RESILIENT INFRASTRUCTURE
June 1–4, 2016

METHODS OF COMPARING EXTREME LOAD EFFECTS BASED ON WEIGH-IN-MOTION DATA

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ABSTRACT

The estimation of extreme load effects caused by vehicles is of critical importance in the evaluation and design of bridge structures. Two methods for estimating extreme load effects over the service life of bridges are commonly cited in literature: (1) the use of a fitted probability distribution based on statistical data to extrapolate the extreme load effects on a probability plot, or (2) the application of Monte Carlo simulation to generate representative truck data over a bridge's lifespan such that maximum load effect values can then be determined directly. In this paper, results obtained using the two aforementioned methods are presented including their advantages and disadvantages in the context of the analysis of rural bridges in Saskatchewan. For this purpose, estimated load effects are based on truck data recorded over a period of one year at several weigh-in-motion (WIM) stations located on Saskatchewan highways. The conducted analyses are based on a typical bridge type common to rural Saskatchewan. It was found that the Monte Carlo simulation approach resulted in more reliable extreme load effect estimations, along with providing other information that is of value in the development of new truck loading models.

Keywords: extreme load effects; WIM data; Monte Caro simulation; rural bridges.

1. INTRODUCTION

The basic equation defining the ultimate limit state in the load and resistance factor (LRFD) design method is:

[1] $\phi R_n \geq \gamma_Q Q_n$, as illustrated in 0:

Where: R_n, Q_n= nominal resistance and nominal (or design) load effect, respectively. $R_n = \delta_R \mu_R$; $Q_n = \delta_Q \mu_Q$.
μ_R, μ_Q= mean value of resistance and load effect, respectively
ϕ, γ_Q, δ_R, δ_Q= resistance factor, load factor, bias factor of resistance and of load effect, respectively

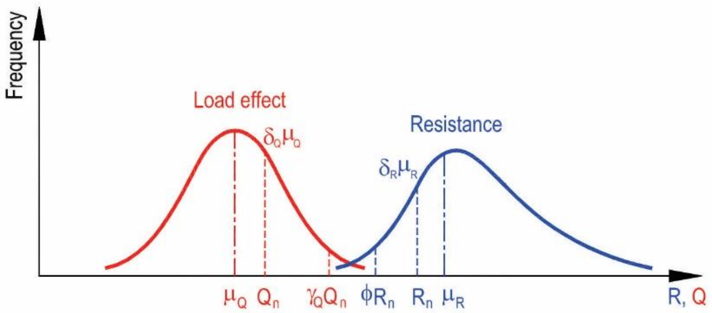

Figure 1: Relationship between mean, nominal and factored values of load effect and resistance

In defining design truck loads in bridge codes, the mean value of the load effect (μ_Q) is not based on one specific truck type experienced routinely by the bridges in question; in fact, load effects caused by typical truck types would be much lower than those due to the nominal loads used for design purposes. This is illustrated in Figure 2, where peak bending moments in a typical 6 m bridge caused by actual truck traffic observed at a WIM station located near St. Dennis, Saskatchewan, are compared with those generated by the design (CL625) truck as defined by the Canadian Highway Bridge Design Code (CHBDC) (CSA 2014). From the histogram plotted on the left of Figure 2, it is evident that the ratio between observed and design peak truck load effects over the observation period was consistently well below unity; when this same data is plotted on the inverse cumulative distribution function (CDF) plot shown on the right of Figure 2, the mean value of the observed-to-design peak load effect is seen to be approximately equal to 0.4. For the purposes of structural design, however, only the largest load effect values are generally of interest in establishing design requirements. In the CHBDC and American Association of State Highway and Transportation Officials (AASHTO) LRFD Bridge (AASHTO 2012) codes, for example, the mean nominal load effect μ_Q was extrapolated from observed truck data to correspond to a design return period of 75 years. Therefore, the load effect (red) curve in 0 does not represent the distribution of daily traffic load effects but, rather, the projected maximum load effects over a specified (for example, annual) period.

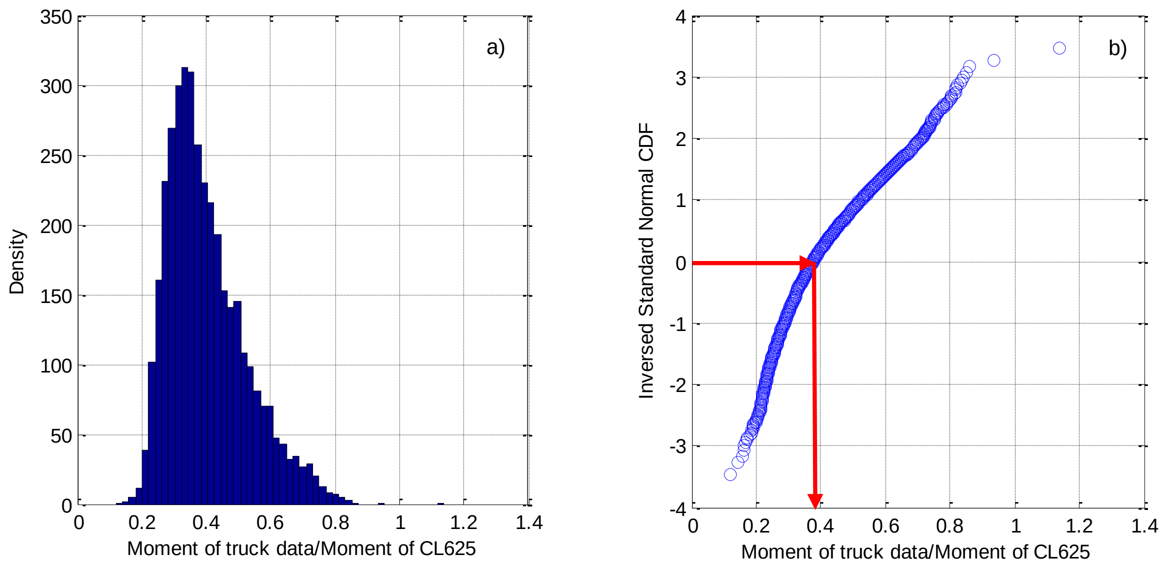

Figure 2: Histogram (a) and probability plot on normal probability paper (b) of truck load effect.

The estimation of lifetime load effects in bridge structures under truck loads has been considered by many researchers. Depending on factors such as the method of collecting truck data, the total number of trucks in a data set, the location of the collected data, etc. the methods applied to estimate the extreme truck load effects have varied.

As an example of one common method, Nowak and his colleagues (Nowak 1994, Nowak and Hong 1991, Nowak, 1993, Nowak and Collin 2000) applied a fitted distribution model based on statistical distributions of load effects for traffic data observed over relatively short periods. In this approach, extrapolation on normal probability plots (based on an inverse cumulative distribution function of the standard normal variable, Φ^{-1}) is used to determine the mean extreme value of the maximum load effects (bending moments and shear) for different time periods, such as 50 years or 75 years, from the short-period survey data. 0 shows a typical example of the extrapolation method used by Nowak and his co-workers incorporating statistical data from truck surveys of nearly 10,000 heavy trucks on Ontario highways carried out by the Ontario Ministry of Transportation in the 1970s (Nowak and Hong 1991). For the extrapolation procedure, it was assumed that live load effects could be described by a normal probability distribution function and that the target return period was equal to the nominal bridge lifetime (75 years). The mean maximum value of load effect was extrapolated corresponding to the bridge lifetime.

The other common approach involves the use of Monte Carlo simulation to determine extreme load effects over the lifetime of a bridge. For example, this method was employed in several studies by O'Brien, Enright and their colleagues (Enright and Caprani 2011, O'Brien, Enright and Getachew 2010, Enright and O'Brien 2012). As shown in 0, the Monte Carlo simulation method was used to generate truck data for four locations in Europe which were then used directly to determine extreme load effects for a return period of 1000 years.

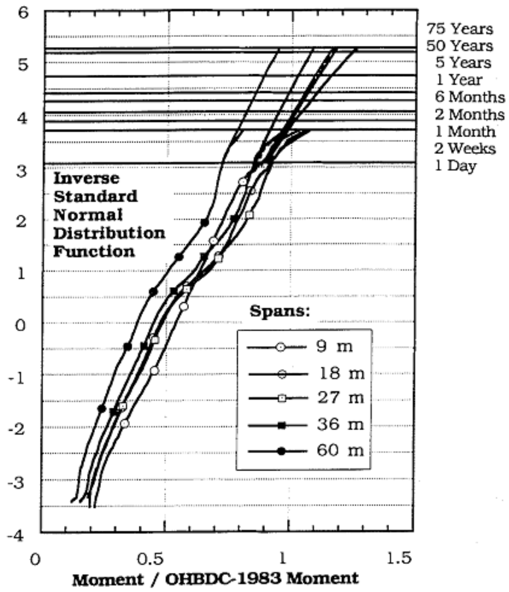

Figure 3: Extrapolation for extreme maximum moment in girder bridge (Nowak, 1994)

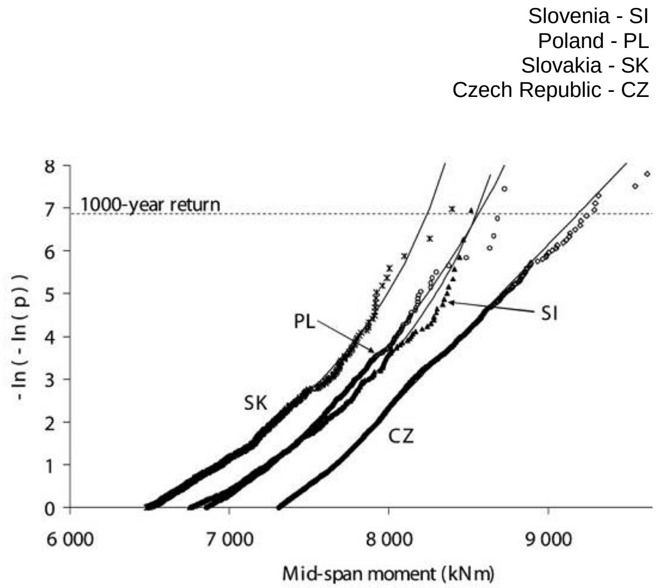

Figure 4: Extrapolation for extreme maximum moment in girder bridge (Enright and O'Brien, 2012)

The extrapolations of Nowak et al. were based on the following equations:

[2] $F_{max}(x) = [F(x)]^n$

[3] $f_{max}(x) = nf(x)[F(x)]^{n-1}$

in which:

F(x), f(x) - CDF and PDF functions of load effect from the parent distribution of observed truck data, respectively

$F_{max}(x)$, $f_{max}(x)$ - CDF and PDF functions, respectively, of the extreme load effect in a statistical order size, n, in period, T (years)

Examples of PDF and CDF distributions for various return periods are shown in 0, based on an assumed normally distributed parent population using average daily traffic (ADT) values typical of rural Saskatchewan.

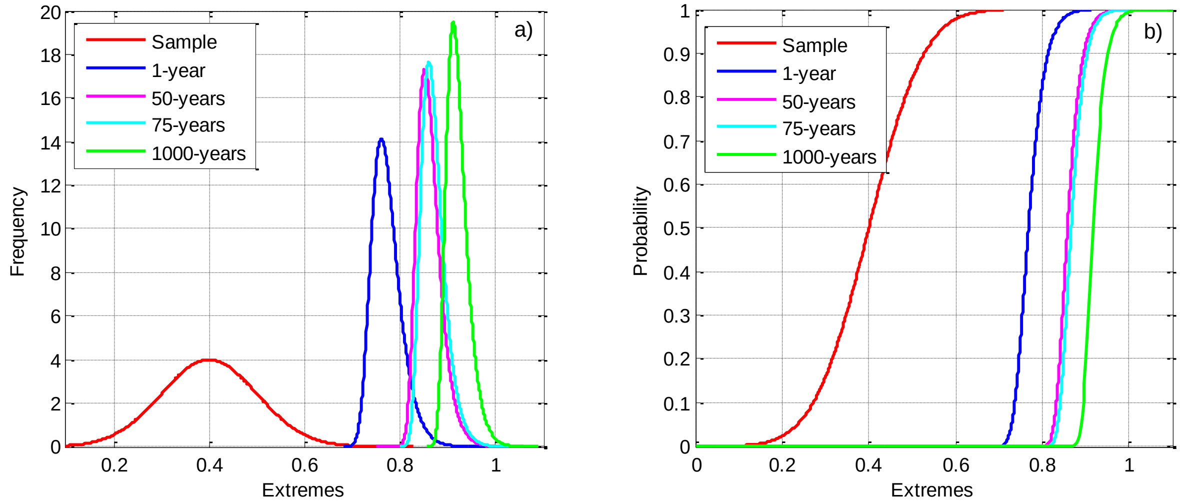

Figure 5: Plots of extreme load effects corresponding to different time-periods: (a) PDF, and (b) CDF

From a theoretical point of view, the assessment of the best (or better) method for determining extreme values in statistics is not well defined. As a result, to aid in the selection of an appropriate method for use with WIM truck data in Saskatchewan, a series of numerical experiments were carried out to compare extreme predictions using the two approaches described above. The present study focused on a prevalent heavy truck configuration allowed to travel on Saskatchewan roads, namely a 5-axle configuration 7 vehicle (a tractor with a single steering axle and a tandem drive combined a semi-trailer with a tandem).

2. WEIGH-IN-MOTION DATA IN SASKATCHEWAN

The WIM technique has been developed to a point where and it can provide accurate results for a traffic survey. WIM systems include sensors embedded in a roadway that measure and record axle weights and configurations, vehicle speeds, etc. In recent years, the WIM technique has been implemented for collecting traffic data in Saskatchewan, with 12 WIM stations located throughout the province to obtain detailed data of traffic on major highways (see Table 1 below for specific locations). In this study, WIM data collected over a period of one year will be used as the main source of information on traffic characteristics with the assumption that this data is accurate and reliable, and that it can be used to obtain the representative data of large truck traffic on rural roads.

Table 1. Names and locations of WIM stations in Saskatchewan

No	Names	Locations	Notes
1	Grand Coulee	Highway 1	
2	Fleming	Highway 1	
3	Alsask	Highway 7	
4	Roche Percee	Highway 39	
5	Maple Creek West	Highway 1	
6	Lang	Highway 39	
7	Farley	Highway 14	
8	Maidstone	Highway 16	
9	Maymont	Highway 16	
10	Bethune	Highway 11	
11	Bredenbury	Highway 16	
12	St. Denis	Highway 5	

Sample PDF and CDF plots of peak midspan bending moments based on a representative 6.1 m simple-span bridge are shown in 0, considering data from truck configuration 7-5 axle truck events observed during 2013 at the WIM station near Maidstone, SK. For this case, a lognormal probability distribution is seen to provide the best fit as compared to peak calculated load effects. On the other hand, similar plots shown in 0 representing load effects based on a WIM data from a station near Maple Creek, SK, suggest that a normal probability distribution provides a better fit for that location.

Since the best-fit distributions did tend to vary based on the WIM station location, it was not possible at this stage to identify a single distribution that adequately describes the load effects for all the locations considered. Because of the observed differences in probability distributions, truck data from each WIM station was processed independently in the current study. It can be stated, though, that the data at the majority of weigh stations did not result in normally distributed peak load effect distributions, as assumed by Nowak and his co-workers. This finding is significant since the estimation of extreme values for a given parameter is influenced strongly by the form of the underlying probability distribution.

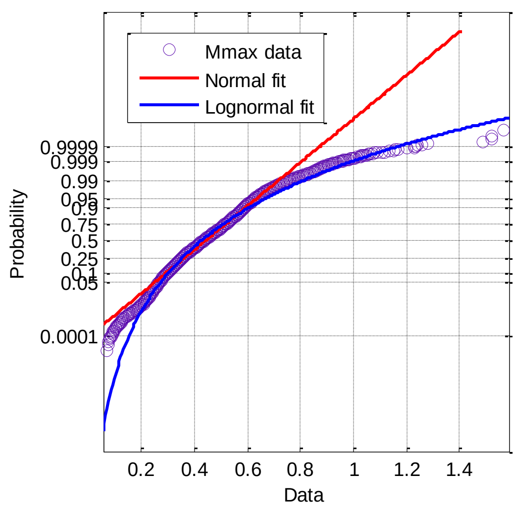

Figure 6: Probability plot of ratio of maximum moment of truck data/CL-625 at midspan of bridge. (6.1m bridge span; Truck configuration 7-5axles, based on WIM data from Maid Stone, 2013, SK)

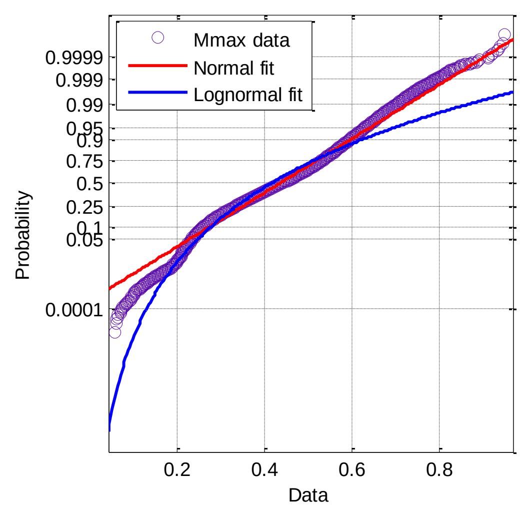

Figure 7: Probability plot of ratio of maximum moment of truck data/CL-625 at midspan of bridge. (6.1m bridge span; Truck configuration 7-5axles, based on WIM data from Maple Creek, 2013, SK)

3. EXPERIMENTS TO DETERMINE MAXIMUM LIFETIME LOAD EFFECTS

Based on the statistical characteristics inferred from the WIM data, a series of numerical experiments were performed to compare various approaches for estimating extreme values of peak load effects in bridges over various time periods. In this exercise, peak midspan bending moments in a 6.1 m simple span bridge were normalized by the corresponding response caused by the CL625 design truck as defined in the CHBDC to obtain a normalized response ratio. Extreme values of this normalized response ratio were then generated using the methodologies described below.

3.1. Experiment 1

In the first experiment, it was assumed that the normalized response ratio could be represented as a continuous variable with a normal probability distribution defined by a mean value of $\mu = 0.4$ and a standard deviation of $\sigma = 0.1$ (close to observed values from the WIM data at several locations in Saskatchewan). Using this assumed normal distribution, a set of data was randomly generated to represent the total number large truck events over a 1,000 year period at a bridge site experiencing approximately 6,000 truck events annually (i.e. a total number of N=6,000,000 random truck loading events were generated); this data set was then assumed to represent the total population of lifetime truck events. From this population, a random sample was drawn of a size equal to the expected number of truck events in one year (i.e., n=6,000) which were treated as sample of measured data. The six different methods described below were then used to produce estimates of the extreme load effect over time periods of 1 year, 50 years and 75 years.

- **Exact Method**: From the exact function of the continuous normal variable, extreme value theory was applied to construct the PDF and CDF functions of extreme values for the different periods. The mean value and coefficient of variation (COV) of the extreme values were determined from these functions using numerical methods.

- **Fitted Method**: From a distribution function fitted to the set of sample data from one year, extreme value theory was used to construct the PDF and CDF functions of extreme values for different periods. As in the exact method, the mean and COV of each function were calculated by numerical methods.
- **Extrapolated Exact Method**: From the exact function of the continuous normal variable, a straight line was plotted on normal probability paper to determine the extreme values at intersection points corresponding to the periods.
- **Extrapolated Fitted**: Using the distribution fitted to the randomly selected annual sample (n=6,000 events), a straight line was plotted on normal probability paper and used to determine the extreme values at intersection points corresponding to the periods of interest.
- **Sample Method:** Multiple randomly selected samples of size, n, corresponding to different periods (where n=6,000 x number of years in the sampling period) were taken from the total population of truck events. The maximum normalized response ratio (maxima) was extracted from each sample and used to determine the mean value and COV of the peak response ratio across all the samples.
- **Monte Carlo Simulation**: Using a probability distribution fitted to data from a random selected annual sample (n=6,000 events), the Monte Carlo Simulation method was applied to generate new samples of a sizes corresponding to the periods of interest. The maximum normalized response ratio (maxima) was extracted from each sample and used to determine the mean value and COV of the peak response ratio across all the samples.

Sample results from experiment 1 are illustrated in 0 and 9. 0 shows the results from the Extrapolated Fitted method; the graph on the right-hand side of this figure shows an expanded view of the data at the upper end of the plot. The fitted normal distribution (green line) based on the annual sample is plotted on normal probability paper and extrapolated to horizontal lines representing different periods (1 year, 50 years, 75 years or 1000 years) to find corresponding mean extreme values. 0, on the other hand, shows plots of PDF functions of both the annual sample (red line) and the 75-year extreme values (blue line). Graphs are provided for data based on the exact continuous function on the left of 0 and the Monte Carlo Simulation method on the right; here, a generalized extreme value (GEV) distribution has been fit to the Monte Carlo simulated data. It is evident that distributions based on the Monte Carlo Simulation data closely resemble those derived using the exact probability functions.

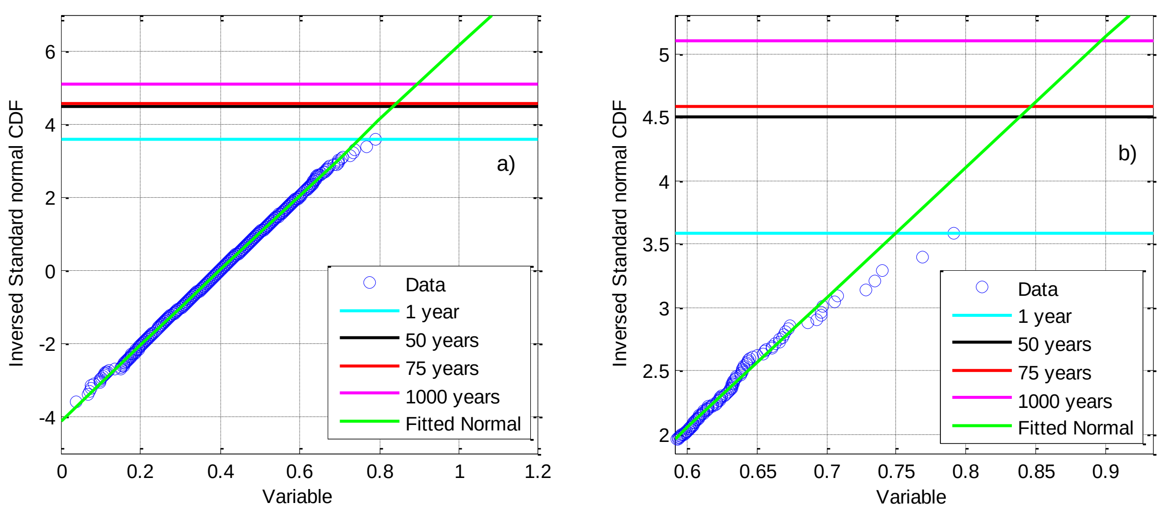

Figure 8: Extrapolated fitted method of extreme value from a basic sample data

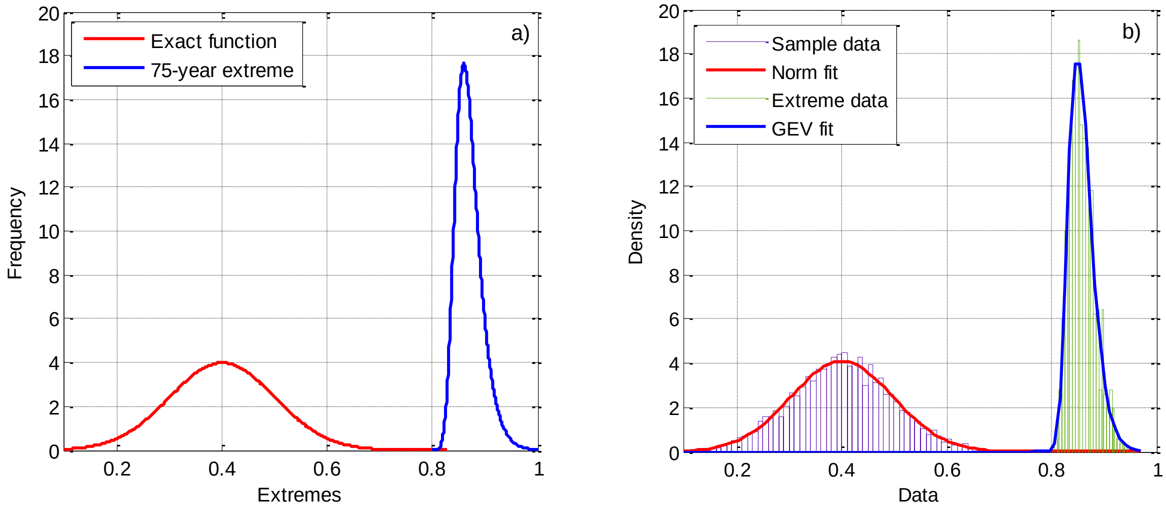

Figure 9: PDF of sample and 75-year extreme of continuous function and Monte Carlo simulation

Detailed results from Experiment 1 listing mean extreme values of the normalized response ratio and their associated coefficients of variations (COV) for the six methods are showed in 0 and 3; since the two extrapolation methods are graphically based, estimates for the COV could not be determined for those methods. Numbers shown in parenthesis represent the difference between the result for a given method and the corresponding value from the Exact method. It is apparent that, if the distribution for the load effect variable is known exactly, all of the methods considered produce very similar results, with mean extreme value estimates differing from the exact value by less than 1.5% in most cases, and 3% overall. Even under these conditions, however, the Monte Carlo Simulation method is seen to provide extreme value estimates that are closer to exact values than the Extrapolated Fitted method, which is significant since those are the two methods that can be applied to real traffic data for which the exact distribution and total population of possible events are unknown.

Table 2: Comparison of mean extreme values of the normalized response ratio (μ) for Experiment 1

No	Extreme value	Exact	Fitted	Extrap Exact	Extrap Fitted	Sample	MC Simul
1	1 year	0.7724	0.7626 (1.27%)	0.7588 (1.76%)	0.7494 (2.98%)	0.7702 (0.28%)	0.7628 (1.24%)
2	50 years	0.8618	0.8496 (1.42%)	0.8504 (1.32%)	0.8385 (2.70%)	0.8540 (0.91%)	0.8498 (1.39%)
3	75 years	0.8702	0.8578 (1.42%)	0.8589 (1.30%)	0.8468 (2.69%)	0.8617 (0.98%)	0.8586 (1.33%)

Table 3: Comparison of coefficients of variation (COV) of extreme normalized response for Experiment 1

No	COV value	Exact	Fitted	Extrap Exact	Extrap Fitted	Sample	MC Simul
1	1 year	0.04052	0.03993 (1.46%)	-	-	0.03992 (1.48%)	0.04117 (1.60%)
2	50 years	0.03013	0.02973 (1.33%)	-	-	0.03031 (0.60%)	0.02998 (0.05%)
3	75 years	0.02937	0.02899 (1.29%)	-	-	0.02947 (0.34%)	0.02992 (0.58%)

3.2. Experiment 2

In reality, the true form of the probability distribution describing a specific load effect, along with the parameters needed to fully define any given distribution, are unknown. Therefore, the accuracy of the extreme value estimate will depend on the selection of the type of distribution fitted to the observed data, and to the parameters used to define that distribution. In some cases, in fact, a best fit model cannot be found that provides consistently close agreement to observed data, introducing the possibility of greater errors in the estimation of the extreme value of that load effect. In Experiment 2, therefore, it was assumed that the probability distribution of the load effect was unknown, but that the distributions of selected variables required to define the truck load (i.e., axle spacing and weights) were known or assumed.

A population of 5-axle truck events experienced by the bridge over a 1,000 year period (with N=6,000,000) was generated based on the assumed parameter distributions shown in 0, which were selected based on typical trends in observed WIM data. From this population, a sample representing observed data for a one year period (with n=6,000) was randomly selected. Methods similar to those described in Experiment 1 were subsequently applied to obtain estimates of extreme values of normalized response ratios over the different periods of interest. In the Fitted and Extrapolated Fitted methods, the one year sample was used to calculate normalized response ratios (as defined for Experiment 1) for each truck event within sample. In the Monte Carlo Simulation, the one year sample was used to generate other truck event samples. In the Sample method, samples with sizes corresponding to the different periods were selected randomly from the population; since the exact distribution of the load effects was unknown in this case, the Exact and Extrapolated Exact methods could not be used.

Table 4: Probability distribution parameters describing 5-axle truck loading events for Experiment 2

No	Names of variables	Parameters: μ, σ, MP*	Model of distribution
1	Axle distance: L_{12}	μ_1=5.2 (m); σ_1=0.35 (m); MP_1=70% μ_2=3.8 (m); σ_2=0.25 (m); MP_2=30%	Bi-modal normal
2	Axle distance: L_{23}	μ_1=1.31 (m); σ_1=0.018 (m); MP_1=60% μ_2=1.38 (m); σ_2=0.022 (m); MP_2=33% μ_3=1.52 (m); σ_3=0.048 (m); MP_3=7.0%	Tri-modal normal
3	Axle distance: L_{34}	μ_1=10.8 (m); σ_1=0.32 (m); MP_1=40% μ_2=7.60 (m); σ_2=1.52 (m); MP_2=28% μ_3=10.2 (m); σ_3=0.69 (m); MP_3=32%	Tri-modal normal
4	Axle distance: L_{45}	μ_1=1.83 (m); σ_1=0.016 (m); MP_1=2% μ_2=1.24 (m); σ_2=0.012 (m); MP_2=80% μ_3=1.48 (m); σ_3=0.10 (m); MP_3=18%	Tri-modal normal
5	Axle weight: W_1	μ=4.7 (T); σ=0.93 (T)	Normal
6	Axle weight: W_2	μ_{log}=1.58 (T); σ_{log}=0.28 (T)	Lognormal
7	Axle weight: W_3	μ_{log}=1.56 (T); σ_{log}=0.29 (T)	Lognormal
8	Axle weight: W_4	μ_{log}=1.38 (T); σ_{log}=0.38 (T)	Lognormal
9	Axle weight: W_5	μ_{log}=1.37 (T); σ_{log}=0.37 (T)	Lognormal

*MP = mixing proportion

Comparisons of the mean extreme values and COV's of the normalized response ratios determined from the various methods in Experiment 2 are provided in the 0 and 6. Since the exact method was unavailable in this experiment, the sample method was used as the benchmark against which other methods were compared since the truck loading event data used in this approach were selected directly from the population with no further simulation or extrapolation required. It is apparent that, when using known distributions to describe truck loading parameters rather than assuming the distribution of load effects directly, the Monte Carlo method still provides estimates of mean extreme values that were within approximately 1% of the benchmark values for all periods of interest, along with COVs within 5% of benchmark values. In contrast, the Fitted method produced mean extreme value estimates that differed from benchmark values by up to 6% and, significantly, COV values that differed from benchmark values by up to 27%.

Table 5: Comparison of mean extreme values of the normalized response ratio (μ) for Experiment 2

No	Extreme values	Exact	Fitted	Extrap Exact	Extrap Fitted	Sample	MC Simul
1	1 year	-	0.9337 (3.08%)	-	0.9093 (0.30%)	0.9058	0.9089 (0.34%)
2	50 years	-	1.0921 (5.93%)	-	1.0725 (4.03%)	1.0308	1.0345 (0.36%)
3	75 years	-	1.1067 (5.98%)	-	1.0874 (4.14%)	1.0442	1.0437 (0.05%)

Table 6: Comparison of coefficient of variation (COV) of extreme normalized response ratio for Experiment 2

No	COV	Exact	Fitted	Extrap Exact	Extrap Fitted	Sample	MC Simul
1	1 year	-	0.05847 (18.24%)	-	-	0.04945	0.04795 (3.03%)
2	50 years	-	0.04119 (20.26%)	-	-	0.03401	0.03341 (1.76%)
3	75 years	-	0.03978 (27.30%)	-	-	0.03125	0.03280 (4.96%)

3.3. Experiment 3

It is further recognized that the probability distributions of parameters defining the truck loading model are often unclear. From the available WIM data at different stations in Saskatchewan, it was found that that, in most cases, the distribution model for axle-distances could reasonably be fitted to normal, bi-modal normal, or tri-modal normal distributions, while axle-weight distribution models were similar to lognormal or normal or generalized extreme value or gamma distributions. However, it can be surmised that the upper tail of the axle-weight probability distribution and the lower tail of axle-distance distribution, in particular, will exert a significant influence on extreme value estimates of the load effects in short bridge structures.

To compare the sensitivity of extreme value estimates to the selection of distribution models for the parameters defining the truck load, Monte Carlo simulations similar to those performed in Experiment 2 were repeated using truck loading parameter distributions described in 0. In the first Monte Carlo simulation, the distributions for all nine truck loading parameters were selected to be of a different form than what was considered to be the optimal fit to observed data (i.e., those used in Experiment 2). In the second Monte Carlo simulation, only the axle-weight parameters were assumed to have distributions that differed from optimal. In all cases, non-optimal distributions were selected such that they would produce conservative results. In the following discussion, the data used for the Sample and Fitted method remain unchanged from Experiment 2.

Table 7: Probability distribution parameters describing 5-axle loading events for Experiment 3

No	Names of variables	Exact models of distribution	Fitted models in MC Simul 1	Fitted models in MC Simul 2
1	Axle distance: L_{12}	Bi-modal normal	Normal	Bi-modal normal
2	Axle distance: L_{23}	Tri-modal normal	Bi-modal normal	Tri-modal normal
3	Axle distance: L_{34}	Tri-modal normal	Bi-modal normal	Tri-modal normal
4	Axle distance: L_{45}	Tri-modal normal	Bi-modal normal	Tri-modal normal
5	Axle weight: W_1	Normal	Gamma	Gamma
6	Axle weight: W_2	Lognormal	GEV	GEV
7	Axle weight: W_3	Lognormal	GEV	GEV
8	Axle weight: W_4	Lognormal	GEV	GEV
9	Axle weight: W_5	Lognormal	GEV	GEV

Comparing the results for Experiment 3 provided in 0 and 9 with those from Experiment 2, it can be concluded that the best fit forms of the truck loading parameter probability distributions had a negligible effect on the mean extreme value estimates, as well as a small effect on the associated COV values. This result suggests that extreme value predictions obtained using the Monte Carlo Simulation method are relatively insensitive to uncertainties in the form of the truck loading parameter distributions.

Table 8: Comparison of mean extreme values (μ)

No	Extreme values	Fitted	Extrap Fitted	Sample	MC Simul 1	MC Simul 2
1	1 year	0.9337 (3.08%)	0.9093 (0.30%)	0.9058	0.9164 (1.17%)	0.9129 (0.78%)
2	50 years	1.0921 (5.93%)	1.0725 (4.03%)	1.0308	1.0405 (0.94%)	1.0365 (0.55%)
3	75 years	1.1067 (5.98%)	1.0874 (4.14%)	1.0442	1.0483 (0.39%)	1.0498 (0.54%)

Table 9: Comparison of coefficient of variation (COV)

No	COV	Fitted	Extrap Fitted	Sample	MC Simul 1	MC Simul 2
1	1 year	0.05847 (18.24%)	-	0.04945	0.04977 (0.65%)	0.05266 (6.49%)
2	50 years	0.04119 (20.26%)	-	0.03401	0.03390 (0.32%)	0.03326 (2.21%)
3	75 years	0.03978 (27.30%)	-	0.03125	0.03022 (3.29%)	0.03248 (3.94%)

4. SUMMARY AND CONCLUSIONS

Numerical simulations were carried out to compare extreme value estimates for critical load effects over the lifetime of short-span bridges based on WIM truck loading data obtained from 12 locations on the Saskatchewan highway system. Of particular interest was the comparison between the accuracy and reliability of extreme value estimates derived using two approaches commonly cited in the literature, namely graphical extrapolation methods using probability distributions fitted to short-term data and methods based on Monte Carlo simulation.

If the precise form of the underlying probability distribution for the critical load effect (peak midspan bending moments, in this case) was known or assumed, all methods considered generated similar estimates of the mean extreme value, as well as comparable levels of variability in that estimate. If, on the other hand, the load effect distribution was assumed to be unknown, but, instead, the probability distributions of the parameters defining the governing truck load model were known or assumed, the Monte Carlo method was found to provide a slightly more accurate result, as well as featuring substantially less variability in the extreme value estimates. Since this second scenario is the more realistic approximation of the situation in practice, the Monte Carlo method appears to be the preferred approach based on the results of this study.

In addition, it was found that the accuracy and reliability of extreme value predictions based on the Monte Carlo simulation method were relatively insensitive to the precise form of the assumed truck loading model parameter distributions, as long as those distributions provided a reasonable fit to the observed data. This apparent robust nature of the Monte Carlo simulation results is thought to be very beneficial, given the variability in the traffic data observed at the different WIM stations considered.

ACKNOWLEDGEMENTS

This research project is being funded by the VIED Fellowship of Ministry of Education of Vietnam, the Saskatchewan Association of Rural Municipalities, Saskatchewan Ministry of Highways and Infrastructure and the University of Saskatchewan.

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