \ddot{\boldsymbol{q}}_s\right)\right)|_{\boldsymbol{\theta}=\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}}\, \widehat{\boldsymbol{\theta}}_{DIDIM}^{it}. && (15)\end{aligned}$$

As explained in [5], by keeping the same bandwidth and stability margin for the simulated system as those of the real one, it can be assumed that the simulated tracking error is close to the real one, that is to say

$$\begin{aligned}&\left(\boldsymbol{q}_s(t, \widehat{\boldsymbol{\theta}}_{DIDIM}^{it}), \dot{\boldsymbol{q}}_s(t, \widehat{\boldsymbol{\theta}}_{DIDIM}^{it}), \ddot{\boldsymbol{q}}_s(t, \widehat{\boldsymbol{\theta}}_{DIDIM}^{it})\right) && (16)\\ &\quad \approx \left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), \ddot{\boldsymbol{q}}(t)\right),\end{aligned}$$

for any $\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}$ and at any time, leading to

$$\boldsymbol{\Delta}_{\boldsymbol{\tau}_s, \widehat{\boldsymbol{\theta}}} \approx \boldsymbol{\Phi}\left(\boldsymbol{q}_s, \dot{\boldsymbol{q}}_s, \ddot{\boldsymbol{q}}_s\right). \tag{17}$$

Consequently, considering $\boldsymbol{X}$, it comes out

$$\begin{aligned}\boldsymbol{\Psi}^{it} &= \boldsymbol{X}\left(\boldsymbol{q}_s(\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}), \dot{\boldsymbol{q}}_s(\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}), \ddot{\boldsymbol{q}}_s(\widehat{\boldsymbol{\theta}}_{DIDIM}^{it})\right) && (18)\\ &= \boldsymbol{Z}(\widehat{\boldsymbol{\theta}}_{DIDIM}^{it}).\end{aligned}$$

By using the approximated input sensitivity (17) with the criterion (12) and the GN algorithm, it comes out after a straightforward calculation:

$$\widehat{\boldsymbol{\theta}}_{DIDIM}^{it+1} = \boldsymbol{Z}(\widehat{\boldsymbol{\theta}}_{DIDIM}^{it})^{+}\boldsymbol{y}(\tau). \tag{19}$$

Therefore, the DIDIM method is based on the approximation that the matrix of basis functions is not dependent on the physical parameters. In system identification this assumption is referred to as Pseudo-Linear Regression (PLR), see Eq. (7.112) in [16]. According to the same reference, PLR is derived from [17]. At last, it can be stated that the DIDIM method developed in the field of robot identification is a specific case of the CLIE method, relying on the PLR assumption and optimised with a Gauss-Newton algorithm.

C. Comparison

As illustrated with (10) and (19), the IDIM-IV and the DIDIM methods are very similar. They indeed rely on the simulation of the DDM and their estimations are iterative. In practice, they use the same convergence criterion and the same initialisation, which is based on CAD values.

Assuming that there is no modelling error and that the IDM is well specified, we obtain

$$\boldsymbol{Z}(\widehat{\boldsymbol{\theta}}^{it}_{DIDIM}) = \boldsymbol{Z}(\widehat{\boldsymbol{\theta}}^{it}_{IV}) = \boldsymbol{X}_{nf}, \tag{20}$$

where $\boldsymbol{X}_{nf}$ is the noise free component of $\boldsymbol{X}$. Equivalently, we have at each sampling time: $\boldsymbol{\zeta}_{F_p}(t, \widehat{\boldsymbol{\theta}}^{it}_{IV}) = \boldsymbol{\zeta}_{F_p}(t, \widehat{\boldsymbol{\theta}}^{it}_{DIDIM}) = \boldsymbol{\phi}^{nf}_{F_p}(t)$, where $\boldsymbol{\phi}^{nf}_{F_p}$ is the noise free component of $\boldsymbol{\phi}_{F_p}$. It is indeed assumed that the filtered observation matrix can be divided such as $\boldsymbol{\phi}_{F_p} = \boldsymbol{\phi}^{nf}_{F_p} + \widetilde{\boldsymbol{\phi}}_{F_p}$. The noise free and noisy parts are uncorrelated, i.e. $\mathbb{E}\left[\left(\boldsymbol{\phi}^{nf}_{F_p}\right)^T \widetilde{\boldsymbol{\phi}}_{F_p}\right] = 0$, with $\mathbb{E}\left[\widetilde{\boldsymbol{\phi}}_{F_p}\right] = 0$. Assuming there is enough sampling point, i.e. N large enough, it comes out

$$\mathbb{E}\left[\left(\boldsymbol{\phi}^{nf}_{F_p}\right)^T \widetilde{\boldsymbol{\phi}}_{F_p}\right] \approx \frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i)\right)^T \widetilde{\boldsymbol{\phi}}_{F_p}(t_i) \tag{21}$$

and

$$\begin{aligned}
\widehat{\boldsymbol{\theta}}^{it+1}_{IV} &= \left[\boldsymbol{Z}(\widehat{\boldsymbol{\theta}}^{it}_{IV})^T\boldsymbol{X}\right]^{-1}\boldsymbol{Z}(\widehat{\boldsymbol{\theta}}^{it}_{IV})^T\boldsymbol{y}(\tau) \\
&= \boldsymbol{Q}^{-1}_{\boldsymbol{\phi}_{F_p}}\left[\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\zeta}^T_{F_p}(t_i, \widehat{\boldsymbol{\theta}}^{it}_{IV})\boldsymbol{\tau}_{F_p}(t_i)\right] \\
&\approx \boldsymbol{Q}^{-1}_{\boldsymbol{\phi}^{nf}_{F_p}}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i)\right)^T\boldsymbol{\tau}_{F_p}(t_i)\right] \\
&= \left[\boldsymbol{X}^T_{nf}\boldsymbol{X}_{nf}\right]^{-1}\boldsymbol{X}^T_{nf}\boldsymbol{y}(\tau) \\
&= \widehat{\boldsymbol{\theta}}^{it+1}_{DIDIM},
\end{aligned} \tag{22}$$

with

$$\begin{aligned}
\boldsymbol{Q}_{\boldsymbol{\phi}_{F_p}} &= \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\zeta}^T_{F_p}(t_i, \widehat{\boldsymbol{\theta}}^{it}_{IV})\boldsymbol{\phi}_{F_p}(t_i) \\
&= \frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i)\right)^T\left[\boldsymbol{\phi}^{nf}_{F_p}(t_i) + \widetilde{\boldsymbol{\phi}}_{F_p}(t_i)\right] \\
&\approx \frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i)\right)^T\boldsymbol{\phi}^{nf}_{F_p}(t_i) = \boldsymbol{Q}_{\boldsymbol{\phi}^{nf}_{F_p}}.
\end{aligned}$$

Therefore, if there is no modelling error, the DIDIM and IDIM-IV methods are perfectly equivalent. It is worth noting that the DIDIM estimates given by (19) is the IV solution raised in [14] equation (3.43b) page 38. If we follow the authors' point of view, the DIDIM approach can be considered as a sort of *bootstrap* IV variant.

The difference between those methods lies in the use of the observation matrix, $\boldsymbol{X}$, for the IDIM-IV method. That may be seen as a drawback since it requires the measurement of the position signals, $\boldsymbol{q}_m$, and the careful bandpass filtering detailed in Section III-A, in order to obtain the joint velocities and accelerations. As stressed in [5], the DIDIM method has the advantage of requiring only the measured torques.

However, the use of $\boldsymbol{X}$ can make the IDIM-IV method less sensitive to modelling errors, in the simulation model, than the DIDIM method. We do not consider modelling errors in the dynamic model but in the rest of the closed-loop system. In fact, if we made an error in $\boldsymbol{Z}$, that error would also be present in $\boldsymbol{X}$ due to the instrument construction and, consequently, both the IDIM-IV and DIDIM estimates would be biased. In other words, we consider here that the simulator is biased but not $\boldsymbol{X}$. The simulator error can be located in the controller used for the simulation for instance. If there was such an error, the auxiliary model would be biased and, therefore, also the simulated signals. Hence the assumption (20) does not stand anymore and the simulation errors are introduced such as

$$\begin{aligned}
\boldsymbol{\zeta}_{F_p}(t, \widehat{\boldsymbol{\theta}}^{it}_{IV}) &= \boldsymbol{\phi}^{nf}_{F_p}(t) + \boldsymbol{d}^{it}_{IV}(t), \\
\boldsymbol{\zeta}_{F_p}(t, \widehat{\boldsymbol{\theta}}^{it}_{DIDIM}) &= \boldsymbol{\phi}^{nf}_{F_p}(t) + \boldsymbol{d}^{it}_{DIDIM}(t).
\end{aligned} \tag{23}$$

Whatever the method used, one assumption can be made concerning the $(n \times 1)$ simulation error vector $\boldsymbol{d}$: it is uncorrelated with the measurement noise, i.e. $\mathbb{E}\left[\left(\boldsymbol{d}^{it}\right)^T \widetilde{\boldsymbol{\phi}}_{F_p}\right] = 0$ and $\mathbb{E}\left[\left(\boldsymbol{d}^{it}\right)^T \widetilde{\boldsymbol{\tau}}_{F_p}\right] = 0$, with $\widetilde{\boldsymbol{\tau}}_{F_p}$ the noisy component of $\boldsymbol{\tau}_{F_p}$. This assumption seems logical because the simulator is noise-free, as explained in Section IV-A. From those definitions, two cases can be considered according to $\boldsymbol{d}$.

The first case is when expectation of the error is null, i.e. $\mathbb{E}\left[\boldsymbol{d}^{it}_{IV}\right] = \mathbb{E}\left[\boldsymbol{d}^{it}_{DIDIM}\right] = 0$. Hence, it appears for IDIM-IV solution:

$$\begin{aligned}
\widehat{\boldsymbol{\theta}}^{it+1}_{IV} &= \boldsymbol{Q}^{-1}_{\boldsymbol{\phi}_{F_p}}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i) + \boldsymbol{d}^{it}_{IV}(t_i)\right)^T\boldsymbol{\tau}_{F_p}(t_i)\right] \\
&\approx \boldsymbol{Q}^{-1}_{\boldsymbol{\phi}^{nf}_{F_p}}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i)\right)^T\boldsymbol{\tau}_{F_p}(t_i)\right] \\
&= \left[\boldsymbol{X}^T_{nf}\boldsymbol{X}_{nf}\right]^{-1}\boldsymbol{X}^T_{nf}\boldsymbol{y}(\tau),
\end{aligned} \tag{24}$$

with

$$\begin{aligned}
\boldsymbol{Q}_{\boldsymbol{\phi}_{F_p}} &= \frac{1}{N}\sum_{i=1}^{N}\left[\boldsymbol{\phi}^{nf}_{F_p}(t_i) + \boldsymbol{d}^{it}_{IV}(t_i)\right]^T\left[\boldsymbol{\phi}^{nf}_{F_p}(t_i) + \widetilde{\boldsymbol{\phi}}_{F_p}(t_i)\right] \\
&\approx \frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i)\right)^T\boldsymbol{\phi}^{nf}_{F_p}(t_i) = \boldsymbol{Q}_{\boldsymbol{\phi}^{nf}_{F_p}}.
\end{aligned}$$

Concerning the DIDIM method, there is a persistent term

$$\widehat{\boldsymbol{\theta}}^{it+1}_{DIDIM} \approx \left(\boldsymbol{Q}^{nf}_{\boldsymbol{\phi}_{F_p}} + \boldsymbol{dQ}\right)^{-1} \left[\frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{\phi}^{nf}_{F_p}(t_i)\right)^T \boldsymbol{\tau}_{F_p}(t_i)\right]$$
$$= \left[\boldsymbol{X}^T_{nf}\boldsymbol{X}_{nf} + \boldsymbol{dX}^T\boldsymbol{dX}\right]^{-1}\boldsymbol{X}^T_{nf}\boldsymbol{y}(\tau), \qquad (25)$$

with

$$\boldsymbol{dQ} = \frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{d}^{it}_{DIDIM}(t_i)\right)^T \boldsymbol{d}^{it}_{DIDIM}(t_i) \approx cov\left(\boldsymbol{d}^{it}_{DIDIM}\right).$$

Equation (24) shows that the IDIM-LS method is robust against a modelling error located in the simulator with a null expected value. In the opposite, (25) shows that the DIDIM method is sensitive to such an error and its estimates are biased.

The second case is when the simulation error has a non zero expectation, i.e. $\mathbb{E}\left[\boldsymbol{d}^{it}_{IV}\right] \neq 0$ and $\mathbb{E}\left[\boldsymbol{d}^{it}_{DIDIM}\right] \neq 0$. In this case, both methods are biased and so are the estimated parameters.

V. EXPERIMENTAL RESULTS

A. Experimental Setup

Experiments are performed with the industrial robot Stäubli® TX40 that is a serial manipulator composed of six rotational joints; see Figure 1. There is a coupling between the joints 5 and 6 that adds two parameters: fv_{m6} and fc_{m6}, which are respectively the viscous and dry friction coefficient of the motor 6. The SYMORO+ software is used to automatically calculate the customized symbolic expressions of models [1]. The robot has 60 base dynamic parameters and from those 60 base parameters, only 28 are well identified with good relative standard deviations. These 28 parameters define a set of essential parameters which are enough to describe the dynamics. This set was validated with a F-statistic, as shown in [6]. We present only those parameters.

The reference trajectories are trapezoidal velocities (also called smoothed bang-bang accelerations). Since $cond(\boldsymbol{\phi}_{F_p}) = 200$, the reference trajectories excite well the base parameters. The joint positions and control signals are stored with a measurement frequency $f_m = 5$ kHz. For the IDIM-LS method, the filters cut-off frequencies are tuned according to [5]: $\omega_{f_q} = 5\omega_{dyn} = 50$ Hz and $\omega_{F_p} = 2\omega_{dyn} = 20$ Hz respectively for the Butterworth and the decimate filters. The maximum bandwidth for joint 6 is indeed $\omega_{dyn} = 10$ Hz.

B. Nominal Identification

In a first time, we illustrate the behaviour of the methods in the nominal case. That is to say that the auxiliary model is unbiased. This auxiliary model is composed of the robot dynamic model, the actual controller, the gains of the actuators and the gear ratios. The DDM is simulated thanks to a Simulink® model integrated with the *ode45* solver. The three methods are initialized with acceptable CAD values. The essential estimated parameters and relative standard deviation are summarized in Table I. The IDIM-LS and the DIDIM methods respectively converged in 3 and 7 iterations.

Fig. 1. TX40 Stäubli robot

TABLE I
ESTIMATED PARAMETERS AND RELATIVE STANDARD DEVIATIONS - NOMINAL CASE

Param.	IDIM-LS	IDIM-IV	DIDIM
$zz1_r$	1.25 (1.04 %)	1.25 (1.08 %)	1.25 (1.23 %)
fc_1	6.90 (1.76 %)	6.86 (1.82 %)	7.11 (2.36 %)
fv_1	8.07 (0.55 %)	8.07 (0.55 %)	7.99 (0.75 %)
$xx2_r$	-0.48 (2.29 %)	-0.48 (2.45 %)	-0.48 (3.06 %)
$xz2_r$	-0.15 (3.69 %)	-0.15 (4.46 %)	-0.15 (6.01 %)
$zz2_r$	1.09 (0.86 %)	1.09 (0.88 %)	1.09 (1.12 %)
$mx2_r$	2.18 (1.92 %)	2.23 (2.43 %)	2.22 (3.32 %)
fc_2	7.89 (1.41 %)	8.02 (1.38 %)	8.35 (1.85 %)
fv_2	5.65 (0.88 %)	5.55 (0.89 %)	5.45 (1.24 %)
$xx3_r$	0.14 (7.35 %)	0.13 (7.86 %)	0.13 (8.97 %)
$zz3_r$	0.12 (6.92 %)	0.12 (7.34 %)	0.11 (8.47 %)
$my3_r$	-0.59 (2.05 %)	-0.59 (2.05 %)	-0.59 (2.11 %)
ia_3	0.09 (7.68 %)	0.09 (8.13 %)	0.09 (8.61 %)
fc_3	6.27 (1.77 %)	6.34 (1.75 %)	6.47 (1.59 %)
fv_3	1.98 (1.69 %)	1.97 (1.68 %)	1.94 (1.58 %)
mx_4	-0.03 (23.2 %)	-0.03 (21.2 %)	-0.03 (18.8 %)
ia_4	0.03 (13.3 %)	0.03 (12.1 %)	0.03 (7.78 %)
fc_4	2.44 (4.92 %)	2.40 (4.94 %)	2.42 (2.43 %)
fv_4	1.11 (3.01 %)	1.12 (2.93 %)	1.11 (1.48 %)
$my5_r$	-0.03 (14.8 %)	-0.03 (12.7 %)	-0.03 (11.0 %)
ia_5	0.04 (10.7 %)	0.04 (11.5 %)	0.04 (9.78 %)
fc_5	2.97 (3.76 %)	2.99 (3.66 %)	2.99 (2.92 %)
fv_5	1.83 (2.35 %)	1.82 (2.30 %)	1.82 (1.84 %)
ia_6	0.01 (25.1 %)	0.01 (29.9 %)	0.01 (15.7 %)
fc_6	0.27 (52.4 %)	0.27 (51.2 %)	0.25 (32.6 %)
fv_6	0.66 (3.18 %)	0.65 (3.24 %)	0.65 (1.59 %)
fc_{m6}	1.87 (5.18 %)	1.87 (5.08 %)	1.92 (3.44 %)
fv_{m6}	0.61 (2.72 %)	0.61 (2.70 %)	0.60 (1.67 %)

As expected, the three methods estimate equivalent parameters with comparable standard deviations. That is confirmed by the satisfactory relative errors provided in Table II. Furthermore, the estimated values are consistent with previous work on this robot arm [6]. Those observations are consistent with the work of [5] and [3]: the IDIM-IV and the DIDIM methods are appropriate to identify industrial robots.

TABLE II

RELATIVE ESTIMATION ERRORS

Case	IDIM-LS	IDIM-IV	DIDIM
Nominal	5.75 %	5.59 %	5.57 %
Biased simulator	5.75 %	5.66 %	6.71 %

TABLE III

ESTIMATED PARAMETERS AND RELATIVE STANDARD DEVIATIONS - BIASED SIMULATOR

Param.	IDIM-LS	IDIM-IV	DIDIM
$zz1_r$	1.25 (1.04 %)	1.25 (0.84 %)	1.08 (1.21 %)
fc_1	6.90 (1.76 %)	6.86 (1.82 %)	7.30 (2.46 %)
fv_1	8.07 (0.55 %)	8.07 (0.48 %)	6.93 (0.81 %)
$xx2_r$	-0.48 (2.29 %)	-0.48 (1.92 %)	-0.36 (3.44 %)
$xz2_r$	-0.15 (3.69 %)	-0.15 (3.58 %)	-0.13 (6.31 %)
$zz2_r$	1.09 (0.86 %)	1.09 (0.74 %)	0.95 (1.17 %)
$mx2_r$	2.18 (1.92 %)	2.25 (1.99 %)	1.73 (3.80 %)
fc_2	7.89 (1.41 %)	8.05 (1.39 %)	8.40 (1.99 %)
fv_2	5.65 (0.88 %)	5.54 (0.79 %)	4.74 (1.35 %)
$xx3_r$	0.14 (7.35 %)	0.14 (6.37 %)	0.10 (10.32 %)
$zz3_r$	0.12 (6.92 %)	0.12 (5.32 %)	0.14 (5.73 %)
$my3_r$	-0.59 (2.05 %)	-0.59 (1.68 %)	-0.59 (2.12 %)
ia_3	0.09 (7.68 %)	0.09 (6.41 %)	0.06 (11.66 %)
fc_3	6.27 (1.77 %)	6.37 (1.74 %)	6.49 (1.77 %)
fv_3	1.98 (1.69 %)	1.96 (1.50 %)	1.73 (1.76 %)
mx_4	-0.03 (23.2 %)	-0.03 (18.2 %)	-0.03 (15.7 %)
ia_4	0.03 (13.3 %)	0.03 (11.0 %)	0.03 (7.89 %)
fc_4	2.44 (4.92 %)	2.41 (4.95 %)	2.47 (2.40 %)
fv_4	1.11 (3.01 %)	1.12 (2.70 %)	1.01 (1.51 %)
$my5_r$	-0.03 (14.8 %)	-0.04 (11.3 %)	-0.03 (12.0 %)
ia_5	0.04 (10.7 %)	0.04 (9.66 %)	0.03 (10.3 %)
fc_5	2.97 (3.76 %)	2.98 (3.67 %)	2.92 (3.00 %)
fv_5	1.83 (2.35 %)	1.83 (2.02 %)	1.64 (1.82 %)
ia_6	0.01 (25.1 %)	0.01 (25.0 %)	0.01 (14.2 %)
fc_6	0.27 (52.4 %)	0.28 (50.1 %)	0.27 (30.1 %)
fv_6	0.66 (3.18 %)	0.65 (2.85 %)	0.57 (1.60 %)
fc_{m6}	1.87 (5.18 %)	1.86 (5.08 %)	1.90 (3.49 %)
fv_{m6}	0.61 (2.72 %)	0.61 (2.37 %)	0.53 (1.68 %)

C. Identification with a Biased Simulator

In this section, we highlight the difference between the IDIM-IV and DIDIM methods. Thus, an error is introduced in the auxiliary model: the gear ratios are taken equal to 90% of their actual values. Table III summarizes the estimated parameters and the relative standard deviations. There is technically no change for the IDIM-LS method, but its estimates are recalled for comparison. The IDIM-IV and the DIDIM methods respectively converged in 3 and 7 iterations.

At a first glance, from Table II, the difference is not so significant. However, with a closer scrutiny, it appears that the inertia parameters are poorly estimated with the DIDIM method, like $zz1_r$, $zz2_r$ or ia_3 for instance. Compared with the parameters previously estimated, there is a gap of almost 20 %. On the other hand, the IDIM-IV estimates are still consistent. That illustrates the impact of a biased simulator on the estimation when the simulation error has a zero expectation.

VI. CONCLUSIONS

In this paper, two robot identification methods, based on the simulation of the DDM, were compared on a six DOF industrial robot: the DIDIM method and the IDIM-IV method. We obtain the following results.

- Both methods are iterative and can share the same convergence criterion and initial values;
- The DIDIM method does not require the same careful bandpass filtering as the IDIM-IV method to construct the observation matrix;
- The IDIM-IV method appears to be more robust to a modelling error located in the auxiliary model, but not in the identified dynamic model.

Finally, we show that each method as its advantages and drawbacks. The practitioner should be aware of that to make the appropriate choice depending on its studied system.

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