Collective reputation with stochastic production and unknown willingness to pay for quality

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Collective Reputation with Stochastic Production and Unknown Willingness to Pay for Quality

Abstract

In many cases consumers cannot observe a single firm's investment in environmental quality or safety, but only the average quality of the industry. The outcome of the investment is stochastic since firms cannot control perfectly the technology or external factors that may affect production. In addition, firms do not know consumers' valuation of quality. We characterize the solution of the firms' investment game and show that the value of stopping investments when firms are already investing in quality can be negative when the free-riding incentives dominate. The existence of systematic uncertainty on the outcome of investment slows down investment in quality, compared to a situation without uncertainty. The uncertainty on consumers' willingness to pay for quality can speed up or slow down investment. We also obtain the counterintuitive result that information acquisition may decrease the overall level of quality.

J.E.L. Classification: C73, D92, Q52.

Keywords: collective reputation, dynamic game, real options, stochastic quality.

1 Introduction

In markets where the consumer cannot trace the product to a specific producer, consumer demand is based on average quality of the industry. All producers will share a collective reputation of the good regardless of individual production quality.[1] Sometimes, the consumer assesses products based on a geographic "savoir-faire", and demand is driven by such notions as Italian shoes or Scottish whiskey. Agricultural markets, in particular, have many examples of products that share a given location of production but where the consumer cannot trace the specific product that is bought to a particular producer in the area, such as Florida grapefruits, Parmesan cheese or Idaho potatoes. The European Protected Geographical Indication and Designation of Origin labels, as well as the state product labels in the U.S. are all based on a common reputation for the product reflecting its origin and use of a common input, such as "terroir" for French wines. But even beyond agricultural markets, examples can be found in the case of industries where reputation depends more on the intrinsic characteristics of the production chain rather than on the individual behaviour, as is the case of the safety reputation for the oil and gas industry. Consumer concern about oil extraction, for instance, seems to be more related to the technology itself rather than to the reputation of a given supplier. Following industrial pollution in-

[1]This can be the case of experience goods, for which the consumer cannot assess quality ex ante, but it can also be the case for credence goods (Darby and Karni, 1973), i.e., goods for which quality characteristics depend on production processes that are difficult to verify.

cidents, the entire industry gets tainted by a bad reputation and examples of this are also common in the chemical industry (see Barnett and King, 2012).

In all these cases, there is a collective supply of some input (effort, feedstock, level of care, etc.) by all participants in the industry that generates positive externalities for each firm in the industry. These externalities affect firms' returns, depending on consumers' ability to effectively evaluate and integrate into their own demand the consequences of firms' investment. In response to this problem, minimum quality standards can be implemented. Such policy is common for products of the same geographic origin, such as the designated origin labels. Inspection and monitoring costs as well as moral hazard still impose limits on these solutions. Even if we assume perfect compliance with a minimum quality standard, the question at hand is what incentives do firms have to increase investment to go beyond the standard?

In the examples above, investments in quality accumulate over time. The impact of investment on firms' returns, however, can be assumed to be stochastic for two main reasons. First, the outcome of the investment can depend on the realization of some external factor that is not under control of the single firm. This is the case of the protected designation of origin labels, for which a disease or weather conditions might impact the quality of the feedstocks. It might also be the case of a systemic event that affects the industry's reputation as regards safety or reliability. The second source of uncertainty stems from the fact that investment is beneficial for the firms only to the extent that consumers realize the positive improvement in quality or safety. Since consumers cannot monitor actual quality investments, they can only rely on their own valuation of quality, which may differ from the actual level, and is unknown to the firms. As a result, we model firms' investments as subject to uncertainty on consumers' valuation of quality, in addition to the uncertainty on the outcome of the investment.

The collective nature of production implies a classic free-riding problem that may undermine investment in quality. We investigate, in a dynamic game framework, whether firms would invest in quality in such a setting and the factors that affect the investment decision. Moreover, assuming that quality accumulation is good per se, we investigate under which conditions releasing information on consumers' willingness to pay (WTP) for quality, i.e., reducing uncertainty on the demand side, allows firms to start a virtuous circle of quality accumulation. Given the strategic interaction setting, when the industry starts from a low level of quality accumulation (which might be conceived as a baseline quality), investing in quality would raise the return on investment but also increase the free-riding incentive. We characterize under which conditions reducing uncertainty induces firms to invest in quality, considering both the case in which firms may acquire information themselves and when there is a third party that discloses the information to the firms.

Our main findings are the following: a) Since quality is a public good for firms, free riding can delay quality accumulation. The value of stopping the investments when firms are already investing in quality can be negative. b) Without systemic uncertainty on quality, the game reduces to a standard investment decision and firms behave as if they were a single monopolist. c) Consumers' WTP for quality speeds up (delays) investments whenever consumers' preferences are positively (negatively) biased towards the industry. The impact of consumers' WTP is non-linear: there are positive returns of consumers' WTP on quality accumulation; there exists a threshold of quality, however, such that whenever quality is below it firms never find it optimal to invest in quality regardless of the WTP for quality. d) When quality is at a low level it is never optimal for firms to acquire information about consumers' WTP, regardless of the cost of doing so. e) An external body, or consortium, would reduce the probability of firms investing in quality by disclosing information on consumers' WTP. Interestingly enough, in an empirical study of the wine industry, Castriota and Delmastro (2015) find that firms can be stuck in a bad reputation - low quality trap. In a different theoretical setting, our model supports this finding as a result of too low a quality standard set by the external body or consortium.

The novelty of our paper is to model the interaction of two sources of stochasticity in a dynamic game of collective good production: stochastic quality accumulation and stochastic consumer valuation. The paper thus extends the existing literature on collective reputation, that has studied agricultural commodities in particular, for which it is a frequent problem, and other products that are not easily distinguishable ex ante (Winfree and McCluskey, 2005; McQuade, Salant and Winfree, 2016), but always in a deterministic framework.[2] The model we develop extends the existing analysis by allowing the quality level to be stochastic both in the level of the investment and in the impact that quality has on firms' demand.[3] Some recent work studies the interaction between individual and collective reputation (Costanigro, Bond and McCluskey, 2012), which has been shown to be relevant empirically in the sense that a stronger individual reputation makes the payoff to the collective reputation higher (Gergaud, Livat and Warzynski, 2012). Here we focus on the dynamic game between firms and their collective production, in order to have clear-cut results on the impact of the two sources of uncertainty on the free-riding problem. With this limitation we obtain analytical results, whereas Costanigro, Bond and McCluskey (2012) have to rely on numerical simulations. We share with Claude and Zaccour (2009) the specification of how reputation shifts aggregate demand and the collective build-up of a stock of quality. The novelty here is to introduce the stochastic nature of quality. Recent work on stochastic quality, on the other hand, normally assumes that a firm can signal the quality of its product (high or low) such that its own reputation matters, not the collective reputation. In Board and Meyer-ter-Vehn (2013) reputation is defined as consumers' belief that the

[2]Levin (2009) introduced a stochastic version of the seminal Tirole (1996) model of collective reputation where the workers' cost of effort evolves randomly and there are noisy signals of a worker's type. In these models the workers' types are imperfectly observed and collective reputation arises through beliefs, and can be expressed as the fraction of workers with good reputation. By comparison, there are no individual reputations in our model and collective reputation is a pure public good.

[3]Static analyses by Fleckinger (2007), McQuade, Salant and Winfree (2016) and Rouvière and Soubeyran (2011) focus on the impact of market structure and of regulation, such as minimum quality standards.

product quality is high. Dilme (2014) and Bohren (2014) study the cumulative effect of past investments on current quality in a stochastic game, but also here, in a game between one firm and multiple consumers. Fishman et al. (2014) provides a recent extension of a model with collective reputation and updating of consumer beliefs. They introduce endogenous membership in a collective brand, and show how branding can provide consumers with better information and hence increase the incentives for the high quality firms to invest. Our contribution is to explicitly consider the dynamic game between firms. The novelty is to focus on the firms' collective game and their incentives to invest under two sources of uncertainty: stochastic production, and uncertainty on consumers' valuation of quality. On the other hand, we do not allow firms to send signals to consumers through advertising and labelling, since these approaches have been thoroughly explored in the literature (see the survey by Dranove and Jin, 2010).

Methodologically, the paper that is the most closely related to ours is Wirl (2008) who studies a continuous time stochastic dynamic game among firms that create a stock externality that is subject to uncertainty over time. By comparison to that model we share the context of a dynamic game among firms but we develop the model to include aspects of information availability that are absent in such dynamic games. In particular, we introduce uncertainty also on consumers' valuation of the quality of the good and analyse the value to firms of acquiring that information as well as the impact of information disclosure on the probability to invest in quality.

The paper is structured as follows. The model is introduced in Section 2. Section 3 characterizes the solution to the firms' quality investment game in the benchmark case when there is no information on consumers' valuation of quality and compares it to the case of full information. In Section 4 we investigate whether it is ex ante optimal to obtain information on consumers' valuation of quality. We analyze two cases: the firm's decision problem on information acquisition and the case of an external body that aims at increasing aggregate quality by disclosing information. Conclusions and indications for future research follow in Section 5. All proofs are presented in the Technical Appendices.

2 The model

We consider a continuous time version of an industry composed of $n > 1$ risk-neutral identical firms that produce a good whose market price at time $t \geq 0$ is affected by the current level of reputation R_t of the industry as a whole. That is, as in Winfree and McCluskey (2005) and Claude and Zaccour (2009), the level of reputation determines the location of the demand curve. We assume that, although the firms may invest in product quality to increase their collective level of reputation, they are unable to assess exactly how consumers value such an investment and convert product quality into reputation.

We also make the following assumptions about the industry. Each firm produces one unit of output per unit of time to be sold in a market where the price depends positively on the willingness to pay for aggregate quality of the entire industry.[4] Assuming consumers' linear preferences, we use the following inverse demand function:[5]

$$p(R_t) = a_t + R_t \tag{1}$$

where $a_t > 0$ is an additive shock that accounts for the relative strength of the market demand. This additive shock is assumed to be identically and independently distributed (*i.i.d.*) over time with expected value $E(a_t) = a$.

Even if consumers are willing to pay a higher price for higher quality, firms do not know their valuation ex ante. This relation is linear for simplicity:

$$R_t = (1 + \theta)Q_t \tag{2}$$

where Q_t is the aggregate stock of quality at time $t \geq 0$ and θ is the unknown time-invariant parameter that represents consumers' specific WTP for product quality. Since there should be no reason to suppose ex ante that consumers have either positive or negative bias towards the quality of the en-

[4] We thus deliberatively abstract from firms' other strategic interactions, such as the quantity choice, in order to focus on the provision of quality as a collective good problem.

[5] For instance, we could assume, as in Claude and Zaccour (2009), a simple linear inverse demand curve:

$$p = A + R - C(nq)$$

where p is the output price per unit, nq is the total output, A is an exogenous shock to demand and C is a non-negative constant representing the slope of the linear demand function. Setting $a = A - C(nq)$ we would obtain the expression (1) in the text.

tire industry, we assume that θ is distributed over the interval $\theta \in [-1, +1]$, with density $f(\theta)$ and $\int_{-1}^{+1} \theta f(\theta) d\theta = 0$.[6]

Each firm may improve the level of accumulated quality of the industry by investing each time an amount k_{it}, $i = 1, 2...n$, in quality. We assume that the stock of quality evolves over time according to a geometric Brownian motion which is common knowledge:

$$dQ_t = \Big(\frac{\alpha}{n} \sum_{i=1}^{n} k_{it} \Big) Q_t dt + \sigma Q_t dz_t, \qquad \text{with } Q_{t=0} = Q \tag{3}$$

where σ is the constant instantaneous volatility and dz_t is the increment of the standard Wiener process satisfying $E_0 [dz_t] = 0$, $E_0 [dz_t^2] = dt$.[7] The assumption of a Brownian motion is shared by several analyses of a single firm's investment problem (e.g., Faingold and Sannikov, 2011, Bohren, 2014). In the context of agricultural goods, the quality process used here could be interpreted as normalizing a minimum quality standard to zero, and analyze whether firms have incentives to go beyond the minimum standard of quality.

We further use two simplifying assumptions: First, only a fraction $\alpha \in [0, 1]$ of new investment in quality accumulates over time. Second, the drift term $\big(\frac{\alpha}{n} \sum_{i=1}^{n} k_{it} \big)$ is a linear function of the average investment in quality

[6]The support of θ can be $[\underline{\theta}, \bar{\theta}]$ without affecting the results. But as seen from Equation (2), as long as $Q_t > 0$, $R_t \geq 0$ if and only if $\underline{\theta} \geq -1$.

[7]Equation (3) implies that, starting from Q, the random position of the actual stock Q_t has a lognormal distribution, with mean $\ln Q + (\frac{\alpha}{n} \sum_{i=1}^{n} k_{it} - \frac{1}{2}\sigma^2)t$, and variance $\sigma^2 t$. Since the process is Markovian, at any point in time the value Q_t observed by the firms is the best predictor of the future stock of quality.

at time t. The assumption on average industry quality is used also in a deterministic framework in Winfree and McCluskey (2005) and in McQuade, Salant and Winfree (2016) who use a quantity-weighted average of the qualities sold by the firms. This assumption is supported by the empirical work by Landon and Smith (1997, 1998).

Substituting (2) in (1), for any given value of θ, the current price depends on the stock of quality accumulated up to t, Q_t, and the current shock a_t:

$$p(Q_t, a_t; \theta) = a_t + (1 + \theta) Q_t. \tag{4}$$

From (4) it is evident that, as both a_t and Q_t are stochastic the firms cannot infer θ from observing the past relations of p_t and Q_t. Even though, given the structure of the price, low realizations of p_t for some high realizations of Q_t could make firms more pessimistic about θ and vice versa, this could be a temporary situation and therefore unable to provide information on the true "long-term" WTP of consumers.[8]

In order to apply the real option approach to the investment decision, we assume that each firm invests at a fixed level or does not invest at all, i.e.,

[8] Since p_t, a_t and Q_t are cointegrated, a sequential learning process could be set in the following way: defining $X_t \equiv \frac{p_t - a_t}{Q_t}$, each firm observes over time a sequence of realizations of a random variable $X_t = (1 + \theta) + \varepsilon_t$, where ε_t is a sequence of "noise" terms. If ε_t are independent and $N(0, 1)$, by the strong law of large numbers, the value of θ would eventually be revealed, i.e., almost surely $\lim_{t \to \infty} \sum X_t / t \to 1 + \theta$. This implies that asymptotically a learning process would reveal the true value of θ. However, we are not interested in this paper to evaluate the time asymptotically or studying how firms could estimate θ in finite time. We simply assume that by paying ϕ firms can acquire in finite time a knowledge of θ that all of them can fully rely on.

$k_{it} \in \{0, 1\}$, for all i and $t \geq 0$. Therefore, at each point in time the firm has the choice (strategy) to make a sunk investment in quality or not at cost $c(1) = c > 0$ and $c(0) = 0$ respectively. This assumption is often used in non-cooperative continuous time versions of the Rubinstein game and allows to obtain a closed form solution of the investment process (Huisman, 2001; Wirl, 2008, 2009).[9]

Finally, the analysis is conducted using feedback stationary Markov strategies: $k_{it} = \Psi_i(Q_t)$, for $i = 1, 2...n$, and $t \geq 0$. This implies that the players' actions at each period t depend only on the accumulated stock of quality up to t, i.e., Q_t is a sufficient statistic on the basis of which firms can condition their investment decisions at time t. In addition the symmetry implies that $k_{it} = \Psi(Q_t)$ for all i and the best reply to Ψ^{-i} is a Nash equilibrium. Note that since the strategy equation is derived by the firm's non-cooperative intertemporal optimization (see Section 3 below), each firm behaves optimally for all values of Q_t regardless of whether this stock was on or off the equilibrium path (Basar and Olsder, 1995).[10]

[9] In order to make the model more realistic, we could also introduce quality depreciation, for instance, by assuming that quality lifetime follows a Poisson process. This would mean that, over any short period dt, there would be a given probability that Q_t got reduced or even completely cancelled. Since the effect of depreciation would make irreversibility weaker, none of the results of the paper would be affected by such an assumption.

[10] It is worth to note that, even if it is not possible to exclude a priori the existence of time-dependent Nash equilibria, stationary strategies are the 'natural' choice in this context where firms are symmetric and invest a constant amount over time (Dangl and Wirl, 2004).

3 Incomplete information vs full information

Assuming profit maximizing behaviour, each firm chooses the level of investment in quality that maximizes the expected present value of its stream of profits over time, subject to the state equation (3). In Section 3.1, we first solve the firms' problem with incomplete information. In Section 3.2, we solve the problem under full information and compare the equilibrium with information on consumers' willingness to pay for quality and without.

3.1 Incomplete information

The incomplete information model is our benchmark model, in which we assume that information on θ cannot be acquired neither ex ante nor ex post. For negligible production costs, if we denote the discount rate by $r > \alpha$, firm i's objective is:

$$V_i(Q) = \max_{k_{it}\in\{0,1\}} \boldsymbol{E}\left[\int_0^\infty e^{-rt}[p(Q_t, a_t;\theta) - ck_{it}]dt\right] \text{ for } i = 1, 2...n \qquad (5)$$

$$\text{such that (3) and (4) hold,}$$

where $\boldsymbol{E}(.)$ is the expectation with respect to θ, $\{a_t, t \geq 0\}$ and $\{Q_t, t \geq 0\}$ Since $E(\theta) = 0$ and Equation (4) is linear, the problem (5) reduces to:

$$V_i(Q;0) = \max_{k_{it}\in\{0,1\}} E_0\left[\int_0^\infty e^{-rt}[p(Q_t, a;0) - ck_{it}]dt\right] \text{ for } i = 1, 2...n \qquad (6)$$

where $V_i(Q;0)$ indicates the firm's value without information on θ, $p(Q_t, a; 0) = E(a_t) + (1 + E(\theta))Q_t$, and $E_0(.)$ is the expectation taken at time $t = 0$ with respect to $\{Q_t, t \geq 0\}$.[11]

By the linearity of (4), the sufficient condition for a stationary Markov perfect equilibrium is given by the Bellman equation for the firm's non-cooperative intertemporal optimization:[12]

$$\begin{aligned} rV_i &= \max_{k_i \in \{0,1\}} \left[\frac{1}{2}\sigma^2 Q^2 V_i'' + \frac{\alpha}{n}\Big(k_i + \sum_{j \neq i}^{n} k_j\Big) Q V_i' + p(Q, a; 0) - ck_i \right] \\ \text{for } i &= 1, 2...n \end{aligned} \tag{7}$$

where V_i' and V_i'' stand for the first and second derivatives with respect to Q respectively. Recall that because of the binary action space and firms' homogeneity assumption, $V_i = V$ for all i. Standard arguments lead to a solution for (7) taking the following form (see Dixit and Pindyck, 1994, and Technical Appendix A):

$$V(Q;0) = \begin{cases} V_0(Q;0) & \text{for } Q < \hat{Q} \\ V_1(Q;0) & \text{for } Q > \hat{Q} \end{cases}$$

where $V_0(Q;0)$ and $V_1(Q;0)$ are the firm's value when it is not investing and when it is investing in quality respectively, and $\hat{Q}$ is the stock of quality

[11] In our framework the absolute integrability condition $E\left[\int_0^\infty e^{-rt} \mid p(Q_t, a_t; \theta) \mid dt\right] < \infty$ is satisfied by the assumption that $r > \alpha$. (The proof is available from the authors upon request.)

[12] From now onward we drop the time index for notational convenience, whenever it is not explicitly needed.

achieved by the industry that triggers the investment. If firms find themselves in a range of quality such that $Q < \hat{Q}$, they would not invest in quality, i.e., would choose $k_i = 0$. This simply means that the overall stock of quality would evolve over time following a purely stochastic path with no drift term (see Equation (3)). If, on the contrary, the overall level of quality was such that $Q > \hat{Q}$, firms would find it optimal to invest, i.e., $k_i = 1$. Recall that the stochastic nature of Q is such that firms change their strategy whenever Q crosses $\hat{Q}$. This might happen several times. In other words, even if firms were in a region where they would find it optimal to invest in quality, it might happen that at a certain point in time Q falls below $\hat{Q}$, which implies that firms would stop investing (and vice versa).

If $\hat{Q}$ exists, then by substituting the two strategies into the Bellman equation (7) the latter reduces to:

$$rV_0 = \frac{1}{2}\sigma^2 Q^2 V_0'' + p(Q, a; 0) \qquad \text{for } Q < \hat{Q} \tag{8}$$

and

$$rV_1 = \frac{1}{2}\sigma^2 Q^2 V_1'' + \alpha Q V_1' + p(Q, a; 0) - c \;\; \text{for } Q > \hat{Q} \tag{9}$$

Provided that $r > \alpha$, solving the problem [8-9] yields the following:

Proposition 1 *In the quality investment game, the following holds true:*

1) The investment rule is:

$$\hat{Q} = \frac{(\beta_1 - \gamma_2)\frac{rn}{\alpha} - \gamma_2\beta_1}{(\beta_1 - \gamma_2) - \beta_1(\gamma_2 - 1)\frac{\alpha}{r-\alpha}} c > 0 \tag{10}$$

2) The value of the firm is:

$$V(Q;0) = \begin{cases} A_0 Q^{\beta_1} + \frac{a+Q}{r} & \textit{for } Q < \hat{Q} \\ B_1 Q^{\gamma_2} + \frac{a}{r} + \frac{Q}{r-\alpha} - \frac{c}{r} & \textit{for } Q > \hat{Q} \end{cases} \tag{11}$$

where:

$$A_0 = \left[\frac{nc}{\alpha} - \frac{\hat{Q}}{r}\right] \frac{\hat{Q}^{-\beta_1}}{\beta_1} > 0 \tag{12}$$

and

$$B_1 = \left[\frac{nc}{\alpha} - \frac{\hat{Q}}{r-\alpha}\right] \frac{\hat{Q}^{-\gamma_2}}{\gamma_2} = \begin{cases} > 0 & if \;\; \beta_1 < n/(n-1) \\ \leq 0 & if \;\; \beta_1 \geq n/(n-1) \end{cases} \tag{13}$$

and: $\beta_1 = \frac{1}{2} + \sqrt{(\frac{1}{2})^2 + \frac{2r}{\sigma^2}} > 1$, $\gamma_2 = (\frac{1}{2} - \frac{\alpha}{\sigma^2}) - \sqrt{(\frac{1}{2} - \frac{\alpha}{\sigma^2})^2 + \frac{2r}{\sigma^2}} < 0.$

Proof. See Technical Appendix A ■

The interpretation of the value of the firm in Equation (11) is straightforward. In the range where $Q < \hat{Q}$, the term $\frac{a+Q}{r}$ indicates the present value of selling the output when each firm is not investing in quality, while $A_0 Q^{\beta_1}$ is the increment in a firm's value due to the opportunity to invest in quality in the future. This latter term is always positive as it accounts for two positive effects: 1) the benefits that each firm receives by its own investment in quality and 2) the benefits each firm expects to obtain by the rivals' future investment in quality. In the quality-investment game that firms are playing, the usual prisoners' dilemma trade-off applies. On the one hand, investing in quality is positive since it increases revenues, but on the other hand each firm is tempted to free ride on the others, given the positive industry spillovers. In a static game, each player, expecting the free-riding behaviour of the other firms, could anticipate the rivals' moves choosing not to undertake its own investment. However, in our dynamic setting this course of action is neutralized by the firms' decision to wait longer before investing in order to be assured that it is worthwhile doing it once and for all. Moreover, by inspecting the proof (See Technical Appendix A), we can see that $\frac{\partial \hat{Q}}{\partial n} > 0$. This is also an interesting result that is coherent with our interpretation. The higher the number of firms in the game, the higher the risk of free riding and therefore the more the firms wait in order to be sure that there would be no such free riding. In the range where $Q > \hat{Q}$, the term $\frac{a}{r} + \frac{Q}{r-\alpha} - \frac{c}{r}$ represents the present value of selling the output when each firm commits to invest forever, while $B_1 Q^{\gamma_2}$ is the value to suspend investment in the future. While in general it is true that the opportunity to suspend investment adds value to the firms (Dixit and Pindyck 1994, ch.8) since it allows to avoid fixed costs, in our case, when the quality falls, this term can become negative.[13] The interpretation of this result is as follows: if there were no option to stop investing, the value of the firm would be given by just $\frac{a}{r} + \frac{Q}{r-\alpha} - \frac{c}{r}$. However,

[13]Interestingly enough, such a result is similar to the one obtained by Wirl (2008) in his study of a dynamic game among firms contributing to the same aggregate externality (Wirl, 2008, Proposition 6, p. 108).

the option to stop investing is coupled with the free-riding problem. When $Q > \hat{Q}$, firms are investing, and therefore the negative spillover generated by the free-riding problem could be so harsh that it may depress the firms' value. This would occur because the potential cost savings stemming from the option to stop investing could be outweighed by the costs spurred by the lack of enforcement of the collective decision. As stated in the Proposition this effect would be particularly important when the number of firms (which is a proxy for the dimension of the industry) is high, i.e., $\beta_1 \geq n/(n-1)$.

Corollary. *When σ vanishes, the game reduces to a standard investment decision in which firms invest provided that marginal revenue covers marginal cost.*

The intuition of this result is the following. The investment rule in (10) can be rewritten as:

$$\hat{Q} = \frac{\frac{(\beta_1-\gamma_2)}{\gamma_2\beta_1}\frac{rn}{\alpha} - 1}{\frac{(\beta_1-\gamma_2)}{\gamma_2\beta_1} - \frac{\alpha}{r-\alpha} + \frac{\beta_1}{\beta_1\gamma_2}\frac{\alpha}{r-\alpha}} c > 0$$

Since both $\lim_{\sigma\to 0}\frac{(\beta_1-\gamma_2)}{\gamma_2\beta_1} = \lim_{\sigma\to 0}\frac{\beta_1}{\gamma_2\beta_1} = 0$, the above trigger becomes:

$$\alpha\hat{Q}\Big|_{\sigma=0} = (r-\alpha)c \qquad (14)$$

Equation (14) has a very natural interpretation. When uncertainty on quality vanishes, the n firms, being all equal, behave as if they were a single monopolist. The left-hand side indicates the value of the marginal unit of investment by this monopolist; the right-hand side is the user-cost of this unit. The evolution of Q becomes perfectly predictable and the firms' optimal investment rule reduces to the Marshallian rule, i.e., invest as long as the marginal revenue covers the marginal cost.

By comparing $\hat{Q}$ to the trigger that arises when there is no game, i.e., under no uncertainty on quality, we can see that the trigger for investment under uncertainty is higher: $\hat{Q}{-}\hat{Q}\big|_{\sigma=0} > 0$. This implies that the opportunistic free-riding behaviour induces firms to postpone the investment decision compared to the case without uncertainty on the aggregate stock of quality, in which firms behave as a monopolist.

3.2 Full information

Consider now the case of full information, i.e., firms can acquire the information on consumers' WTP for quality (θ) without paying any cost. In such a setting, by firms' homogeneity, Equation (5) becomes:

$$V_i(Q;\theta) = \max_{k_{it}\in\{0,1\}} E_0\left[\int_0^{\infty} e^{-rt}[p(Q_t,a;\theta) - ck_{it}]dt\right] \text{ for } i = 1, 2...n \quad (15)$$

where $p(Q_t, a; \theta) = E(a_t) + (1 + \theta)Q_t$, and $E_0(.)$ is the expectation taken at time $t = 0$ with respect to $\{Q_t, t \geq 0\}$. From (15) it is easy to prove that:

Proposition 2 *Under full information, 1) The optimal trigger is:*

$$\tilde{Q} = \frac{\hat{Q}}{(1+\theta)} \tag{16}$$

2) and the value of the firm becomes:

$$V(Q;\theta) = \begin{cases} \tilde{A}_0 Q^{\beta_1} + \frac{a+(1+\theta)Q}{r} & \text{for } Q < \tilde{Q} \\ \tilde{B}_1 Q^{\gamma_2} + \frac{a}{r} + \frac{(1+\theta)Q}{r-\alpha} - \frac{c}{r} & \text{for } Q > \tilde{Q} \end{cases} \tag{17}$$

where:

$$\tilde{A}_0 = \left[\frac{nc}{\alpha} - \frac{\tilde{Q}}{r}\right] \frac{\tilde{Q}^{-\beta_1}}{\beta_1} \quad \text{and} \quad \tilde{B}_1 = \left[\frac{nc}{\alpha} - \frac{\tilde{Q}}{r-\alpha}\right] \frac{\tilde{Q}^{-\gamma_2}}{\gamma_2} \tag{18}$$

Proof. Straightforward from Technical Appendix A ■

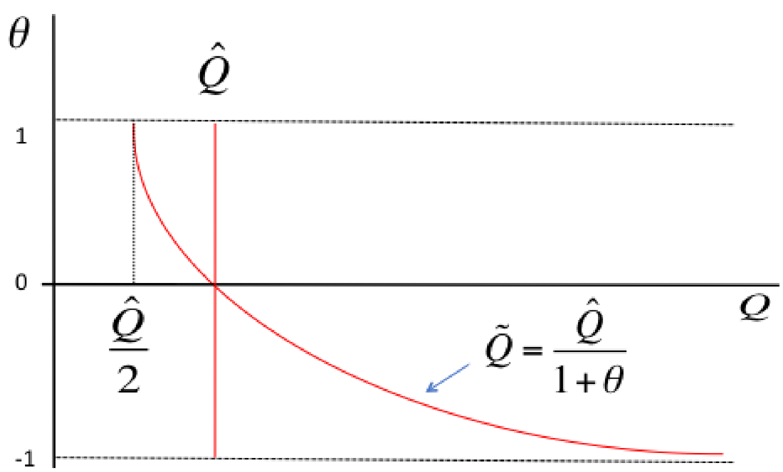
Note that the firms' value function described above (17) maintains the same structure and characteristics of the case discussed above. In particular, the option value of investing ($\tilde{A}_0$) and of halting investment ($\tilde{B}_1$) can be negative or positive, according to the parameters of the game. In addition, by (16), it is immediate to show that:

$$\begin{aligned} \tilde{Q} &\geq \hat{Q} \quad \text{for } \theta \in [-1, 0] \\ \tilde{Q} &< \hat{Q} \quad \text{for } \theta \in (0, +1] \end{aligned}$$

This implies that the information on consumer WTP for quality (θ) may foster investment in quality compared to the case when there is no information acquisition, i.e., $\tilde{Q} < \hat{Q}$, if the unknown parameter for consumer WTP for quality is such that, once revealed, it shows a positive bias in consumers' valuation of quality, i.e., if $\theta > 0$. On the contrary, the information on θ might slow down the investment in quality, i.e., $\tilde{Q} > \hat{Q}$, if it is revealed that consumers' WTP is lower than its expected value, i.e., if $\theta < 0$. Obviously, it is never optimal to invest in quality if the WTP for quality by consumers is the worst possible one, $\theta \rightarrow -1$ (i.e. $\tilde{A}_0 = 0$ and $\tilde{Q} = \infty$). Figure 1 describes the difference in terms of the stock of quality that triggers the investment in quality with information compared to the case when there is no information acquisition, as a function of consumers' valuation (θ).

We can see in Figure 1 that the trigger of the investment in the full information case coincides with the one for the no information case only if consumers have no bias (either positive or negative) in their valuation of quality, i.e., $\theta = 0$. On the contrary, a positive (negative) bias induces firms to invest earlier (later), as expected. The trigger has a lower boundary when the parameter θ is positive, due to the lowest boundary of any investment trigger, namely $\frac{\hat{Q}}{2}$. Given the structure of the game, when the accumulated quality level is low (below $\frac{\hat{Q}}{2}$), the positive spillover of the quality is so low that firms find it optimal not to invest in quality even if they knew that consumers have the highest possible valuation of their investment. On the contrary, a negative bias in consumers' valuation delays the investment decision even if

Figure 1: $\tilde{Q}$ as a function of θ



the aggregate quality level is extremely high. In the extreme case, for a fully negative bias firms would never invest. In fact, it is as if firms' investment decisions were penalized more by a negative bias on behalf of consumers towards their quality than by a positive one, which might induce a more cautious behaviour by firms.[14]

4 Information acquisition and disclosure

We have considered so far the firms' optimal choice of investing in quality for the case of non observable marginal value of quality and perfectly observable marginal value of quality, respectively. We study now whether obtaining information on consumers' willingness to pay (θ) by either acquiring it or having it revealed by a trustworthy third party can start firms' quality accumulation, whenever its level is so low and the free-riding incentive so high that it impedes investing in quality. On the basis of the result shown above, this translates into considering the optimal investment problem under uncertainty for aggregated quality levels below $\frac{\hat{Q}}{2}$. We consider two cases. First, the information on θ can be acquired by firms themselves by paying each a fee ϕ. Given that we intend to focus on the choice to acquire information *per se*, and do not want to mix the investment choice with other possible gaming behaviour, we assume that no free riding is possible in the information acqui-

[14]This result agrees with Bernanke's (1983) Bad News Principle, for which relative uncertainty asymmetrically influences firms' decision processes, since unfavourable events have a higher impact on investment decisions than favourable ones.

sition choice. We show below that the existence of the fee is not crucial for the results. Second, we consider the impact that free information disclosure to firms can have on the probability of starting quality accumulation. This case can be interpreted as the case of a consortium of the industry that could credibly certify the veracity of the information.

4.1 Firms' information acquisition

Consider the case of costly information acquisition. We assume that firms can choose when to acquire the information. That is to say, firms are producing and investing without knowing θ and ask themselves if there is a level of quality $Q_{\mathbf{T}}$ (i.e., a learning time T), such that it is worthwhile spending ϕ to gain information on θ. There exist therefore two triggers that have to be compared: one is the trigger of the investment choice without knowing θ and the other one is the trigger of the information acquisition choice. The first is still $\hat{Q}$ and call the second Q_I. As explained before, we consider only the case where $Q_I \in [Q, \hat{Q})$. Let us define the value of the firm with information acquisition[15] at $t = 0$ as:

$$E_0 \left[\int_0^T e^{-rt}[p(Q_t, a; 0) - ck_{it}]dt + \int_T^{\infty} e^{-rt}[p(Q_t, a; \theta) - ck_{it}]dt - e^{-rT}\phi \right]$$

i.e.,

$$V_i(Q; 0) + E_0 \left[e^{-rT} \left[(V(Q_T; \theta) - V(Q_T; 0) - \phi) \right] \right]$$

[15] We thank an anonymous reviewer for pointing out this.

Since ex-ante θ is not known and we are looking for Q_T in the range $(0, \hat{Q})$, we can take the expected value with respect to θ:

$$V_i(Q;0) + E_0 \left[\left(E_\theta \left[V(Q_T;\theta) \right] - V(Q_T;0) - \phi \right) e^{-rT} \right] \tag{19}$$

Maximizing Equation (19) with respect to T is equivalent to maximizing the term between square brackets, that can be interpreted as the value of information acquisition (Murto, 2014):

$$I(Q) = \max_T E_0 \left[\left(E_\theta \left[V(Q_T;\theta) \right] - V(Q_T;0) - \phi \right) e^{-rT} \right] \tag{20}$$

The firms' problem will consist of finding the time T that maximizes $I(Q)$. T, the optimal learning time, is defined as:

$$T(Q) = \inf \left\{ t \geq 0 \mid Q_T = Q_I, \text{ and } E_\theta \left[V(Q_I;\theta) \right] - V(Q_I;0) = \phi + I(Q_I) \right\} \tag{21}$$

By direct inspection of (21), we can see that it is not necessarily optimal to buy the information on θ immediately even if $\phi = 0$. Even if information acquisition was costless, the opportunity cost represented by the option to wait $I(Q)$ may induce the firms to wait longer before deciding to discover the true value of θ. Once θ is known the real trigger is $\tilde{Q}$, which can be much higher than $\hat{Q}$. Thus, when the current status of quality Q is very far from $\tilde{Q}$ the value of information is negative and the firms delay the moment at which information acquisition is worthwhile.

The solution of problem (20) can be summarized as:

Proposition 3 *If the trigger of the information acquisition Q_I exists, it belongs to the interval $[\frac{\hat{Q}}{2}, \hat{Q})$.*

Proof. See Technical Appendix B ■

We stress that the result of Proposition 3 is independent of the cost of acquiring information (ϕ). Firms find themselves in a low-quality trap: when the initial level of quality is low, i.e., below $\frac{\hat{Q}}{2}$, they are locked in a period of inertia in which they do not find it optimal to acquire information even if it were costless to do so. Therefore, firms would not undertake any effort to get to know consumers' WTP for quality when the initial level of quality is too low. This resembles the bad reputation trap of Castriota and Delmastro (2015).

4.2 The Consortium's information disclosure choice

The result of the previous section is particularly interesting from the perspective of a consortium, or industrial association, that knows that firms are trapped and aims at maximizing the probability of quality investment by its members. Examples include the case of consortia that are formed as public entities with a formal mandate to guarantee members' quality on behalf of consumers, as is the case of agricultural producer organizations (Bouamra-Mechemache and Zago, 2015; Zago, 1999),[16] or consortiums such

[16]Originally introduced in 1996 in the fruit and vegetable sector (Council Regulation 2200/96), the EU Regulation 1308/2013 extended the use of Producer Organisations to

as the parmesan cheese consortium (Consorzio Parmigiano Reggiano). Another example of more voluntary character is the initiative of trade associations created to foster common quality and safety standards, such as Responsible Care in the chemical industry (King and Lenox, 2000). Thus, we consider here the case of an external body that maximizes the probability that the firms invest in quality by finding the optimal release time for the information.

Let $\tilde{P}\left(\tilde{Q}(\theta);Q\right)$ and $\hat{P}\left(\hat{Q};Q\right)$ be the probabilities of investing when θ is revealed and when it is unknown, respectively. In other words, starting at Q in the interior of the range $(0,\tilde{Q}(\theta)]$ and $(0,\hat{Q}]$, $\tilde{P}$ and $\hat{P}$ indicate the probability that the stock of quality reaches the triggers $\tilde{Q}(\theta)$ and $\hat{Q}$, respectively. Obviously, $\hat{P}\left(\hat{Q};Q\right) < 1$ when $Q < \hat{Q}$ and $\hat{P}\left(\hat{Q};Q\right) = 1$ for $Q \geq \hat{Q}$.

We suppose that the consortium does not know θ but can acquire information on consumers' WTP for quality and reveal it immediately to the firms.[17] As our objective is not to model the consortium's welfare function, we assume that the benefit of early investment in quality is always higher than the cost of acquiring information and communicating it to the firms. The consortium has to decide the optimal time to acquire the information, i.e., whether it is best to acquire the information right away and reveal it to

all agricultural sectors.

[17] We do not allow firms to know ex ante that the consortium has the possibility to release such information - in other words, no signalling game is allowed between the consortium and the firms. We also assume that once the information is acquired it is immediately visible to all firms.

all members, or wait for firms to invest in quality by themselves and then disclose the information on θ. Hence, the consortium's problem consists of identifying the stock Q that maximizes the following difference:[18]

$$\Delta(P(Q)) = \int_{-1}^{\tilde{\theta}(Q)} \tilde{P}\left(\tilde{Q}(\theta); Q\right) f(\theta) d\theta + \int_{\tilde{\theta}(Q)}^{+1} 1 f(\theta) d\theta - \hat{P}\left(\hat{Q}; Q\right) \quad \text{for } Q < \hat{Q} \tag{22}$$

where $\tilde{\theta}(Q) = \frac{\hat{Q}}{Q} - 1$. We can claim the following

Proposition 4 *It is never (strictly) optimal to disclose the information on θ. (The consortium would be indifferent to do so only when $Q = \frac{\hat{Q}}{2}$.)*

Proof. See Technical Appendix C ■

The above proposition clearly shows that for the consortium it is never optimal to acquire and disclose the information on consumers' WTP for quality. In fact, it is always strictly worse, in the sense that the probability of investing in quality, once the information is revealed, is lower for all possible levels of accumulated quality, with just one specific exception, namely, that exact level of quality for which it would be indifferent to reveal the information since it would have no impact on the probability of early investment in quality. The rationale of such a result is the following. When firms are already investing in quality, there is no point in revealing information that might just do harm, in the sense of inducing firm to stop investing should they come to know that they have adopted quality too early. What about

[18] For coherence with the preceding section, we consider only the case of $Q < \hat{Q}$. Proposition 4 holds also for $Q > \hat{Q}$ as proved in Fontini et al. (2013).

when firms are not investing? If Q is too low, i.e., below $\frac{\hat{Q}}{2}$, it is not optimal to disclose information since this would not induce any change in investment strategy. Therefore, it might be optimal to disclose information only when quality is in the range for which firms might find it profitable to invest, i.e., when $\frac{\hat{Q}}{2} \leq Q$. Recall that there would be early investment in quality only if θ was in the range for which it delivered "good news", in the sense that once discovered θ happened to be larger than its expected value. On the contrary, it would yield "bad news" if it was lower than 0. Recall that, without any information, the firm would invest at $Q = \hat{Q}$. Let Q be in the range of $\frac{\hat{Q}}{2} \leq Q < \hat{Q}$. As Q reaches $\hat{Q}$ from below, the set of possible good news, which would induce early investment in quality, namely, those values of θ included in the range $0 < \theta < \theta(Q)$, shrinks compared to the set of bad news. In other words, as Q reaches the investment trigger, firms are getting closer to the moment in which they would start investing anyhow. Revealing consumer WTP for quality would therefore do only harm to early investment in quality, because the set of possible results of the information disclosure on the basis of which the firms would postpone their investment decision becomes relatively larger than the set of those cases in which they would anticipate the investments. Obviously, with θ symmetrically distributed, there is just one value of Q for which those sets are equivalent, that is, when the level of quality is at $Q = \frac{\hat{Q}}{2}$.

5 Conclusion and Policy Implications

We set up a model of collective production of quality where firms face two sources of randomness: one related to the overall level of quality, because of the occurrence of random events that affect production, and the other related to uncertainty on consumers' true willingness to pay for quality. The model aims at capturing a situation where what matters for firms is not just their own actual investment but also other firms' investment and the willingness to pay by consumers for aggregate quality when consumers cannot separately assess the quality of each firm.

First we analyze the firms' optimal choice of investment in quality without information on consumers' valuation of industry quality and determine the option value related to its investment in quality. There are two opposing effects at work: on the one hand the positive impact on demand of consumers' willingness to pay for aggregate quality gives incentives for investment in quality, on the other hand free riding among firms makes a firm wait until going ahead with its investment. A positive option value exists depending on which effect prevails. Second, we analyze whether relieving the uncertainty on consumers' willingness to pay for quality fosters quality accumulation. This is done first by investigating whether firms themselves have an incentive to acquire this information, when the initial situation is one of low aggregate quality. We show that it is never optimal for a firm to invest in acquiring this information regardless of its cost. We then go onto investigate whether a third party, such as a public body or industry consortium, can increase the probability of investment in quality when firms are not investing in quality. We show that even if the information disclosure were costless it would have a negative impact on the probability of early investment in quality by the firms. Information disclosure is more likely to do harm to quality investment in the sense that it would induce firms to wait longer than they would without information on consumers' willingness to pay for quality.

The latter result has an interesting policy implication. When industry quality starts from a low level, or the quality standard is set at a low level, firms are trapped in an inertia area in which quality evolves over time on a purely random basis: only a positive random shift of quality that is high enough would induce firms to start investing themselves in quality. In such a setting, a policy of reducing uncertainty on consumers' willingness to pay for quality would not be effective. A consortium or a producer organization that aims at stimulating quality accumulation would be more effective if it invested directly in quality, or obtained subsidies for financing firms' investments.

Notice, however, that information disclosure might have a positive impact from a social welfare point of view. This happens when consumers' perception of quality is close to -1 (the lowest possible), in which case the investment in quality would be pointless (since it would never be perceived as such from consumers). Therefore, revealing the true θ might be regarded as positive since it would put an end to fruitless investments.[19]

[19] We thank an anonymous reviewer for pointing this out.

Our results depend on the following assumptions: the stochastic evolution of quality over time is a random walk, and the distribution of willingness to pay for quality has zero mean. Far from being ad hoc assumptions, we believe that they allow us to develop the model through a neutral and meaningful set up, which does not introduce any artificial exogenous bias on the willingness to pay for quality by consumers, on the one hand, and which guarantees that quality can be accumulated only by means of deliberate action by firms through investment, on the other hand. The latter assumption, in particular, seems to describe well those situations of collective reputation building, like the protected geographical labels that we referred to in the introduction.

The model is only a first attempt at investigating the role of information in the framework of stochastic evolution of quality. Alternative modelization of quality could be explored, for example, how firms' investment decisions change when their action may reduce the variance of quality, instead of the average level of quality as in the present model. We leave such extensions to future research.

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A Technical Appendix A

The general solution to the differential equations (8) and (9) takes the form:[20]

$$V_0(Q;0) = A_0 Q^{\beta_1} + B_0 Q^{\beta_2} + \frac{a+Q}{r} \quad \text{for } Q < \hat{Q}$$

and

$$V_1(Q;0) = A_1 Q^{\gamma 1} + B_1 Q^{\gamma_2} + \frac{a}{r} + \frac{Q}{r-\alpha} - \frac{c}{r} \text{ for } Q > \hat{Q}$$

where $\beta_1, \gamma_1 > 1$ and $\beta_2, \gamma_2 < 0$ are the roots of the characteristic equations $\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1)-r = 0$ and $\Phi(\gamma) = \frac{1}{2}\sigma^2\gamma(\gamma-1)+\alpha\gamma-r = 0$ respectively. A_0, B_0, A_1, B_1 are four constants to be determined. Note that under $Q < \hat{Q}$ the first and second terms stand for the value of the option to switch to investment. However, since the value of the option vanishes as $Q \to 0$, we set $B_0 = 0$. Similarly, under $Q > \hat{Q}$ the option to suspend investment is valueless as $Q \to \infty$ and then we set $A_1 = 0$. To find the constants A_0, B_1 and the optimal trigger $\hat{Q}$ we impose a matching value condition and smooth pasting at $\hat{Q}$:

$$A_0 \hat{Q}^{\beta_1} + \frac{a+\hat{Q}}{r} = B_1 \hat{Q}^{\gamma_2} + \frac{a}{r} + \frac{\hat{Q}}{r-\alpha} - \frac{c}{r} \tag{23}$$

$$A_0 \beta_1 \hat{Q}^{\beta_1 - 1} + \frac{1}{r} = B_1 \gamma_2 \hat{Q}^{\gamma_2 - 1} + \frac{1}{r-\alpha} \tag{24}$$

[20] See Dixit and Pindyck (1994, chapters 6 and 7) for a thorough discussion.

and the incentive constraint[21]

$$A_0\beta_1\hat{Q}^{\beta_1} + \frac{\hat{Q}}{r} = \frac{c}{\alpha - \alpha\frac{n-1}{n}} \tag{25}$$

Solving the system [23-25] yields the following:

$$\hat{Q} = \frac{(\beta_1 - \gamma_2)\frac{rn}{\alpha} - \gamma_2\beta_1}{(\beta_1 - \gamma_2) - \beta_1(\gamma_2 - 1)\frac{\alpha}{r-\alpha}} c > 0 \tag{26}$$

and:

$$\hat{A}_0 = \left[\frac{nc}{\alpha} - \frac{\hat{Q}}{r}\right]\frac{\hat{Q}^{-\beta_1}}{\beta_1}$$

$$\hat{B}_1 = \left[\frac{nc}{\alpha} - \frac{\hat{Q}}{r-\alpha}\right]\frac{\hat{Q}^{-\gamma_2}}{\gamma_2}$$

[21]This condition follows from the maximization of Equation (7). Each firm i will invest $k = 1$ if

$$\frac{1}{2}\sigma^2 Q^2 V'' + \frac{\alpha}{n}\left(1 + \sum_{j=1,j\neq i}^{n} k_j\right) QV' + p(Q;0) - c \geq \frac{1}{2}\sigma^2 Q^2 V'' + \frac{\alpha}{n}\left(\sum_{j=1,j\neq i}^{n} k_j\right) QV' + p(Q;0)$$

which at $\hat{Q}$, reduces to

$$\begin{aligned} \frac{\alpha}{n}\left(1 + \sum_{j=1,j\neq i}^{n} k_j\right)\hat{Q}V' - c &= \frac{\alpha}{n}\left(\sum_{j=1,j\neq i}^{n} k_j\right)\hat{Q}V' \\ \hat{Q}V' &= \frac{c}{\frac{\alpha}{n}\left(1 + \sum_{j=1,j\neq i}^{n} k_j\right) - \frac{\alpha}{n}\left(\sum_{j=1,j\neq i}^{n} k_j\right)} \end{aligned}$$

Then by symmetry we obtain

$$\hat{Q}V\prime = \frac{c}{\alpha - \alpha\frac{n-1}{n}}$$

The marginal gain of investing one more unit in quality should be equal to the marginal cost.

Then, the condition for $A_0 > 0$ is $\hat{Q} < n\frac{r}{\alpha}c$, or $\frac{(\beta_1-\gamma_2)\frac{rn}{\alpha}-\gamma_2\beta_1}{(\beta_1-\gamma_2)-\beta_1(\gamma_2-1)\frac{\alpha}{r-\alpha}} < n\frac{r}{\alpha}$, which can be simplyfied into $-\gamma_2 < -(\gamma_2-1)\frac{nr}{r-\alpha}$, which is always satisfied. On the other hand, the condition for $B_1 > 0$ is $\hat{Q} > n\frac{r-\alpha}{\alpha}c$, or $\frac{(\beta_1-\gamma_2)\frac{rn}{\alpha}-\gamma_2\beta_1}{(\beta_1-\gamma_2)-\beta_1(\gamma_2-1)\frac{\alpha}{r-\alpha}} > n\frac{r-\alpha}{\alpha}$, which can be simplified into $\gamma_2\alpha(\beta_1(n-1)-n) > 0$; this is satisfied for $\beta_1 < n/(n-1)$.

Q.E.D.

B Technical Appendix B

Let us calculate $E_\theta[V(Q;\theta)] - V(Q;0)$. In particular, since (16) is monotonic, we write:

$$E_\theta[V(Q;\theta)] - V(Q;0) = \\ \begin{cases} \int_{-1}^{\tilde{\theta}(Q)} V_0(Q;\theta)f(\theta)d\theta + \int_{\tilde{\theta}(Q)}^{+1} V_1(Q;\theta)f(\theta)d\theta - V_0(Q;0) & \frac{\hat{Q}}{2} < Q < \hat{Q} \\ \int_{-1}^{+1} V_0(Q;\theta)f(\theta)d\theta - V_0(Q;0) & \text{for } Q < \frac{\hat{Q}}{2} \end{cases} \tag{27}$$

where $\tilde{\theta}(Q) = \frac{\hat{Q}}{Q} - 1$[22]. The first line of Equation (27) shows that, for any given stock Q, the ex-ante value $E_\theta[V(Q;\theta)]$ is formed by two terms. The

[22] To be precise, since $\frac{d\tilde{Q}(\theta)}{d\theta} < 0$ for all $\theta \in [-1,+1]$, by inverting (16) we get:

$$\tilde{\theta}(Q) = \begin{cases} -1 & \text{when } \tilde{Q}(-1) = \infty \\ \frac{\hat{Q}}{Q} - 1 & \text{for } \frac{\hat{Q}}{2} < Q < \infty \\ +1 & Q \leq \tilde{Q}(+1) = \frac{\hat{Q}}{2} \end{cases}$$

first integral indicates the firm's value when the revealed value of θ is so low that it is not optimal to invest in quality, while the second integral reflects the case where θ is found sufficiently high to induce the firm to invest in quality.

Assuming that there exists a value $Q_I \in [Q, \hat{Q}]$ beyond which each firm decides to coordinate spending on acquiring information on θ, the solution of (20) must solve the following Bellman equation:

$$rI = \frac{1}{2}\sigma^2 Q^2 I'' \qquad \text{for } Q < Q_I < \hat{Q} \tag{28}$$

where I' and I'' stand for the first and second derivatives with respect to Q respectively. The general solution for I takes the form:

$$I(Q) = M_0 Q^{\beta_1} + M_1 Q^{\beta_2} \quad \text{for } Q < Q_I$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equations $\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) - r = 0$. However, since the value of the option to acquire the information vanishes as $Q \to 0$ we set $M_1 = 0$.

Assume $\phi \geq 0$. To find the constant M_0 and the optimal trigger Q_I we impose a matching value condition and smooth pasting at Q_I. Using (27), we have

$$
\begin{aligned}
M_0 Q_I^{\beta_1} &= \int_{-1}^{\tilde{\theta}(Q_I)} V_0(Q_I;\theta) f(\theta) d\theta \\
&\quad + \int_{\tilde{\theta}(Q_I)}^{+1} V_1(Q_I;\theta) f(\theta) d\theta - V_0(Q_I;0) - \phi
\end{aligned} \tag{29}
$$

and:

$$
\begin{aligned}
M_0 \beta_1 Q_I^{\beta_1 - 1} &= \frac{d\tilde{\theta}(Q_I)}{dQ_I} \left[V_0(Q_I;\tilde{\theta}(Q_I)) f(\tilde{\theta}(Q_I)) \right] \\
&\quad + \int_{-1}^{\tilde{\theta}(Q_I)} V_0'(Q_I;\theta) f(\theta) d\theta \\
&\quad - \frac{d\tilde{\theta}(Q_I)}{dQ_I} \left[V_1(Q_I;\tilde{\theta}(Q_I)) f(\tilde{\theta}(Q_I)) \right] \\
&\quad + \int_{\tilde{\theta}(Q_I)}^{+1} V_1'(Q_I;\theta) f(\theta) d\theta - V_0'(Q_I;0)
\end{aligned} \tag{30}
$$

Since by (17) $V_0(Q_I;\tilde{\theta}(Q_I)) = V_1(Q_I;\tilde{\theta}(Q_I))$, then (30) reduces to:

$$
\begin{aligned}
M_0 Q_I^{\beta_1} &= \int_{-1}^{\tilde{\theta}(Q_I)} \frac{Q_I}{\beta_1} V_0'(Q_I;\theta) f(\theta) d\theta \\
&\quad + \int_{\tilde{\theta}(Q_I)}^{+1} \frac{Q_I}{\beta_1} V_1'(Q_I;\theta) f(\theta) d\theta - \frac{Q_I}{\beta_1} V_0'(Q_I;0)
\end{aligned}
$$

Substituting in (29), we get:

$$
\begin{aligned}
& \int_{-1}^{\tilde{\theta}(Q_I)} V_0(Q_I;\theta) f(\theta) d\theta \\
& + \int_{\tilde{\theta}(Q_I)}^{+1} V_1(Q_I;\theta) f(\theta) d\theta - V_0(Q_I;0) - \phi \\
= \; & \int_{-1}^{\tilde{\theta}(Q_I)} \frac{Q_I}{\beta_1} V_0'(Q_I;\theta) f(\theta) d\theta \\
& + \int_{\tilde{\theta}(Q_I)}^{+1} \frac{Q_I}{\beta_1} V_1'(Q_I;\theta) f(\theta) d\theta - \frac{Q_I}{\beta_1} V_0'(Q_I;0)
\end{aligned}
$$

Substituting in for $V_0(Q_I;\theta)$ and $V_1(Q_I;\theta)$, and their derivatives gives:

$$
\begin{aligned}
& \int_{-1}^{\tilde{\theta}(Q_I)} \left[\tilde{A}_0(\theta) Q_I^{\beta_1} + \frac{a + (1+\theta) Q_I}{r} \right] f(\theta) d\theta \\
& + \int_{\tilde{\theta}(Q_I)}^{+1} \left[\tilde{B}_1(\theta) Q_I^{\gamma_2} + \frac{a}{r} + \frac{(1+\theta) Q_I}{r - \alpha} - \frac{c}{r} \right] f(\theta) d\theta \\
& - \left[\hat{A}_0 Q_I^{\beta_1} + \frac{a + Q_I}{r} \right] - \phi \\
= \; & \int_{-1}^{\tilde{\theta}(Q_I)} \frac{Q_I}{\beta_1} \left[\tilde{A}_0(\theta) \beta_1 Q_I^{\beta_1 - 1} + \frac{(1+\theta)}{r} \right] f(\theta) d\theta \\
& + \int_{\tilde{\theta}(Q_I)}^{+1} \frac{Q_I}{\beta_1} \left[\tilde{B}_1(\theta) \gamma_2 Q_I^{\gamma_2 - 1} + \frac{(1+\theta)}{r - \alpha} \right] f(\theta) d\theta \\
& - \frac{Q_I}{\beta_1} \left[\hat{A}_0 \beta_1 Q_I^{\beta_1 - 1} + \frac{1}{r} \right]
\end{aligned}
$$

or:

$$\frac{\alpha Q_I}{r(r-\alpha)}\int_{\tilde{\theta}(Q_I)}^{+1}(1+\theta)f(\theta)d\theta = \frac{\beta_1}{\beta_1-1}\left[\phi+\frac{c}{r}\left[1-F(\tilde{\theta}(Q_I))\right]\right] - \frac{\beta_1-\gamma_2}{\beta_1-1}\left\{\int_{\tilde{\theta}(Q_I)}^{+1}\tilde{B}_1(\theta)Q_I^{\gamma_2}f(\theta)d\theta\right\} \tag{31}$$

Recall that the expression above should hold for $Q_I < \hat{Q}$. We can show that if Q_I existed, it would belong to the interval $(\frac{\hat{Q}}{2}, \hat{Q})$. This can be seen by noting that if we look for Q_I that tends to $\frac{\hat{Q}}{2}$ from above, i.e., $\tilde{\theta}(Q_I) \simeq +1$, the condition (31) is never satisfied. This means that, since the initial condition $\frac{\hat{Q}}{2} < Q < \hat{Q}$ must be true whatever is the value of Q_I, it is never optimal to acquire information about the true value of θ, i.e., $Q < Q_I$ and thus $Q_I \in (\frac{\hat{Q}}{2}, \hat{Q})$. Note, however, the exception when the initial condition is $Q = \frac{\hat{Q}}{2}$ and $\phi = 0$ in which case Q_I would be undetermined.

Consider now the case when $Q \leq \frac{\hat{Q}}{2}$. Equation (27) becomes:

$$\begin{aligned} E_\theta\left[V(Q;\theta)\right] - V(Q;0) &= \int_{-1}^{+1} V_0(Q;\theta)f(\theta)d\theta - V_0(Q;0) \\ &= \int_{-1}^{+1}[V_0(Q;\theta) - V_0(Q;0)]f(\theta)d\theta \end{aligned}$$

Recalling that $V_0(Q;\theta) = \tilde{A}_0 Q^{\beta_1} + \frac{a+(1+\theta)Q}{r}$ and $V_0(Q;0) = A_0 Q^{\beta_1} + \frac{a+Q}{r}$, we can write $E_\theta\left[V(Q;\theta)\right] - V(Q;0) = S(\theta)Q^{\beta_1}$, where $S(\theta) = \left[\int_{-1}^{+1}\Delta A(\theta)f(\theta)d\theta\right]$

and $\Delta A = \tilde{A}_0 - A_0$ is given by:

$$\Delta A(\theta) = \tilde{A}_0 - A_0 = \left[\frac{nc}{\alpha} - \frac{\tilde{Q}}{r}\right]\frac{\tilde{Q}^{-\beta_1}}{\beta_1} - \left[\frac{nc}{\alpha} - \frac{\hat{Q}}{r}\right]\frac{\hat{Q}^{-\beta_1}}{\beta_1}$$

and $\tilde{Q} = \frac{\hat{Q}}{(1+\theta)}$. To find the constant and the optimal trigger Q_I we impose a matching value condition and smooth pasting at Q_I.

$$M_0 Q_I^{\beta_1} = S(\theta) Q_I^{\beta_1} - \phi$$

and:

$$M_0 \beta_1 Q_I^{\beta_1 - 1} = S(\theta) \beta_1 Q_I^{\beta_1 - 1}$$

Defining $Y(Q) = [M_0 - S(\theta)]\, Q^{\beta_1}$ we should distinguish two cases:

1. $S(\theta) > 0$. In this case there always exists a positive constant M_0 such that $Y(Q) > 0$, then $\max Y(Q) \rightarrow Q_I = \frac{\hat{Q}}{2}$.

2. $S(\theta) < 0$. In this case it is never optimal to invest. *Q.E.D.*

C Technical Appendix C

Proof of Proposition 4:

Assume $Q < \hat{Q}$. Recall that when Q is between 0 and $\frac{\hat{Q}}{2}$, it is never optimal to invest in quality. Next, consider the case when $\frac{\hat{Q}}{2} \leq Q < \hat{Q}$. Recall that Q, defined in Equation (3), is our stochastic process. We calculate the probability that a level $\hat{Q}$ is hit starting from a generic starting value Q_t. Since

$$d\ln[Q_t] = \mu dt + \sigma dz_t,$$

where $\mu = (\alpha - \frac{1}{2}\sigma^2)$, the probability $\Pr(Q_\tau = \hat{Q} \mid Q_t)$ is given by (Cox and Miller, 1965, p.212; Harrison, 1985, p. 43):

$$\Pr\left(\hat{Q}, Q\right) = \begin{cases} 1 & \text{if } 2\frac{\alpha}{n}\sum_{i=1}^{n} k_{it}/\sigma^2 \geq 1 \\ \left(\frac{\hat{Q}}{Q}\right)^{(2\frac{\alpha}{n}\sum_{i=1}^{n} k_{it}/\sigma^2)-1} & \text{if } 2\frac{\alpha}{n}\sum_{i=1}^{n} k_{it}/\sigma^2 < 1 \end{cases} \tag{32}$$

Starting at Q in the interior of the range $(0, \hat{Q}]$, after a "sufficient" long interval of time the process is sure to hit the trigger $\hat{Q}$ if the trend is positive and sufficiently large with respect to the uncertainty. However, if $\frac{\alpha}{n}\sum_{i=1}^{n} k_{it}$ is positive but low with respect to the uncertainty or it is negative, the process may drift away and never hit $\hat{Q}$. Taking into account that the firms never invest below $\hat{Q}$, applying the expression in (32) to Equation (22) with $\frac{\alpha}{n}\sum_{i=1}^{n} k_{it} = 0$ we obtain:

$$\begin{aligned} \Delta(P(Q)) &= \int_{-1}^{\tilde{\theta}(Q)} \left(\frac{Q}{\tilde{Q}(\theta)}\right) f(\theta)d\theta + \int_{\tilde{\theta}(Q)}^{+1} 1 f(\theta)d\theta - \left(\frac{Q}{\hat{Q}}\right) \\ &= \left[1 - F(\tilde{\theta}(Q))\right]\left[1 - \left(\frac{Q}{\hat{Q}}\right)\right] + \left(\frac{Q}{\hat{Q}}\right)\int_{-1}^{\tilde{\theta}(Q)} \theta f(\theta)d\theta \end{aligned} \tag{33}$$

We see that $\Delta(P(\frac{\hat{Q}}{2})) = 0$ and

$$\frac{\partial \Delta(P(Q))}{\partial Q} = \left(\frac{1}{\hat{Q}}\right)\left[\int_{-1}^{\tilde{\theta}(Q)} \theta f(\theta) d\theta - \left[1 - F(\tilde{\theta}(Q))\right]\right] < 0, \text{ for all } \frac{\hat{Q}}{2} < Q < \hat{Q}\ .$$

Then $Q = \frac{\hat{Q}}{2}$ is the single root of Equation (33). *Q.E.D.*