CORE

# EFFECT OF GEOMETRIC IMPERFECTIONS ON THE CAPACITY OF CONICAL STEEL TANKS UNDER HYDRODYNAMIC PRESSURE 

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#### Abstract

Liquid tanks in the form of truncated steel cones are commonly used for liquid storage in North America and in other locations. The main cause of failure, for conical steel tanks in particular, which was identified in most of the failure cases, is the buckling of the tank's shell at locations of maximum compressive stress. Being constructed of steel, geometric imperfections in the conical tank walls will exist and their amplitude will be dependent on the quality controls applied by the builder. Such geometric imperfections play an important role in defining the buckling capacity of shell structures in general. Some studies found in the literature assessed the effect of geometric imperfections on the buckling capacity of steel tanks. However, most of these studies focused on hydrostatic pressure and not on hydrodynamic pressure that is induced on the tank walls when the tank base is subjected to either horizontal or vertical ground excitations. In this study, an expression for the critical imperfection wave length is obtained and the effect of the geometric imperfections' amplitude on the buckling capacity of conical steel tanks is assessed numerically under hydrodynamic pressure due to horizontal and vertical ground excitations. The study is conducted numerically through non-linear static pushover analysis using an in-house finite element model that accounts for the geometric and material nonlinear effects.


Keywords: conical steel tanks, buckling, non-linear static, geometric imperfections, hydrodynamic pressure

## 1. INTRODUCTION

Conical-shaped steel liquid tanks are used for fluid storage in industrial facilities or for water supply. A typical conical tank consists of a steel vessel resting on a supporting structure. The conical vessel is constructed of prefabricated steel panels welded together circumferentially and longitudinally. In some cases, a conical tank vessel is elevated above the ground by a reinforced concrete shaft. Two common configurations for conical steel tanks exist: (1) A pure truncated cone as shown in Figure 1a, (2) A combined conical tank with a cylindrical cap as shown in Figure 1b.

In spite of the fact that some failures of conical steel tanks occurred during the last decades, most of the previous studies and design specifications focused on cylindrical steel tanks. The most common failure mode for conical steel tanks is in the form of shell instability due to the relatively narrow wall thickness. The only seismic design guidelines for conical steel tanks found in some specifications (AWWA 2005, API 2005, and Eurocode 1998) are those based on using an equivalent cylinder approach despite the fact that the state of stresses under hydrostatic pressure for cylindrical tanks is not similar to that for conical tanks due to the inclination of the tank walls.
To better understand the resulting stresses for conical tanks, the volume of the contained liquid is divided into vol. 1 and vol. 2 as shown in Figure 2. The first, is transferred directly to the tank base, while the second, is resting on the tank's inclined walls. Due to the inclination of the walls, compressive meridional stresses $\sigma_{\mathrm{m}}$ are developed in addition to tensile hoop stresses $\sigma_{h}$ meridionally and circumferentially, respectively, through the tank shells. Compressive stresses $\sigma_{\mathrm{m}}$ are maximum near the tank base due to the reduction in the tank radius in addition to the increase of the fluid volume resting on the tank walls, i.e., vol.2. These compressive stresses are very critical for steel tanks as they might lead to shell instability.


Figure 1: (a) Pure conical tank, (b) Combined conical tank
Motivated by the collapse of a conical steel water tower in Belgium, Vandepitte et al. (1982) tested a large number of small-scale conical tank models experimentally under hydrostatic pressure. The objective was to develop a set of design charts for different base restraining conditions and geometric imperfection levels. In 1990, a conical steel water tower collapsed in Fredericton, Canada when it was filled with water for the first time. The tank failed as the amplitude of the geometric imperfections was underestimated (Vandepitte 1999). For the design of conical steel tanks under hydrostatic pressure, El Damatty et al. (1999) and Sweedan and El Damatty (2009) provided a simplified design approach that takes into account geometric imperfections and the existence of an upper cylindrical cap.


Figure 2: Stresses induced due to inclination of the wall
It is important to understand the seismic behaviour of liquid storage tanks as any failure to such structures might have serious consequences in addition to the structural damage. Many studies were carried out in order to understand the seismic behaviour of cylindrical steel tanks either assuming the tank walls to be rigid (Housner 1957 and Housner 1963) or taking into consideration the effect of wall flexibility in the form of fluid-structure interaction (Veletsos 1974; Haroun and Housner 1982). It was concluded that the flexibility of cylindrical tank walls amplifies the tank's response and must be accounted for.

The induced hydrodynamic pressure on a liquid storage tank's walls due to a horizontal ground excitation is divided into two components known as the impulsive and sloshing components. The impulsive component corresponds to the smaller amount of liquid, which moves with the walls of the tank. As a result, it has a maximum value near the tank base. The long period sloshing component corresponds to the upper limit of the liquid undergoing sloshing. In general, the impulsive pressure is the most critical unless the liquid in the tank is extremely shallow. When a conical steel tank is subjected to earthquake excitation, hydrodynamic pressure is induced on the tank walls. In the case of horizontal excitation, the induced hydrodynamic pressure will amplify both $\sigma_{m}$ and $\sigma_{h}$ on one side of the tank and reduce them on the other side based on the direction of the ground excitation, while the induced hydrodynamic pressure due to the vertical excitation will amplify or reduce both $\sigma_{\mathrm{m}}$ and $\sigma_{\mathrm{h}}$ in an axisymmetric manner based on the direction of the ground excitation, i.e., either upwards or downwards. El Damatty et al. (1997 b, c) conducted the first study to assess the behaviour of conical tanks under seismic loading where a coupled shell element-boundary element formulation was developed to simulate the fluid-structure interaction for both horizontal and vertical excitations where a fluid added mass matrix that can be incorporated into a nonlinear dynamic analysis routine was derived. Jolie et al. (2013)
assessed using the equivalent cylinder approach found in the literature (AWWA 2005, API 2005, and Eurocode 1998) when analyzing conical tanks subjected to horizontal ground excitations. It was shown that the base shear is wellpredicted by the Eurocode, while it is over estimated by the AWWA and API. In contrast, the three design codes under-estimated the overturning moment due to ignoring the effect of the vertical component of the hydrodynamic forces caused by the horizontal excitation when assuming the tank walls to be vertical.

Early studies of the seismic behaviour of cylindrical liquid tanks often ignored the effect of the vertical excitations as most of the structures are relatively stiff in the vertical direction. However, it has been observed that the maximum amplitude of the vertical excitation component can exceed the peak horizontal amplitude especially near the center of an earthquake. Vertical excitation is transmitted to a horizontal hydrodynamic loading acting on the tank walls. As a result, tensile hoop stresses are amplified and might lead to an inelastic buckling of the shell. Marchaj (1979) attributed the failure of metallic tanks during past earthquakes to the lack of consideration of the vertical excitation in their design. Veletsos and Kumar (1984) and Haroun and Tayel (1985) showed that the flexibility of cylindrical tank walls amplifies the tank's response and must be accounted for in the case of vertical excitations as well. Jolie et al. (2014) assessed the importance of considering the vertical component of ground excitations when designing conical steel tanks. Their results showed that the vertical ground excitation has a considerable effect on the increase of the meridional wall stresses compared to those resulting from hydrostatic pressure, especially in high seismic hazard regions emphasizing the importance of vertical excitation consideration.

The aim of this study is to determine the capacity of conical steel tanks under hydrodynamic pressure due to the horizontal and vertical components of a ground excitation. The conical tank's capacity, which is expressed in terms of total base shear and total vertical force corresponding to horizontal and vertical excitations, respectively, is obtained using a finite element model through non-linear static pushover analysis. Regarding horizontal excitation, two base shear capacities corresponding to both the impulsive and the sloshing hydrodynamic pressures are obtained. Geometric imperfections are incorporated into the finite element model in order to study their effect on the capacity of the conical steel tanks. The base shear capacities for the different levels of the geometric imperfections are represented in charts for different tank geometries.

## 2. HYDRODYNAMIC PRESSURE

### 2.1 Horizontal Excitation

Hydrodynamic pressure is induced on the tank walls and floor during seismic excitation acting on a conical tank. The total hydrodynamic pressure can be divided into two components: impulsive pressure $\mathrm{P}_{\mathrm{IH}}$ and sloshing pressure $\mathrm{P}_{\mathrm{SH}}$. The sloshing component is a long period component relative to that for the impulsive one and, hence, the two components can be decoupled in the analysis. The impulsive component associated with the hydrodynamic pressure for a conical tank containing an ideal fluid is given by El Damatty et al (1997b) as in equation [1] below:
[1] $\quad P_{I H}(r, \theta, z, t)=\sum_{n=1}^{N_{2}} \sum_{i=1}^{N_{1}} A_{i n}(t) I_{n}\left(\alpha_{i} r\right) \cos \left(\alpha_{i} z\right) \cos (n \theta)$
where $\mathrm{A}_{\text {in }}$ ( t ) is an amplitude function of time, $\mathrm{I}_{\mathrm{n}}$ are the modified Bessel's functions of the first kind, $\alpha_{i}=(2 i-1) \pi / 2 h$, and $t$ is the time. The term, $\cos \left(\alpha_{i} z\right)$, represents the distribution of the hydrodynamic pressure for mode $i$ in the vertical Z direction, while the term, $\cos (\mathrm{n} \theta)$, represents the distribution of the hydrodynamic pressure for mode n in the circumferential direction where n is the wave number. Coordinates $\mathrm{r}, \theta$, and z in addition to the tank dimensions are shown in Figure 2. As a result of the decoupling between the liquid sloshing modes and the shell vibration modes, the sloshing component $\mathrm{P}_{\mathrm{S}}(\mathrm{r}, \theta, \mathrm{z}, \mathrm{t})$ can be evaluated assuming that the tank walls are rigid. Based on this assumption, El Damatty et al. (2000) derived an expression for the fundamental sloshing component of the hydrodynamic pressure as follows:

$$
\begin{equation*}
\mathrm{P}_{\mathrm{SH}}(\mathrm{r}, \theta, \mathrm{z}, \mathrm{t})=\mathrm{B}(\mathrm{t}) \rho_{\mathrm{F}} \mathrm{~J}_{1}\left(\mathrm{k}_{1} \mathrm{r}\right) \cosh \left(\mathrm{k}_{1} \mathrm{z}\right) \cos (\theta) \tag{2}
\end{equation*}
$$

where $J_{1}\left(k_{1} r\right)$ is the Bessel's function of the first kind of order one; $B(t)$ is the arbitrary function of time, and $\rho_{F}$ is the fluid density. A procedure to evaluate the constant $\mathrm{k}_{1}$ was discussed in detail by El Damatty et al. (2000). For a conical tank subjected to a horizontal excitation, a resulting base shear Q and a corresponding overturning moment M will act on the tank walls just above the tank base. Based on the distributions of the different circumferential
hydrodynamic pressure modes, the total base shear will result from the $\cos \theta$ pressure mode only as shown in Figure 3a.

### 2.2 Vertical Excitation

On the other hand, when a conical tank is subjected to a vertical ground excitation, the tank is subjected to accelerations resulting in a hydrodynamic pressure acting on the tanks walls and base in addition to the existing hydrostatic pressure. The hydrodynamic pressure produced by the vertical excitation in a conical tank containing an ideal fluid is given by El Damatty et al. (1997b) as follows:

$$
\begin{equation*}
P_{D}(r, \theta, z, t)=\sum_{i=1}^{N_{1}} A_{i o}(t) I_{o}\left(\alpha_{i} r\right) \cos \left(\alpha_{i} z\right)+\ddot{G}_{v}(t)\left(1-\frac{z}{h}\right) \tag{3}
\end{equation*}
$$

where $A_{i o}(t)$ are the amplitude functions of time, $I_{o}$ are the modified Bessel's functions of the first kind, and $\ddot{G}_{v}(t)$ is the vertical ground excitation. For a conical tank subjected to a vertical excitation, a resulting total normal force N will act just above the tank base. Due to the axisymmetric distribution of the resulting hydrodynamic pressure as shown in Figure 3b, neither a total base shear nor an overturning moment will result from the vertical excitation.


Figure 3: (a) Horizontal excitation $\operatorname{Cos} \theta$ pressure mode; (b) Vertical excitation $\operatorname{Cos} 0$ pressure mode

## 3. FINITE ELEMENT MODEL

In this study, three-dimensional numerical models are developed for conical steel tanks using the finite element method. The numerical model is based on a consistent 13 noded subparametric triangular shell element as shown in Figure 4a, which was developed by Koizey \& Mirza (1997). This element has the advantages of being free of the spurious shear modes, i.e., locking phenomenon observed in isoparametric shell elements when used in modelling thin shell structures. El Damatty et al. (1997d) extended the formulation of this shell element to include both geometric and material non-linearities. Accordingly, this model can be used to predict both elastic and inelastic buckling. Due to the symmetry of the horizontal axis in both loading and geometry, only half of the cone is modelled and used in the analysis. A mesh sensitivity analysis was performed in order to determine the mesh size that can accurately capture the expected buckling. It is found that a mesh of 512 triangular elements as shown in Figure 4b is sufficient to accurately capture the buckling waves near the tank base. The length of the elements is not uniform as a finer mesh is used near the base of the tank due to the stress concentration at this location where buckling is expected to occur. The tanks are assumed to be hinged at the base along the circumference and to be free at the top.

## 4. METHOD OF ANALYSIS

In this study, a non-linear static analysis is used to obtain the load-carrying capacity of the conical tanks, which is conducted by increasing the load value incrementally until reaching failure in the form of buckling or yielding of the steel vessel. The load increase is achieved using an increasing load factor, which is multiplied by the applied hydrodynamic pressure load pattern. The non-linearity in the analysis comes from the inclusion of both geometric and material non-linearity in the finite element model previously discussed. To include both the hydrostatic and the hydrodynamic pressure in the analysis, two load factors are used. The first is $\mathrm{P}_{\mathrm{HS}}$, which corresponds to the hydrostatic pressure; while the second is $\mathrm{P}_{\mathrm{HD}}$, which corresponds to the hydrodynamic pressure. The analysis starts with a value of the load factor $\mathrm{P}_{\mathrm{HD}}$ equaling zero and then the load factor $\mathrm{P}_{\mathrm{HS}}$ is increased incrementally until it reaches the actual value of the hydrostatic pressure acting on the tank. After this stage, the value of $\mathrm{P}_{\mathrm{HS}}$ is kept constant and the value of $\mathrm{P}_{\mathrm{HD}}$ begins at zero and increases until failure occurs.

The hydrodynamic pressure load pattern used follows the distributions described in Eqs. 1, 2, and 3 corresponding to the cases of impulsive capacity due to the horizontal excitation, the sloshing capacity due to the horizontal excitation, and the impulsive capacity due to the vertical excitation, respectively. A conical steel tank is considered to have failed whenever one of the following two failure criteria is met: (1) Yielding failure when the tank shell yields before buckling instability takes place, or (2) Buckling failure when the tank shell suffers instability before yielding, i.e., elastic buckling. As a result, the base shear capacity of a conical steel tank in the current study represents the base shear value just before the yielding or buckling of the tank vessel.


Figure 4: (a) Coordinates and degrees of freedom for consistent shell element; (b) Finite element mesh for half cone

## 5. GEOMETRIC IMPERFECTIONS

Conical steel tanks are normally constructed from curved panels welded together in both circumferential and longitudinal directions. As a result, geometric imperfections will exist and will play an important role in determining the capacity of conical steel tanks for liquids and might lead to failure if not estimated correctly as in the case of the collapsed conical steel water tower in Fredericton (Vandepitte 1999). A commonly used model for simulating the geometric imperfections $\mathrm{W}(\mathrm{s})$, as shown in Figure 5, is described as:

$$
\begin{equation*}
\mathrm{W}(\mathrm{~s})=\mathrm{W}_{0} \sin \left(\frac{2 \pi \mathrm{~s}}{\mathrm{~L}_{\mathrm{l}}}\right) \cos (\mathrm{n} \theta) \tag{4}
\end{equation*}
$$

where $\mathrm{w}_{0}$ is the imperfection amplitude, $\mathrm{L}_{\mathrm{I}}$ is the imperfection wavelength, s is a coordinate measured along the generator of the vessel, and n is an integer defining the circumferential wavelength of the imperfection shape. According to Vandepitte et al. (1982), a conical tank with a ratio $\mathrm{w}_{0} / \mathrm{L}_{\mathrm{I}}$ less than 0.004 is considered to be a good cone while a conical tank with a ratio $\mathrm{w}_{0} / \mathrm{L}_{\mathrm{I}}$ ranging from 0.004 to 0.01 is considered to be a poor cone. The geometric imperfections are incorporated in the finite element model discussed in section 3 in the form of initial strains.


Figure 5: Assumed imperfection shape along the generator of the tank walls

Under the effect of hydrostatic pressure only, Vandepitte et al. (1982) used experimental results to obtain an expression for the critical buckling wave length, $L_{C R}$. Regarding the circumferential distribution of the imperfections along the surface of the vessel, El Damatty et al. (1997a) have shown that an axisymmetric distribution, i.e., n=0, leads to a minimum buckling capacity of pure conical tanks. This is due to the presence of hydrostatic pressure, which tends to force the structure to buckle in an axisymmetric mode; consequently an imperfection in the shape matching this mode is the most critical.

In order to determine the critical imperfection shape, i.e., $\mathrm{L}_{\mathrm{I}}$ and n , leading to the minimum capacity of conical steel tanks, the same analogy identified in the latter studies is followed. As the hydrostatic pressure has an axisymmetric distribution, and the hydrodynamic pressure corresponding to the horizontal excitations is in the form of the $\cos \theta$ mode, a distribution of geometric imperfections corresponding to $n=0$ or $n=1$ will lead to the minimum buckling capacity of the conical steel tanks when subjected to both hydrodynamic and hydrostatic pressures. The same twophase loading procedure discussed in section 4 is repeated twice for a group of 60 conical steel tanks of practical dimensions; one with the inclusion of axisymmetric imperfections in the tanks and the other with an antisymmetric distribution (i.e., $n=1$ ) with the same buckling wave length $\mathrm{L}_{\mathrm{I}}$ recommended by Vandepitte et al. (1982). It is found that a value of $\mathrm{n}=0$ will always lead to a lower buckling capacity for the conical steel tanks. This means that the hydrostatic pressure loading governs the buckling capacity of the conical tanks due to the initiation of the buckling waves only during the hydrostatic pressure loading phase.

It must be mentioned that the inclusion of the axisymmetric geometric imperfections in the non-linear static analyses results in an increase of the buckling capacity in some of the studied cases, especially with $\theta_{\mathrm{v}}=30$, in comparison to that of perfect tanks. This is due to the fact that the hydrodynamic pressure in the form of the $\cos \theta$ mode acts in the same direction as the hydrostatic pressure in one half of the tank and in the opposite direction in the other half, which might delay the failure due to the buckling. In order to make the geometric imperfections more critical and avoid increasing the buckling capacity of the tanks, a geometric imperfection pattern with $\mathrm{n}=0$ is used but only on the side where the hydrodynamic pressure is acting in the same direction as the hydrostatic pressure, i.e., $\theta=0$ to 90 .

In order to determine the critical imperfection wave length $L_{C R}$, several analyses are performed for each of the 60 conical tanks with different imperfection wave lengths, and that which leads to a minimum buckling capacity is considered to be the critical imperfection wave length, $\mathrm{L}_{\mathrm{CR}}$. A similar expression to that proposed by Vandepitte et al. (1982) is assumed. The effect of a variation in the tank height on the critical imperfection wave length is found to be insignificant as noted by Vandepitte et al. (1982). Using regression analysis as shown in Figure 6, the final expression is:

$$
\begin{equation*}
\mathrm{L}_{\mathrm{cr}}=4.03 \sqrt{\mathrm{R}_{\mathrm{b}} \mathrm{t}_{\mathrm{w}} / \cos \theta_{\mathrm{v}}} \tag{5}
\end{equation*}
$$



Figure 6: Relationship between critical imperfection wave length and the parameter $\sqrt{R t / \cos \theta_{v}}$
A similar procedure is followed in order to estimate the critical imperfection wave length in the case of combined hydrostatic and hydrodynamic pressures due to vertical excitation. It is found that the same critical imperfection wavelength expression, i.e., Eq. 5, will lead to the minimum buckling capacity as the buckling waves initiate during the
initial hydrostatic pressure phase similar to that in the case of horizontal excitation. An axisymmetric geometric imperfections' distribution over the tank walls will be the most critical as both the hydrostatic and the hydrodynamic pressures have axisymmetric distributions and consequently force the tank walls to buckle in an axisymmetric manner.

## 6. CONICAL STEEL TANK CAPACITIES

In this section, the effect of geometric imperfections on the capacity of the conical steel tanks when subjected to both hydrostatic and hydrodynamic pressure due to horizontal and vertical excitations is studied. After deriving an expression for the critical imperfection wave length Lcr, an axisymmetric imperfection pattern based on Eq. 5 is applied to the tanks. Two levels of imperfections are studied: the first with $\mathrm{w}_{0}=0.004 \mathrm{~L}_{\text {cr }}$ to represent good tanks and the second with $\mathrm{W}_{0}=0.01 \mathrm{~L}_{\text {cr }}$ to represent poor tanks.

A group of 75 tanks of practical dimensions was chosen for this study with $\mathrm{R}_{\mathrm{b}}$ ranging from 4.0 m to 6.0 m , h ranging from 5.0 m to 9.0 m , and $\theta_{\mathrm{v}}=30^{\circ}, 45^{\circ}, 60^{\circ}$ with a steel yield stress of 300 MPa . The tanks were preliminary designed under hydrostatic pressure based on the simplified method proposed by Sweedan and El Damatty (2009) assuming the use of good tanks regarding the level of geometric imperfections.

### 6.1 Horizontal Excitation

Using the non-linear static analysis procedure described in section 4, the capacity of conical steel tanks corresponding to impulsive or sloshing hydrodynamic pressure due to horizontal excitations is obtained. The capacity of a conical steel tank can be represented by the base shear value just before the yielding or buckling of the tank walls. The base shear capacity for the impulsive component $\mathrm{V}_{\mathrm{I}}$ and the sloshing component $\mathrm{V}_{\mathrm{S}}$ are represented in the form of the unitless parameter $\mathrm{VR}_{\mathrm{b}} / \mathrm{Wh}$ where W is the weight of the contained fluid. Figures 7a to 7c show how the impulsive base shear capacity represented by $\mathrm{V}_{\mathrm{I}} \mathrm{R}_{\mathrm{b}} / \mathrm{Wh}$ changes with the slenderness parameter $h / \mathrm{R}_{\mathrm{b}}$. Each subplot shows this variation for the two levels of geometric imperfections considered (i.e., both good and poor) corresponding to a specific value for the angle $\theta_{\mathrm{v}}$. In order to estimate the reduction in the base shear capacity due to the inclusion of geometric imperfections, the capacity $\mathrm{V}_{\mathrm{I}} \mathrm{R}_{\mathrm{b}} / \mathrm{Wh}$ for perfect tanks is plotted on the same charts. Regarding the governing failure mode for perfect tanks, the general trend is that the probability for a yielding failure to occur is higher when the angle $\theta_{\mathrm{v}}$ is increased. In the case of imperfect conical steel tanks, the probability of inelastic buckling taking place increases with the higher geometric imperfections' amplitude.

In the case of $\theta_{\mathrm{v}}=30$, it is observed that an imperfection with an amplitude of 0.004 L or less has no remarkable effect on the normalized base shear capacity for the tanks with $h / R_{b}$ being less than 1.1 . In the case where an imperfection in amplitude equals 0.01 L , the reduction in the normalized base shear capacity increases with a higher $\mathrm{h} / \mathrm{R}_{\mathrm{b}}$. For $\theta_{\mathrm{v}}=45$, it is found that an imperfection with an amplitude of 0.004 L or less does not have a remarkable effect on the normalized base shear capacity for tanks with $\mathrm{h} / \mathrm{R}_{\mathrm{b}}$ less than 1.0 while for an imperfection amplitude of 0.01 L , the reduction in the normalized base shear capacity is almost the same regardless of the value $h / R_{b}$. Finally, for $\theta_{v}=60$, the reduction in the normalized base shear capacity decreases for the higher $h / R_{b}$ values, and this is valid for the two levels of imperfections studied.

As discussed in section 2, another component of the hydrodynamic pressure that acts on the tank walls when subjected to a horizontal excitation is the convective pressure, which results from the sloshing on the water surface. Since the natural frequencies of the impulsive and sloshing vibration modes are well separated from one another, the analysis for each can be performed separately and then combined using the appropriate combination rule. A similar analysis procedure as that done for the impulsive component is repeated for the sloshing component. Since the impulse and connective hydrodynamic pressures act simultaneously, the effect of a geometric imperfection on the sloshing base shear capacity of the conical steel tanks is studied using the same imperfection pattern discussed earlier in the impulsive case. Figures 8 a to 8 c show how the sloshing base shear capacity represented by $\mathrm{V}_{\mathrm{S}} \mathrm{R}_{\mathrm{b}} / \mathrm{Wh}$ changes with the slenderness parameter $h / R_{b}$. Each subplot shows this variation for the two levels of geometric imperfections considered (i.e., good and poor) corresponding to a specific value for the angle $\theta_{\mathrm{v}}$ in addition to that in the case of perfect tanks. As for the governing failure mode of the group of conical steel tanks considered, the same observations as those made in the case of impulsive pressure are also valid in the case of sloshing pressure.

In the case of $\theta_{\mathrm{v}}=30$, an imperfection with an amplitude 0.004 L or less has no effect on the tank's normalized base shear capacity. On the other hand, in the case of where the imperfection amplitude equals 0.01 L , the reduction in the
normalized base shear capacity increases with a higher $h / R_{b}$. When $\theta_{v}=45$, it is found that an imperfection with an amplitude 0.004 L or less has no remarkable effect on the normalized base shear capacity for tanks with an $h / R_{b}$ less than 1.8 ; while for an imperfection amplitude of 0.01 L , the reduction in the normalized base shear capacity is higher for a larger $h / R_{b}$ until it reaches a value of 2 . Finally, for $\theta_{v}=60$ with an imperfection amplitude of 0.004 L , the reduction in the normalized base shear capacity is higher with a larger $h / R_{b}$ until a value of 2 is reached. A significant reduction in the normalized base shear capacity is observed in the case of a 0.01 L imperfection amplitude.

### 6.2 Vertical Excitation

Using the non-linear static analysis procedure described in section 4, the capacity of the conical steel tanks corresponding to the impulsive hydrodynamic pressure due to vertical excitations is obtained. The capacity of a conical steel tank can be represented by the total vertical force value at failure. The total vertical force for the impulsive component $N_{I}$ is represented in the form of the unit-less parameter $N h / W_{c} R_{b}$ in charts, where $W_{c}$ is the cylindrical part weight of the contained fluid.

Figures 9a to 9c show how the total vertical force capacity represented by $\mathrm{Nh} / \mathrm{W}_{\mathrm{c}} \mathrm{R}_{\mathrm{b}}$ changes with the slenderness parameter $h / R_{b}$. Each subplot shows this variation for the two considered levels of geometric imperfections, i.e., good and poor, corresponding to a specific value for the angle $\theta_{\mathrm{v}}$ in addition to the case of perfect tanks. For the group of conical steel tanks considered, the tank shell was found to yield before instability took place regardless of the level of geometric imperfections similar to what was noted by El Damatty et al. (1997a) for the case of conical steel tanks under hydrostatic pressure, which has an axisymmetric distribution similar to the case of the hydrodynamic pressure due to vertical excitations. The reduction in the normalized total vertical force capacity $\mathrm{Nh} / \mathrm{W}_{\mathrm{c}} \mathrm{R}_{\mathrm{b}}$ is found to increase as the tank walls become more inclined. The average percentage of the reduction for good tanks is found to be $40 \%$, $53 \%$, and $63 \%$ for $\theta_{\mathrm{v}}=30,45$, and 60 , respectively. For poor tanks, the average percentage of the reduction is found to be $69 \%, 83 \%$, and $95 \%$ for $\theta_{\mathrm{v}}=30,45$, and 60 , respectively.

## 7. CONCLUSIONS

In this study, the effect of geometric imperfections on the capacity of conical steel tanks subjected to horizontal and vertical excitations is assessed. First, the critical geometric imperfection distribution in both the meridional and the circumferential directions that leads to the minimum tank capacity is determined. Finally, the tanks' capacities corresponding to both the impulsive and the sloshing hydrodynamic pressure components are obtained using nonlinear static analysis where the geometric imperfections are incorporated within the nonlinear finite element model in the form of initial strains. For the critical imperfection, an axisymmetric distribution for the geometric imperfections is found to yield the lowest tank capacity corresponding to the impulsive hydrodynamic pressure due to both the horizontal and the vertical excitations. This is related to the presence of hydrostatic pressure, which tends to force the structure to buckle in an axisymmetric mode. Consequently, an imperfect shape matching this mode is the critical one. Regarding the reduction in the impulsive base shear capacity due to the inclusion of geometric imperfections, the following observations are made: (1) For tanks with $\theta_{\mathrm{v}}=30$, it is observed that an imperfection with an amplitude of 0.004 L or less has no remarkable effect on the normalized impulsive base shear capacity for tanks with an $h / R_{b}$ less than 1.1. In the case of an imperfection amplitude equal to 0.01 L , the reduction in the normalized base shear capacity increases with a higher $h / R_{b}$; (2) For tanks with $\theta_{v}=45$, it is found that an imperfection amplitude of 0.004 L or less has no remarkable effect on the normalized impulsive base shear capacity for tanks with an $h / R_{b}$ less than 1.0 ; while for an imperfection amplitude of 0.01 L , the reduction in the normalized base shear capacity is almost the same regardless of the value of $h / R_{b} ;$ (3) For tanks with $\theta_{v}=60$, the reduction in the normalized base shear capacity decreases for higher $h / R_{b}$ values for both good and poor tanks.

With regard to the reduction in the sloshing base shear capacity due to the inclusion of geometric imperfections, the following observations are made: (1) For tanks with $\theta_{\mathrm{v}}=30$, an imperfection with an amplitude of 0.004 L or less has no effect on the normalized base shear capacity. On the other hand, when the imperfection amplitude equals 0.01 L , the reduction in the normalized base shear capacity increases with a higher $h / R_{b}$; (2) For tanks with $\theta_{v}=45$, it is found that an imperfection with an amplitude of 0.004 L or less has no remarkable effect on the normalized base shear capacity for tanks with an $h / R_{b}$ less than 1.8 ; while for an imperfection amplitude of 0.01 L , the reduction in the normalized base shear capacity is higher for a larger $h / R_{b}$ until a value of 2 is reached; (3) For tanks with $\theta_{v}=60$ with an imperfection amplitude of 0.004 L , the reduction in the normalized base shear capacity is larger with a higher $\mathrm{h} / \mathrm{R}_{\mathrm{b}}$
until a value of 2 is reached. A significant reduction in the normalized base shear capacity is observed in the case of 0.01L imperfection amplitude.

For impulsive pressure due to vertical excitations, the reduction in the normalized total vertical force capacity $\mathrm{Nh} / \mathrm{W}_{\mathrm{c}} \mathrm{R}_{\mathrm{b}}$ is found to increase as the tank walls become more inclined. The average percentage of the capacity reduction for good tanks is found to be $40 \%, 53 \%$, and $63 \%$ for $\theta_{v}=30,45$, and 60 , respectively. For poor tanks, the average percentage of the reduction is found to be $69 \%, 83 \%$, and $95 \%$ for $\theta_{v}=30,45$, and 60 , respectively. Finally, for the governing failure mode for perfect tanks subjected to hydrodynamic pressure due to horizontal excitation, the general trend is that the probability of yielding failure occurring is higher when the angle $\theta_{\mathrm{v}}$ is increased. In the case of imperfect conical steel tanks, the probability of inelastic buckling taking place increases with a higher geometric imperfection amplitude. On the other hand, the tank shell is found to yield before instability takes place regardless of the level of the geometric imperfections in the case of vertical excitations.


Figure 7: Effect of level of geometric imperfections on the normalized impulsive base shear capacity: (a) $\theta_{v}=30$, (b) $\theta_{v}=45$, and (c) $\theta_{v}=60$


Figure 8: Effect of level of geometric imperfections on the normalized sloshing base shear capacity: (a) $\theta_{\mathrm{v}}=30$, (b) $\theta_{\mathrm{v}}=45$, and (c) $\theta_{\mathrm{v}}=60$


Figure 9: Effect of level of geometric imperfections on the normalized impulsive vertical force capacity: (a) $\theta_{\mathrm{v}}=30$, (b) $\theta_{\mathrm{v}}=45$, and (c) $\theta_{\mathrm{v}}=60$

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