WINNERLESS COMPETITION IN NEURAL DYNAMICS; CLUSTER SYNCHRONISATION OF COUPLED OSCILLATORS.

Submitted by

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Abstract

Systems of globally coupled phase oscillators can have robust attractors that are heteroclinic networks. Such a heteroclinic network is generated, where the phases cluster into three groups, within a specific regime of parameters when the phase oscillators are globally coupled using the function $g(\varphi) = -\sin(\varphi + \alpha) + r\sin(2\varphi + \beta)$. The resulting network switches between 30 partially synchronised states for a system of $N = 5$ oscillators. Considering the states that are visited and the time spent at those states a spatio-temporal code can be generated for a given navigation around the network. We explore this phenomenon further by investigating the effect that noise has on the system, how this system can be used to generate a spatio-temporal code derived from specific inputs and how observation of a spatio-temporal code can be used to determine the inputs that were presented to the system to generate a given coding. We show that it is possible to find chaotic attractors for certain parameters and that it is possible to detail a genetic algorithm that can find the parameters required to generate a specific spatio-temporal code, even in the presence of noise. In closing we briefly explore the dynamics where $N > 5$ and discuss this work in relation to winnerless competition.

Contents

Overview

Chapter 1 presents background material and a general introduction to neural models from a mathematical point of view. We introduce more traditional spiking models such as the Hodgkin-Huxley model and highlight recent advances with work relating to winnerless competition models. A short history of the use of phase oscillators in the field of neural dynamics is detailed in Chapter 2. The importance of the Kuramoto Model is focussed upon and we look into the derivation of the Kuramoto model which justifies the use of phase oscillators in synchronisation theory, and thus, neural dynamics. Chapter 2 also touches upon modifications that have been made to the Kuramoto model to expand its uses into a number of fields - such as in laser arrays and synchronisation in chemical systems.

In Chapter 3 we properly introduce the generalisation of the Hansel, Mato and Meunier model that is the focus of the rest of this thesis. This chapter explains the derivation of the model and briefly explores the wide range of dynamics that can be witnessed for different parameter regimes. The generalised Hansel, Mato and Meunier model exhibits cluster state dynamics and a type of winnerless competition. Chapter 4 shows that navigation around the generated network of saddle nodes can be driven by inputs and that the system is robust in the presence of noise. In this chapter we also investigate excitable dynamics that can be found using a different set of parameters for the model. Chapter 5 discusses the appearance of chaos in a related system of 5 globally coupled oscillators. This chapter investigates the presence of chaos that emerges for a specific regime of parameters and we show that there exist periodic orbits where the parameters are close to those that produce chaos.

In Chapter 6 we show that our system can be used to generate a spatio-temporal code from low amplitude inputs in the form of detuning of the oscillators. We show that in the case of $N = 5$ oscillators the spatiotemporal coding can be used to resolve all of the information that relates the individual inputs to each other, providing a long enough time series is observed.

Chapter 7 shows that it's possible to train a system of globally coupled phase oscillators using genetic evolution to generate a desired coding. We developed a system that represents inputs to our model, in the form of detunings, as genotypes that are evolved to produce a desired spatio-temporal coding sequence. Finding a suitable method and function to rank the fitness of each genotype was challenging and we present one possible function. Finally we discuss how this function could be futher refined.

In Chapter 8 we explore the effect of increasing the number of oscillators in our model above five. We show that for $N = 7$ oscillators there exists a range of parameters that produces robust cluster-state dynamics. With $N = 7$ oscillators the number of possible cluster states rises dramatically and the system remains robust to noise. This chapter poses questions about how larger systems could produce complex models of winnerless competition that might be used to encode information.

Chapter 9 details the software that was written for running numerical simulations that proved invaluable in exploring the model described in this thesis. This chapter gives an overview of how the software works and how it was expanded to include functionality for evaluating genetic evolution with globally coupled oscillators.

Finally, in Chapter 10, we speculate on methods that could be used to expand the software and the model to work with a greater number of oscillators. We consider the parallels that our system draws with classical winnerless competition models and discuss what further investigation could take place if there was more time available for the project. Lastly we look into how this model could be used with regards to specific applications.

Part of Chapter 2 appeared in a tutorial that won a student prize at the SIAM Conference 2007 and appears on SIAM's Dynamical Systems Web [59]. Chapters 3 and 4 and the introduction to Chapter 5 appeared in the SIAM Journal on Applied Dynamical Sysytems and was jointly written with Peter Ashwin, Gábor Orosz and Stuart Townley [8]. Chapter 6 appeared in Physical Review E and was jointly written with Peter Ashwin [61].

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