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ADDRESSING THE ISSUES OF MODAL IDENTIFICATION USING TENSOR DECOMPOSITION

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ABSTRACT

Modal identification has been an indispensable tool for condition assessment of critical civil infrastructure. Recently several signal processing techniques including time-frequency analysis have shown significant success in addressing wide range of challenges in modal identification of flexible structures. In a parallel development, tensor decomposition is explored as an attractive and versatile system identification tool that can use even a limited number of vibration sensors to estimate the modal parameters under ambient excitations. In this paper, the performance of tensor decomposition is evaluated for modal identification of a building model under a multitude of earthquake excitations.

Keywords: Modal identification; tensor decomposition; nonstationary excitation; stationary durations; PGA; earthquake

1. INTRODUCTION

As large infrastructure and building age, they may be subjected to significant seismic and climatic events that cause deterioration in structural elements. Structural health monitoring (SHM) (Doebling et al. 1996) is an important tool to help predict and reduce the risk of a failure that has the potential to cause property loss, injurers or loss of life. SHM has evolved from input-output analysis to output-only analysis, due to the large costs of equipment and feasibility of measuring inputs for large scale civil structures. Modal identification (Maia and Silva 2001) is one of the key components of SHM where useful modal parameters (like frequency and mode shapes) are extracted from vibration measurements and subsequently used for condition assessment. This paper examines the potential of tensor-decomposition based modal identification using vibration data under earthquake excitation.

Time-frequency and time-scale analysis methods employing wavelet transform (Guo and Kareem 2015), Hilbert transform (Huang et al. 1998), empirical mode decomposition (EMD) (Darryll and Liming 2006; Sadhu 2015) and blind source separation (BSS) techniques (Sadhu 2013) are explored towards modal identification under nonstationary excitations. In parallel, tensor decomposition based methods (also known as parallel factor (PARAFAC) decomposition) have also garnered significant attention in the area of modal identification. In tensor decomposition methods, a covariance tensor is constructed from vibration measurements under multiple lags and is subsequently decomposed into co-variances of hidden sources using multilinear algebra tools such as alternating least squares (ALS) (Smilde et al. 2004; Mokios et al. 2006). Owing to the matricization operation among various lags, tensor decomposition-based methods are versatile and can be undertaken for both complete as well as partial measurement cases.

Antoni and Chauhan (Antoni and Chauhan 2011) used alternating least squares (ALS) to solve tensor decomposition associated with modal identification using a limited number of sensors. The underdetermined source separation capability of PARAFAC decomposition was recently explored to identify the modal parameters of high-rise building (Abazarsa et al. 2013; Mcneill 2012; Abazarsa et al. 2015) and a structure equipped with tuned-mass damper (Sadhu et al. 2014) using limited sensor measurements. PARAFAC decomposition is recently integrated with wavelet packet transform (WPT) to improve the source separation capability where mode-mixing in the WPT coefficients is alleviated

using PARAFAC decomposition (Sadhu et al. 2013, Sadhu et al. 2015). However, none of the above studies investigate the performances of PARAFAC decomposition under earthquake excitation. This is the motivation of the proposed research where the PARAFAC decomposition is studied under a wide range of nonstationary ground motions and the resulting performances are discussed.

2. BASICS OF PARALLEL FACTOR DECOMPOSITION

Tensor representation of a higher dimensional signal enables use of multilinear algebra tools that are more powerful than linear algebra tools. A vector is a first-order tensor, whereas a matrix is second-order tensor. In general, a *p*-th order tensor is written as:

[1] $\quad \underline{\mathbf{H}} = h_{ijk...p} \in \Re^{n_1 \times n_2 \times n_3 \times .... \times n_p}$.

A third-order tensor is primarily decomposed into a sum of outer products of triple vectors (Bro 1997):

[2] $\quad \underline{\mathbf{H}} = \sum_{r=1}^{R} \alpha_r \circ \beta_r \circ \gamma_r \iff \underline{H}_{ijk} = \sum_{r=1}^{R} \alpha_{ir}\beta_{jr}\gamma_{kr}$

where "$\circ$" denotes outer product with $i \in [1\ \mathrm{I}]$, $j \in [1\ \mathrm{J}]$ and $k \in [1\ \mathrm{K}]$. Here R is the number of rank-1 tensors present in $\underline{\mathbf{H}}$. This is also defined as trilinear Model of $\underline{\mathbf{H}}$, where $\underline{\mathbf{H}} = [\bar{\alpha}, \bar{\beta}, \bar{\gamma}]$, Each triple vector product is a rank-1 tensor, namely PARAFAC component. Eq. 2 represents the summation of R such PARAFAC components that fits the higher order tensor *H* (Bro 1997; Lathauwer and Castaing 2008). The technique was introduced simultaneously in two different independent works: canonical decomposition (CANDECOMP) (Carroll and Chang 1970) and PARAFAC analysis (Harshman 1970). The details of this method are not repeated herein and can be found in above references.

3. EQUIVALENCE OF MODAL IDENTIFICATION WITH PARAFAC DECOMPOSITION

Consider a linear, classically damped, and lumped-mass n_s-degrees-of-freedom (DOF) dynamical system, subjected to an excitation force, *F*(t).

[3] $\quad \mathbf{M\ddot{x}}(t) + \mathbf{C\dot{x}}(t) + \mathbf{Kx}(t) = \mathbf{F}(t)$

where, **x** is a vector of displacement coordinates at the DOFs. M, C, and K are the mass, damping and stiffness matrices of the multi-degree-of-freedom system. The solution to Eq. 3 can be expressed in terms of modal superposition of vibration modes with the following matrix form:

[4] $\quad \mathbf{x} = \mathbf{\Psi}\nu$

The covariance matrix $\mathbf{C_x}(\tau_k)$ of vibration measurements x evaluated at time-lag τ_k can be written as:

[5] $\quad \mathbf{C_x}(\tau_k) = E\left\{\mathbf{x}(n)\mathbf{x}^T(n-\tau_k)\right\} = \mathbf{MC_s}(\tau_k)\mathbf{M}^T$

where,

[6] $\quad \mathbf{C_s}(\tau_k) = E\left\{\mathbf{s}(n)\mathbf{s}^T(n-\tau_k)\right\}$.

For any general n_s-DOF dynamical model, above equation can be simplified as (Sadhu et al. 2013):

[7] $\quad C_{ijk}^x = \sum_{r=1}^{n_s} m_{ir}m_{jr}C_{kr}^s \iff \mathbf{C^x} = \sum_{r=1}^{n_s} \mathbf{m_r} \circ \mathbf{m_r} \circ \mathbf{C_r^s}$.

Considering the similarity between Eq. 2 and Eq. 7, it is seen that by decomposing the third order tensor C^x into n_s number of PARAFAC components (i.e., modal responses), the mixing matrix can be estimated. By using PARAFAC decomposition of C_x, the resulting solutions yield the mixing matrix M = [m$_1$, m$_2$, m$_3$, …, m$_{ns}$] and the autocorrelation

function C_r^s for r = 1, 2, 3, …, n_s from which the natural frequency and damping of the individual modal responses can be estimated.

4. NUMERICAL ILLUSTRATION

A dynamical system with 5 degree-of-freedoms is selected to demonstrate the performance of PARAFAC method under base excitations. The natural frequencies of the model are 0.91, 3.4, 7.1, 10.7 and 12.7 Hz, respectively. The model is subjected to a suite of ground motions and the resulting vibration responses are processed through PARAFAC to extract the modal parameters under a wide range of nonstationary excitations. Table 1 shows six typical ground motions selected for preliminary study along with the detailed information of peak ground acceleration (PGA) and ground motion duration (T). The extent of non-stationarity is characterized by the ratio of stationary duration (T_s) (Trifunac and Brady 1975) and T. T_s (Trifunac and Brady 1975) may be computed using the time interval containing the energy envelope between 5 and 95 percent of the total energy of an earthquake. An earthquake is considered stationary when a majority of the energy is released between 5 and 95 percent of the total earthquakes duration. On the other hand, when this ratio attains lower value say, ≤ 0.3, the earthquake (i.e., NR and PF) is extremely non-stationary in nature in time-domain.

Table 1: Details of example ground motions

Earthquake	PGA (g)	T (s)	T_s/T
El Centro (EC), 1940	0.004	50.0	0.50
Northridge (NR), 1994	0.009	60.0	0.12
Imperial Valley (IV), 1940	0.36	53.8	0.47
Kern County (KC), 1952	0.16	54.4	0.63
Parkfield (PF), 1966	0.37	44.0	0.21
San Fernando (SF), 1971	0.02	68.7	0.67

Fig. 1 shows the Fourier spectra of the excitations revealing both wideband and narrowband characteristics with respect to the modal frequencies of the model. For example, other than EC and KC, the energies of the ground motions are distributed in a very narrow frequency range. Furthermore, the example excitations cover a broad range of PGA values with $0.01 - 0.37$g. Therefore, these ground motions form a perfect test bed to validate the PARAFAC method. Fig. 2 shows the Fourier spectra of top floor measurements of the building under example ground motions. As evident from Fig. 1 that most of the energies of the earthquakes are distributed within 0-8 Hz, mostly the first three modes (i.e., 0.91, 3.4 and 7.1 Hz) of the building models are excited and showed up in the vibration spectra. Therefore, the modal identification is restricted to only first three modes for brevity.
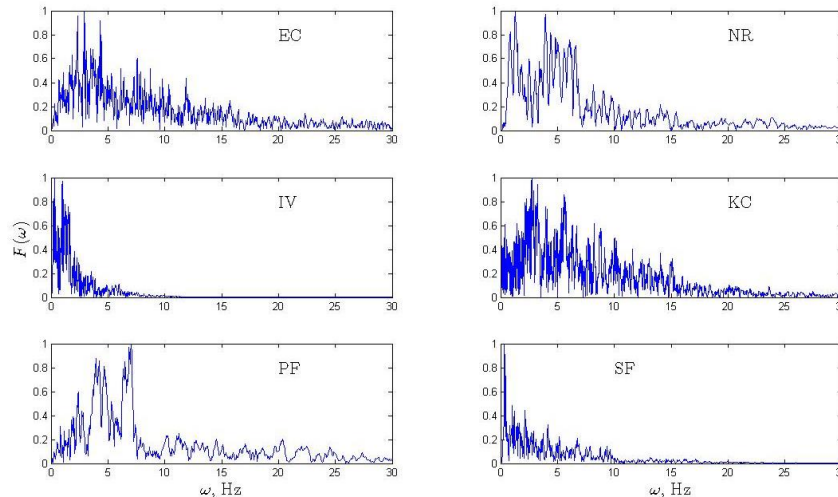

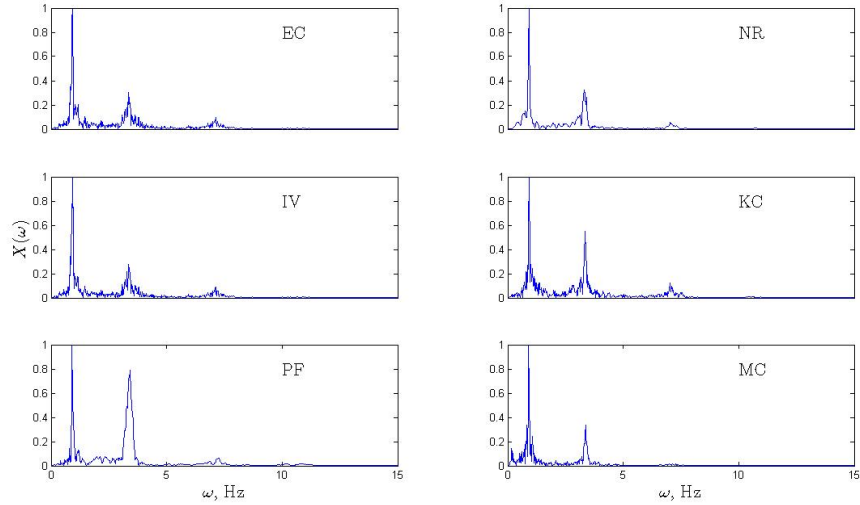Fig. 1: Fourier Spectra of example earthquakes

Fig. 2: Fourier spectra of top floor response of the building under six example earthquakes

The floor vibration responses resulting from each of the earthquakes are processed through the PARAFAC method. In this section, all the floor measurements are used and covariance matrices with lag up to 30 seconds are utilized. The resulting modal responses are shown in Fig. 3, 4 and 5. It may be noted except under SF, PARAFAC separates out the target modes under all ground motions. In fact, as shown in Fig. 1 and 2, SF can be considered as a very narrowband excitation that excites only a few structural frequencies. The results reveal that PARAFAC method can be considered as a versatile modal identification method for both stationary and nonstationary earthquake excitations.

The effectiveness of PARAFAC method depends on the lag parameter and the number of sensors used. In order to evaluate the performance of PARAFAC under different lag parameters, a suite of different lags is undertaken. Fig. 6 shows the performance of the identified modal parameters using three different lags which reveals its insensitivity towards its choice of lag parameters.
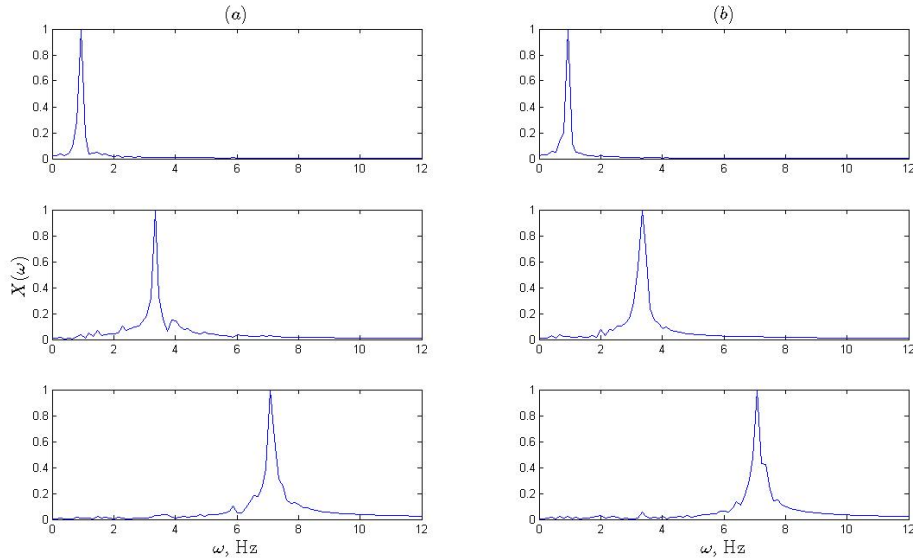

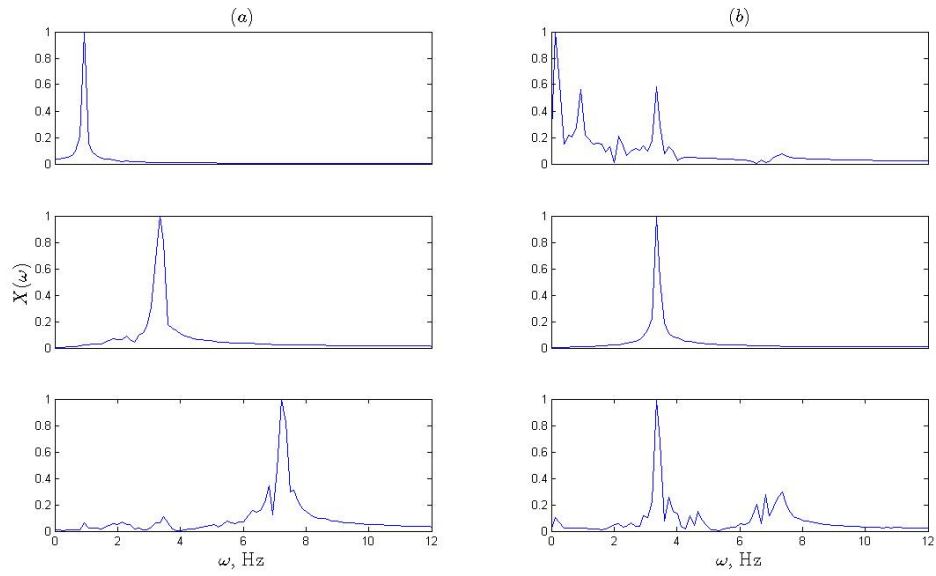Fig. 3: Fourier spectra of identified modal responses under (a) EC and (b) NR

Fig. 4: Fourier spectra of identified modal responses under (a) PF and (b) SF
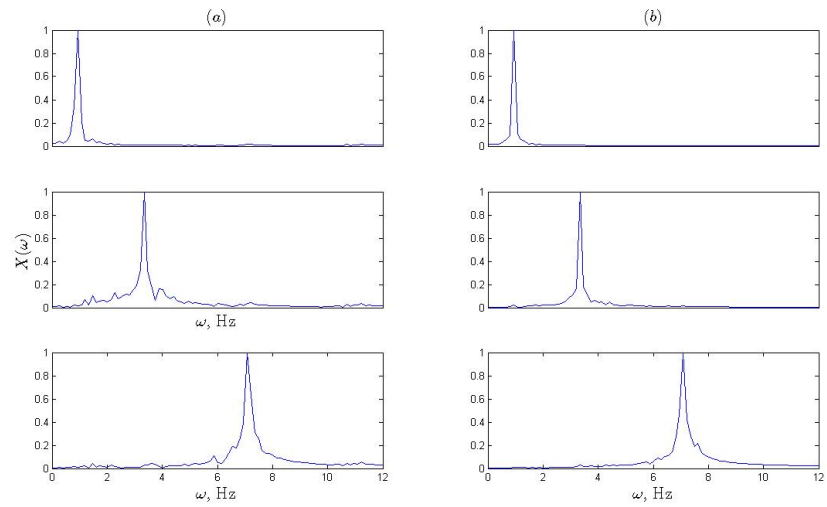

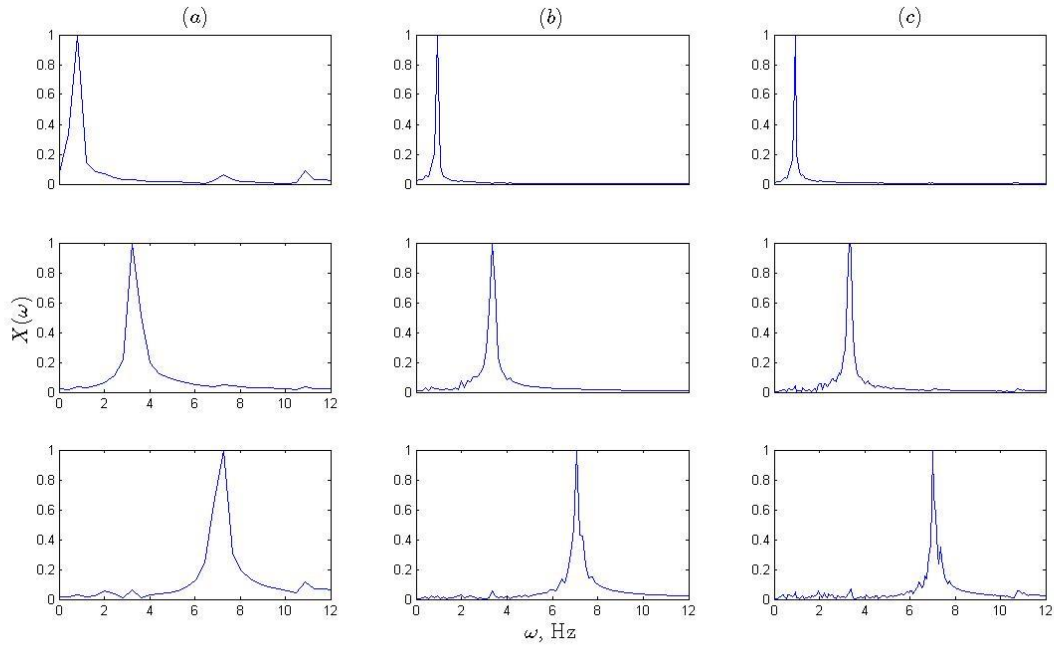Fig. 5: Fourier spectra of identified modal responses under (a) IV and (b) KC

Fig. 6: Fourier spectra of identified modal responses under NR using a lag of (a) 500, (b) 1500 and (c) 3000

A database of more than 1000 accelerograms is now considered to validate the performance of PARAFAC under wide range of nonstationarity and sensor combinations. The accelerograms considered in the database were recorded during 15 earthquake events in western U.S.A between 1931 and 1984 (Trifunac and Brady 1975) and during the 1994 Northridge earthquake. The records are chosen in such a way that the suite has a balanced distribution of records in terms of magnitude, epicentral distance, strong motion duration, and geologic site conditions. All records have peak ground accelerations (PGAs) greater than 0.1g, and have magnitude M ranging from 4.5 to 6.9, epicentral distance R from 4 to 62 km, strong motion duration T_s from 1.8 to 42 s, and site conditions from alluvium to rock.

Fig. 6 shows the identification results of the PARAFAC method with the PGA values of earthquake under full sensor and fewer sensor case using five different sensor combinations. The results show that the performance of PARAFAC method is insensitive to the location of partial sensors.
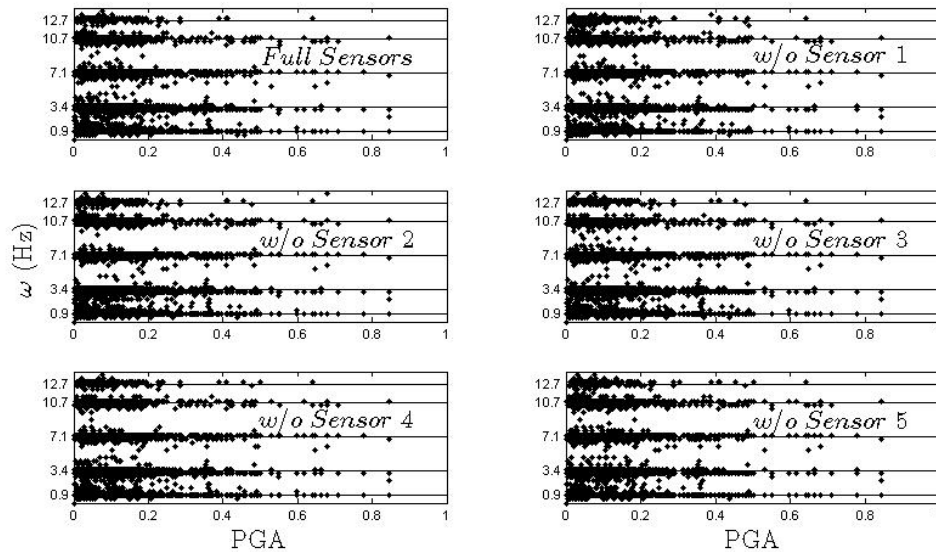

Fig. 6: Identification results w.r.t. PGA values under different sensor combinations

5. CONCLUSIONS

In this paper, PARAFAC decomposition is explored as a possible tool for modal identification under earthquake excitations. The method is validated using a suite of ground motions covering wide range of nonstationary characteristics. The effects of lag parameter and rank order selection of PARAFAC decomposition are also investigated for a numerical model under complete and limited sensor measurements.

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