Contributed Paper

What Is the Slope of this Pitch?

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The FIFA Quality Concept (2012) requires a slope lower than 1% for soccer fields (<0.5% recommended). On the other hand, the use of a single constraint is not sufficient to capture the singularities and local anomalies of the pitch. This paper presents a procedure able to (1) reconstruct the entire pitch surface and (2) analyze its shape. The method can be intended as a multi-step methodology where three-dimensional coordinates are captured with accurate and rapid surveying techniques, a model of the pitch is extracted with automated techniques for data interpolation based on non-uniform rational B-spline curves and surfaces, and raster algebra provides a detailed and exhaustive visualization of slope distribution and profile curvature.

I. Introduction

Sports fields have a slope to move water to sideline areas, where it can be collected and drained away. Good drainage, that is, the removal of excess water by gravity and artificial means, is a fundamental requirement in the construction of sports fields. Different sports require different slope configurations. Football and soccer pitches have a central crown for the whole length of the field so that goal areas are the highest and drier parts. Guidelines require a predefined slope threshold from the crown in the center of the field to the sideline areas. Baseball fields have a configuration where the high point of the field is the pitcher's mound. Tennis courts slope in one plane with directions side-to-side, corner-to-corner, or end-to-end. Softball fields slope in a single plane from the back stop and to the outfield fences.

This paper illustrates a procedure for the measurement of the slope of a soccer pitch, which is an essential parameter for a playable surface. The FIFA Quality Concept[1] requires a slope lower than 1% (<0.5% recommended), and the FIFA Laws of the Game[2] allows for pitches in adult matches to be 64–73 m wide. A field of minimum width and 0.5% slope has a 0.16-m crown, whereas the maximum width corresponds to an approximate 0.18 m crown. Surface regularity is also checked with the 3-m straight edge conditions, which must be less than 10 mm.

The ideal cross section of a pitch is a smooth curve whose tangent is a horizontal line in the middle, whereas the maximum value is reached toward the sideline (*Figure 1*). This means that the slope is not a constant value, and only an "average slope" can be estimated as *Rise* over *Run*. On the other hand, this simple estimate along a profile does not ensure the respect of the maximum slope condition. In addition, the computation is carried out only with two elevation values (center and sideline) without additional information about the internal variability of the slope. In other words, the slope is not a constant. It changes based upon the location of the two points being used.

The procedure presented in this paper is a new methodology able to show the full slope distribution for the entire field. The method shows the real shape of existing sports fields. It can be intended as a procedure to inspect the external layer (the playable surface) in order to determine global or local geometric anomalies. It also provides an easy-to-understand graphical visualization of results, which facilitates the inspection of irregular parts. The method is based on three-dimensional (3D) measurements collected by modern measurement techniques and advanced surface interpolation algorithms that provide a continuous representation of the pitch through non-uniform rational basis-spline[3] (NURBS). A real case study is illustrated and discussed: a soccer pitch in Milan. However, the method can be easily extended to other sports fields where accurate measurements are required.

II. Data Acquisition

Different measurement techniques can be used to evaluate the slope of a pitch. The

Figure 1. Ideal cross section of a pitch with a central crown

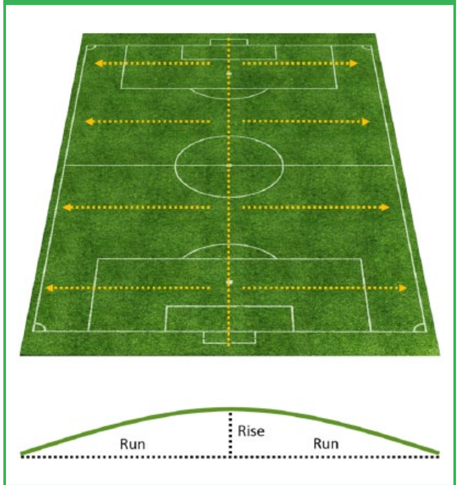


Figure 2. Instruments used for data acquisition: total station, prism, pole and base

method used in this work relies on trigonometric leveling with total station, that is, the measurement of the slope distance and vertical angle. Slope distance is measured using electromagnetic distance measurers, whereas the vertical (zenith) angle provides the elevation of points and the reduction in slant distance to the horizontal. The horizontal angle is used to complete the estimate of 3D point coordinates X_{TS}, Y_{TS}, Z_{TS} and makes total station powerful instruments for rapid and accurate contouring of any type of sports fields.

Data acquisition is carried out with a reflector on a pole with a constant height, which is moved on the pitch to obtain a set of 3D coordinates. A small rectangular plate with a conic connection is placed on the ground surface (the top of the turf) to provide a stable base. The total height of the reflector (pole and base) has no influence on the final shape because the height can be assumed as a bias (*Figure 2*).

The system is very simple and rapid, and the computation of 3D coordinates X_{TS}, Y_{TS}, Z_{TS} can be carried with the relationships

$$\begin{aligned} X_{TS} &= S \sin\varphi \sin\vartheta \\ Y_{TS} &= S \sin\varphi \cos\vartheta \end{aligned} \qquad (1)$$

$$Z_{TS} = S\cos\varphi + (1-k)\frac{(S \sin\varphi \sin)^2}{2R^2}$$

where S is the slope distance, φ and ϑ are vertical and horizontal angles, k is the refraction index, and R is the Earth's radius (6378 km).

Measurements are provided in the instrumental reference system. In the case of short distances (less than 100 m), the effects of Earth curvature and atmospheric refraction can be ignored for their small magnitude (less than 1 mm) when compared with the expected accuracy of the survey (±2 mm).

Once the instrument has been properly oriented, measurements can be taken with the automated search of the prism. It is highly recommended to carry out both direct circle and reverse circle readings to remove the effect of systematic errors (biases) with the average based on Bessel formula.

III. Surface Reconstruction

The second part of the work is the reconstruction of the surface. The case study described in this paper is a soccer field in Milan (Italy). Data acquisition was carried out with a first-order total station Leica TS30 able to measure distances with a precision of $\sigma_d = \pm 0.6$ mm and angles with a precision $\sigma_{\vartheta,\varphi a} = \pm 0.5''$. The instrument is set up on a geodetic tripod in midfield. A set of 3D coordinates is measured by means of the retro-reflective prism placed on a pole with fixed height, obtaining a set of point coordinates X_{TS}, Y_{TS}, Z_{TS} with a rather homogeneous distribution in the field and spatial precisions better than ±2 mm (*Figure 3*).

In all, 256 points were measured and divided into three categories:

- Drawing points for reconstructing lines;
- Reference points used for the geometric reconstruct of the surface;
- Check points for the evaluation of the geometric accuracy of the final surface.

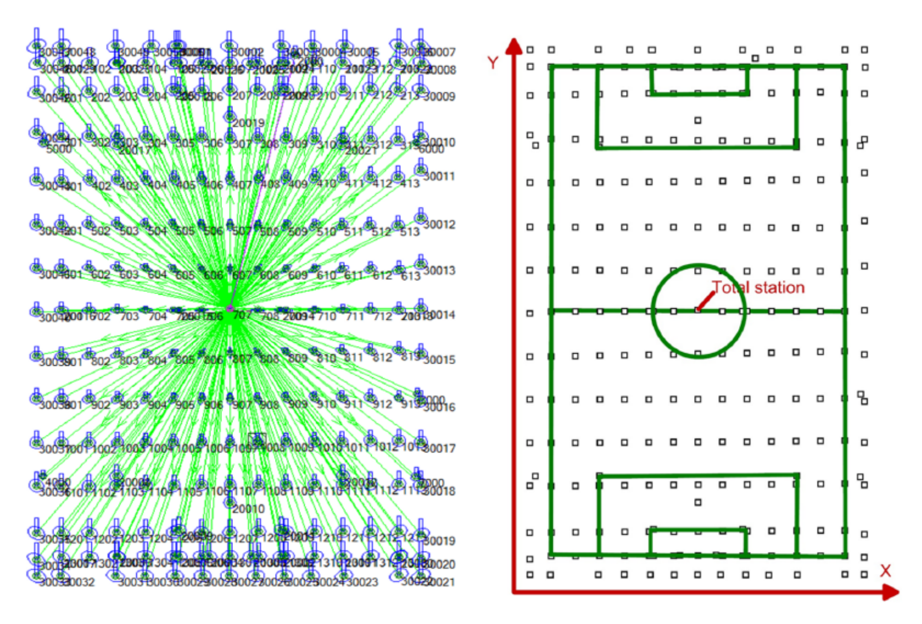
The approach for surface reconstruction is based on a curve network based on NURBS curves, which are given by the set of reference points.

A NURBS curve is a vector-valued piecewise rational polynomial function of the form

$$\mathbf{C}(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i \mathbf{P}_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \qquad (2)$$

where $\{w_i\}$ are the weights, $\{\mathbf{P}_i\}$ are the control points, and $\{N_{i,p}u)\}$ are pth-degree B-spline basis functions defined by the recursive Cox-de Boor form[4]

Figure 3. Measurement scheme and point distribution



$$N_{i,0}(u)=\begin{cases}1 & \text{if } u_i \le u \le u_{i+1}\\ 0 & \text{otherwise}\end{cases}$$

$$N_{i,p}(u)=\frac{u-u_i}{u_{i+p}-u_i}N_{i,p-1}(u)+\frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}}N_{i+1,p-1}(u) \tag{3}$$

and U is a non-decreasing sequence of real numbers whose elements are called knots, which form a knot vector $U = \{0,\ldots, 0, u_{p+1},\ldots, u_{m-p-1}, 1,\ldots,1\}$.

NURBS curves of degree 2 were used to generate a 7×11 network in space [$\mathbf{C}_k(u)$, $\mathbf{C}_l(v)$] that interpolates 77 reference points. The curves in one direction cross all curves in the other direction.

A generic NURBS surface of degree (p, q) in the directions (u, v) is defined as

$$\mathbf{S}(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)N_{j,q}(v)w_{i,j}\mathbf{P}_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)N_{j,q}(v)w_{i,j}} \tag{4}$$

where $\{w_{i,j}\}$ are the weights and $\{N_{i,p}(u)\}$ and $\{N_{j,q}(v)\}$ are the B-spline basis functions defined on the knot vectors

$$U=\{0,\ldots,0,u_{p+1},\ldots,u_{r-p-1},1,\ldots,1\}$$
$$V=\{0,\ldots,0,v_{q+1},\ldots,v_{s-q-1},1,\ldots,1\} \tag{5}$$

where $r = n + p + 1$ and $s = m + q + 1$.

The reconstruction of the surface is carried out with the NURBS network [$\mathbf{C}_k(u)$, $\mathbf{C}_l(v)$] and a new NURBS surface [$\mathbf{S}(u,v)$, which interpolates the profiles in space so that [$\mathbf{C}_k(u) = \mathbf{S}(u,v_k)$ $(0 \le k \le K$ and $\mathbf{C}_l(v) = \mathbf{S}(u_l,v)$ $0 \le l \le L$.

Given a network of curves in space, there are infinite surfaces that contain the curve network. A Gordon[5] surface is an efficient solution to the fitting problem. It is based on the sum of three surfaces

$$\begin{aligned}\mathbf{S}(u,v)&=\sum_{l=0}^{s}\mathbf{C}_l(v)\alpha_l(u)+\\&\sum_{k=0}^{r}\mathbf{C}_k(u)\beta_k(v)-\\&\sum_{l=0}^{s}\sum_{k=0}^{r}\mathbf{Q}_{l,k}\alpha_l(u)\beta_k(v)\\&=\mathbf{S}_1(u,v)+\mathbf{S}_2(u,v)-\mathbf{T}(u,v)\end{aligned} \tag{6}$$

where $\{\alpha_l(u)\}_{l=0}^{s}$ and $\{\beta_k(v)\}_{k=0}^{r}$ are any two sets of blending functions that satisfy the constraints

$$\alpha_l(u_i)=\begin{cases}0 & if\ l\ne i\\ 1 & if\ l=i\end{cases}\quad \beta_l(v_i)=\begin{cases}0 & if\ k\ne i\\ 1 & if\ k=i\end{cases} \tag{7}$$

The solution can also be seen as the sum of three particular surfaces. The first $\mathbf{S}_1(u,v)$ and the second $\mathbf{S}_2(u,v)$ contain all $\mathbf{C}_l(v)$ and $\mathbf{C}_k(u)$. The third function $\mathbf{T}(u,v)$ (i.e. a tensor product) contains the intersection points between the curves.

$\mathbf{S}_1(u,v)$and $\mathbf{S}_2(u,v)$ have remarkable properties as they are skinned functions. Skinning can be defined as the procedure to create a surface given a set of curves $\{\mathbf{C}_k(u)\}$ in the u direction with respect to a blending direction v. Shown in *Figure 4* is the curve network estimated from reference points (black points) and the Gordon surface visualized with an increment of the scale in the vertical direction.

IV. Accuracy Evaluation and Slope Estimation

An accurate reconstruction of the real surface is a fundamental requirement of the proposed approach. The NURBS surface was converted into a 2.5D raster (grid) representation $Z=Z(X, Y)$ to simplify the next phases of the work. The use of raster (grid) representations is a common solution in cartographic applications for the availability of efficient algorithms based on raster algebra. The NURBS surface can be easily converted into a Digital Elevation Model (DEM)[6] with a sub-millimeter discrepancy from the original NURBS surface.

A set of 44 check points (red points in *Figure 3*) extracted from total station data was used to evaluate surface accuracy as follows[7]

$$RMS=\sqrt{\frac{\sum(Z_{TS}-Z_R)^2}{n}} \tag{8}$$

The Root Mean Square (*RMS*) represents the discrepancy along the vertical direction between total station data and surface. Z_{TS} is the elevation measured by total station, Z_R is the corresponding value of the interpolated grid surface $Z=Z(X, Y)$, and $n=44$ is the number of points. The independent set of check points (i.e. points that are not used in the estimation of the surface) gave a final *RMS* of ±3 mm, confirming the good metric accuracy of the interpolated raster surface. The achieved value, similar to the nominal precision of total station data, is acceptable and the

Figure 4. NURBS curve network with reference points (black) and check points (red). The final NURBS surface that shows some geometric irregularities

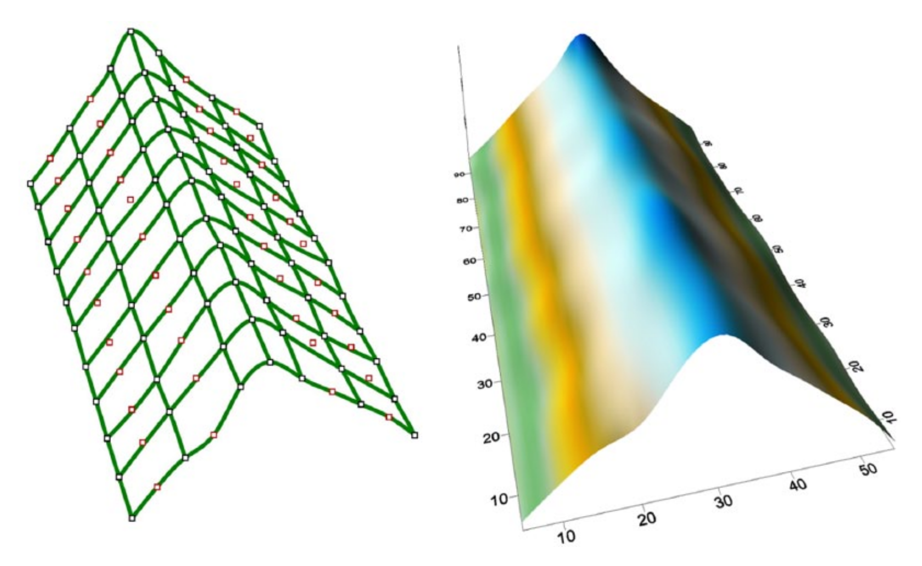
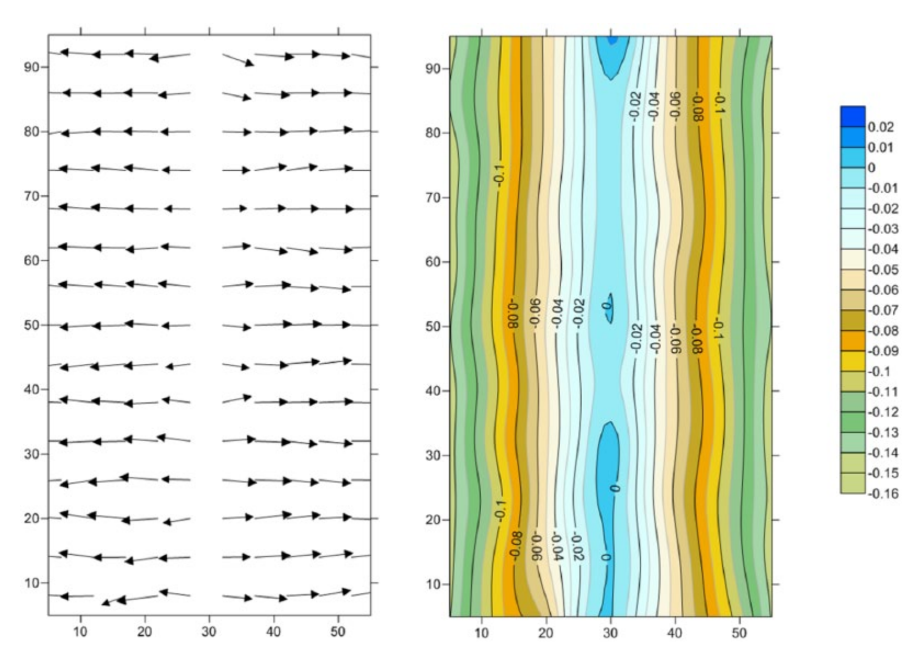


Figure 5. Slope vectors and contour lines for the pitch



raster surface can be used to estimate slope distribution.

A preliminary visual inspection of the surface can be carried out with slope vectors and contour lines (*Figure 5*) derived from the raster surface. The analysis of slope vectors shows two main directions from the center line. This is a very important result and can be highlighted only with a technique that extends the reconstruction to the entire pitch.

The inspection of contour lines shows a symmetrical trend from the center of the pitch to the sidelines. The elevation along both center line and sidelines is rather uniform.

A very detailed visualization of the real slope can be carried out by extending the analysis to the entire surface. Slope can be intended as the direction of steepest descent or ascent at that point. From a mathematical point of view, the slope s at a point (X,Y) is the magnitude of the gradient at that point

$$s(X,Y)=\sqrt{\left(\frac{\partial Z}{\partial X}\right)^2+\left(\frac{\partial Z}{\partial Y}\right)^2} \tag{9}$$

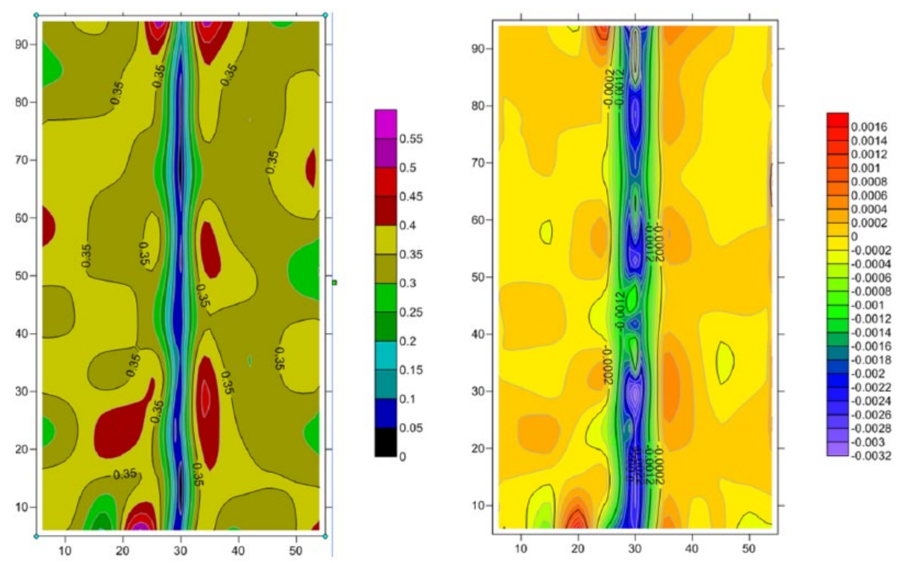
The visualization in *Figure 6* (left) shows isolines of constant steepest slope. It underlines a value lower than 0.6% for the entire pitch. Large values can be found close to the edges of the field, where there is a lack of total station data, especially in external areas. This can be intended as "boundary effects" for the lack of data, where the interpolation of the surface is usually worse.

Another important parameter is the curvature (the slope of the slope), especially the profile curvature K, which determines the downhill or uphill rate of change in slope in the gradient direction. K can be estimated as follows

$$K=\frac{\dfrac{\partial^2 Z}{\partial X^2}\left(\dfrac{\partial Z}{\partial X}\right)^2+2\dfrac{\partial^2 Z}{\partial X\partial Y}\dfrac{\partial Z}{\partial X}\dfrac{\partial Z}{\partial Y}+\dfrac{\partial^2 Z}{\partial Y^2}\left(\dfrac{\partial Z}{\partial Y}\right)^2}{\left[\left(\dfrac{\partial Z}{\partial X}\right)^2+\left(\dfrac{\partial Z}{\partial Y}\right)^2\right]\left[1+\left(\dfrac{\partial Z}{\partial X}\right)^2+\left(\dfrac{\partial Z}{\partial Y}\right)^2\right]^{3/2}} \tag{10}$$

Figure 6 (right) shows isolines of constant rate of change of steepest slope across the surface. This operation is similar to the computation of the second directional derivative. However, additional information can be obtained because K automatically determines the downhill direction at each point on the surface. Then, the rate of change of slope along that direction at that point can be easily estimated. Negative values are convex upward (concave) and indicate accelerated flow of water over

Figure 6. Slope distribution (left) and profile curvature (right)



the surface. Positive values are concave upward (convex) and show slowed flow over the surface.

The results illustrated in *Figure 6* show that the pitch has a slope compatible with the FIFA regulation. Slope is quite uniform notwithstanding some minor anomalies between the center and sidelines. However, the variation in curvature of the anomalies is not sufficient to generate large water stagnation which, in sports terms, is a requirement for a playable surface.

V. Conclusion

This paper presented a methodology able to provide an advanced visualization of slope distribution along with additional information (slope directions, curvature, etc.). The proposed method can be intended as a procedure for inspection or quality check, notwithstanding the method can help pitch installers to obtain the slope profile required. In this second case, an initial model of the surface provides a set of 3D points that can be physically materialized via staking. On the other hand, the proposed methodology is more like an inspection technique after the construction work because pitch installers today use automated methods based on laser equipment.

Although a specific case study was illustrated and discussed, the method can be easily extended to different sports fields without changing measurement tools and processing algorithms. In fact, the total station allows the direct estimation of 3D coordinates of virtually any kind of ground surface, whereas NURBS surfaces are able to digitally reconstruct both simple (e.g. planar) or complex surfaces by changing the parameters of the function (degree, knot vector, weights, and control points).

The procedure provides several integrated results not only limited to an “average slope” estimated as Rise over Run. Contour lines, slope distribution, profile curvature, and slope direction can be derived in a fully automated way from a set of accurate measurements. A digital surface can be rapidly generated with NURBS functions, and then simple visualizations based on raster maps allow for an efficient interpretation of numerical results.

Slope, slope variations, and local or global anomalies can be numerically evaluated to plan further activities in the case of strong deviations from existing regulations. Additional considerations in terms of time and cost reveal that the proposed solution is faster and more complete than traditional methods based on leveling or portable bubbles. In the case of the soccer pitch, the direct measurement of 3D coordinates by means of a total station and the use of advanced algorithms for surface interpolation and raster map generation took an overall measurement/processing time limited to couple of working days. In other words, the method can provide a 3D model of the surface, whose accuracy is provided with an additional set of check points acquired during the survey. This is a fundamental requirement that ensures the robustness of the proposed methodology, especially in the establishment of quality parameters.

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