Radiation Damping in Atomic Photonic Crystals

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The force exerted on a material by an incident beam of light is dependent upon the material's velocity in the laboratory frame of reference. This velocity dependence is known to be difficult to measure, as it is proportional to the incident optical power multiplied by the ratio of the material velocity to the speed of light. Here we show that this typically tiny effect is greatly amplified in multilayer systems composed of resonantly absorbing atoms exhibiting ultranarrow photonic band gaps. The amplification effect for optically trapped ^{87}Rb is shown to be as much as 3 orders of magnitude greater than for conventional photonic–band-gap materials. For a specific pulsed regime, damping remains observable without destroying the system and significant for material velocities of a few ms^{-1}.

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The force of radiation pressure is dependent upon the velocity of the body being pushed. For a perfectly reflecting mirror, this velocity dependence appears as a kinetic friction term in the equations of motion for the mirror and is due to the reduction in photon flux and frequency as observed in the material's rest frame, relative to the laboratory frame. This phenomenon was predicted some time ago by Braginsky and Makunin [1], where it was observed that the oscillatory motion of such a mirror connected to a wall via a spring would be damped in proportion to the power density of the incident beam. More recently, there was revived interest in this effect in reference to the precise interferometry experiments required to detect gravitational waves (e.g., the Laser Interferometer Gravitational Wave Observatory and Virgo projects) [2]. However, as stated in Ref. [3], to observe these velocity-dependent terms, even in the case of a perfectly reflecting metallic mirror, the most favorable parameters lead to a laser power density so great that the mirror would be unlikely to remain intact. Therefore the question must be asked as to whether there are other physical systems where the fundamental velocity dependence of the force of radiation pressure could possibly be observed.

Here we examine the radiation pressure experienced by a one-dimensional multilayered atomic structure [4], with incident radiation of a frequency close to an atomic transition. This is done with the help of the Maxwell stress tensor [5], which enables us to arrive at an exact expression for the pressure exerted by a light pulse and in turn to assess the feasibility of measuring radiation damping with such an atomic structure. For suitable choices of the atomic multilayer period within this frequency window, an array of trapped ^{87}Rb atoms is known to exhibit an ultranarrow photonic band gap, with a width on the order of gigahertz [6–8]. Because of the very high sensitivity of the optical response (transparency or reflection) of such a multilayer to the frequency of the incident radiation, we observe that the velocity dependence of the force of radiation pressure is greatly enhanced and, in addition, can take either sign. This enhancement is 3 orders of magnitude above that recently predicted for conventional dielectric photonic crystals [3]. We note that our results are not specific to trapped ^{87}Rb but apply equally well to any system exhibiting a photonic band gap on the same frequency scale.

The rate of transfer of four-momentum, $dP^{\mu}_{\mathrm{MAT}}/dt$, to a medium which is possibly dispersive and absorbing may be calculated from the electromagnetic fields in the vacuum region outside the medium as $dP^{\mu}_{\mathrm{MAT}}/dt = -\int_{\partial\mathrm{MAT}} T^{\mu j}_{\mathrm{FIELD}} dS_j$, where the surface element dS_j points outward from the material surface [5] and where the relevant components of the energy-momentum tensor are $T^{0i} = c\epsilon_0(\boldsymbol{E}\times\boldsymbol{B})_i$ and $T^{ij} = \epsilon_0[\delta_{ij}(\boldsymbol{E}^2 + c^2\boldsymbol{B}^2)/2 - E_iE_j - c^2B_iB_j]$. We start by considering a planar material slab at *rest*, upon which a beam of linearly polarized radiation of cross sectional area A is normally incident. Radiation of frequency ω enters the material through the surface at $x = x_1$ and exits through a similar surface at $x = x_2$, with complex reflection and transmission amplitudes $r(\omega)$ and $t(\omega)$, respectively. Averaging over a time interval $\Delta t \gg \omega^{-1}$ yields the average four-force experienced by the slab at rest:

$$\left\langle \frac{dP^{\mu}_{\mathrm{MAT}}}{dt} \right\rangle = \frac{A\epsilon_0 E_0^2}{2} \begin{pmatrix} [1 - R(\omega) - T(\omega)] \\ [1 + R(\omega) - T(\omega)]\hat{\boldsymbol{x}} \end{pmatrix}, \qquad (1)$$

where $R(\omega) = |r(\omega)|^2$ and $T(\omega) = |t(\omega)|^2$. Upon Lorentz transforming (1), one obtains the corresponding expression for the slab in *motion* with velocity $\boldsymbol{V} = V\hat{\boldsymbol{x}}$ in the lab frame. In terms of the lab frame (*primed*) quantities, one has $\omega = \sqrt{(1 - V/c)/(1 + V/c)}\,\omega' = \eta\omega'$, $E_0 = \eta E_0'$, and the average four-force becomes

$$\left\langle \frac{dP^{\mu}_{\rm MAT}}{dt} \right\rangle' = \frac{A\epsilon_0(\eta E_0')^2}{2\sqrt{1-(\frac{V}{c})^2}} \begin{pmatrix} \left[[1-R(\eta\omega')-T(\eta\omega')]+\frac{V}{c}[1+R(\eta\omega')-T(\eta\omega')]\right] \\ \left[[1+R(\eta\omega')-T(\eta\omega')]+\frac{V}{c}[1-R(\eta\omega')-T(\eta\omega')]\right]\hat{x} \end{pmatrix} \simeq \frac{P'}{c} \begin{pmatrix} \left[F^0_{(0)}-\frac{V}{c}F^0_{(1)}\right] \\ \left[F^1_{(0)}-\frac{V}{c}F^1_{(1)}\right]\hat{x} \end{pmatrix}, \qquad (2)$$

where $P' = A\epsilon_0 c E_0'^2/2$ is the incident radiation mean power. The first expression in (2) has a rather involved dependence on the velocity as it stems from R and T, that depend on the velocity through the Doppler effect. A characteristic velocity range for cold atoms experiments is such that $\eta \simeq 1 - V/c$, so that R and T can be expanded to the leading order in V/c around the lab frame frequency as $R(\eta\omega') \simeq R(\omega') - (\omega' V/c)R^{(1)}(\omega')$ and $T(\eta\omega') \simeq T(\omega') - (\omega' V/c)T^{(1)}(\omega')$, respectively. This yields the simpler expression on the right-hand side of (2), where we set

$$F^0_{(0)} = 1 - R(\omega') - T(\omega'),$$

$$F^0_{(1)} = 1 - 3R(\omega') - T(\omega') - \omega'[R^{(1)}(\omega') + T^{(1)}(\omega')],$$

$$F^1_{(0)} = 1 + R(\omega') - T(\omega'),$$

$$F^1_{(1)} = 1 + 3R(\omega') - T(\omega') + \omega'[R^{(1)}(\omega') - T^{(1)}(\omega')].$$

For nondispersive materials, $F^1_{(0)}$ and the first three terms in $F^1_{(1)}$ are clearly the only contributions to the three-force. These are *positive* and for a lossless medium of fixed reflectivity R_0 reduce to the familiar results [1,2], respectively, for the pressure force $2R_0P'/c$ and the radiation pressure damping ("friction") $-4R_0P'v/c^2$. The latter is typically much smaller than the former, and for nondispersive mirrors, where $R(\eta\omega') \sim R(\omega')$ and $T(\eta\omega') \sim T(\omega')$, velocities as large as $V \sim 10^6\ \mathrm{ms}^{-1}$ are needed to observe a few percent shift in the velocity-dependent contribution to the force, corresponding to *damping*.

For dispersive materials, on the other hand, we have an extra contribution to $F^1_{(1)}$ whose sign depends on the relative strength of the reflectivity and transmissivity gradients $R^{(1)}(\omega')$ and $T^{(1)}(\omega')$. When R and T change significantly over a frequency range $\Delta\omega \ll \omega'$, this contribution could be substantial and become the dominant term in $F^1_{(1)}$ whose sign, unlike other terms in the force expression in (2), can then become positive or negative. Thus, as a consequence of the exchange of photon momentum, in the relevant mechanical equation of motion of the slab a friction force of the form $-\gamma V\hat{x}$ appears with γ positive or negative corresponding to, respectively, either *damping* or *amplification* [3,9]. Materials with optical reflectivity changes $\Delta R \sim 1$ on the scale of megahertz, i.e., $\omega'\Delta R/\Delta\omega \sim 10^8$, would yield an appreciable shift in the force even at $V \sim \mathrm{ms}^{-1}$, and periodic structures of trapped two-level atoms separated by vacuum fit quite well this parameter range [6].

We then specialize in what follows to a stack of alternating refractive indices $n_a(\omega) \simeq 1$ and $n_b(\omega) \simeq \sqrt{1 + 3\pi\mathcal{N}/(\delta - i)}$, respectively, with thicknesses a and b, where $\mathcal{N} = N/(V\lambda\!\!\!^{-3}_0)$ denotes the scaled density of atoms and $\delta = (\omega_0 - \omega)/\gamma_e$ the scaled detuning from the atomic resonance transition $\omega_0 = c/\lambda\!\!\!^{-}_o$ of width γ_e [10]. Unlike traditional dielectric photonic crystals, these multilayered atomic structures behave as resonantly absorbing Bragg reflectors [11] and exhibit two pronounced photonic stop bands when the Bragg scattering frequency is not too far from the atomic resonance ω_o: One develops from the polaritonic stop band with one edge at ω_o, and the other one corresponds to the usual stop band associated with the Bragg frequency [6]. At a given frequency ω' the radiation pressure is solely determined by the multilayer optical response, which is here calculated by using a transfer matrix approach [6] and then employed in Fig. 1 to plot both force components $F^1_{(0)}$ and $F^1_{(1)}$ in the spectral region between the two gaps. In particular, Fig. 1(b) displays the velocity-dependent contribution to the force, $F^1_{(1)}$, which

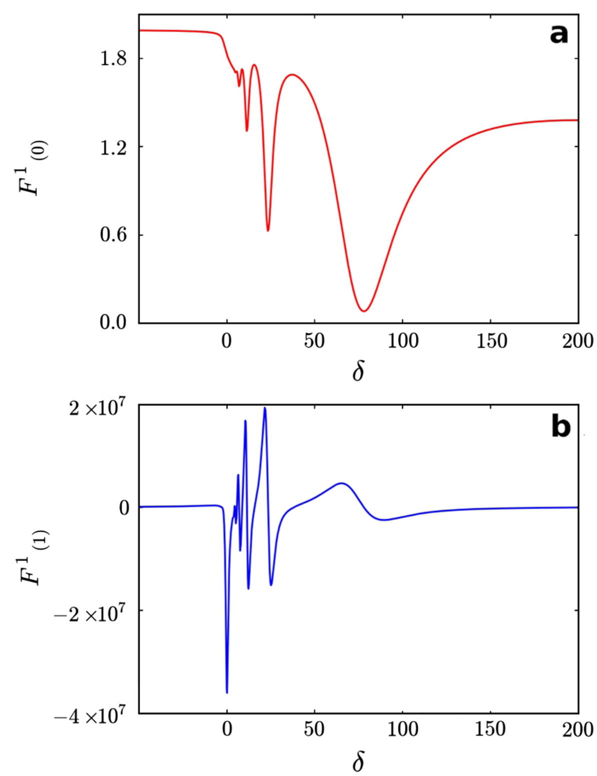

FIG. 1 (color online). Normalized x component of the force $\vec{F}$ as a function of detuning around resonance. The two panels show the force velocity-independent ($F^1_{(0)}$) and velocity-dependent ($F^1_{(1)}$) parts.

largely arises from the term proportional to $R^{(1)}(\omega') - T^{(1)}(\omega') = 2R^{(1)}(\omega') + A^{(1)}(\omega')$. Radiation force values can scale with V/c by as much as $|F^1_{(1)}| = 1 \times 10^7$ (ignoring the region very close to resonance) so that an appreciable 10% force shift is possible with velocities of $\sim 3\ \mathrm{ms}^{-1}$, that are within experimental reach [12]. This amounts to an enhancement of the velocity-dependent pressure component that is roughly 3 orders of magnitude larger than that anticipated with dielectric photonic crystals [3] and hinges on the very high sensitivity of the atomic array optical response to the frequency of the incident radiation [3]. In addition, depending on whether the difference $R^{(1)}(\omega')$-$T^{(1)}(\omega')$ is positive or negative, alternating cooling and heating cycles become easily accessible in the appropriate spectral region. The above analysis holds to first order in V/c, yet the conclusions are not fundamentally altered if all orders of V/c are retained.

This is now shown, for the case of a Gaussian light *pulse*, rather than a monochromatic plane light wave, a situation that turns out to be relevant when measuring, e.g., optomechanical effects of radiation pressure associated with a light pulse [7,13]. The momentum exchanged between the atomic multilayer described above and a pulse of central frequency ω_c and half width at half maximum $\mathcal{L}\sqrt{\ln(2)}$, with $\omega_c \mathcal{L}/c \gg 1$, is here calculated by keeping all orders of V/c and in the *impulsive* approximation, i.e., when the atoms are not significantly displaced during the passage of the pulse. The pulse electric field at the entrance (x_1) and rear (x_2) surface of the multilayer can be written, respectively, as a superposition of its incident and reflected $e^{i\omega(x_1/c-t)} + r(\omega)e^{-i[\omega(x_1/c+t)-\phi_r(\omega)]}$ parts and as its transmitted $t(\omega)e^{i\omega(x_2/c-t)}$ part. The net integrated four-momentum imparted by the pulse as seen in the lab (primed) frame is shown to be

$$\Delta P^{\mu\prime} = \frac{\epsilon_0 A \eta}{2\sqrt{1-(V/c)^2}} \int_0^\infty d\omega' |\boldsymbol{\xi}(\eta\omega')|^2 \begin{pmatrix} \mathcal{P}^{0'} \\ \mathcal{P}^{1'}\hat{\boldsymbol{x}} \end{pmatrix}, \quad (3)$$

where $\boldsymbol{\xi}(\omega) \simeq \sqrt{\mu_0 \bar{N}\hbar\omega_c \mathcal{L}/(A\sqrt{\pi/2})}\hat{\boldsymbol{y}}e^{-[\mathcal{L}(\omega-\omega_c)/2c]^2}$ and $\bar{N}$ is the pulse average number of photons. In terms of the lab frame (primed) quantities, one has $\mathcal{L} = \mathcal{L}'/\eta$; $\omega_c = \eta\omega'_c$; $\mathcal{P}^{0'} = 1 - R(\eta\omega') - T(\eta\omega') + (V/c)\times[1 + R(\eta\omega') - T(\eta\omega')]$; and $\mathcal{P}^{1'} = 1 + R(\eta\omega') - T(\eta\omega') + (V/c)[1 - R(\eta\omega') - T(\eta\omega')]$. This recovers well known results for a transparent lossless slab [14] in the limit for which $V/c \to 0$ ($\eta \to 1$). The plane wave limit in (2) also emerges when $\mathcal{L} \to \infty$ [15]. Using Eq. (3) and the multilayer optical response functions employed to draw Fig. 1, we illustrate in Fig. 2 the momentum transfer $\Delta\vec{P}' \cdot \hat{\boldsymbol{x}}$ in units of the total momentum contained in the incident pulse in the lab frame, $\bar{N}\hbar k'_c$. They substantiate the findings of the previous section: Figure 2 shows a difference in the radiation pressure force per pulse of the order of -15% at $\delta_c \simeq 65$ relative to a stationary structure ($V = 0$), while a difference of $\sim +5\%$ is shown to occur away from resonance, at $\delta_c \simeq 100$, where the absorption $A \approx 1\%$. It is worth emphasizing that the different signs of the velocity-dependent contribution correspond, respectively, to damping and amplification of the motion of the multilayer Bragg mirror.

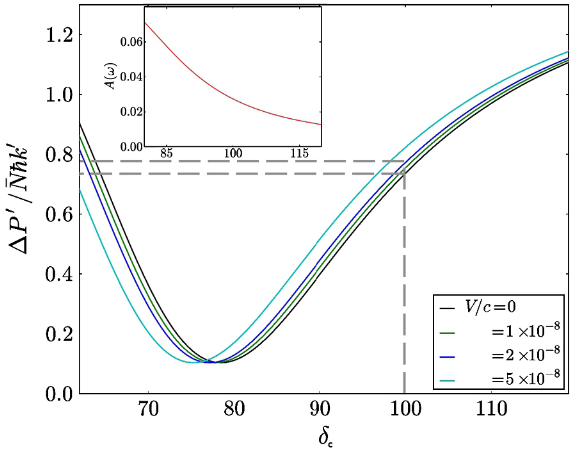

FIG. 2 (color online). Normalized impulse $\Delta\vec{P}' \cdot \hat{\boldsymbol{x}}$ acquired from a 3-m-long Gaussian pulse impinging upon a multilayered atomic structure. The detuning δ_c between the pulse carrier frequency (ω_c) and atomic resonance (ω_0) is in units of γ_e. The inset shows the absorption $A = 1 - R - T$.

Such tunable radiation force effects are best investigated in optical lattices, which cool and localize atoms at the lattice sites [16]. For a sufficiently long 1D optical lattice, the (Bragg) mirror may be envisaged as being made of an array of disks spaced by half the wavelength λ of the confining optical lattice $U(x)$ and filled with atoms in the vibrational ground state of the lattice wells. The disks thickness b is essentially given by the rms position spread around the minima of the potential $U(x)$, while the transverse size $D \simeq \sqrt{4N_0(a+b)/\pi L\rho b}$ is determined by the in-well atomic density ρ and the number of atoms N_0 loaded into a trap of length L. For typical densities $\rho \sim 10^{12}\ \mathrm{cm}^{-3}$ and filling factors $b/(a+b) \simeq b/a \simeq 0.05$, transverse sizes $D \sim 30\ \mu\mathrm{m}$ are obtained for $N_0 \sim 10^6$ when $L \simeq 2$ cm. This sets the incident beam waist w_0 and in turn its Rayleigh range $x_R = \pi w_o^2/\lambda_o$, so that for waists $w_0 \sim 25$–$80\ \mu\mathrm{m}$ one readily has $x_R \sim 0.25$–2.5 cm, that compares with the overall mirror length L. After loading the atoms into a 1D optical lattice, whose counterpropagating beams are taken to be far (red-) detuned from resonance [17], the lattice is further set into motion, dragging along the atom stack. Such a motion may be achieved by changing the relative frequency detuning $\Delta\omega$ of the two laser beams, which corresponds to a lattice velocity $v = \Delta\omega/2k$, where k is the average wave number; velocities on the ms^{-1} range can be reached this way [12].

Upon passage of a light pulse, the transferred momentum $\Delta\vec{P}'$ induces coherent oscillations in the center of mass of the atomic wave packet in the lattice wells [19]. This in turn induces a periodic redistribution of the power difference between the two counterpropagating lasers beams forming the optical lattice that can be measured [16]. In particular, every half a cycle of the atomic wave-packet oscillations, the momentum of the atomic ensemble changes by $2\Delta\vec{P}'$; this corresponds to a coherent scattering of a number of photons $\Delta N_{\rm ph}$ from one of the two counterpropagating laser beams to the other given by $\Delta N_{\rm ph} = |\Delta\vec{P}'|/\hbar k$.

We finally address the problem of the fragility of the atomic lattice structure upon the passage of the pulse in the $\delta_c \simeq 100$ region of interest (see Fig. 2) where $R \simeq 0.6$ and $A \approx 1\%$. There are two things to worry about here: (i) the collective motion of the atoms induced by the reflection of the photons from the lattice and (ii) the heating of the trapped atoms due to absorption followed by spontaneous emission. To address (i), consider a single lattice site. The trapping potential observed by an atom in this site will be $U_0\sin^2(2\pi x/\lambda)$, approximately equivalent to a harmonic trap with a frequency, $\omega_h = (2\pi/\lambda)\times\sqrt{2U_0/m}$. For an initially stationary atom, experiencing a sudden impulse $2\hbar k$, an oscillation of an amplitude $\Delta x = 2\hbar k/m\omega_h$ will be set up in the well. Hence, for a trap strength (U_0) of 10^3 the recoil energy, each photon reflection induces a wave-packet oscillation of amplitude 0.01λ. Choosing a pulse energy corresponding to 10 photons per atom, around $\delta_c \simeq 100$ where $\Delta\vec{P}'/\bar{N}\hbar k \simeq 0.8$ (see Fig. 2), one sets oscillations of amplitude of about 0.04λ, comparable to those actually measured in Ref. [16] and lasting for several periods. On the other hand, as regards (ii), for the same number of photons per atom and $A \approx 1\%$, the absorption and subsequent spontaneous emission would increase the stack thermal energy by about one-tenth of the recoil energy per atom. This is much smaller than both the kinetic energy of the ordered motion due to radiation pressure and the typical initial thermal energy of cold atom samples.

All-optically tailorable light pressure damping and amplification effects can here be attained in the absence of a cavity, which makes multilayered atomic structures not only interesting in their own right but also amenable to a new optomechanical regime. The large per-photon pressures that can be observed compare in fact with state of the art micro- [20] and nano- [21] optomechanical resonators yet involving (atomic) masses that are various orders of magnitude smaller.

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