State Space Estimation Method for the Identification of an Industrial Robot Arm

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Abstract: In this paper, we study the identification of industrial robot dynamic models. Since the models are linear with respect to the parameters, the usual identification technique is based on the Least-Squares method. That requires a careful preprocessing of the data to obtain consistent estimates. In this paper, we carefully detail this process and propose a new procedure based on Kalman filtering and fixed interval smoothing. This new technique is compared to usual one with experimental data considering an industrial robot arm. The obtained results show that the proposed technique is a credible alternative, especially if the system bandwidth is unknown.

Keywords: Robots identification; System identification; Closed-loop identification; Least-squares identification; Parameter identification; Kalman filters

1. INTRODUCTION

The usual method for robot identification is based on the Least-Squares (LS) technique and the Inverse Dynamic Identification Model (IDIM). The IDIM indeed allows expressing the input torque as a linear function of the physical parameters thanks to the modified Denavit and Hartenberg (DHM) notation. Therefore, the IDIM-LS method is a really practical solution, which explains its success, see (Gautier et al., 2013) and the references given therein. However this method needs a well-tuned band pass filtering to get the derivatives of the joint positions. Thus, it requires a good a priori knowledge of the system to tune adequately the filters. That may be an issue for the early tests of a system, especially if there is no access to the key design parameters, as with a robot bought "off-the-shelf".

This article has two aims. Firstly, it develops the usual process of robot identification. Secondly, the new technique proposed in (Brunot et al., 2016) is tested on an industrial robot arm. This technique has indeed already proven to be a suitable solution for a prototype robot with one degree of freedom. Its principle is to avoid relying on a priori knowledge of the system. For this work, the author designates by "a priori knowledge" the values of the parameters, which are known or guessed prior to the identification. In any case, the model structure is assumed to be known.

As it will be seen, the main part of the work consists in differentiating the position signal to construct the regressors (see Section 2 for a proper definition) for the LS method. In many fields, the problem of differentiating numerical signals was raised. In the domain of continuous-time system identification, it has been successfully dealt by different techniques like the generalized Poisson moment functional (GPMF) in (Rao and Unbehauen, 2006), the State Variable Filters (SVF) in (Mahata and Garnier, 2006) or the Refined Instrumental Variable (RIV) in (Garnier et al., 2007). For further reading on the topic, see e.g. (Garnier et al., 2003). Nevertheless, those attractive methods require either the system to be linear in the states, in order to have a self-tuned filtering (RIV), or the user to provide the bandwidth for the filter (GPMF and SVF). As it will be seen, for a robot, the regressors are non-linear in the states. Hence, those techniques do not fulfil the requirements of our study. Therefore, it would be worth to look at other fields to find a technique which does not require a priori knowledge of the system and which can handle non-linearities in the states.

The plan of this article is as follows. Firstly, the usual method for robot identification is presented. Secondly, the new solution based on a Kalman filter and a fixed interval smoother is introduced. Afterwards, the techniques are compared with experimental data from a Stäubli TX40 industrial robot arm. Two cases are considered: first, a good a priori knowledge on the system which allows a good bandpass filtering; second, an inadequate bandpass filtering due to a lack of knowledge concerning the robot. Finally, concluding remarks are expressed.

2. LEAST-SQUARES FOR ROBOT IDENTIFICATION

2.1 Inverse Dynamic Identification Model

If a robot with n moving links is considered, the vector $\boldsymbol{\tau}(t)$ contains the inputs of those links, which are the applied forces or torques. The signals $\boldsymbol{q}(t)$, $\dot{\boldsymbol{q}}(t)$ and $\ddot{\boldsymbol{q}}(t)$ are respectively the $(n \times 1)$ vectors of generalized joint positions, velocities and accelerations. With respect to the Newton's second law it comes out:

$$\boldsymbol{M}\left(\boldsymbol{q}(t)\right)\ddot{\boldsymbol{q}}(t) = \boldsymbol{\tau}(t) - \boldsymbol{N}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)\right) \qquad (1)$$

where, $\boldsymbol{M}\left(\boldsymbol{q}(t)\right)$ is the $(n \times n)$ inertia matrix of the robot, and $\boldsymbol{N}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)\right)$ is the $(n \times 1)$ vector modelling the disturbances or perturbations. Those perturbations contain the friction forces, gravity effects and other non-linearities depending on the studied robot. Experience has shown that those disturbances are, in the vast majority of cases, linear in the parameters, but not in the states. Therefore, it appears to be very convenient for the identification to consider the Inverse Dynamic Model (IDM). The IDM is described by 2, where: the input torque is the dependent (or observation) variable; $\boldsymbol{\phi}$ is the $(n \times b)$ matrix of regressors (or independent variables); $\boldsymbol{\theta}$ is the $(b \times 1)$ vector of base parameters to estimate.

$$\boldsymbol{\tau}_{idm}(t) = \boldsymbol{\phi}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), \ddot{\boldsymbol{q}}(t)\right)\boldsymbol{\theta} \tag{2}$$

Because of perturbations coming from measurement noise and modelling errors, the actual torque $\boldsymbol{\tau}$ differs from $\boldsymbol{\tau}_{idm}$ by an error $\boldsymbol{v}$. The Inverse Dynamic Identification Model (IDIM) is given by

$$\boldsymbol{\tau}(t) = \boldsymbol{\tau}_{idm}(t) + \boldsymbol{v}(t) = \boldsymbol{\phi}\left(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t), \ddot{\boldsymbol{q}}(t)\right)\boldsymbol{\theta} + \boldsymbol{v}(t). \tag{3}$$

2.2 Least-Squares Equation

The model described by 3 can straightforwardly be extended to the vector-matrix form:

$$\boldsymbol{u}_m = \begin{bmatrix} \boldsymbol{\tau}(t_1) \\ \vdots \\ \boldsymbol{\tau}(t_{N_s}) \end{bmatrix} = \boldsymbol{X}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})\boldsymbol{\theta} + \boldsymbol{e}_{LS}. \tag{4}$$

where, $\boldsymbol{u}_m$ is a $(N_t \times 1)$ vector constructed with the measured signals, $\boldsymbol{X}$ is a $(N_t \times b)$ matrix composed of the regressors and $\boldsymbol{e}_{LS}$ is a $(N_t \times 1)$ vector of error terms, with $N_t = nN_s$ and N_s the number of sampled points considered. It is assumed that $\boldsymbol{X}$ is full rank, i.e. $rank(\boldsymbol{X}) = b$, and that $N_t \gg b$, to have an over-determined system of equations. From 4, the Least-Squares (LS) estimates and their associated covariance matrix are given by (see e.g. (Gautier et al., 2013)):

$$\widehat{\boldsymbol{\theta}}_{LS} = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\boldsymbol{u}_m \tag{5}$$

$$\boldsymbol{\Sigma}_{LS} = \sigma^2\left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1} \tag{6}$$

$$\widehat{\sigma}^2 = \frac{1}{N_t - b}||\boldsymbol{u}_m - \boldsymbol{X}\widehat{\boldsymbol{\theta}}_{LS}||^2 \tag{7}$$

From a theoretical point of view, the LS estimates 5 are unbiased if the error has a zero mean and if the regressors are uncorrelated with the error, see relations 8.

$$E[\boldsymbol{e}_{LS}] = 0, \quad E[\boldsymbol{X}^T\boldsymbol{e}_{LS}] = 0 \tag{8}$$

The covariance matrix given by Eq. 6 assumes that $\boldsymbol{X}$ is deterministic and that $\boldsymbol{e}_{LS}$ is homoscedastic i.e. $var(\boldsymbol{e}_{LS}(t)) = \sigma^2$, for each t. It is assumed that those two assumptions hold. However, systems considered in this article operate in closed-loop, since they are unstable in open-loop. In that case, the assumption given by 8 does not hold (Van den Hof, 1998). This partly explains why a tailor-made pre-filtering of the data is done in practice

2.3 States Estimation by Tailor-Made Filtering

This part emphasizes the classical technique used in robots identification to construct the regressors matrix $\boldsymbol{X}$. Since the regressors vectors are function of the states, the work mainly consists in estimating the velocity and the acceleration from the measured position. As described in (Gautier, 1997) or more recently in (Gautier et al., 2013), the data pre-processing is divided in four sequential steps. Those steps are influenced by the sampling frequency, noted ω_s. This frequency is usually chosen 100 times larger than the natural frequency of the highest mode, $\omega_{dyn} = \omega_s/100$, which must be modelled, in order to satisfy the Nyquist rule.

Step 1. The first step consists in reconstructing the missing data, or, more practically, to compute the derivatives of the measured position. It is usually done thanks to numerical differentiation (centred scheme). Prior to this, to avoid amplification of the noise at high frequency, a low-pass filtering is undertaken. This filter is applied forward and backward to avoid phase lag introduction. It is a Butterworth filter, whose order is $n_d + 2$. Where n_d is the desired derivative order, which is usually equal to two. The issue is to choose the cutting frequency of the filter, ω_q, to have $\widehat{\dot{\boldsymbol{q}}}(t) = \dot{\boldsymbol{q}}(t)$ and $\widehat{\ddot{\boldsymbol{q}}}(t) = \ddot{\boldsymbol{q}}(t)$ over the frequency range of the system. The rule of thumb is to take it as $2\omega_{dyn} \leq \omega_q \leq 10\omega_{dyn}$. It obviously requires knowledge about the system.

Step 2. The observation matrix is constructed with the estimated signals from the previous step: $\boldsymbol{X}\left(\widehat{\boldsymbol{q}}, \widehat{\dot{\boldsymbol{q}}}, \widehat{\ddot{\boldsymbol{q}}}\right)$.

Step 3. A filter is then applied to all signals. The objective is to remove high frequencies perturbations in the dependent variable measurements (generally, the input torque). To be consistent, this filter is also applied to the independent variables. Its cut-off frequency, ω_f, is chosen at about $\omega_f \geq 2\omega_{dyn}$.

Step 4. After the previous step, the signals do not contain any information above ω_f. Therefore, they are re-sampled at a lower frequency (down-sampling). This frequency is usually taken equal to ω_f.

In practice, three elements are worth noting. First, the filters frequencies may be defined taking into account the excitation signal spectrum instead of ω_{dyn}. The second element is that, with MatLab©, the two last steps are performed simultaneously with the decimate function. The last element is that the described methodology is a rule of thumb. It only provides approximate relations or intervals. The choice relies on the practitioner skills. This is why another way is investigated for users without solid background in robotic identification in order to perform the step 1. The decimate filter is still considered for steps 3 and 4 to have a fair comparison with the classical technique by taking into account the same number of data points.

3. KALMAN FILTER AND INTEGRATED RANDOM WALK

3.1 The State Space Model: IRW

Many methods have been developed to deal with the numerical differentiation issue; see e.g. (Dridi et al., 2010). Nonetheless, the goal is here to suggest a practical and straightforward technique. Therefore, the study will focus on the well-known Kalman filter technique, in a discrete time framework. This technique is summarized in (Norton, 1975) or (Young, 2000). It allows an off-line estimation of the states without using the dynamic model, unlike High Gain observers for instance. Equation 9 defines the state vector of state space model, relation 10 is the state equation and relation 11 is the observation equation. Considering our robot velocity estimation, y would be the measured position of link j, i.e. $\boldsymbol{q}_j$.

$$\boldsymbol{x}(k) = \begin{bmatrix} x(k) \\ \Delta x(k) \end{bmatrix} \tag{9}$$

$$\boldsymbol{x}(k) = \boldsymbol{A}\boldsymbol{x}(k-1) + \boldsymbol{D}\boldsymbol{\eta}(k-1) \tag{10}$$

$$y(k) = \boldsymbol{h}(k)\boldsymbol{x}(k) + e(k) \tag{11}$$

With,

$$\boldsymbol{A} = \begin{bmatrix} \alpha & \beta \\ 0 & \gamma \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} \delta & 0 \\ 0 & \kappa \end{bmatrix} \tag{12}$$

$\boldsymbol{h}$ is the row observation vector. $\boldsymbol{\eta}$ is the state noise, assumed to be white and zero mean, with covariance matrix $\boldsymbol{Q}_\eta$ (diagonal). The measurement noise e is also zero mean and white. Its covariance is written σ_e^2. This model, developed in (Young, 2012), is named Generalized Random Walk (GRW). Many variants exist depending on the choice of the hyper-parameters $[\alpha \quad \beta \quad \gamma \quad \delta \quad \kappa \quad \boldsymbol{Q}_{\eta_{11}} \quad \boldsymbol{Q}_{\eta_{22}}]$. For this study, the *Integrated Random Walk* (IRW: $\alpha = \beta = \gamma = \kappa = 1$, $\delta = 0$ and $\boldsymbol{h} = [1 \quad 0]$) will be considered. In that case, since $\delta = 0$, the term $\boldsymbol{Q}_{\eta_{11}}$ has no influence. Therefore, it will be equal to $\boldsymbol{Q}_{\eta_{22}}$ in order to preserve the definite-positive property of the covariance matrix. Finally, the only remaining hyper-parameter is $\boldsymbol{Q}_{\eta_{22}}$. As it will be seen later, its value may be estimated thanks to a Maximum Likelihood (ML) optimization.

3.2 The Kalman and FIS Equations

From the model previously described, a specific Kalman filter is implemented. First of all, it is associated with a Fixed Interval Smoother (FIS) to take advantage of the off-line process. Secondly, the filter and smoother equations are modified to avoid the knowledge of the observation noise variance, σ_e^2. In a classical Kalman Filter, this information is indeed required, like the covariance of the state noise, $\boldsymbol{Q}_\eta$. Instead, all the equations are written as functions of the Noise Variance Ratio (NVR), which is defined by $\boldsymbol{Q}_{nvr} = \boldsymbol{Q}_\eta / \sigma_e^2$. The algorithm described in (Young, 2012) is summarized below.

Prediction step:

$$\widehat{\boldsymbol{x}}(k|k-1) = \boldsymbol{A}\widehat{\boldsymbol{x}}(k-1) \tag{13}$$

$$\boldsymbol{P}(k|k-1) = \boldsymbol{A}\boldsymbol{P}(k-1)\boldsymbol{A}^T + \boldsymbol{D}\boldsymbol{Q}_{\boldsymbol{nvr}}\boldsymbol{D}^T \tag{14}$$

Correction step:

$$\widehat{\boldsymbol{x}}(k|k) = \widehat{\boldsymbol{x}}(k|k-1) + \boldsymbol{g}(k)[y(k) - \boldsymbol{h}(k)\widehat{\boldsymbol{x}}(k|k-1)] \tag{15}$$

$$\boldsymbol{g}(k) = \boldsymbol{P}(k|k-1)\boldsymbol{h}(k)[1 + \boldsymbol{h}(k)\boldsymbol{P}(k|k-1)\boldsymbol{h}^T(k)]^{-1} \tag{16}$$

$$\boldsymbol{P}(k|k) = \boldsymbol{P}(k|k-1) - \boldsymbol{g}(k)\boldsymbol{h}(k)\boldsymbol{P}(k|k-1) \tag{17}$$

$$\boldsymbol{P}^*(k|k) = \widehat{\sigma}_e^2 \boldsymbol{P}(k|k) \tag{18}$$

Smoothing step:

$$\widehat{\boldsymbol{x}}(k|N_s) = \boldsymbol{A}^{-1}\left[\widehat{\boldsymbol{x}}(k+1|N_s) + \boldsymbol{D}\boldsymbol{Q}_{\boldsymbol{\eta}}\boldsymbol{D}^T\boldsymbol{\lambda}(k)\right] \tag{19}$$

$$\boldsymbol{\lambda}(k-1) = \left[\boldsymbol{I} - \boldsymbol{P}^*(k|k)\frac{\boldsymbol{h}^T(k)\boldsymbol{h}(k)}{\widehat{\sigma}_e^2}\right]^T \left(\boldsymbol{A}^T\boldsymbol{\lambda}(k) - \frac{\boldsymbol{h}^T(k)}{\widehat{\sigma}_e^2}\left[y(k) - \boldsymbol{h}(k)\boldsymbol{A}\widehat{\boldsymbol{x}}(k-1|k-1)\right]\right) \tag{20}$$

with $\boldsymbol{\lambda}(N_s) = 0$

$$\boldsymbol{P}^*(k|N_s) = \boldsymbol{P}^*(k|k) + \boldsymbol{P}^*(k)\boldsymbol{A}^T\boldsymbol{P}^*(k+1|k)^{-1} \left[\boldsymbol{P}^*(k+1|N_s) - \boldsymbol{P}^*(k+1|k)\right]\boldsymbol{P}^*(k+1|k)^{-1}\boldsymbol{A}\boldsymbol{P}^*(k|k) \tag{21}$$

The observation noise covariance, σ_e^2, is estimated at the end of the filtering process in order to obtain the state covariance matrix, $\boldsymbol{P}^*$, for the smoothing process. By defining n_x the size of the state vector ($n_x = 2$ for the IRW), the estimation is given by:

$$\begin{aligned} \widehat{\sigma}_e^2 &= \frac{1}{N_s - n_x}\sum_{k=n_x+1}^{N_s}\frac{(y(k) - \boldsymbol{h}(k)\widehat{\boldsymbol{x}}(k|k-1))^2}{1 + \boldsymbol{h}(k)\boldsymbol{P}(k|k-1)\boldsymbol{h}^T(k)}, \\ &= \frac{1}{N_s - n_x}\sum_{k=n_x+1}^{N_s}\frac{\varepsilon^2(k)}{\nu(k)}. \end{aligned} \tag{22}$$

In the time domain, the first order derivative of the signal is then approximated as follows $\frac{dx}{dt}(t_k) \approx \frac{\widehat{\Delta x}(k)}{t_{k+1} - t_k}$, with $\widehat{\Delta x}(k)$ the second term of the estimated state vector $\widehat{\boldsymbol{x}}(k|N_s)$. Similarly, $\boldsymbol{x}$ could be augmented with $\Delta^2 x$ in order to estimate the second order derivative. From a practical point of view, this algorithm is implemented in the function *irwsm* of the CAPTAIN Toolbox, developed by a team of Lancaster University; see (Taylor et al., 2007) and http://captaintoolbox.co.uk.

3.3 Hyper-Parameters Optimization

As it has been said, the user does not have to provide the observation noise covariance to *irwsm* contrary to a classical Kalman filter. It remains the issue of the hyper-parameters and more specifically of the NVR. Fortunately, the toolbox provides also a function called *irwsmopt* which estimates the hyper-parameters maximizing the likelihood of the prediction error, $\varepsilon(k)$, defined in 22. For further information, see e.g. (Durbin and Koopman, 2012). Finally, this toolbox allows the user to process the data from a system without a priori knowledge about it. Obviously, it does not prevent him to be vigilant on the results.

4. EXPERIMENTAL RESULTS

4.1 Robot Description

To illustrate the methods previously described, we consider the Stäubli TX40 robot, see Figure 1. This is a serial manipulator composed of six rotational joints. There is a coupling between the joints 5 and 6. Thus, two additional parameters are added: fv_{m6} and fc_{m6}, which are respectively the viscous and dry friction coefficient of the motor 6. The robot has 60 base dynamic parameters. The SYMORO+ software is used to automatically calculate the customized symbolic expressions of models (Khalil and Dombre, 2004), which allows to construct the columns of $\boldsymbol{X}$ in Eq. 4. The reference trajectories are trapezoidal (also called smoothed bang-bang accelerations). Since $cond(\boldsymbol{X}) = 200$, the reference trajectories excite well the base parameters. The joint positions and control signals are stored with a sampling frequency $f_s = 5kHz$.

The identification is performed with experimental data. For the IDIM-LS method, the filters cut-off frequencies are

Fig. 1. Stäubli TX40

tuned according to (Gautier et al., 2013): 50Hz and 10Hz respectively for the Butterworth and the decimate filters. With respect to the rules given in section 2, it implies that $\omega_{dyn} = 5Hz$. From an identification point of view, three methods are compared. The first one is the classical approach, with Butterworth filters, described in Section 2 and will be named by "Classical". The second method is the irwsm implemented in the CAPTAIN Toolbox, described in Section 3, and will be referred as "IRWSM 1". The last one is a variant of the irwsm where the GRW model contains three states, which allows estimating directly the second derivative without calling the algorithm twice. This approach will be named "IRWSM 2". The state matrices are then defined by relations 23.

$$\boldsymbol{A}_{irwsm1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \boldsymbol{A}_{irwsm2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \tag{23}$$

4.2 Robot Identification with Good a priori Knowledge

Table 1 summarizes the results of the identification from the experimental data. From 60 base parameters, only 28 are well identified with good relative standard deviations, as explained in (Janot et al., 2014). Those parameters define a set of essential dynamic parameters. The three methods almost estimate the same parameters. Those estimated values are satisfactory since they are similar to those found in previous studies on this robot; see e.g. (Janot et al., 2014). Their relative errors are equivalent and can be considered as satisfactory, see Table 2.

This first identification proves that methods based on the *irwsm* are able to provide as good estimates as the classical one. However, in this case, we assumed good a priori knowledge. In other words, the system bandwidth was well-known and the filters were adequately tuned. That can be an issue especially with the first identification

Table 1. Estimated parameters and relative standard deviations for the good a priori knowledge case

Param.	Classical	IRWSM 1	IRWSM 2
$zz1_r$	1.24 (1.31 %)	1.24 (1.32 %)	1.24 (1.33 %)
fc_1	6.90 (2.19 %)	6.85 (2.23 %)	6.85 (2.26 %)
fv_1	8.07 (0.69 %)	8.09 (0.69 %)	8.09 (0.69 %)
$xx2_r$	-0.48 (2.89 %)	-0.47 (2.94 %)	-0.48 (2.91 %)
$xz2_r$	-0.16 (4.45 %)	-0.16 (4.47 %)	-0.16 (4.44 %)
$zz2_r$	1.09 (1.08 %)	1.09 (1.08 %)	1.09 (1.08 %)
$mx2_r$	2.22 (2.35 %)	2.22 (2.37 %)	2.23 (2.36 %)
fc_2	7.90 (1.75 %)	7.85 (1.78 %)	7.86 (1.77 %)
fv_2	5.62 (1.10 %)	5.65 (1.10 %)	5.64 (1.10 %)
$xx3_r$	0.14 (9.15 %)	0.14 (9.18 %)	0.14 (9.16 %)
$zz3_r$	0.11 (9.01 %)	0.11 (9.16 %)	0.11 (9.08 %)
$my3_r$	-0.59 (2.56 %)	-0.59 (2.56 %)	-0.59 (2.54 %)
ia_3	0.09 (9.26 %)	0.09 (9.30 %)	0.09 (9.38 %)
fc_3	6.26 (2.22 %)	6.25 (2.24 %)	6.26 (2.23 %)
fv_3	1.98 (2.11 %)	1.98 (2.12 %)	1.98 (2.12 %)
mx_4	-0.03 (28.2 %)	-0.03 (30.0 %)	-0.03 (29.9 %)
ia_4	0.03 (16.4 %)	0.03 (16.6 %)	0.03 (16.7 %)
fc_4	2.38 (6.18 %)	2.35 (6.27 %)	2.37 (6.22 %)
fv_4	1.12 (3.66 %)	1.13 (3.64 %)	1.13 (3.65 %)
$my5_r$	-0.03 (17.8 %)	-0.03 (18.1 %)	-0.03 (18.2 %)
ia_5	0.04 (14.3 %)	0.04 (14.7 %)	0.04 (14.8 %)
fc_5	2.94 (4.67 %)	2.93 (4.72 %)	2.93 (4.70 %)
fv_5	1.84 (2.89 %)	1.84 (2.91 %)	1.84 (2.90 %)
ia_6	0.01 (30.9 %)	0.01 (31.5 %)	0.01 (31.5 %)
fc_6	0.29 (59.9 %)	0.25 (70.9 %)	0.25 (69.0 %)
fv_6	0.66 (3.94 %)	0.66 (3.95 %)	0.67 (3.96 %)
fc_{m6}	1.83 (6.57 %)	1.87 (6.48 %)	1.87 (6.46 %)
fv_{m6}	0.61 (3.35 %)	0.61 (3.40 %)	0.61 (3.40 %)

Table 2. Direct comparison relative errors

	Classical	IRWSM 1	IRWSM 2
Good knowledge	5.1 %	5.1 %	5.1 %
Poor knowledge	9.2 %	7.5 %	8.6 %

process for a system totally unknown, like a robot bought off-the-self.

To test the robustness of our proposed methodology, we consider the same data set but assuming $\omega_{dyn} = 20Hz$ due to a lack of knowledge on the system. The relation $\omega_q = 10\omega_{dyn}$ is kept identical, which leads to a 200Hz cut-off frequency for the Butterworth filter. Concerning the decimate filter, we are slightly less restrictive with $\omega_f = 5\omega_{dyn} = 100Hz$.

4.3 Robot Identification with Poor a priori Knowledge

The results of the identification with poor a priori knowledge are summarized in Table 3. IRWSM 1 provides better results than the two others since its has a lower relative error, see Table 2. That is also visible on some estimated parameters like $zz1_r$ or $zz3_r$. This example illustrates the difficulty of the classical IDIM-LS method when the system bandwidth is not well-known and when non-optimal relations are used to tune the filters.

The relative standard deviations provided in Table 3 may seem promising. However, they are not valid since the residuals are not white as it should be the case in theory. Figures 2 and 3 provide the residuals autocorrelation estimations respectively for the good and the poor a priori knowledge cases. The residuals clearly appear to be serially correlated in the poor knowledge case. In practice, this

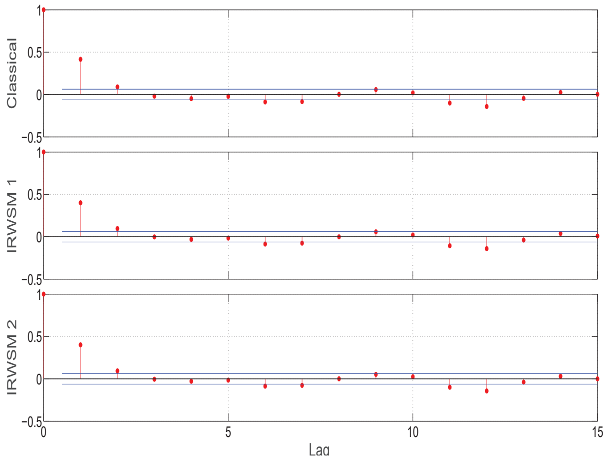

Fig. 2. Residuals autocorrelation for the good a priori knowledge case

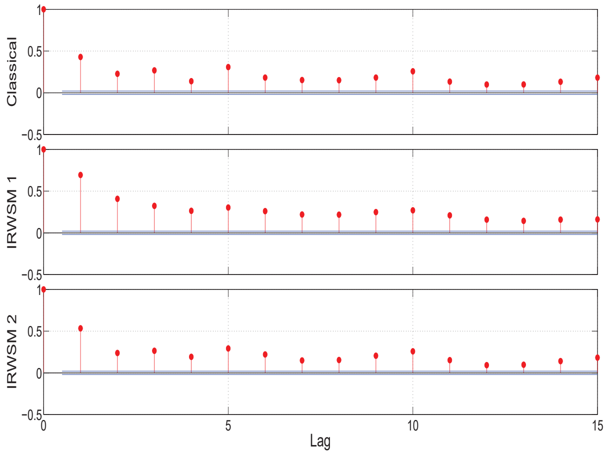

Fig. 3. Residuals autocorrelation for the poor a priori knowledge case

identification process can be viewed as a first step to retrieve the closed-loop dynamics. Subsequently, a second step can be performed to identify the system with filters appropriately tuned in order to obtain valid estimates and standard deviations.

Figure 4 illustrate the torques estimated with the three methods for each link. As expected with the parameters values, the IRWSM 1 (middle column) estimated torques are better. For instance, it is noticeable that this method is slightly less sensitive to the high frequencies ripples about 1s for the links 3, 4 and 6.

One fact is worth noting about this poor knowledge case. The *irwsmopt* algorithm tends to catch all the dynamic of the noisy signal. In other words, it gives too much importance to the covariance of the state noise compared to the one of the measurement noise. A careful visual inspection of the signals, prior to the identification, by the user is therefore required. A NVR equal to 10^{-5} has already proved to be an appropriate choice, see (Brunot et al., 2016). This value of NVR was fixed for both IRWSM methods presented in this section.

This more practical case shows that the IRWSM approach is robust against poor a priori knowledge. However, it

Table 3. Estimated parameters and relative standard deviations for the poor a priori knowledge case

Param.	Classical	IRWSM 1	IRWSM 2
$zz1_r$	1.21 (0.75 %)	1.25 (0.61 %)	1.22 (0.70 %)
fc_1	6.55 (1.30 %)	6.52 (1.06 %)	6.50 (1.22 %)
fv_1	8.18 (0.38 %)	8.19 (0.31 %)	8.20 (0.36 %)
$xx2_r$	-0.48 (1.64 %)	-0.48 (1.34 %)	-0.48 (1.54 %)
$xz2_r$	-0.16 (2.50 %)	-0.16 (2.09 %)	-0.16 (2.37 %)
$zz2_r$	1.03 (0.63 %)	1.09 (0.50 %)	1.05 (0.58 %)
$mx2_r$	2.32 (1.29 %)	2.20 (1.11 %)	2.29 (1.23 %)
fc_2	7.65 (1.01 %)	7.52 (0.84 %)	7.59 (0.95 %)
fv_2	5.71 (0.61 %)	5.77 (0.49 %)	5.74 (0.57 %)
$xx3_r$	0.14 (4.98 %)	0.14 (4.28 %)	0.14 (4.77%)
$zz3_r$	0.05 (11.3 %)	0.12 (4.01 %)	0.07 (7.38 %)
$my3_r$	-0.61 (1.39 %)	-0.58 (1.21 %)	-0.60 (1.32 %)
ia_3	0.13 (3.72 %)	0.09 (4.58 %)	0.11 (3.88 %)
fc_3	5.80 (1.33 %)	5.90 (1.06 %)	5.80 (1.24 %)
fv_3	2.08 (1.13 %)	2.07 (0.92 %)	2.08 (1.05 %)
mx_4	-0.01 (47.6 %)	-0.03 (12.3 %)	-0.02 (27.2 %)
ia_4	0.03 (8.43 %)	0.03 (7.43 %)	0.03 (7.99 %)
fc_4	2.24 (3.68 %)	2.25 (2.98 %)	2.24 (3.44 %)
fv_4	1.17 (1.99 %)	1.15 (1.65 %)	1.16 (1.87 %)
$my5_r$	-0.04 (7.49 %)	-0.03 (8.02 %)	-0.04 (7.46 %)
ia_5	0.04 (6.69 %)	0.04 (6.42 %)	0.04 (6.38 %)
fc_5	2.80 (2.71 %)	2.75 (2.25 %)	2.80 (2.53 %)
fv_5	1.87 (1.59 %)	1.90 (1.28 %)	1.88 (1.49 %)
ia_6	0.01 (14.4 %)	0.01 (14.3 %)	0.01 (13.7 %)
fc_6	0.28 (33.3 %)	0.30 (25.4 %)	0.30 (29.8 %)
fv_6	0.68 (2.18 %)	0.67 (1.77 %)	0.68 (2.03 %)
fc_{m6}	1.75 (3.77 %)	1.73 (3.07 %)	1.73 (3.58 %)
fv_{m6}	0.62 (1.85 %)	0.63 (1.49 %)	0.63 (1.72 %)

requires a careful use with a potential manual selection of the NVR. This estimation can be used as a first step for the design of pre-filters for the Classical method. In practice, the IRWSM 1 solution should be preferred.

5. CONCLUSION

In this paper the usual robot identification methodology is presented. It is based on the well-known Least-Squares method but it requires a careful tailor-made pre-filtering to deal with closed-loop issues and to estimate the non-measured signals. This tailor-made pre-filtering process is summarized. Furthermore, a new pre-filtering methodology is developed. That one is based on a combination of a Kalman filter and a fixed interval smoother. The obtained results suggest that the new method is a suitable alternative when the system bandwidth is not known prior to the identification.

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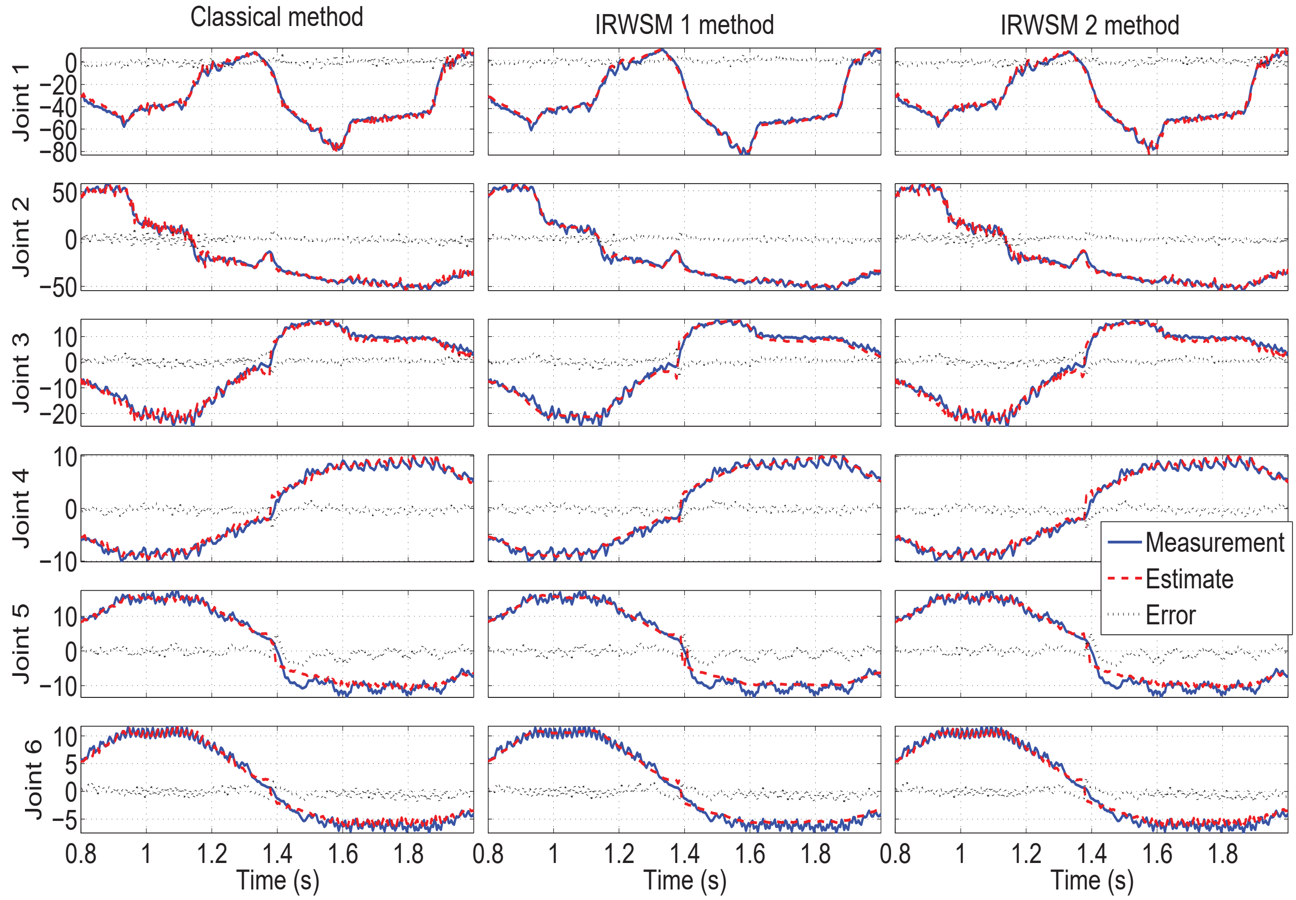

Fig. 4. Comparison of the actual (blue) and estimated (red dashed) torques (Nm), errors (black dotted) - Poor a priori knowledge

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